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Polyhedral Reductions for Petri nets

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Polyhedral equivalence

$$(N_1, m_1) \equiv_E (N_2, m_2)$$

Polyhedral equivalence defines a *linear dependence relation* (E) between the reachable markings of two nets; where E is a Presburger formula, or a system of linear Diophantine equations. This equivalence, denoted $(N_1, m_1) \equiv_E (N_2, m_2)$, preserves enough information in E so that we can rebuild the state space of N_1 knowing only the one of N_2 .

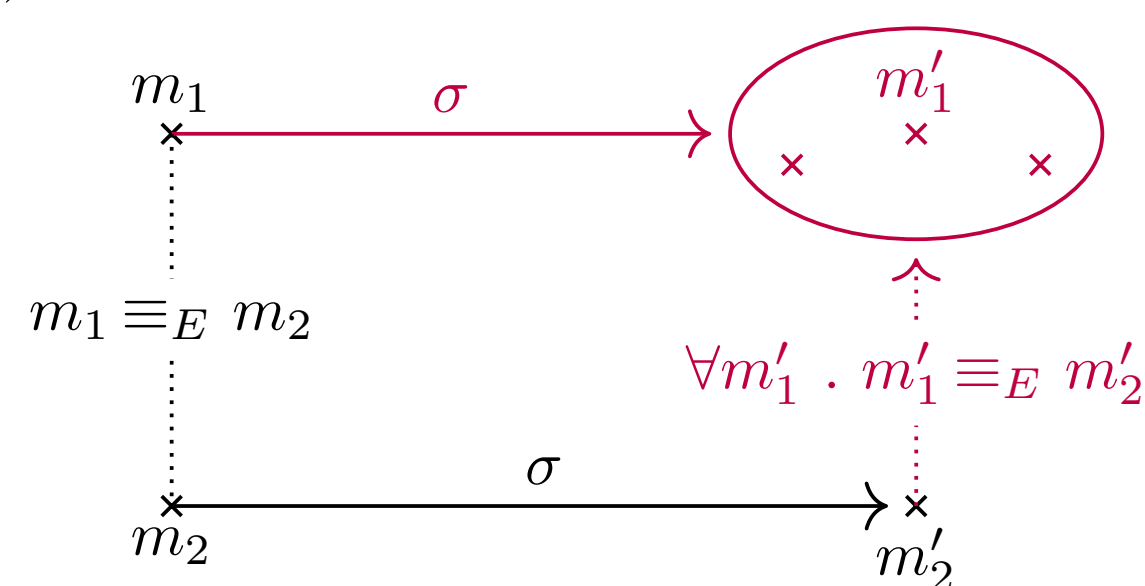
We say that two markings m_1, m_2 are compatible up-to E , denoted $m_1 \equiv_E m_2$, when formula $E \wedge m_1 \wedge m_2$ is satisfiable, with m the conjunction $\bigwedge_{p \in P} p = m(p)$ (e.g., $(p_0 = 5) \wedge (p_1 = 0) \wedge \dots$).

Nets (N_1, m_1) and (N_2, m_2) are E -equivalent when:

- (A1) $E \wedge m$ is satisfiable for all markings m in $R(N_1, m_1)$ or $R(N_2, m_2)$;
- (A2) initial markings are *compatible*: $m_1 \equiv_E m_2$;
- (A3) assume m'_1, m'_2 are markings of N_1, N_2 , such that $m'_1 \equiv_E m'_2$, then m'_1 is reachable iff m'_2 is reachable.

