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Drawing numerable linear orderings

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Numerable linear orderings (S, \leq_S)

- S is a numerable set
- \leq_S is a reflexive, anti-symmetric and transitive relation on S
- any two elements of S are related/comparable
- examples:
 - ω : the ordering of the natural numbers $(0, 1, 2, \dots)$
 - ζ : the ordering of the integers $(\dots, -2, -1, 0, 1, 2, \dots)$
- operations
 - addition: set union and all the elements from a set before the elements from the other
 - product: set product and with lexicographical order
 - $\{a, b\}^*$ with lexical order
- examples:
 - $\omega + \omega$ is $0, 1, 2, \dots, 0', 1', 2', \dots$ ($0'$ is a distinct copy of 0)
 - $\omega \cdot \omega$ is $(0, 0), (0, 1), (0, 2), \dots, (1, 0), (1, 1), (1, 2), \dots, (2, 0), (2, 1), (2, 2), \dots$

Ordinals

- well founded orderings
- examples:
 - $\omega, \omega + \omega, \omega + \omega + \omega, \omega \cdot \omega, \omega \cdot \omega \cdot \omega + \omega \cdot \omega + \omega$
- non-ordinal linear orderings:
 - ζ has the infinite decreasing sequence $(\dots, -2, -1, 0)$
 - $\{a, b\}^*$ with lexical order has the infinite decreasing sequence (\dots, aab, ab, b)

- More on linear orderings \rightsquigarrow Rosenstein [1982]

Decidable

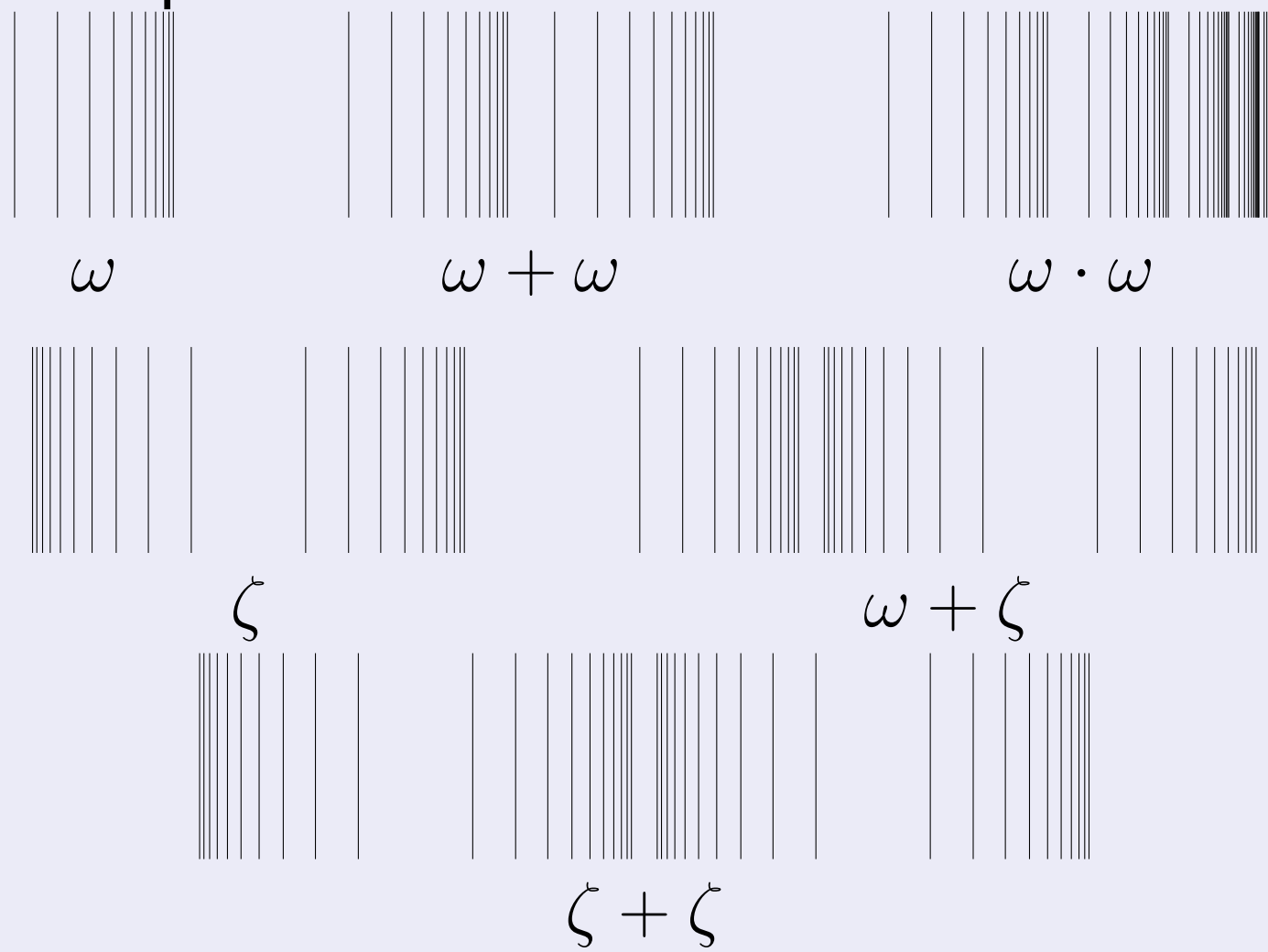
- s_0, s_1, s_2, \dots : an enumeration of the elements of S
- A Turing machine decides the relation

