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To cite this version:
Nicolas Amat, Bernard Berthomieu, Silvano Dal Zilio, Didier Le Botlan. Polyhedral Reductions for Petri nets. Modélisation des Systèmes Réactifs (MSR’23), Nov 2023, Toulouse, France. hal-04355257

HAL Id: hal-04355257
https://hal.science/hal-04355257
Submitted on 20 Dec 2023

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Polyhedral equivalence

\((N_1, m_1) \equiv_E (N_2, m_2)\)

Polyhedral equivalence defines a linear dependence relation \((E)\) between the reachable markings of two nets, where \(E\) is a Presburger formula, or a system of linear Diophantine equations. This equivalence, denoted \((N_1, m_1) \equiv_E (N_2, m_2)\), preserves enough information in \(E\) so that we can rebuild the state space of \(N_1\) knowing only the one of \(N_2\).

We say that two markings \(m_1, m_2\) are compatible up to \(E\) denoted \(m_1 \equiv_E m_2\), when formula \(E \land m_1 \equiv m_2\) is satisfiable, with \(m\) the conjunction \(\bigwedge_{p \in P} p \equiv m(p)\) (e.g., \((p_5 = 5) \land (p_0 = 0) \land \ldots\))

### Nets

\((N_1, m_1)\) and \((N_2, m_2)\) are \(E\)-equivalent when:

1. \(E \land m_2\) is satisfiable for all markings \(m\) in \(R(N_1, m_1)\) or \(R(N_2, m_2)\);
2. initial markings are compatible: \(m_1 \equiv_E m_2\);
3. assume \(m_1', m_2'\) are markings of \(N_1, N_2\) such that \(m_1' \equiv_E m_2'\), then \(m_1'\) is reachable iff \(m_2'\) is reachable.

#### Lemma (Reachability Checking)

Given \(m_2'\) reachable in \(N_2\) then \(m_1' \equiv_E m_2'\) implies \(m_1' \in R(N_1, m_1)\).

#### Lemma (Invariance Checking)

For all \(m_1'\) in \(R(N_1, m_1)\) there is \(m_2'\) in \(R(N_2, m_2)\) such that \(m_1' \equiv_E m_2'\).

We use this property in a SMT-based model-checker with structural reductions. It supports generalized reachability formulas of the form:

\[ A \equiv (p_1 + p_1 \geq 5) \lor (p_2 < p_3) \]

### Counting using SDD

\[ \text{Count} \text{\# instances: } 4471223 \text{ markings, } 19756224 \text{ transitions} \]

### Reachability using SMT

To check if property \(F_1\) is reachable in \(N_1\), it is enough to check its \(E\)-transform \(F_2\) on \(N_2\):

\[ F_2(y) \equiv \exists x \cdot E(x, y) \land F_1(x) \]

### Automatic equivalence checking

The problem of checking whether two nets are equivalent is undecidable. But we have implemented a sound, complete procedure in the case of equivalence between parametric nets \((N_1, C_1) \equiv_E (N_2, C_2)\).

### Some examples

- **SmallOperatingSystem-PT-MT0016DC0008**

- **AirplaneLD-PT-0050**

### Prevalence of reductions in the MCC

The number of instances: 4471223 markings, 19756224 transitions.