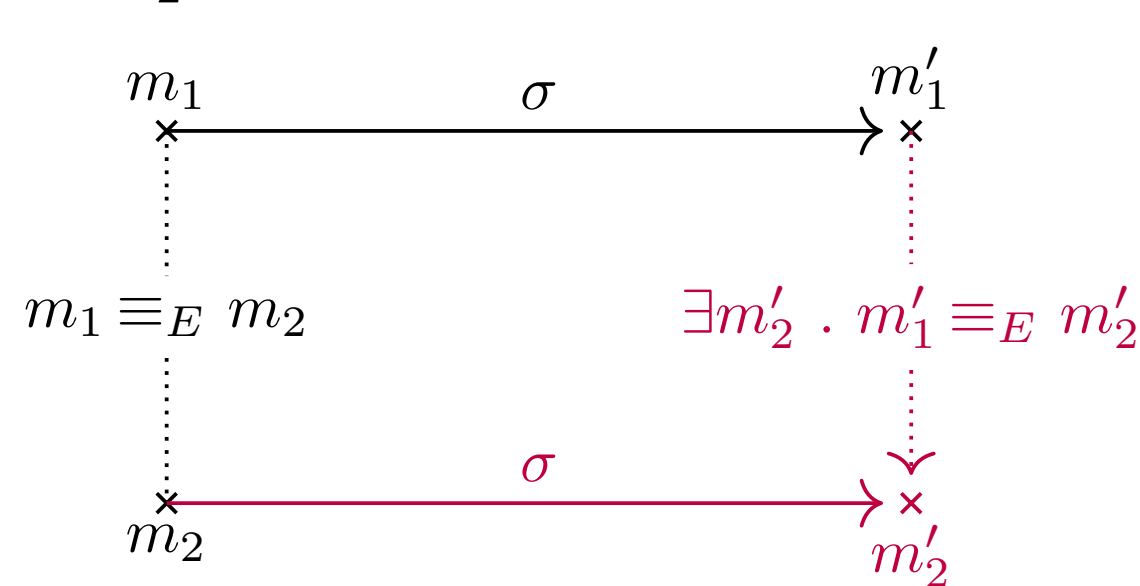
Lemma (Reachability Checking).

Given m'_2 reachable in N_2 then $m'_1 \equiv_E m'_2$ implies $m'_1 \in R(N_1, m_1)$.



Lemma (Invariance Checking).

For all m'_1 in $R(N_1, m_1)$ there is m'_2 in $R(N_2, m_2)$ such that $m'_1 \equiv_E m'_2$.



Counting using SDD



```
$ sift AirplaneLD-PT-0050.pnml
4471223 marking(s), 19756224 transitions(s)
67.569s

$ tedd -q AirplaneLD-PT-0050.pnml
4471223 state(s)
2.691s

$ tedd -q AirplaneLD-PT-4000.pnml
2.1773922960e12 state(s)
33.574s

$ tedd -rg,redundant,compact AirplaneLD-PT-4000.pnml
2.1773922960e12 state(s)
0.000s
```

Reachability using SMT



To check if property F_1 is reachable in N_1 , it is enough to check its E -transform, F_2 , on N_2 .

$$F_2(\mathbf{y}) \triangleq \exists \mathbf{x} . \tilde{E}(\mathbf{x}, \mathbf{y}) \wedge F_1(\mathbf{x})$$

