A Delay-Based Output Feedback Controller for the Active Vibration Damping of a Vibrating Thin Membrane

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***Abstract*— This paper addresses the problem of active vibration control of a flexible axisymmetric membrane. This mechanical system is equipped with two piezoelectric circular chips where one of them works as an actuator, whereas the other is used as a sensor. Both are glued on the membrane, one on each side, and centered according to its axis of symmetry. The design of the proposed controller is based on delayed proportional actions. We exploit a property called *Coexisting Real Roots inducing Dominancy* to an assignment of spectral values in an appropriate region of the complex plane, corresponding to a desired damping. The aim of this work is to examine the performances of the proposed output feedback controller in terms of vibration damping of the main observable and controllable vibrating modes.**

***Index Terms*— Time-delay systems, stability and stabilization, active control of vibrations, partial pole placement**

I. Introduction

Since 1930s and the theory of servomechanisms proposed by Hazen [1], it is commonly accepted that, in most of practical situations, the presence of time-delays in dynamical processes is synonymous with instability and bad behaviors. However, there are cases where the delay in the input can induce a stabilizing effect on the dynamics of the system, and the idea to use the *delay* as a *control parameter* is not new (see, e.g., [2], [3], and the references therein). For instance, at the end of the 70s, [4] introduced the so-called "proportional-minus-delay" (PMD) controller and showed that such a controller can replace the well-known PD-controller (quick responses to input changes) with less sensitivity to high-frequency noise. Such a controller was further exploited in various case studies including, among others, the well-known inverted pendulum (see, for instance, [5], [6], to cite only a few).

In this work, we are exploiting the control structure introduced in [7], whose idea is close to the PMD control mentioned above in the sense that the controller structure is of PMD type in both output/input signals and the delay is used as a design parameter to improve the overall behavior of the closed-loop system. In other words, for constructing the input at a given time value, we need an appropriate past information from both the input and the output. Despite its simplicity in terms of construction, such an infinite-dimensional controller induces unexpected behaviors in closed-loop. In the sequel, we are using such a control law for performing the active vibration control of a flexible axisymmetric membrane.

Regarding the delay as a control parameter, a second important idea concerns the so-called *multiplicity-induced-dominancy* (MID) property [8], [9] and the *coexisting real roots induced dominancy* one [10], [11], where the authors emphasized the existing links between the spectral abscissa[1] of the corresponding characteristic function in the linear case and the multiple real characteristic roots with the maximal multiplicity or the maximal number of real roots [2]. Finally, these approaches and ideas open interesting perspective in using the so-called *partial pole placement* method in controlling dynamical systems.

In the sequel, in order to reduce the peaks of resonance of the first three vibration modes for a flexible axisymmetric membrane, the methodology described in [10] is applied. The corresponding model was firstly introduced for control purposes in [12].

The remaining paper is organized as follows: in Section II, the axisymmetric membrane under consideration is presented and its modelling is briefly explained. Section III provides a description of the PPP-delayed controller design. Some numerical simulations illustrating the efficiency of the proposed method are presented in Section IV. Finally, the paper ends with some concluding remarks.

The notations are standard.

II. System description and its modelling

The considered system, depicted on Fig. 1, is a thin metallic membrane with a clamped circular edge. This disc is embedded into a mobile support that only moves along the z axis. The support is submitted to an unknown motion, characterized by an acceleration denoted $d(t)$ in the sequel. This flexible membrane is equipped with two ceramic-based piezoelectric chips. The thinner one is used as an actuator with a thickness of 0.4 *mm*, whereas the other is used as a sensor with a thickness of 0.7 *mm*. These circular chips are assumed to be perfectly bounded on the disc, one on each side, collocated and centered according to the axis of symmetry of the disc. All the physical parameters of the

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[1]The *spectral abscissa* is defined by the real part of the rightmost characteristic root; see, for instance, [3] for a deeper discussion.

