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# Waves and Velocity Addition

Abhishek Majhi <sup>\*1</sup> and Tiyasa Kar <sup>†2</sup>

<sup>1</sup>Indian Statistical Institute,  
Plot No. 203, B. T. Road, Baranagar,  
Kolkata 700108, West Bengal, India.

<sup>2</sup>Department of Physics, North Carolina State University,  
Raleigh, NC 27695, USA.

## Abstract

In standard practice, velocity addition rules are derived for point particles from Galilean and Lorentz transformations. Such rules are then intuitively applied for waves to explain Doppler effect, following what Doppler did himself originally. However, such application goes without any explanation in terms of equations manifesting such frame dependence of velocity. We show that wave propagation should be modeled with first order, not second order, partial differential equations so as to manifest such frame dependence of wave velocity. From a historical point of view, this work settles the Doppler-Petzval debate that was pivoted to the question of underlying differential equations for Doppler effect. From a modern theoretical physics stand point, this work ignites a foundational debate regarding what can now be considered as “equation for wave propagation”.

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\*abhishek.majhi@gmail.com

†tkar@ncsu.edu

# 1 Introduction: Doppler’s intuition and phenomenology

The standard practice of dealing with frame dependence of velocity of wave propagation and the effect of the motion of the source and of the observer/detector, whether it may be sound, light, hydrodynamic or any other type of wave, relies on our intuition of velocity transformation of a particle under Galilean and Lorentz coordinate transformations. That is, velocity addition for waves is done only at the phenomenological level. The practice started with Doppler [1–8], which got experimental support from Russell [9], Ballot [10] and gave birth to other seminal works like those of Voigt [11, 12] and Mach [13–15] who, especially, changed the course of scientific queries regarding effects of motion on waves at the most elementary level by bringing in the concepts of Mach cone, Mach number, etc. – for example see ref. [16]. Doppler effect and its various other consequences have become a useful tool of the scientists for analyzing the characteristics of waves due to the effect of motion [2–8]. Such a practice is justified by the empirical essence of the resulting formulas such as those of Doppler-Fizeau effect [17, 18], reverse/inverse Doppler effect [19–39], Vavilov-Cherenkov effect [40] leading to various experimental applications in optics, acoustics and hydrodynamics till date. Also, it continues to foster research regarding basic concepts of physics [43–51].

Nevertheless, it can not be denied that it has hitherto not been analyzed whether an equation for wave propagation actually manifests such velocity transformation like that of a particle. That is, when it is written in words “velocity of a wave differs in two frames which are in constant relative velocity with respect to each other”, such a statement is not explicated in terms of equations. To clarify and establish our doubt/skepticism, we may quote Rayleigh from p.154 of ref. [52], who applied Doppler’s intuition [1], as follows: “*The pitch of a sound is liable to modification when the source and the recipient are in relative motion. It is clear, for instance, that an observer approaching a fixed source will meet the waves with a frequency exceeding that proper to the sound, by the number of wave-lengths passed over in a second of time. Thus if  $v$  be the velocity of the observer and  $c$  that of sound, the frequency is altered in the ratio  $c \pm v : c$ , according as the motion is towards or from the source.*” [“ $a$ ” replaced by “ $c$ ” if compared to original] and then, we pose the following question, **Q**, in general for wave propagation.

**Q:** *Let us suppose that there is an equation which explains the propagation of a wave, with velocity  $c$  with respect to the preferred rest frame of the medium (e.g. air, water, optical medium, etc.). Is there an equation that explains the same, with velocity  $c \pm v$  with respect to the observer, where  $\pm v$  is the velocity of the observer with respect to the preferred rest frame of the medium of wave propagation, to which the source is attached?*

On a historical note, we may point out that similar concern was raised by Petzval who asked for the underlying differential equations for the frame dependence of velocity and frequency of waves [3, 5, 8, 53]. Our intent is to search for an answer to **Q**. In course of doing that we reach a conclusion that velocity addition for wave propagation can only be explained through first order partial differential equations (PDEs) and not second order PDEs which is however the present standard understanding. In case the reader is misled by the simplicity of our question and the concerned analyses, we suggest a consultation of Appendix (A) and particularly the references cited therein; we have chosen to exclude such discussions from the main body of this work so as to keep it simple and direct.

## 2 Analysis of propagation under Galilean transformation

In this section we revisit the velocity transformation of a point particle based on Galilean coordinate transformation, along with suitable diagrammatic representations, and then proceed towards a similar analysis to examine the frame dependence of wave velocity with an aim to identify what we can call “equation for wave propagation”.

### 2.1 Revisiting particle propagation

Let us consider two frames  $S$  and  $S'$ , with origins  $O$  and  $O'$  respectively, such that the latter is moving with respect to the former, along  $x$  direction with constant velocity  $v$ . The Galilean transformation of coordinates

are well known (e.g. see ref. [55]) and given by:

$$x' = x - \sigma vt \quad : \sigma = \pm 1, \quad (1)$$

$$y' = y, \quad z' = z, \quad (2)$$

$$t' = t. \quad (3)$$

Here, the values of  $\sigma$  have the following significance:

1. “ $\sigma = +1$ ” signifies “ $S'$  (or  $O'$ ) is moving along increasing/positive  $x$  direction and away from  $S$  (or  $O$ )”. (Figure: 1a)
2. “ $\sigma = -1$ ” signifies “ $S'$ (or  $O'$ ) is moving along decreasing/negative  $x$  direction and towards  $S$  (or  $O'$ )”. (Figure: 1b)

We may clarify a few points as follows which will help when we shall pass on to the scenario of wave propagation. We consider here that the particle is thrown by a source located at  $O$  and it is observed by an observer/detector located at  $O'$ . We note that in case of  $\sigma = -1$ , i.e. when  $O'$  moves towards  $O$ , there must be an initial separation between  $O$  and  $O'$ , say  $a$ . Therefore, we must have  $x' = x + vt - a$  so that the motion of  $O'$  ( $x' = 0$ ) is given by  $x = a - vt$  in  $S$  frame. Thus, in general we must write  $x' = x - \sigma vt + \frac{1}{2}(\sigma - 1)a$  :  $a > 0$ , in place of (1). However, this apparently does not affect the partial derivatives that we are going to consider for the construction of equation for wave propagation. Hence, we continue to ignore this subtlety about the effect of initial configuration on Galilean transformation.