We use this property in a SMT-based model-checker with structural reductions. It supports generalized reachability formulas of the form: $AG((p_1 + p_3 \geq 5) \vee (p_0 < p_4))$

```
$ python3 -m smpt -n SmallOpSys-MT0016.pnml
--methods BMC K-INDUCTION PDR-REACH STATE-EQUATION
--show-time --show-techniques ...
FORMULA RC4 FALSE TECHNIQUES ... BMC TIME 0.04s

$ python3 -m smpt -n SmallOpSys-MT2048.pnml ...
timeout

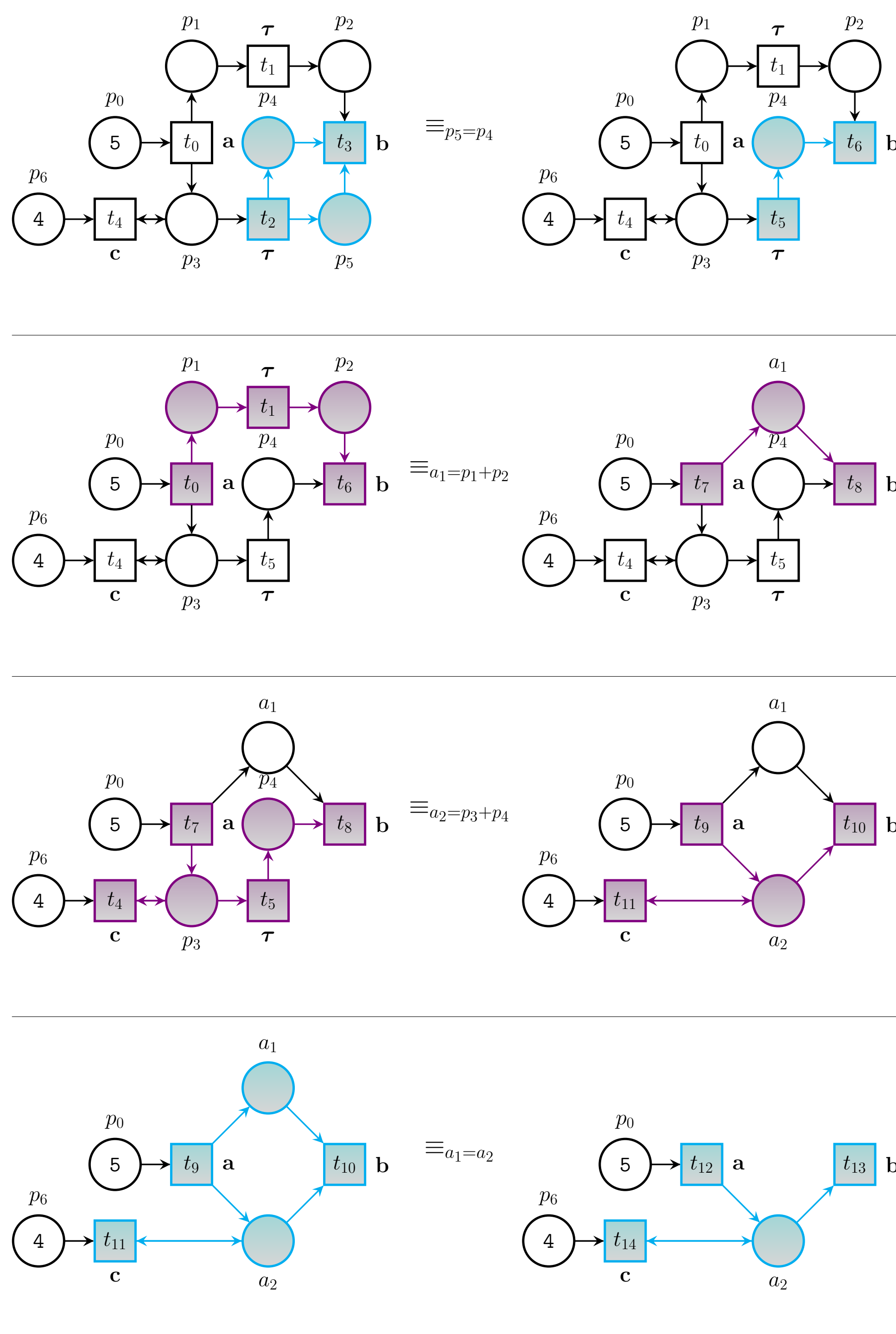
$ same with option --auto-reduce or --mcc
# Reduction time (Full): 0.11s
FORMULA RC4 FALSE TIME 0.05s

$ or with options --project --show-projection
# Reduction time (TFG): 0.09s
# Reduction ratio (TFG) 44%
# Projection of RC4: [ ] - (1 ≤ p0 + p2 + a2) ∧ ...
```

Polyhedral reductions



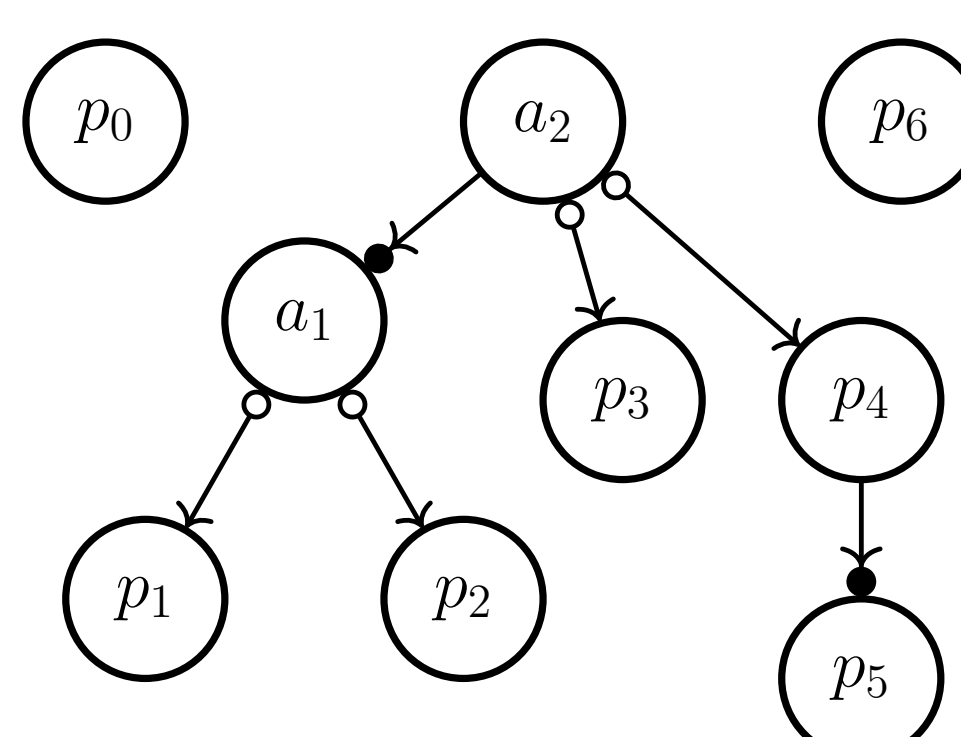
We can compute instances of polyhedral equivalences using structural reductions.