$$i, j \mapsto s_i \leq_S s_j$$

Graphical representation

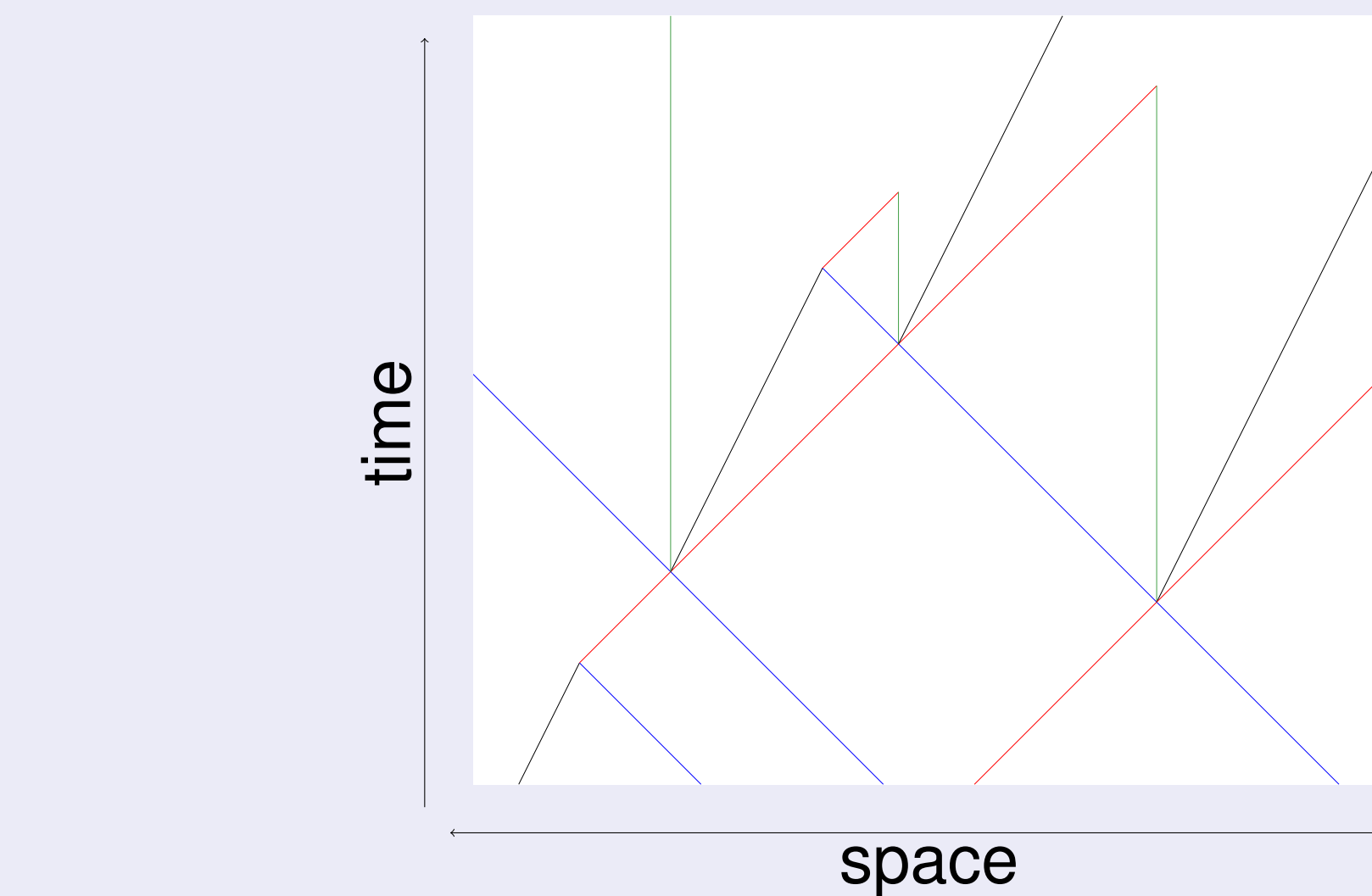
- by parallel vertical lines s.t.:
 - one line per element in S
 - if $x \leq_S y$ then x is on the left of y
 - there is an empty space between x and y ($x \leq_S y$) iff x is the immediate predecessor of y
 - iff y is the immediate successor of x
 - iff $\forall z, x \leq_S z \leq_S y \rightarrow x = z = y$

- examples:

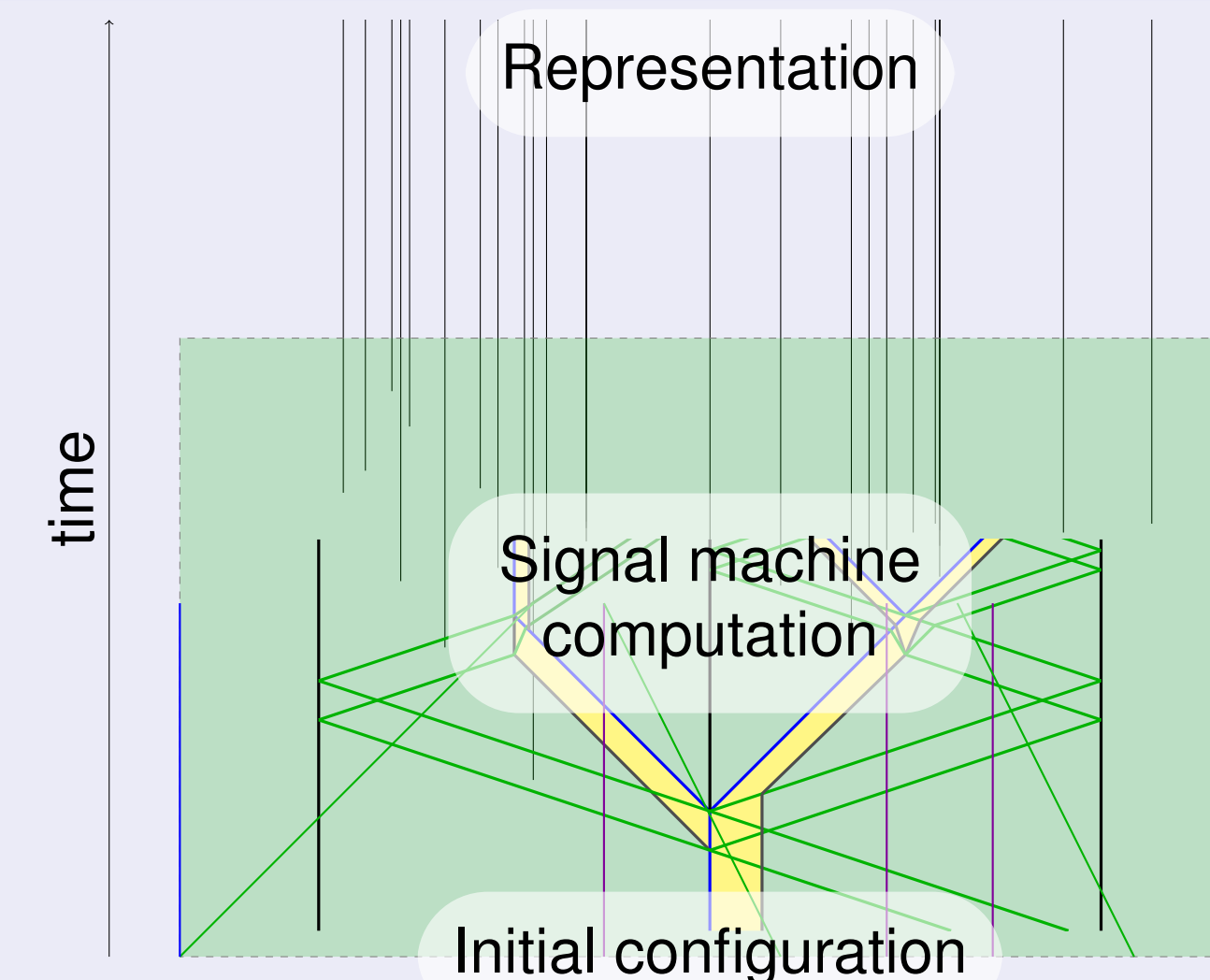


Signal Machines

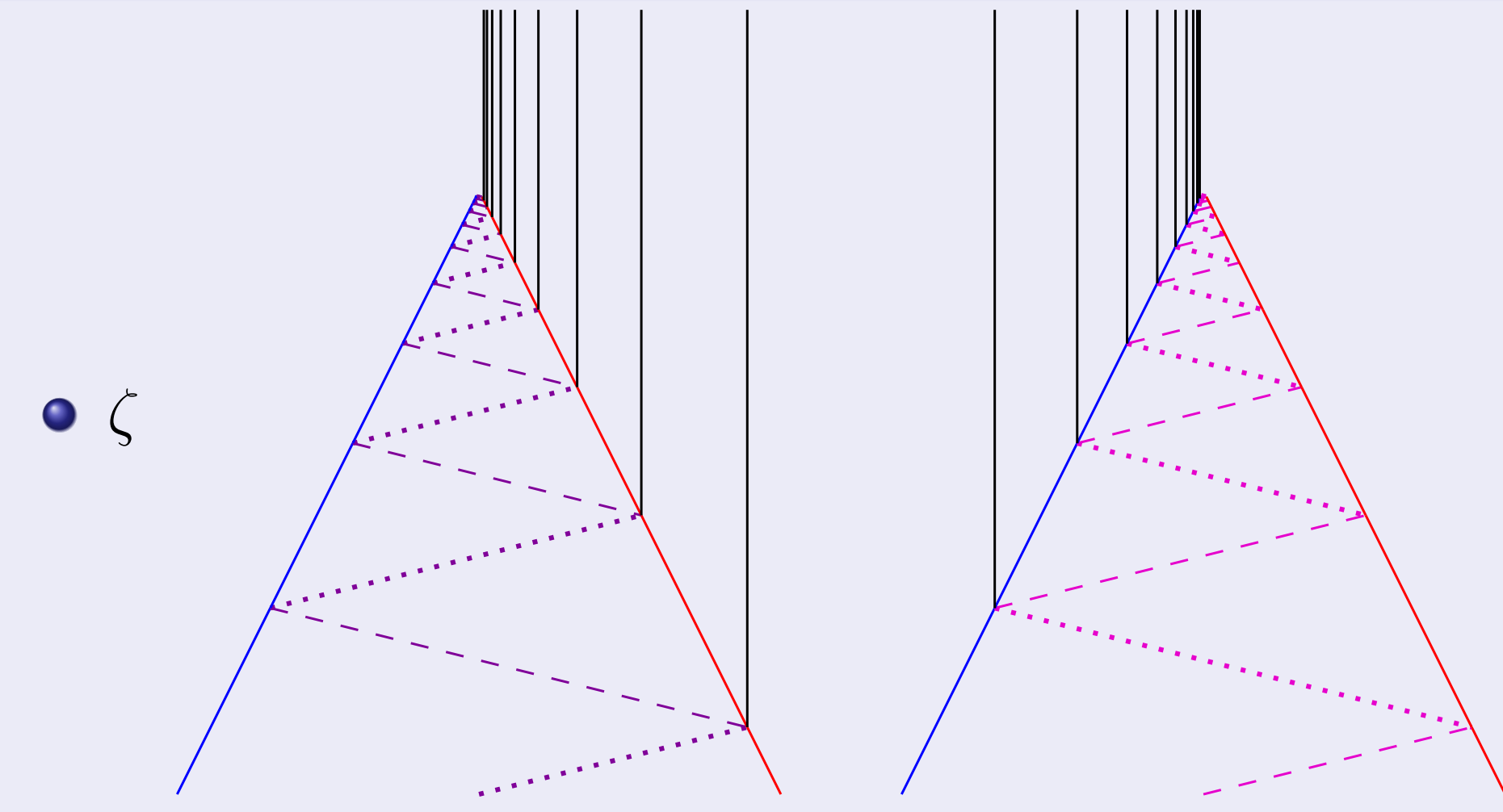
- signals
 - red, black, green, blue
 - constant speed for each kind
- collision rules
 - $\{\text{black}, \text{blue}\} \rightarrow \{\text{red}\}$
 - $\{\text{red}, \text{blue}\} \rightarrow \{\text{blue}, \text{green}, \text{black}, \text{red}\}$
- dynamics \rightsquigarrow space-time diagrams