For the motion of a point particle, the velocity transformation can be realized in a straightforward manner as follows:

$$\frac{dx'}{dt'} = \frac{dx}{dt} - \sigma v \Leftrightarrow \eta c' = \eta c - \sigma v \quad : \eta c' = \frac{dx'}{dt'}, \quad \eta c = \frac{dx}{dt}, \quad (4)$$

where  $\eta c$  and  $\eta c'$  are the velocities of the particle with respect to  $S$  and  $S'$  respectively. Here, “ $\eta = +1$ ” signifies “forward propagating particle”(left to right) and “ $\eta = -1$ ” signifies “backward propagating particle”(right to left). Thus, the velocity transformation can be written as follows:

$$c' = c - \frac{\sigma}{\eta} v. \quad (5)$$

■ When  $\eta = +1$  (i.e. forward propagation) and the observer at  $O'$  is situated on the right side of  $O$  during the course of the phenomenon,

1. for  $\sigma = +1$ , we have  $c' = c - v$ . (Figure: 2a)
2. for  $\sigma = -1$ , we have  $c' = c + v$ . (Figure: 2b)

■ When  $\eta = -1$  (backward propagation) and the observer at  $O'$  is situated on the left side of  $O$  during the course of the phenomenon,

1. for  $\sigma = +1$ , we have  $-c' = -c - v$ . (Figure: 3a)
2. for  $\sigma = -1$ , we have  $-c' = -c + v$ . (Figure: 3b)

■ When  $\eta = -1$  (backward propagation) and the observer at  $O'$  is situated on the left side of  $O$  during the course of the phenomenon,

1. for  $\sigma = +1$ , we have  $-c' = -c - v$ . (Figure: 3a)
2. for  $\sigma = -1$ , we have  $-c' = -c + v$ . (Figure: 3b)

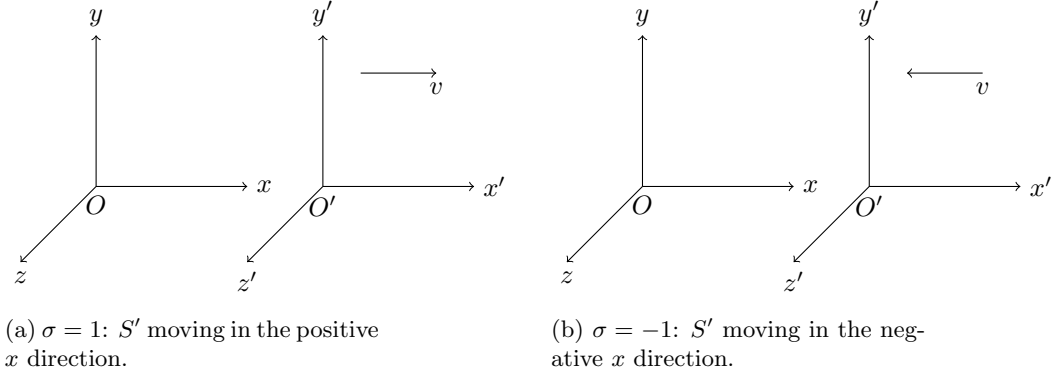


Figure 1: Relative motions of  $S'$  with respect to  $S$ .

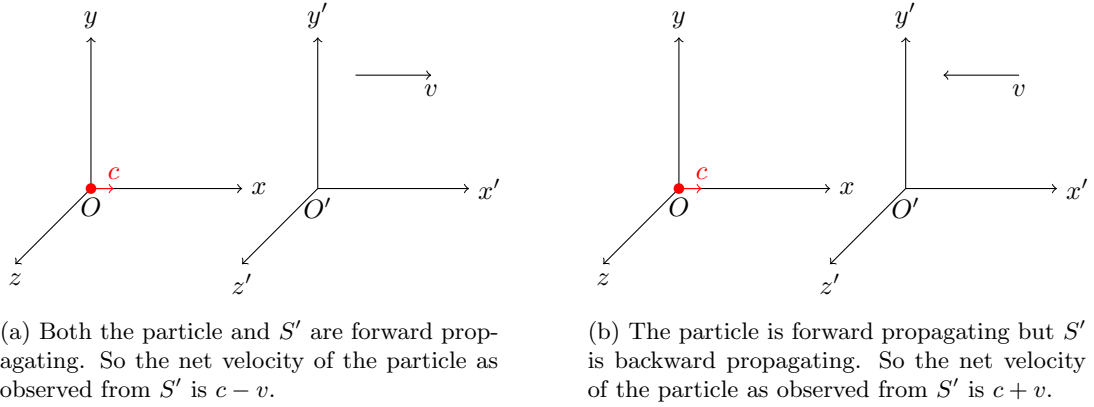


Figure 2: Relative motions of  $S'$  with respect to  $S$  with particle with forward propagating particle with  $S'$  being on the right side of  $S$  for the entire motion

## 2.2 Properties of “equation for wave propagation”

Now, let us consider that  $S$  frame is the preferred frame of rest (e.g. air, water, optical medium, etc.) in which the equation for wave propagation is constructed. The source is fixed with respect to this medium and situated at the origin  $O$  of  $S$ . The observer/detector is situated at the origin  $O'$  of  $S'$  frame, which moves with a constant velocity of magnitude  $v$ , with respect to the source. We expect that the “equation for wave propagation” must have the following two properties.

$P_1$ : It must maintain its form, or remain invariant, under Galilean transformation so that we can actually write “the wave is observed from both  $S$  and  $S'$  frames”.

$P_2$ : It must showcase the velocity transformation corresponding to our explanation that “a wave propagates with velocity  $\eta c$  with respect to  $S$ ” and “a wave propagates with velocity  $\eta c'$  with respect to  $S'$ ”.

## 2.3 Analysis with 1st order PDE

We note that the partial derivatives transform as follows:

$$\frac{\partial}{\partial x} \Leftrightarrow \frac{\partial}{\partial x'}, \quad (6)$$

$$\frac{\partial}{\partial t} \Leftrightarrow -\sigma v \frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}. \quad (7)$$

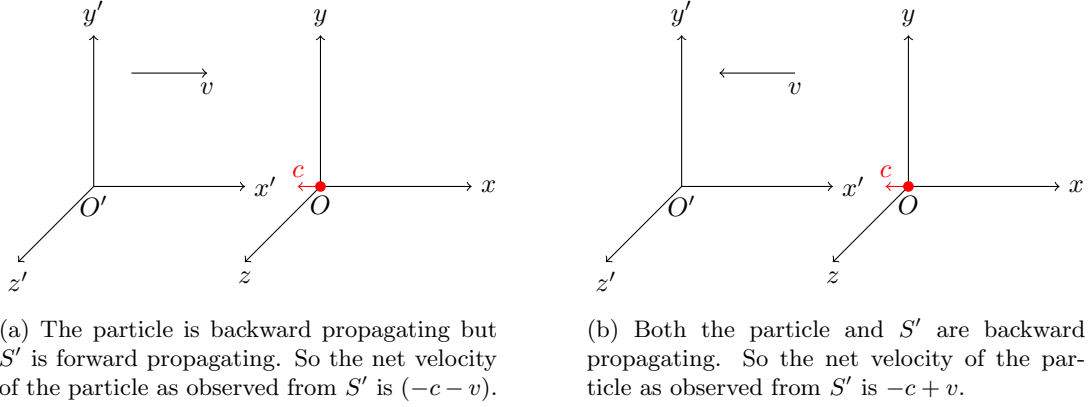


Figure 3: Relative motions of  $S'$  with respect to  $S$  with particle with backward propagating particle with  $S'$  being on the left side of  $S$  for the entire motion