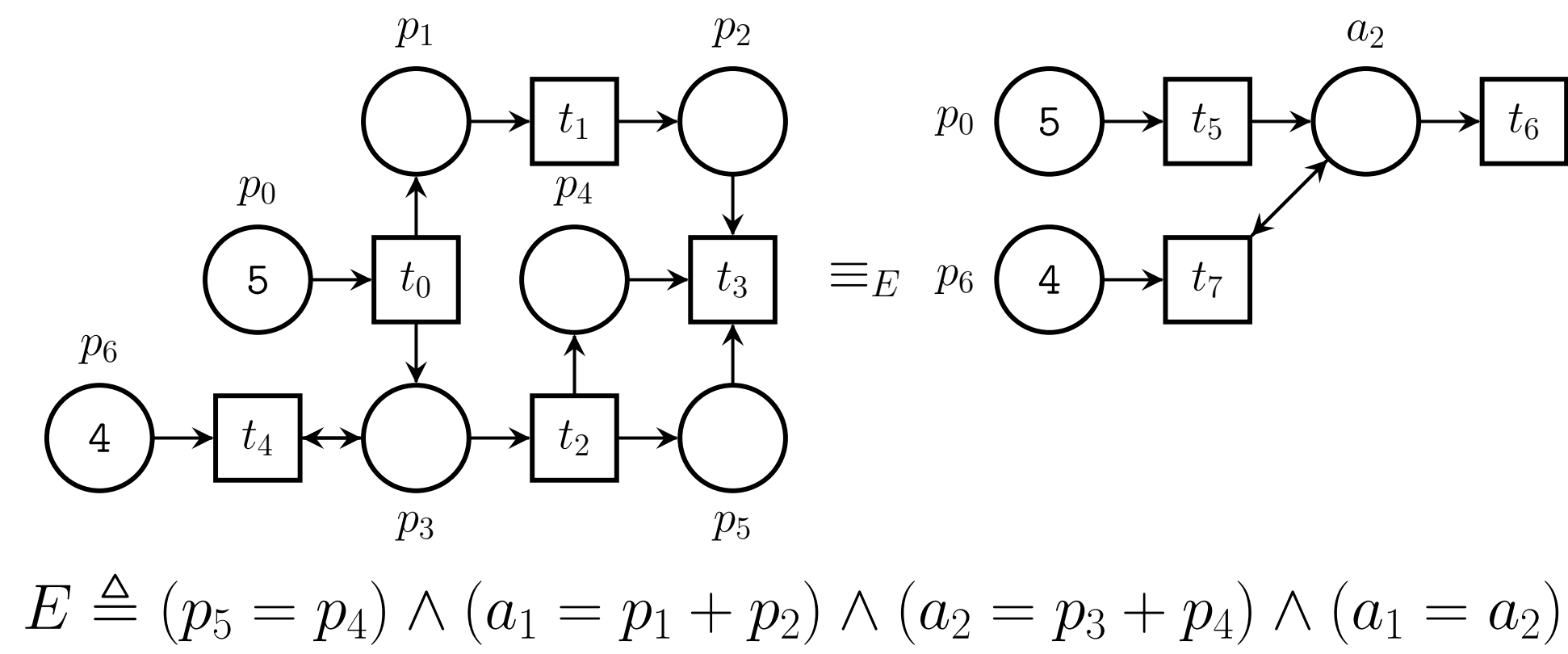
Token Flow Graph

- Octant
- Kong
- Polycount

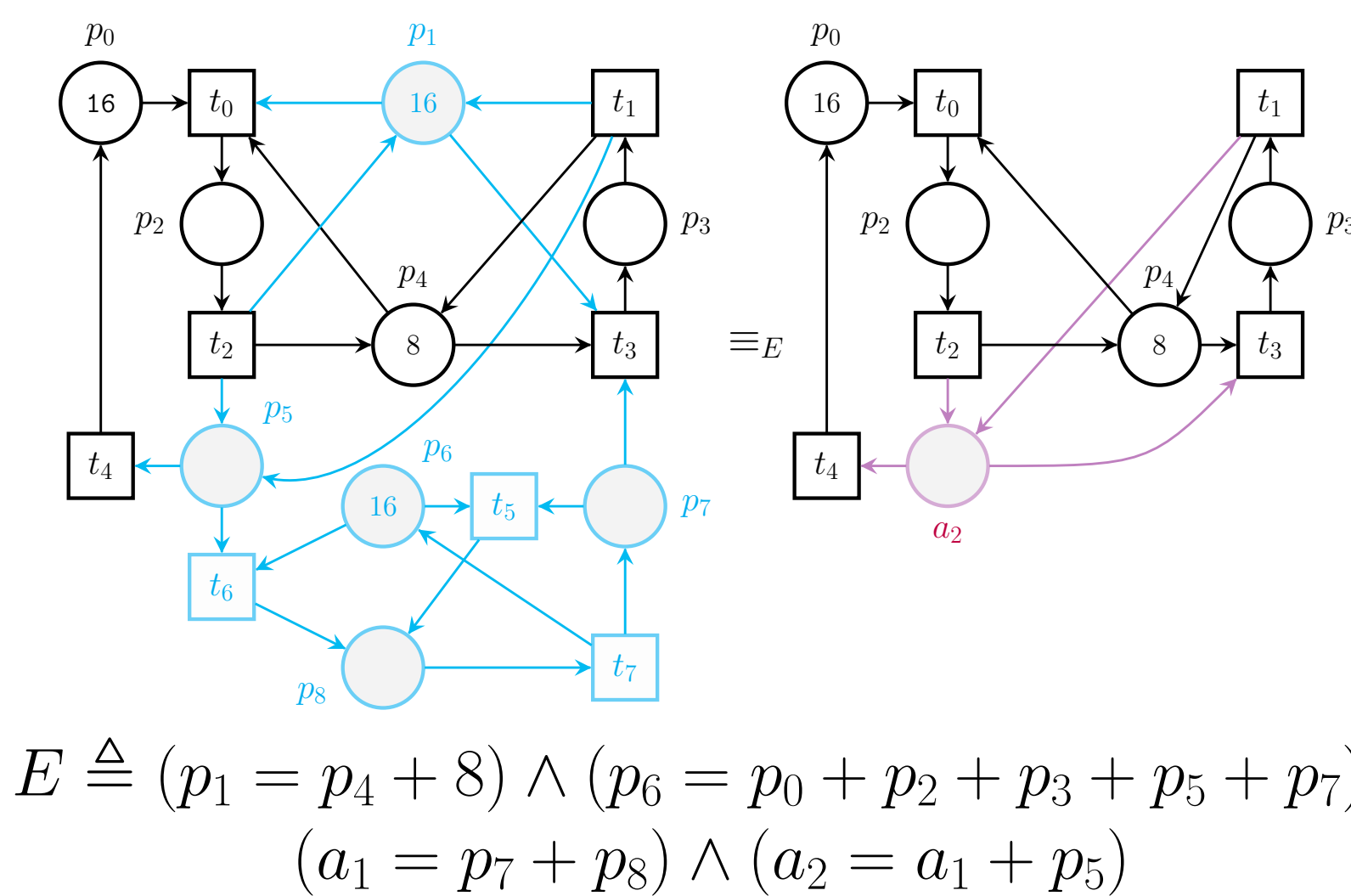
- # $R \vdash p_5 = p_4$
- # $A \vdash a_1 = p_2 + p_1$
- # $A \vdash a_2 = p_4 + p_3$
- # $R \vdash a_1 = a_2$