[2]The maximal real multiplicity or the maximal number of real roots correspond to the degree of the corresponding characteristic function; see, e.g. [8] for the definition and related properties

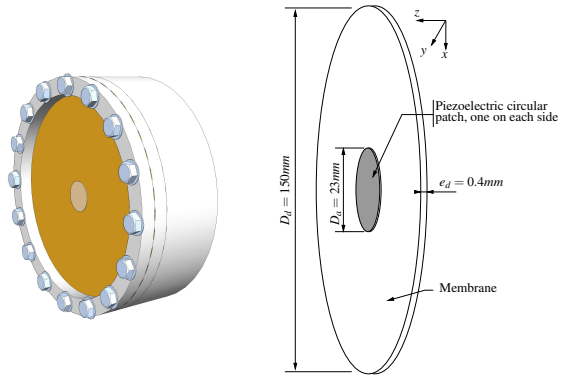


Fig. 1: Axisymmetric flexible membrane (dimensions on the right) inserted in the device which is subjected to vibrations (Computer Aided Design figure on the left).

materials used here can be found in [12] and references therein. The dimensions of the used elements are indicated in Fig. 1.

The control signal (input), noted $u(t)$, is the voltage applied across the piezoelectric actuator. The measured output, noted $y(t)$, is the electric voltage across the piezoelectric sensor. The disturbance signal $d(t)$ is the total acceleration applied to the clamped circular edge of the structure. The controlled output that is considered in this work, noted $z(t)$, is the z component (see Figure 1) of the acceleration at the center of the disc.

The mathematical modeling of the dynamical behavior for such mechanical structures is based on several coupled Partial Differential Equations (PDEs) in space and time-dependent variables. See for example [12] or [13] and references therein, for more details that are out of the scope of this paper. Such equations are usually addressed through numerical methods like the *Finite Element Modeling (FEM)* [14]. This method leads to a set of Ordinary Differential Equations (ODEs) that are linear but with an important number of variables. Because of the numerical feature of this method, it is worth mentioning that the physical parameters of the PDEs are dissolved within numerical coefficients of the corresponding ODEs.

An accurate model in state-space form is derived from the set of ODEs thanks to a model reduction method called *modal analysis*. It describes the inputs-to-outputs dynamical behavior in the bandwidth of interest, say $[0-4000Hz]$ for our system. This model, called the *analysis model*, is of order 12 and includes only the *controllable* and *observable modes*, *i.e.* the axisymmetric ones.

A reduced-order control-oriented model is obtained from the analysis one thanks to a reduction method based on state-variable truncation. This model correspond to the *synthesis model*. It is of order 6 and includes the dynamic of the first three axisymmetric vibration modes.

Figure 2 shows the geometrical shapes of the vibration modes in the synthesis model. The control input u is a scalar signal, as well as the measured output signal. Then, the considered flexible membrane is a single-input-single-output system. The disturbance input d is the total acceleration applied to the clamped edge of the structure. The controlled output z is the vertical total acceleration of the membrane center. The frequency responses of the full and reduced order models, from the input d to the output $d+z$, are shown in Fig. 3.

By denoting the Laplace variable by s, the model based on transfer functions is given by

$$\mathscr{S}\begin{cases} z(s) = \dfrac{N_{dz}(s)}{\psi(s)}d(s) + \dfrac{N_{uz}(s)}{\psi(s)}u(s), \\ y(s) = \dfrac{N_{dy}(s)}{\psi(s)}d(s) + \dfrac{N_{uy}(s)}{\psi(s)}u(s). \end{cases} \tag{1}$$

These polynomials are written as

$$\begin{aligned} N_{dz}(s) &:= \sum_{k=0}^{n_p} n_{dz_k}\, s^k, & N_{uz}(s) &:= \sum_{k=0}^{n_p} n_{uz_k}\, s^k, \\ N_{dy}(s) &:= \sum_{k=0}^{n_p} n_{dy_k}\, s^k, & N_{uy}(s) &:= \sum_{k=0}^{n_p} n_{uy_k}\, s^k, \\ & & \text{and} \quad \psi(s) &:= \sum_{k=0}^{n_p} a_k\, s^k, \end{aligned} \tag{2}$$