Global scheme



Ad hoc constructions

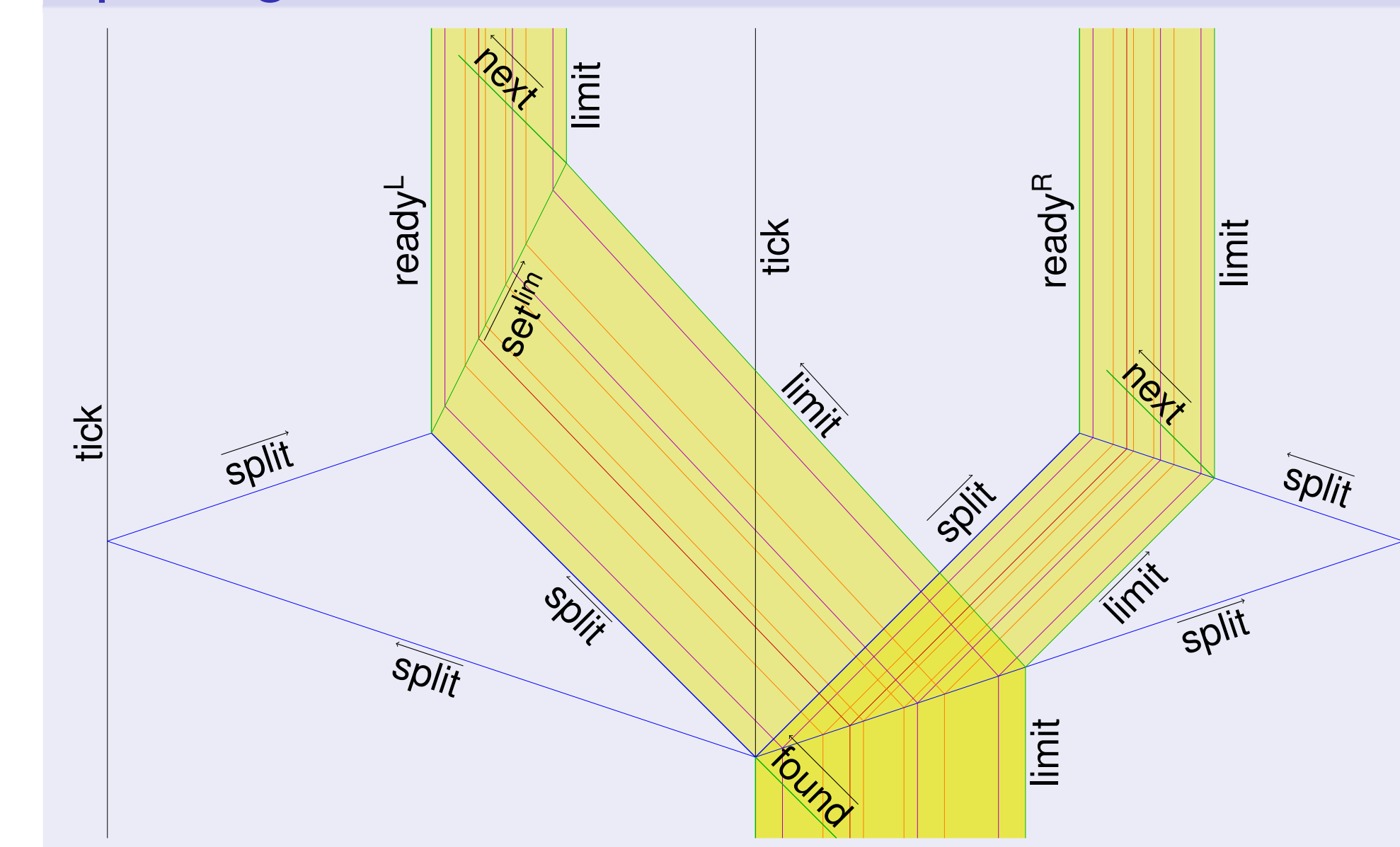


Algorithm

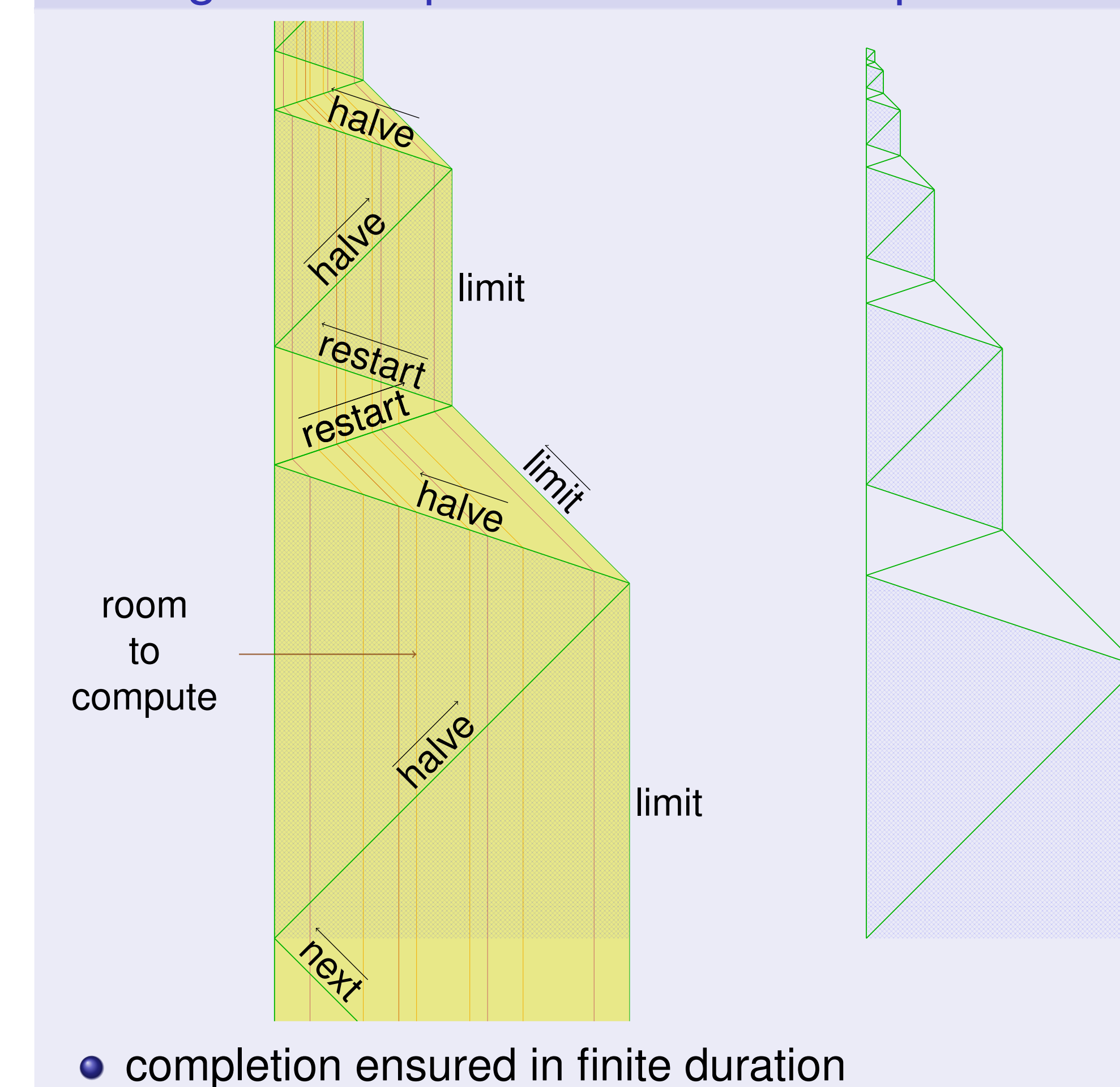
- let i, j be two indices s.t. lines are set
- find least k such that:

$$k \neq i \wedge k \neq j \wedge s_i \leq_S s_k \leq_S s_j$$
- if k is found, the interval is split and the algorithm is restarted on i and k on the left and k and j on the right
- if no such k exists, the computation vanishes
- this is done in finite time

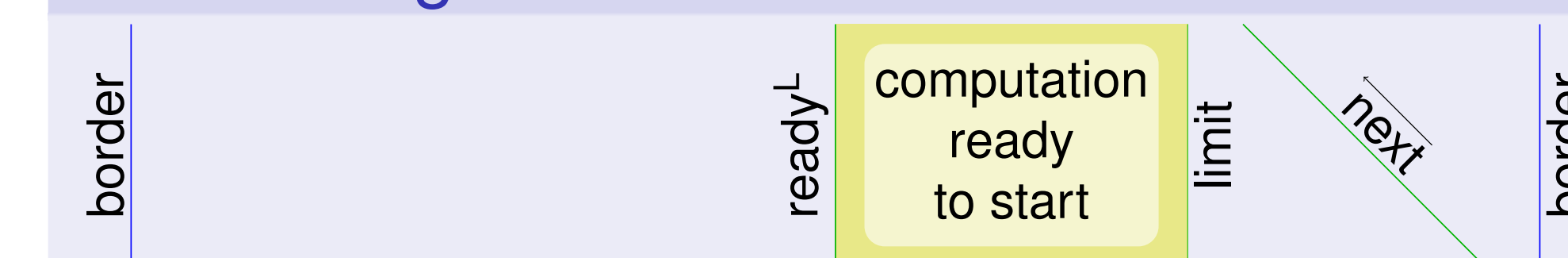
Splitting



Scaling the computation at each step



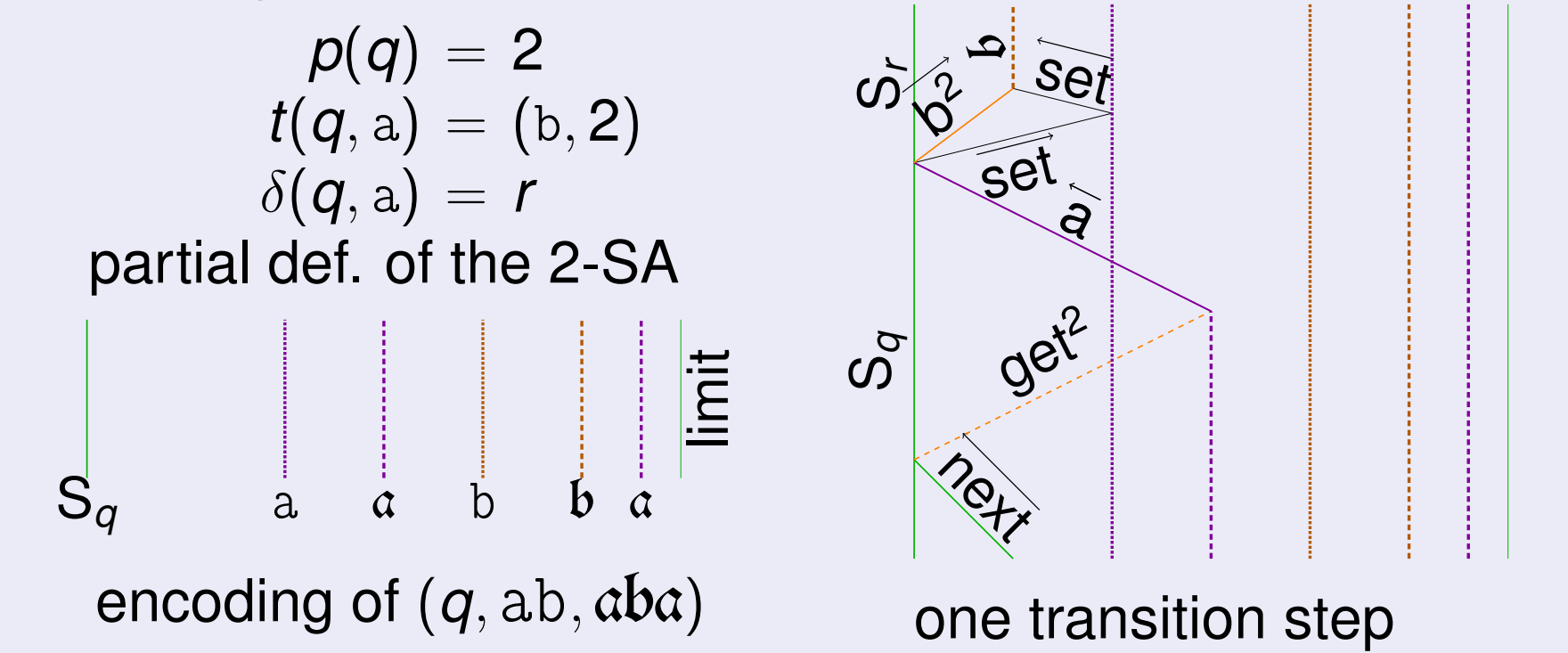
Initial configuration



Working on \mathbb{Q}

There is a rational signal machine able to generate the representation of any decidable countable linear ordering.