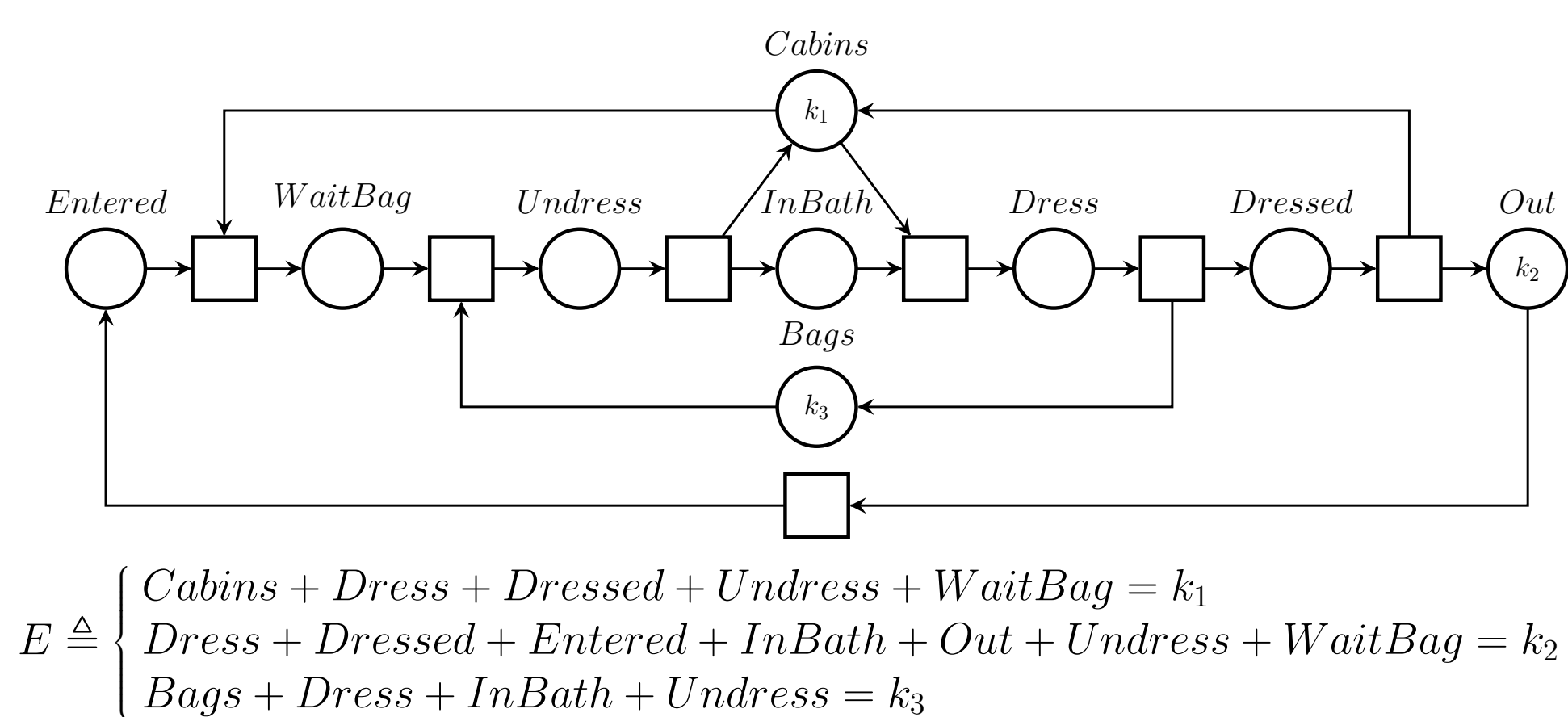
Some examples



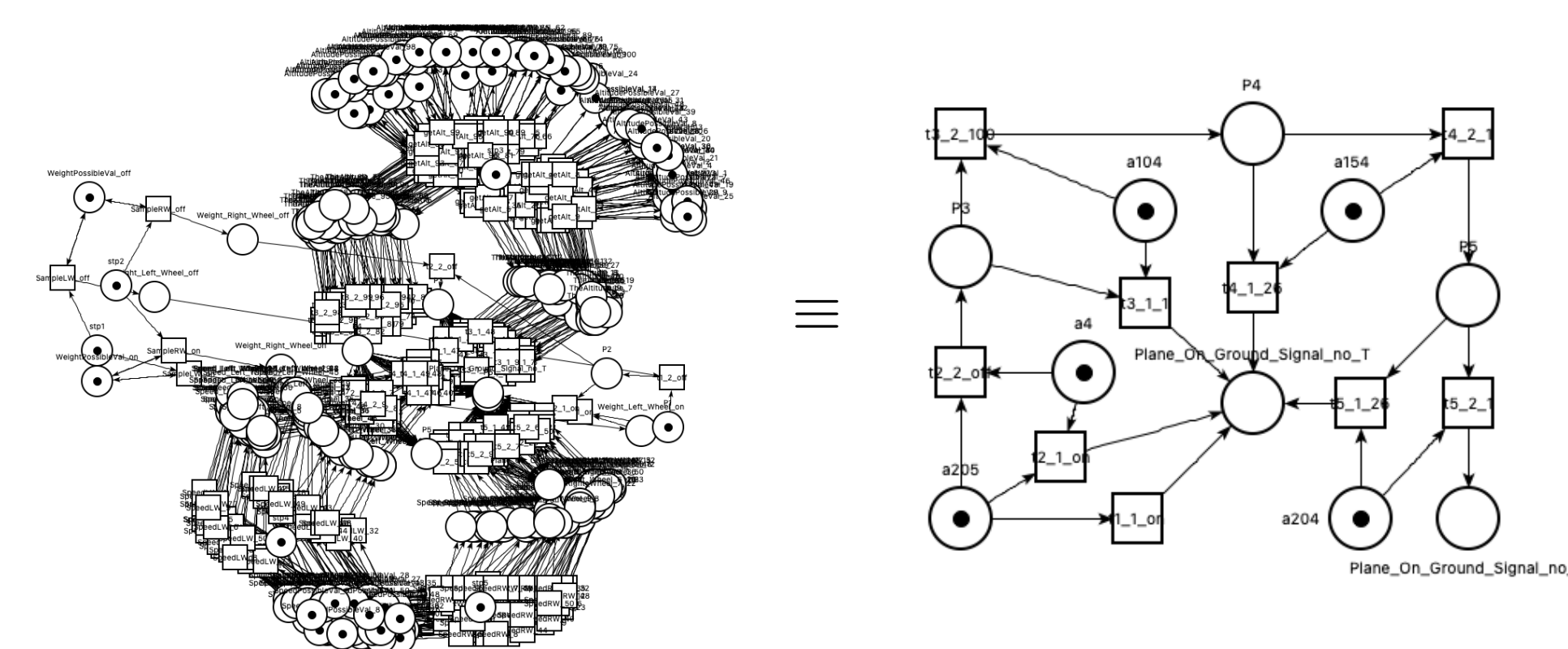