We write the propagating function as follows:

$$f_\eta(x - \eta ct) : \eta = \pm 1, \quad (8)$$

where

- “ $\eta = +1$ ” signifies the “forward propagation/left to right” (along increasing  $x$  direction (Figure: 2)),
- “ $\eta = -1$ ” signifies the “backward propagation/right to left” (along decreasing  $x$  direction (Figure: 3)).

Then, we can construct the following equation in  $S$  frame:

$$\frac{\partial f_\eta}{\partial x} = -\frac{1}{\eta c} \frac{\partial f_\eta}{\partial t}, \quad (9)$$

from which we can read off equations for “forward propagation” and “backward propagation” for  $\eta = +1$  and  $\eta = -1$  respectively, i.e., we write the following correspondence.

$$\text{Forward propagation } (\eta = +1): \quad \frac{\partial f_+}{\partial x} = -\frac{1}{c} \frac{\partial f_+}{\partial t}, \quad (10)$$

$$\text{Backward propagation } (\eta = -1): \quad \frac{\partial f_-}{\partial x} = +\frac{1}{c} \frac{\partial f_-}{\partial t}. \quad (11)$$

The above two equations can represent “equation for wave propagation” on satisfaction of both  $P_1$  and  $P_2$ , which is indeed the case as we see in what follows. Using (6) and (7) we can write the eq.(9) transformed to  $S'$  frame as follows:

$$\frac{\partial f_\eta}{\partial x'} = -\frac{1}{\eta c'} \frac{\partial f_\eta}{\partial t'} : c' = c - \frac{\sigma}{\eta} v. \quad (12)$$

Also, we may note that the phase of the wave must remain invariant under transformation between  $S$  and  $S'$  frames, which leads to the fact that the wavelength remains invariant as well, as follows:

$$x - \eta ct = x' - \eta c' t' \equiv h \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) (x - \eta ct) = 0 \equiv \lambda = \lambda' \text{ for arbitrary } x, t \text{ satisfying } x \neq \eta ct, \quad (13)$$

where  $\lambda$  and  $\lambda'$  are the wavelengths of the wave in  $S$  and  $S'$  frames respectively. We may now calculate the relation between the frequencies of the wave,  $\nu = c/\lambda$  and  $\nu' = c'/\lambda'$ , in  $S$  and  $S'$  frames respectively, to be given by the following relation:

$$\nu' = \nu \left( 1 - \frac{\sigma v}{\eta c} \right). \quad (14)$$

So, from (12) and (14), we have the following cases for different values of  $\eta$  and  $\sigma$ .

■ When  $\eta = +1$  (forward propagation/left to right) and the observer at  $O'$  is situated on the right side of  $O$  during the course of the phenomenon (otherwise a forward propagating wave, originating from the source at  $O$ , won't reach the observer/detector/receiver at  $O'$ ),

1. for  $\sigma = +1$  (i.e. observer moving away from source in the increasing  $x$  direction, Figure: 4a), we have

$$\frac{\partial f_+}{\partial x'} = -\frac{1}{(c-v)} \frac{\partial f_+}{\partial t'} \quad \text{and} \quad \nu' = \nu \left( 1 - \frac{v}{c} \right), \quad (15)$$

2. for  $\sigma = -1$  (i.e. observer moving towards source in the decreasing  $x$  direction, Figure: 4b), we have

$$\frac{\partial f_+}{\partial x'} = -\frac{1}{(c+v)} \frac{\partial f_+}{\partial t'} \quad \text{and} \quad \nu' = \nu \left( 1 + \frac{v}{c} \right). \quad (16)$$

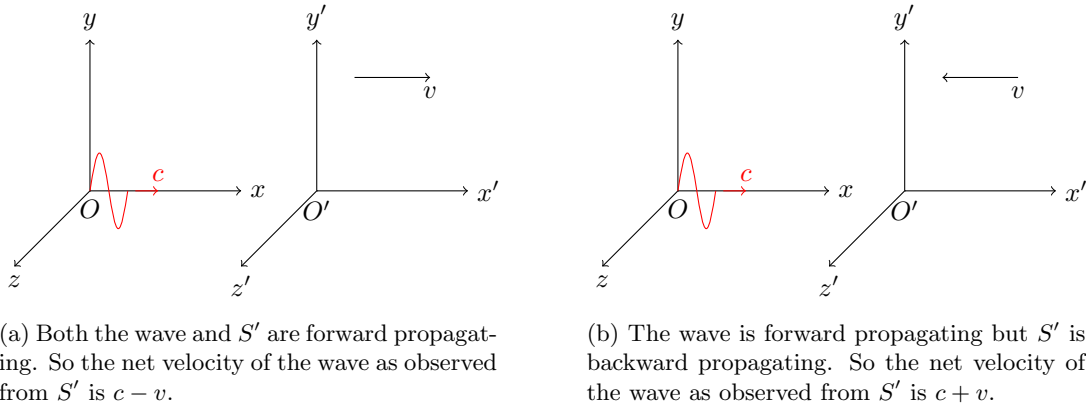


Figure 4: Relative motions of  $S'$  with respect to  $S$  with particle with forward propagating wave with  $S'$  being on the right side of  $S$  for the entire motion

■ When  $\eta = -1$  (backward propagation/right to left) and the observer at  $O'$  is situated on the left hand side of  $O$  during the course of the phenomenon (otherwise a backward propagating wave, originating from the source at  $O$ , won't reach the observer/detector/receiver at  $O'$ ),

1. for  $\sigma = +1$  (i.e. observer moving towards the source in the increasing  $x$  direction, Figure: 5a), we have

$$\frac{\partial f_-}{\partial x'} = \frac{1}{(c+v)} \frac{\partial f_-}{\partial t'} \quad \text{and} \quad \nu' = \nu \left( 1 + \frac{v}{c} \right), \quad (17)$$

2. for  $\sigma = -1$  (i.e. observer moving away from the source in the decreasing  $x$  direction, Figure: 5b), we have

$$\frac{\partial f_-}{\partial x'} = \frac{1}{(c-v)} \frac{\partial f_-}{\partial t'} \quad \text{and} \quad \nu' = \nu \left( 1 - \frac{v}{c} \right). \quad (18)$$

In view of the above analysis, we may conclude that the 1st order PDEs encoded in (12), satisfy the properties  $P_1$  and  $P_2$  and, therefore, can be considered as equations for wave propagation. This mathematically justifies Doppler's intuition of considering frame dependent velocity of waves to explain frequency shifts, as manifested through Rayleigh's statement quoted earlier and the standard phenomenological practice of velocity addition for waves.