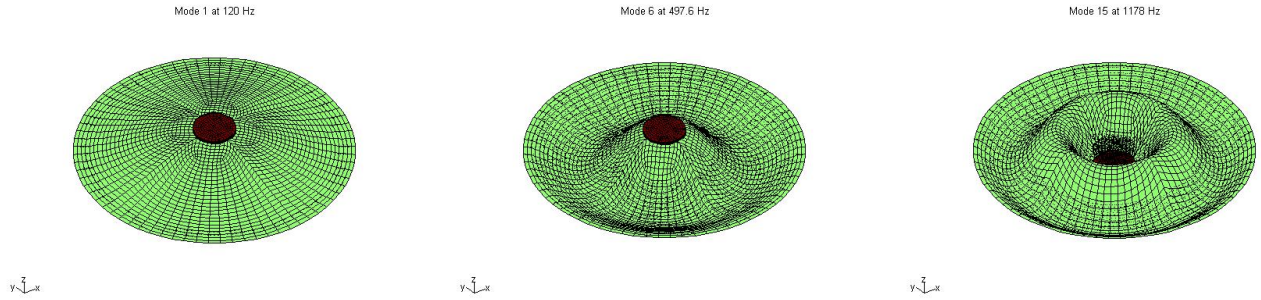
where $a_{n_p} := 1$ is imposed for simplicity. The numerical values of the polynomials' coefficients for the synthesis model in (1) are given in Table I.

n_{dz_6}	-1.1482770092356	n_{uz_6}	-1.08684807395548
n_{dz_5}	$-1.29576709733243 \cdot 10^{2}$	n_{uz_5}	$-1.22644794131004 \cdot 10^{2}$
n_{dz_4}	$-5.92403731382423 \cdot 10^{7}$	n_{uz_4}	$-4.12495030494032 \cdot 10^{7}$
n_{dz_3}	$-3.05369213559935 \cdot 10^{9}$	n_{uz_3}	$-2.00512841757836 \cdot 10^{9}$
n_{dz_2}	$-7.53045748218982 \cdot 10^{14}$	n_{uz_2}	$-2.21049785301237 \cdot 10^{14}$
n_{dz_1}	$-5.3167399068102 \cdot 10^{15}$	n_{uz_1}	$-2.38029330082552 \cdot 10^{15}$
n_{dz_0}	$5.30487731413775 \cdot 10^{14}$	n_{uz_0}	$-1.01734739641052 \cdot 10^{20}$
n_{dy_6}	$-1.48961486915325 \cdot 10^{-3}$	n_{uy_6}	$8.38247723151215 \cdot 10^{-2}$
n_{dy_5}	$-1.68094799392667 \cdot 10^{-1}$	n_{uy_5}	9.45916194730949
n_{dy_4}	$7.50730900417259 \cdot 10^{4}$	n_{uy_4}	$5.8545095338644 \cdot 10^{6}$
n_{dy_3}	$-1.12430891777554 \cdot 10^{6}$	n_{uy_3}	$2.67284842906589 \cdot 10^{8}$
n_{dy_2}	$-3.20992721534292 \cdot 10^{12}$	n_{uy_2}	$5.40386484579556 \cdot 10^{13}$
n_{dy_1}	$4.69023322107588 \cdot 10^{13}$	n_{uy_1}	$5.08699135510151 \cdot 10^{14}$
n_{dy_0}	$1.7174289693222 \cdot 10^{19}$	n_{uy_0}	$2.93494011490626 \cdot 10^{19}$
a_5	$1.1284446931451 \cdot 10^{2}$	a_4	$6.51670383450632 \cdot 10^{7}$
a_3	$2.98444480192078 \cdot 10^{9}$	a_2	$5.72533172685257 \cdot 10^{14}$
a_1	$5.42593932182628 \cdot 10^{15}$	a_0	$3.04635759998437 \cdot 10^{20}$

TABLE I: Numerical values for the polynomials' coefficients in the case of the synthesis model.