SmallOperatingSystem-PT-MT0016DC0008



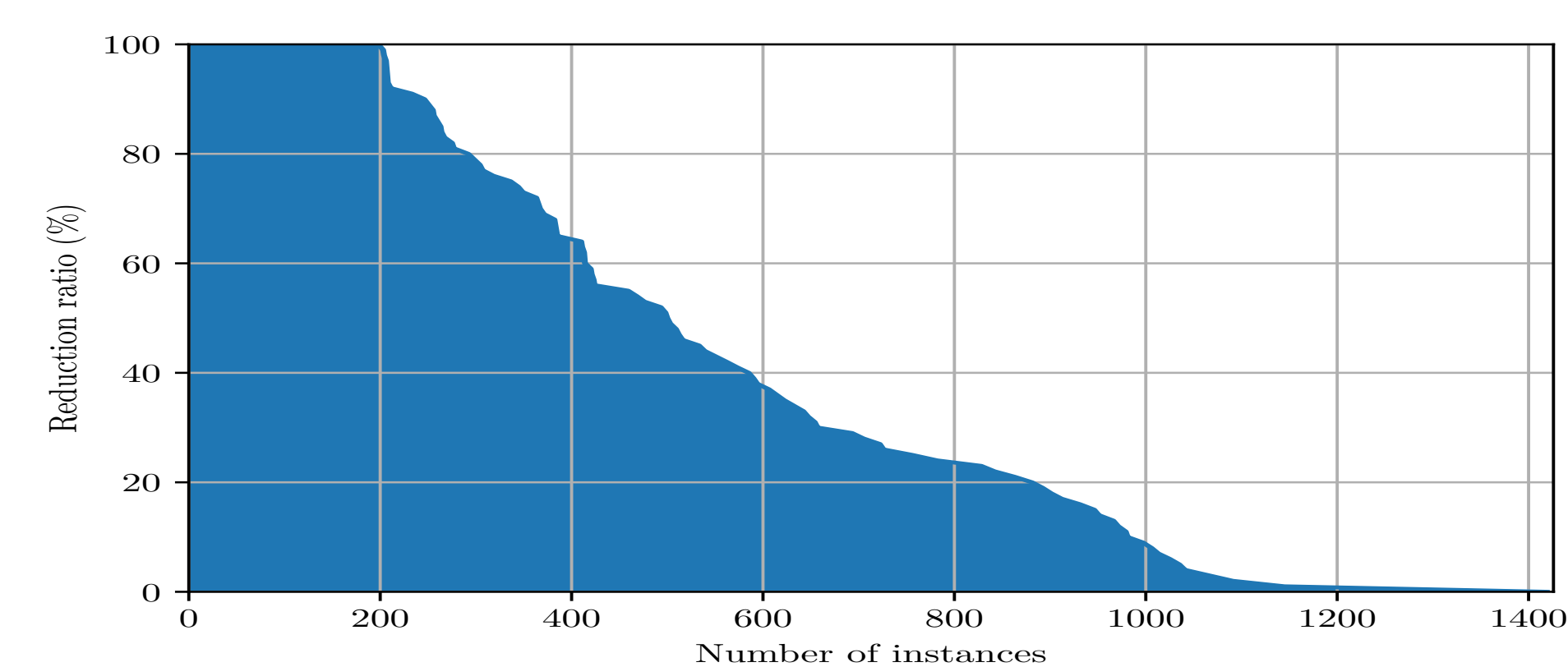
SwimmingPool (fully reduced)