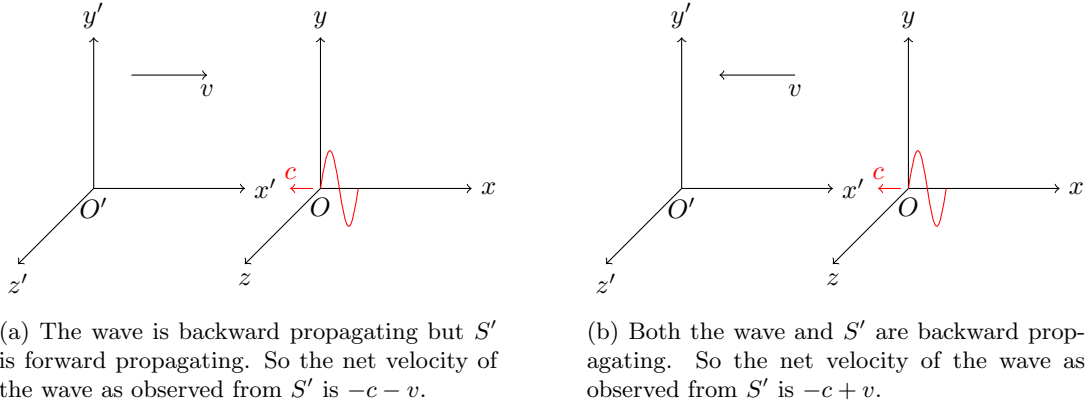


Figure 5: Relative motions of  $S'$  with respect to  $S$  with particle with backward propagating wave with  $S'$  being on the left side of  $S$  for the entire motion

## 2.4 Settling the Doppler-Petzval debate (what could have been....)

We believe, in view of our analysis, it is worth devoting a section concerning the historical encounters between Doppler [3] and Petzval [53] which we may call the Doppler-Petzval debate [3, 5, 8, 53]. A very recent concise account on this can be found in ref. [5], from which we use some quotes in the following remarks. As has been noted in ref. [5], Doppler’s empirical analysis of frequency shift of light was countered with fierce criticism from Petzval on the ground that Doppler’s analysis lacked any mathematical basis as “*all natural phenomena were the manifestations of underlying differential equations*”. Instead of providing a reply based on differential equations, Doppler’s defense relied on the question “*whether an observed phenomenon must be deemed nonexistent if it cannot be derived from differential equations.*” and on the confidence due to the empirical verification of Doppler effect by Russell [9] and Ballot [10] (also see the references cited in ref. [5]). Doppler’s reply was not considered to be satisfactory enough by most of his contemporaries, especially given the socio-scientific authority of Petzval at that time, which resulted in very disappointing, but undeserved, consequences for Doppler. We believe that the analysis, which we have presented above, could have been the appropriate reply by Doppler to “Petzval’s attack” [5], resulting in an avoidance of the unfortunate consequences. Now, it is interesting to note that, since the present analysis appears for the first time in the literature of science (up to the best of our knowledge), it indicates that the Doppler-Petzval debate has remained unsettled and ignored since its conception in the 19th century. Therefore, we may claim that now the Doppler-Petzval debate is settled as per our analyses.

## 2.5 Hearing sound backwards (Rayleigh)

Now, let us put some focus on the following particular fact. On p.154 of ref. [52], Rayleigh explained hearing sound backwards as follows: “*Since the alteration of pitch is constant, a musical performance would still be heard in tune, although in the second case, when  $c$  and  $v$  are nearly equal, the fall in pitch would be so great as to destroy all musical character. If we could suppose  $v$  to be greater than  $c$ , a sound produced after the motion had begun would never reach the observer, but sounds previously excited would be gradually overtaken and heard in the reverse of the natural order. If  $v = 2c$ , the observer would hear a musical piece in correct time and tune, but backwards.*” [“ $a$ ” has been replaced by “ $c$ ” if compared to the original text.] This explanation is now mathematically realizable from eq.(15) and eq.(18) as follows.

■ From (15), for  $v > c$  we have

$$\frac{\partial f_{\pm}}{\partial x'} = \frac{1}{(v - c)} \frac{\partial f_{\pm}}{\partial t'} \quad \text{and} \quad \nu' = -\nu \left( \frac{v}{c} - 1 \right) \quad (19)$$

and for  $v = 2c$  we have



$$\frac{\partial f_+}{\partial x'} = \frac{1}{c} \frac{\partial f_+}{\partial t'} \quad \text{and} \quad \nu' = -\nu. \quad (20)$$

Eq.(19) means the forward propagating wave appears to the observer at  $O'$ , who is situated on the right hand side of  $O$  and moving away from  $O$  (towards positive  $x$  direction), to be propagating backward with velocity of magnitude  $(v - c)$  and frequency  $\nu(v/c - 1)$ , where the negative sign of the frequency signifies that the sound is heard backward or in reverse order. Eq.(20) signifies the same except it is a particular case where the frequency is exactly same as the original in magnitude but the negative sign indicates the backward or reverse order of the sound heard by the observer.

■ From (18), for  $v > c$  we have

$$\frac{\partial f_-}{\partial x'} = -\frac{1}{(v - c)} \frac{\partial f_-}{\partial t'} \quad \text{and} \quad \nu' = -\nu \left( \frac{v}{c} - 1 \right) \quad (21)$$

and for  $v = 2c$  we have

$$\frac{\partial f_-}{\partial x'} = -\frac{1}{c} \frac{\partial f_-}{\partial t'} \quad \text{and} \quad \nu' = -\nu. \quad (22)$$

Eq.(21) means the backward propagating wave appears to the observer at  $O'$ , who is situated on the left hand side of  $O$  and moving away from  $O$  (towards negative  $x$  direction), to be propagating forward with velocity of magnitude  $(v - c)$  and frequency  $\nu(v/c - 1)$ , where the negative sign of the frequency signifies that the sound is heard backward or reverse order. Eq.(22) signifies the same, except that it is a particular case where the frequency is exactly same as the original in magnitude but the negative sign indicates the backward or reverse order of the sound heard by the observer.