III. Control design

The main goal of the sought controller is to reduce the peaks of resonance for the first three controllable/observable modes, by using an *output feedback controller*, without making unstable the vibration modes that are not in the synthesis model. By using the same notations as in [15],



(a) First mode at 120 Hz (b) Second mode at 497.6 Hz (c) Third mode at 1178 Hz

Fig. 2: First three controllable and observable axisymmetric vibration modes. The non-axisymmetric modes are not shown.

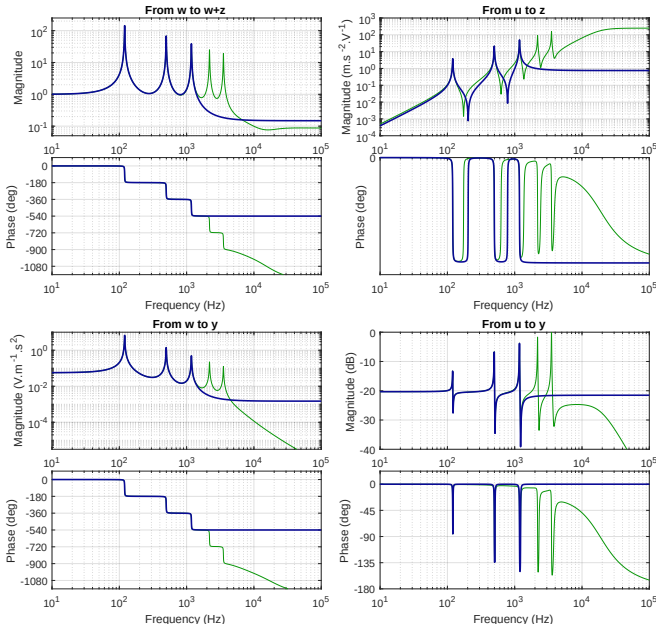


Fig. 3: Frequency responses of the analysis (thin-green) and synthesis (thick-dark blue) models.

the considered system is inserted in the output feedback control structure of Fig. 5, where the reference signal is always equal to zero. A rectangular impulse signal is used for the disturbance input d. The control problem consists in reducing the vibrations generated by the first three modes, when the mobile support imposes a shock to the whole flexible membrane. Let us define the output feedback control law in frequency domain by

$$u(s) := K(s,\tau)\, y(s), \tag{3}$$

where $K(s,\tau)$ stands for the PPP controller. This last is given in Laplace domain by

$$K(s,\tau) := \frac{Q(s,\tau)}{P(s,\tau)} \tag{4}$$

where $Q(s,\tau) := q_0 + q_{r_0}\, e^{-\tau s}$, and $P(s,\tau) := p_0 + p_{r_0}\, e^{-\tau s}$.

After applying the inverse Laplace transform, this control law given in time-domain writes as follows:

$$u(t) = -\frac{p_{r_0}}{p_0} u(t-\tau) + \frac{q_0}{p_0} y(t) - \frac{q_{r_0}}{p_0} y(t-\tau) \tag{5}$$

which is nothing else than an output feedback control law based on proportional actions plus delayed proportional actions, including also past information of the control input.

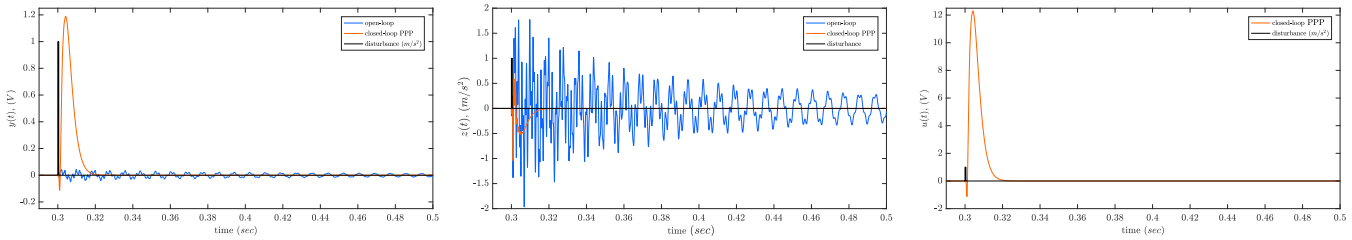
Proposition 1 *There exists a set of parameters q_0, q_{r_0}, p_0, p_{r_0} and $\tau > 0$ such that the controller* (4) *allows to assign three negative dominant poles (spectral values) λ_1, λ_2 and λ_3, for the system* (1) *in closed-loop with the control law* (3)*.*