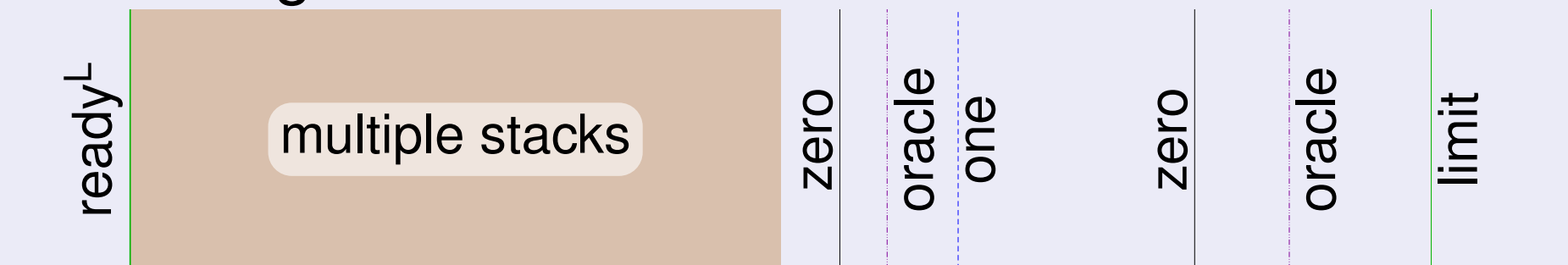
- Simulating multi-stack automaton



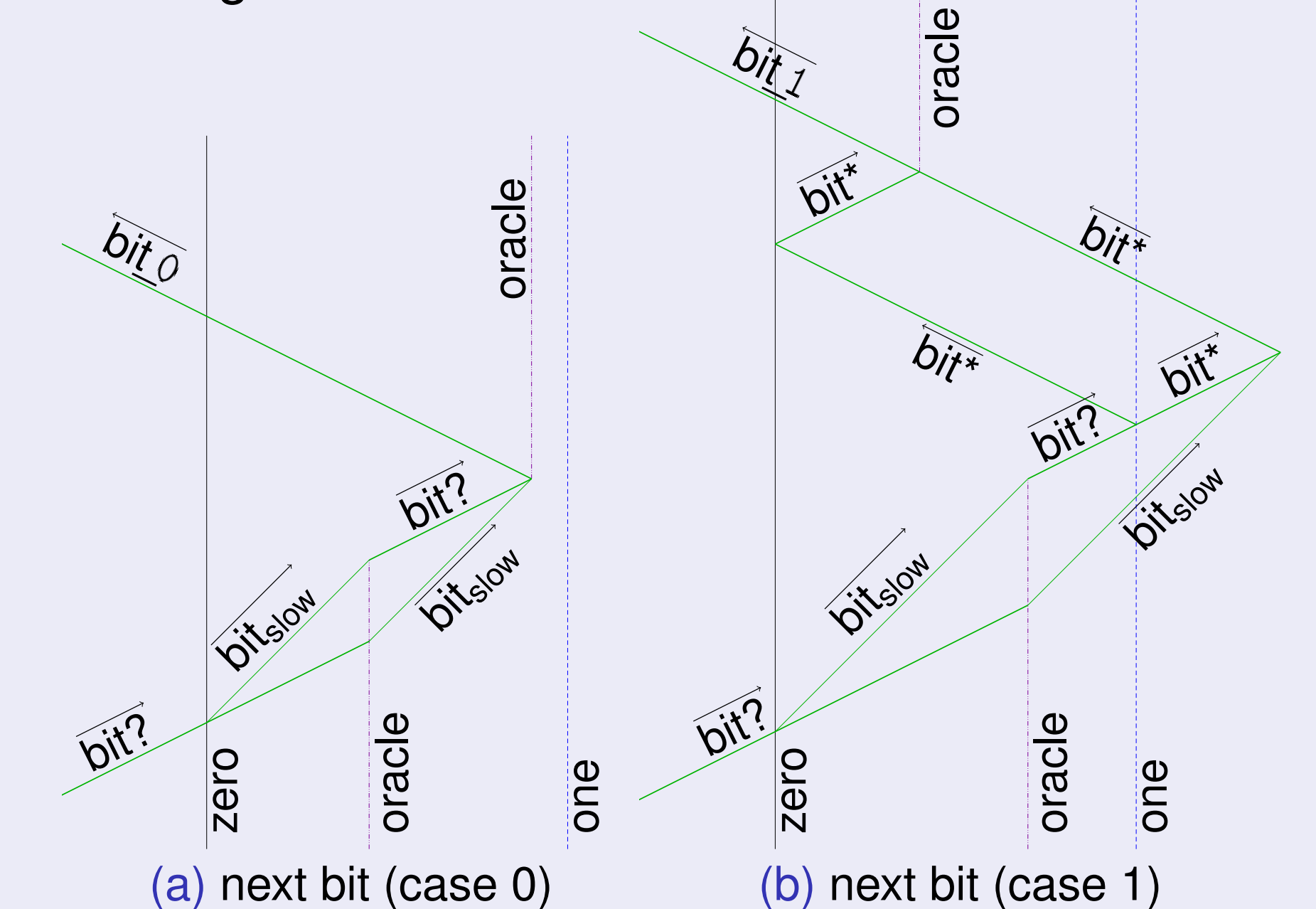
Working on \mathbb{R}

There is a signal machine able to generate the representation of any countable linear ordering.

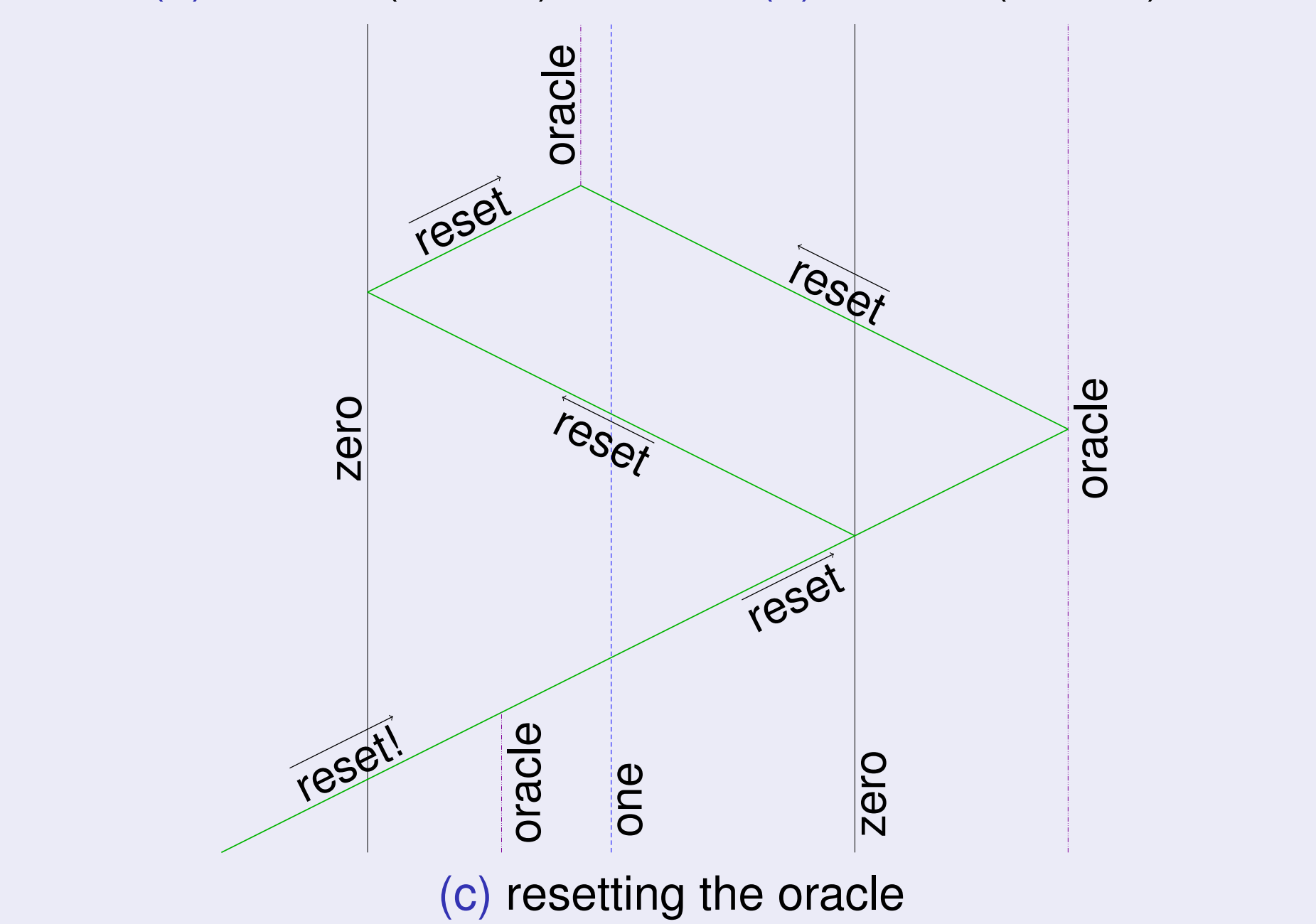
- Encoding an oracle



- Handling the oracle



(a) next bit (case 0) (b) next bit (case 1)



(c) resetting the oracle

References

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- J. G. Rosenstein. *Linear ordering*. Academic Press, 1982.