### 2.5.1 Inverse/Reverse Doppler Effect

The study of inverse/reverse Doppler is a matter of importance both from theoretical and experimental point of view [19–39], which can now be realized directly in terms of PDEs from the above analysis just by considering the case  $v > 2c$ . To see this let us write  $v = 2c + v_e$  where  $v_e > 0$  and the subscript ‘ $e$ ’ stands for ‘excess over  $2c$ ’.

■ For  $v = 2c + v_e : v_e > 0$ , eq.(19) yields the following:

$$\frac{\partial f_+}{\partial x'} = \frac{1}{(v_e + c)} \frac{\partial f_+}{\partial t'} \quad \text{and} \quad \nu' = -\nu \left( \frac{v_e}{c} + 1 \right). \quad (23)$$

Eq.(23) means the forward propagating wave appears to the observer at  $O'$ , who is situated on the right hand side of  $O$  and moving away from  $O$  (towards positive  $x$  direction), to be propagating backward with velocity of magnitude  $(v_e + c)$  and frequency  $\nu(\frac{v_e}{c} + 1)$ , where the negative sign of the frequency signifies that the sound is heard backward or reverse order.

■ For  $v = 2c + v_e : v_e > 0$ , eq.(21) yields the following:

$$\frac{\partial f_-}{\partial x'} = -\frac{1}{(v_e + c)} \frac{\partial f_-}{\partial t'} \quad \text{and} \quad \nu' = -\nu \left( \frac{v_e}{c} + 1 \right). \quad (24)$$

Eq.(21) means the backward propagating wave appears to the observer at  $O'$ , who is situated on the left hand side of  $O$  and moving away from  $O$  (towards negative  $x$  direction), to be propagating forward with velocity of magnitude  $(v_e + c)$  and frequency  $\nu(\frac{v_e}{c} + 1)$ , where the negative sign of the frequency signifies that the sound is heard backward or reverse order.

Therefore, in both the above cases, the observer is moving away from the source, but the frequency is increasing. This is called reverse/inverse Doppler effect, albeit restricted to the situation where the source and the observer are moving away from each other and the source is fixed with the medium. An explanation of inverse Doppler effect, for mutually approaching source and observer, requires few other subtleties to be addressed, regarding which some remarks will follow shortly.

Now, there are two points which are worth noting here.

- For  $v_e = Nc : N = 1, 2, 3, \dots$ , we have  $\nu' = -\nu(N + 1)$  i.e. integral multiple of the original frequency is heard by the observer.
- The observed wave is backward only. That is, the reverse/inverse Doppler effect can only be perceived with the wave propagating in the reverse direction compared to the original direction (w.r.t. the source).

So, we may arrive at the following conclusion. If the alternation of direction of the wave due to the relative velocity of the source and the observer is explained as a type of aberration, as has been done in ref. [41], then inverse Doppler effect is always associated with an aberration effect. It will be interesting to see the consequences of such analysis in case of cosmic microwave observations [42].

□ *Mutually approaching observer and source:* Throughout our analyses we have kept the source to be fixed with the medium. However, in case of the mutual approach of the source and the observer, for explaining inverse Doppler effect, albeit within the Galilean framework, it is necessary for the source to be in motion with respect to the medium. Additionally the situation raises some basic concerns about coordinate transformations and equation for wave propagation. We explain the issues as follows.

The phenomenological formula that we generally deal with in such situation is written as [1]:

$$\begin{aligned}\nu' &= \frac{c + v_r}{c - v_s} \nu \\ &= \frac{1 + \beta_r}{1 - \beta_s} \nu \quad \text{such that} \quad \beta_r = \frac{v_r}{c}, \beta_s = \frac{v_s}{c},\end{aligned}\tag{25}$$

where  $v_r$  is the observer velocity w.r.t. the medium towards the source and  $v_s$  is the source velocity w.r.t. the medium towards the observer. Now, it is clearly visible from (25) that there is a signature change only when  $v_s > c$ , whereas there is no effect of  $v_r$ . However, our common sense suggests that there should be a dependence on  $v_r$  as well (like the case of mutually moving away source and observer). So, the question arises whether (25) is actually an approximation like the following:

$$\begin{aligned}\nu' &= \frac{1}{(1 - \beta_r)(1 - \beta_s)} \nu \\ &\simeq \frac{1 + \beta_r}{1 - \beta_s} \nu \quad \text{for} \quad \beta_r \ll 1,\end{aligned}\tag{26}$$

where the dependence on  $v_r$  ( $\beta_r$ ) is now manifest in a way that indicates a signature change for  $v_r > c$  ( $\beta_r > 1$ ), leading to an inverse Doppler effect, as well. If this is true, then a further question arises whether Galilean coordinate transformation is applicable to handle the situation or we need to think about something else e.g. see refs. [45] for a different perspective on waves and coordinate transformation. Furthermore, refs. [48, 49] are suggestive of the fact that the Doppler formula indeed gets very subtle when the motions of the source, the observer and the medium are involved. Therefore, the case of inverse Doppler effect needs separately devoted attention, especially regarding the coordinate transformation and the structure of the equation for wave propagation itself, which we plan to report elsewhere. In this work, we shall keep our discussion restricted to Galilean transformation and the structure of equation for wave propagation, as our main motto is to identify the equation for wave propagation that is consistent with the prevalent standard coordinate transformations.

## 2.6 Analysis with 2nd order PDE

Now, let us analyze the situation with second order PDE. From (9), we construct the following equation through the standard steps of calculations:

$$\frac{\partial^2 f_\eta}{\partial x^2} = \frac{1}{\eta^2 c^2} \frac{\partial^2 f_\eta}{\partial t^2},\tag{27}$$

which we is generally identified as “equation for wave propagation” in the standard literature. However, this equation does not transform to an “equation for wave propagation” in  $S'$  frame and rather leads to the

following result:

$$\frac{\partial^2 f_\eta}{\partial x'^2} = \frac{1}{(\eta^2 c^2 - \sigma^2 v^2)} \left( \frac{\partial^2 f_\eta}{\partial t'^2} + 2\sigma v \frac{\partial^2 f_\eta}{\partial x' \partial t'} \right). \quad (28)$$

The above equation can not be “equation for wave propagation” because of its failure to satisfy  $P_1$  and  $P_2$ . It is certainly true on mathematical ground that in  $S$  frame, the forward and backward propagating functions and their linear superposition satisfy the second order PDE. However, it fails to retain its form in  $S'$  frame and, therefore, incapable of providing the mathematical support to Doppler’s intuition through a manifestation of velocity addition.