IV. Simulation Results

The PPP method gives the following numerical values for the parameters of the controller in (4) that assigns $\lambda_1 = -500$, $\lambda_2 = -550$ and $\lambda_3 = -600$ as the dominant roots of the characteristic polynomial for $p_0 \simeq 4.461108157$, $p_{r0} \simeq 3.051496534$, $q_0 \simeq 41.28980087$, $q_{r0} \simeq 36.40327852$ and $\tau \simeq 3.358554554 \cdot 10^{-4}$. Figure 6 shows the distribution of the closed-loop poles, with a focus around the three assigned poles λ_1, λ_2 and λ_3. To show the performances of the proposed PPP-controller, the time responses of both output signals in open-loop (blue) and in closed-loop (orange) are depicted in Fig. 4c, when the disturbance d is a rectangular impulse (black), modelling a very brief shock imposed to the whole flexible structure. The time response of the control signal u is also plotted in Fig. 4c. A peak of magnitude roughly equal to 12 V can be observed, a value that is quite realistic for this kind of application.

V. Concluding Remarks

This work has shown the interest of using delays in a control law as a design parameter. The practical application that we have considered is a flexible axisymmetric membrane equipped with a piezoelectric sensor and a piezoelectric actuator in a collocated configuration. For this system, it was expected to achieve an active vibration control thanks to an output feedback controller. This controller have been



(a) Time response of the measured output y. (b) Time response of the controlled output z. (c) Time response of the closed-loop control signal u.

Fig. 4: Time responses: open-loop *vs* closed-loop.

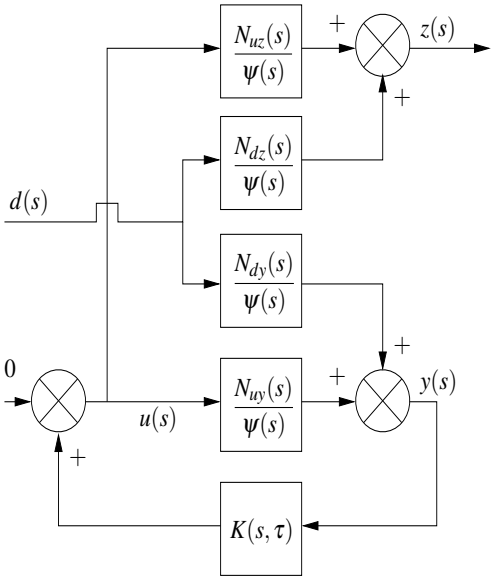


Fig. 5: Feedback control structure.

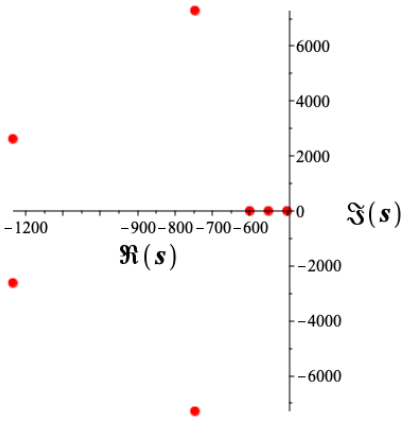


Fig. 6: Distribution of the closed-loop poles when $p_0 \simeq 4.461108157$, $p_{r0} \simeq 3.051496534$, $q_0 \simeq 41.28980087$, $q_{r0} \simeq 36.40327852$, $\tau \simeq 3.358554554 \cdot 10^{-4}$.

designed by using the partial pole placement approach in order to ensure the damping of the first three observable and controllable vibrating modes. The output feedback feature opens the possibility to implement such a controller in an experimental test bench.

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