AirplaneLD-PT-0050

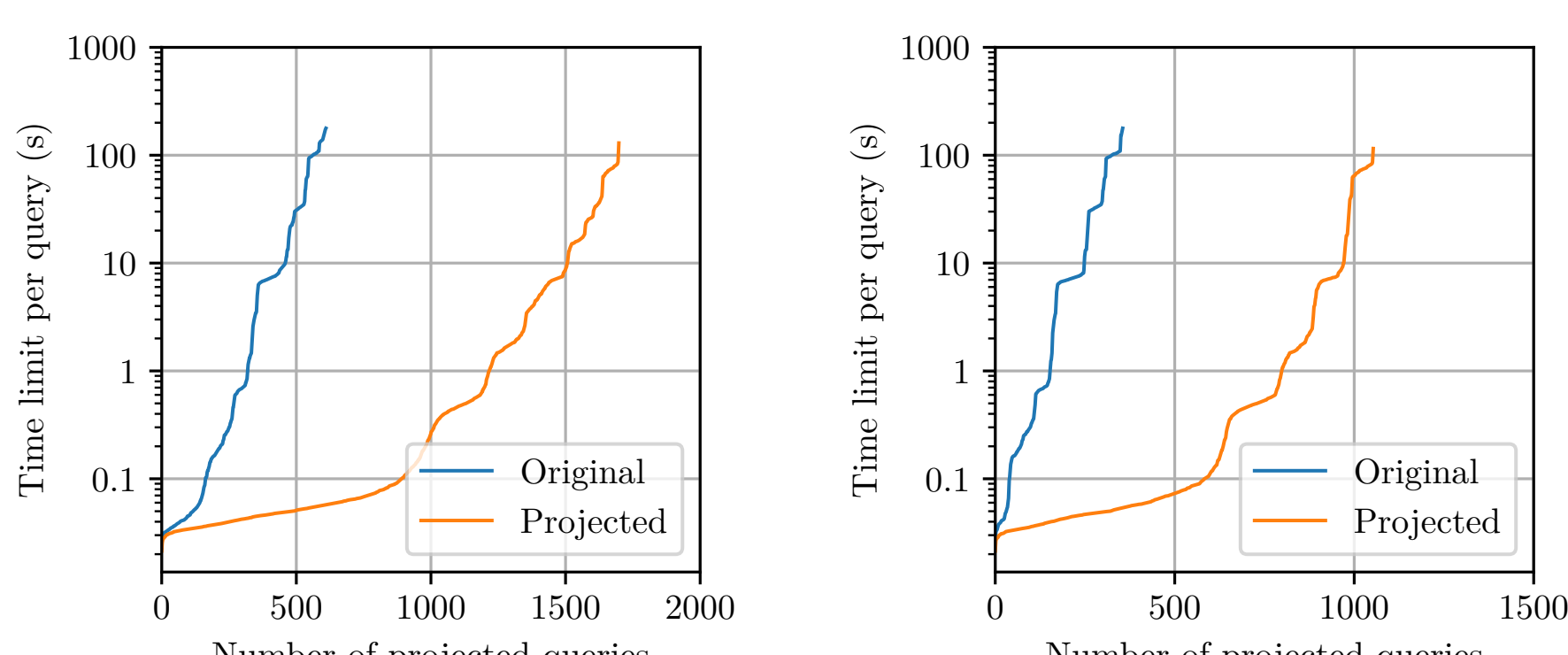


Prevalence of reductions in the MCC



Automatic equivalence checking

The problem of checking whether two nets are equivalent is undecidable. But we have implemented a sound, complete procedure in the case of equivalence between *parametric nets*: $(N_1, C_1) \approx_E (N_2, C_2)$.



$$r \in [0.01, 0.5[\quad r \in [0.5, 1]$$

k -induction with and without reductions