### 3 Analysis of propagation under Lorentz transformation

In this section we do similar analyses with Lorentz transformations. We revisit the velocity transformation for particles and then analyze propagating functions to decide what we may call “equation for wave propagation”.

#### 3.1 Revisiting particle propagation

Let us consider two frames  $S$  and  $S'$ , with origins  $O$  and  $O'$  respectively, such that the latter is moving with respect to the former along  $x$  direction with constant velocity  $v$ . Then the Lorentz transformations that relate these two frames are given as follows [55]:

$$x' = \gamma x - \sigma \gamma v t \quad : \sigma = \pm 1 \quad (29)$$

$$y' = y, \quad z' = z, \quad (30)$$

$$t' = \gamma t - \sigma \gamma \frac{v}{c^2} x. \quad (31)$$

$\sigma$  can be either +1 or -1 with the following significance.

1. “ $\sigma = +1$ ” signifies “ $S'$  (or  $O'$ ) is moving along increasing/positive  $x$  direction and away from  $S$  (or  $O$ )”.
2. “ $\sigma = -1$ ” signifies “ $S'$ (or  $O'$ ) is moving along decreasing/negative  $x$  direction and towards  $S$  (or  $O$ )”.

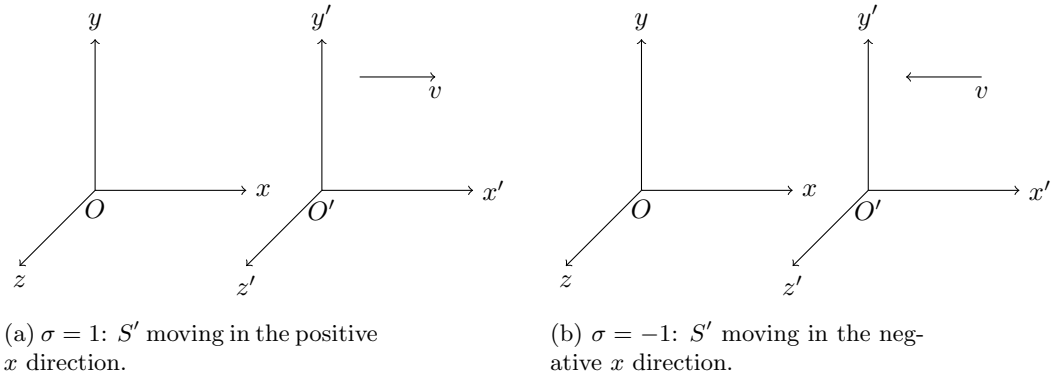


Figure 6: Relative motions of  $S'$  with respect to  $S$ .

The velocity transformation for a point particle goes as follows.

$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - \sigma v}{1 - \frac{\sigma v}{c^2} \frac{dx}{dt}} \implies \eta V' = \frac{\eta V - \sigma v}{1 - \frac{\sigma v}{c^2} \eta V} : \eta V' = \frac{dx'}{dt'}, \eta V = \frac{dx}{dt} \quad (32)$$

$\eta = +1$  implies “forward propagating particle” and  $\eta = -1$  implies “backward propagating particle”. Hence, the velocity transformation can be written as

$$V' = \frac{V - \kappa v}{1 - \kappa \frac{v}{c^2} V}, \quad (33)$$

where  $\kappa = \frac{\sigma}{\eta} = \sigma\eta$ .

■ When  $\eta = +1$  (i.e. forward propagation) and the observer at  $O'$  is situated at the right side of  $O$  during the course of phenomenon,

1. for  $\sigma = +1$ , we have  $V' = \frac{V-v}{1-\frac{vV}{c^2}}$  (Figure: 7a),
2. for  $\sigma = -1$ , we have  $V' = \frac{V+v}{1+\frac{vV}{c^2}}$  (Figure: 7b).

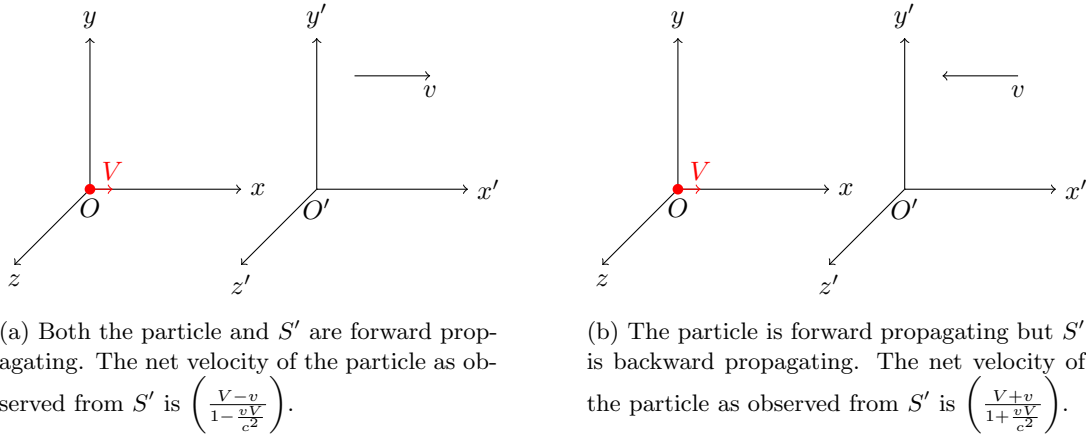


Figure 7: Relative motions of  $S'$  with respect to  $S$  with forward propagating particle with  $S'$  being on the right side of  $S$  for the entire motion.

■ When  $\eta = -1$  (i.e. backward propagation), and the observer at  $O'$  is situated at the left side of  $O$  during the course of phenomenon,

1. for  $\sigma = +1$ , we have  $V' = \frac{-V-v}{1+\frac{vV}{c^2}}$  (Figure: 8a).
2. for  $\sigma = -1$ , we have  $V' = \frac{-V+v}{1-\frac{vV}{c^2}}$  (Figure: 8b).

### 3.2 Properties of “equation for wave propagation”

Now, let us consider that  $S$  frame is the preferred frame of rest in which the “equation for wave propagation” is constructed. The source is fixed with respect to this preferred rest frame and situated at the origin  $O$  of  $S$ . The observer/detector is situated at the origin  $O'$  of  $S'$  frame, which moves with a constant velocity of magnitude  $v$ , with respect to the source. We expect that the “equation for wave propagation” must have the following two properties.

- $L_1$ : It must maintain its form, or remain invariant, under Lorentz transformation so that we can actually write “the wave is observed from both  $S$  and  $S'$  frames”.
- $L_2$ : It must showcase the velocity transformation corresponding to our explanation that “a wave propagates with velocity  $\eta V$  with respect to  $S$ ” and “a wave propagates with velocity  $\eta V'$  with respect to  $S'$ ”.

In what follows, we analyze first order and second order PDEs to find that  $L_1$  and  $L_2$  are only satisfied by the first order PDEs, not second order PDEs, obtained from the usual propagating functions through differentiation.

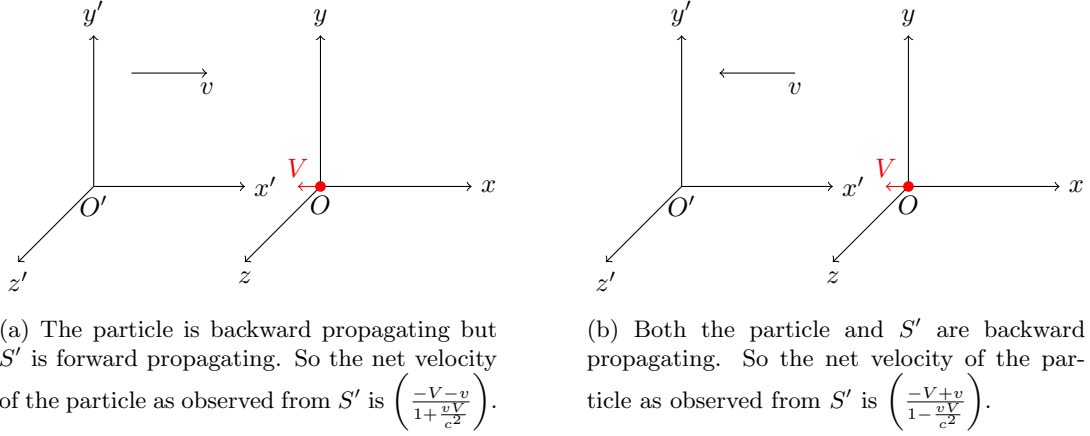


Figure 8: Relative motions of  $S'$  with respect to  $S$  with backward propagating particle with  $S'$  being on the left side of  $S$  for the entire motion.

### 3.3 Analysis with 1st order PDE

The partial derivatives transform as follows.

$$\frac{\partial}{\partial x} \equiv \gamma \frac{\partial}{\partial x'} - \sigma \gamma \frac{v}{c^2} \frac{\partial}{\partial t'} \quad (34)$$

$$\frac{\partial}{\partial t} \equiv -\sigma \gamma v \frac{\partial}{\partial x'} + \gamma \frac{\partial}{\partial t'} \quad (35)$$

The propagating function can be written as follows.

$$f_\eta(x - \eta V t) : \eta = \pm 1. \quad (36)$$

So the propagation equation can be written as,

$$\frac{\partial f_\eta}{\partial x} = -\frac{1}{\eta V} \frac{\partial f_\eta}{\partial t}, \quad (37)$$

where  $\eta = 1$  signifies forward propagation and  $\eta = -1$  signifies backward propagation. The propagation velocity is  $V$  in  $S$  frame. We can write the following correspondence.

$$\text{Forward propagation equation } (\eta = +1): \quad \frac{\partial f_+}{\partial x} = -\frac{1}{V} \frac{\partial f_+}{\partial t}, \quad (38)$$

$$\text{Backward propagation equation } (\eta = -1): \quad \frac{\partial f_-}{\partial x} = +\frac{1}{V} \frac{\partial f_-}{\partial t}. \quad (39)$$

The same wave as viewed in  $S'$ , using (34) and (35), can be written as,

$$\frac{\partial f_\eta}{\partial x'} = -\left(\frac{1 - \eta \sigma \frac{vV}{c^2}}{\eta V - \sigma v}\right) \frac{\partial f_\eta}{\partial t'}. \quad (40)$$

■ When  $\eta = +1$  (i.e., forward propagation or left to right moving) and the observer at  $O'$  is situated on the right side of  $O$  for the entire course of phenomenon,

1. for  $\sigma = +1$  (observer moving away from source in increasing  $x$  direction, Figure: 9a),

$$\frac{\partial f_+}{\partial x'} = -\left(\frac{1 - \frac{vV}{c^2}}{V - v}\right) \frac{\partial f_+}{\partial t'} \quad (41)$$

2. for  $\sigma = -1$  (observer moving towards source in decreasing  $x$  direction, Figure: 9b),

$$\frac{\partial f_+}{\partial x'} = -\left(\frac{1 + \frac{vV}{c^2}}{V + v}\right) \frac{\partial f_+}{\partial t'} \quad (42)$$

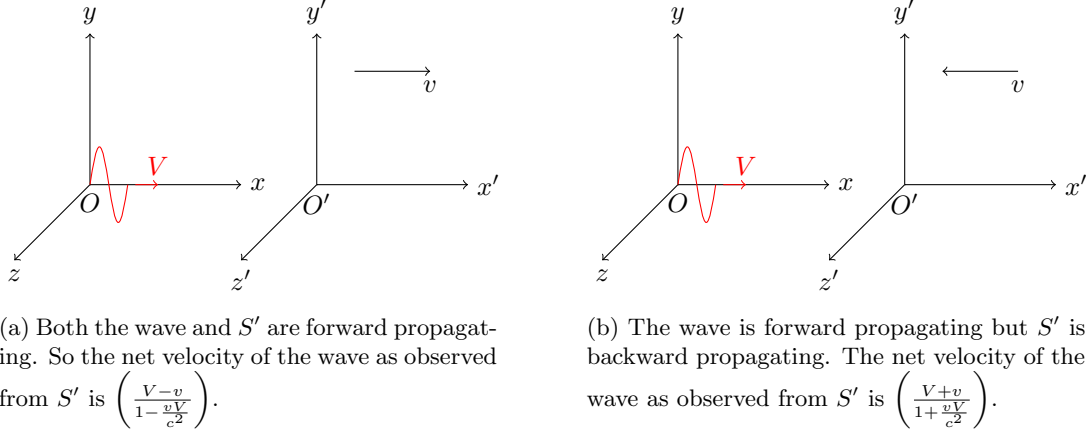


Figure 9: Relative motions of  $S'$  with respect to  $S$  with forward propagating wave with  $S'$  being on the right side of  $S$  for the entire motion

■ When  $\eta = -1$  (i.e, backward propagation or right to left motion) and the observer at  $O'$  is situated in the left hand side of  $O$  during the course of phenomenon,

1. for  $\sigma = +1$  (observer moving towards the source in the increasing  $x$  direction, Figure: 10a),

$$\frac{\partial f_-}{\partial x'} = -\left(\frac{1 + \frac{vV}{c^2}}{-V - v}\right) \frac{\partial f_-}{\partial t'} \quad (43)$$

2. for  $\sigma = -1$  (observer moving away from the source in the decreasing  $x$  direction, Figure: 10b),

$$\frac{\partial f_-}{\partial x'} = -\left(\frac{1 - \frac{vV}{c^2}}{-V + v}\right) \frac{\partial f_-}{\partial t'} \quad (44)$$

It can be seen that when we substitute  $V = c$  in the above cases we get back equations for wave traveling with velocity  $c$ . Further, from the above simple analyses it becomes evident that  $L_1$  and  $L_2$  are mathematically in tandem with the first order PDEs. So, the above first order PDEs, which we have called propagation equations, can now be termed as “equations for wave propagation”.

### 3.4 Analysis with 2nd order PDE

The question remains whether similar analyses can be done with second order PDEs. To begin with, from (37), the following second order PDE can be written.

$$\frac{\partial^2 f_\eta}{\partial x^2} - \frac{1}{V^2} \frac{\partial^2 f_\eta}{\partial t^2} = 0. \quad (45)$$

Using (34) and (35),

$$\frac{1 - \frac{v^2}{V^2}}{1 - \frac{v^2}{c^2}} \frac{\partial^2 f_\eta}{\partial x'^2} - \frac{2\sigma v(\frac{1}{c^2} - \frac{1}{V^2})}{1 - \frac{v^2}{c^2}} \frac{\partial^2 f_\eta}{\partial x' \partial t'} + \frac{\gamma^2}{c^2} \left(\frac{v^2}{c^2} - \frac{c^2}{V^2}\right) \frac{\partial^2 f_\eta}{\partial t'^2} = 0 \quad (46)$$

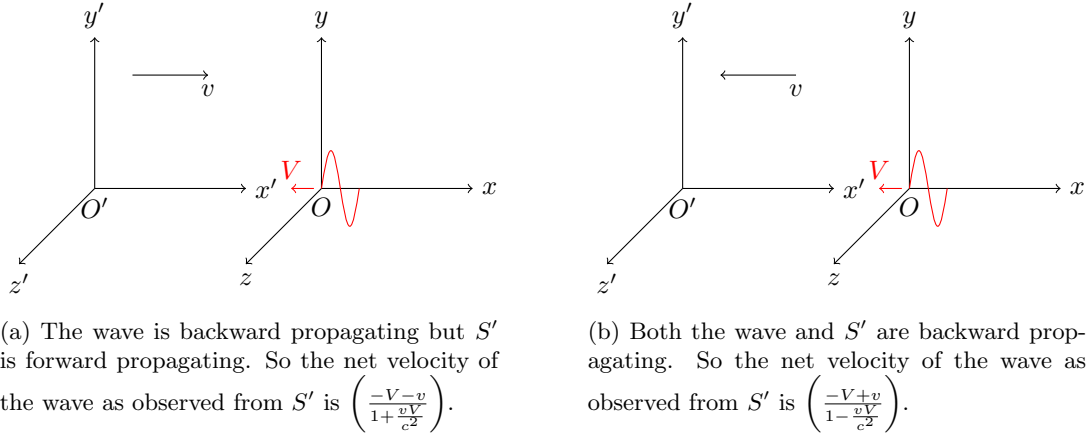


Figure 10: Relative motions of  $S'$  with respect to  $S$  with particle with backward propagating wave with  $S'$  being on the left side of  $S$  for the entire motion.

The above equation can not be “equation for wave propagation” because of its failure to satisfy  $L_1$  and  $L_2$ . It is certainly true on mathematical ground that in  $S$  frame, the forward and backward propagating functions and their linear superposition satisfy the second order PDE. However, it fails to retain its form in  $S'$  frame and, therefore, incapable of providing the mathematical support to Doppler’s intuition through a manifestation of velocity addition.

## 4 Conclusion and Outlook

Based on this work we may conclude that wave equations should be modeled with first order partial differential equations so that Doppler’s effect can be explained in terms of equations. Considering this work as the first step of a new line of investigation concerning velocity transformation and the structure of wave equation, we have restricted our work to one spatial dimension and considered the source to be fixed with the medium wherever applicable. Certainly we plan to investigate the scenarios where the medium, the source and the detector all are in motion and also generalize such investigations for three spatial dimensions.

Further, in view of the present work, we claim that the Doppler-Petzval debate, which has remained ignored and unresolved for nearly two centuries, is now settled. However, such a settlement is not complete as it comes with an association of some unsettling questions regarding some of the standard accepted structures of theoretical physics which, however, may potentially open up new pastures of investigations concerning the foundations of physics. Our work is of utmost significance in at least three different scenarios: (i) light propagation in a material medium where various types of Doppler effect and Vavilov-Cherenkov effect have become relevant topics of research with immediate practical applications (ii) light propagation in cosmic observations where the concept of ether or a preferred rest frame are reemerging from various point of views [66–71] (iii) de Broglie’s phase waves [63, 64] and compliance with the corresponding equations of quantum mechanics e.g. Schroedinger equations [65]. The first two can force us to rethink about the structure of the Maxwell equations and the last one can potentially affect the foundations of quantum mechanics as the Schroedinger equations [65], which are the representatives of de Broglie’s phase waves according to current understanding, come under scrutiny in the process. Indeed the present analysis can provide the motivation to rethink about modeling de Broglie phase waves as far as the correspondence of the wave function with physical reality is concerned [72, 73]. We hope to report further developments along such lines of investigation in the near future.

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# A The principle of relativity, the principle of completeness and frame dependence of wave velocity

The necessity of providing an explanation of the frame dependence of the velocity of a wave in terms of equations can be motivated from two principles of physics, namely, the Principle of Relativity (**PR**) [61] and the Principle of Completeness (**PC**) [62]. We state and explain their respective significance in the present context as follows.

- **PC**: Every element of physical reality must have a counter part in the theory, where physical reality takes the form of experiment and measurement realized through human sense experience.
- **PR**: The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of coordinates in uniform translatory motion.

**PC** is the statement of Einstein-Podolsky-Rosen [62], otherwise discussed only in the context of quantum mechanics and **PR** is just Galileo's principle which was considered as the first postulate by Einstein in ref. [61]. While **PR** is known and widely discussed, **PC** has remained out of focus but holds the potential to radicalize our thinking e.g. ref. [56, 58]. Now, let us quote a few original paragraphs to explain the significance of **PC** and **PR** as far as wave equation is concerned.

Russell, the discoverer of solitary waves or solitons in modern terminology, can be quoted from p1 of ref. [54] as follows, which concerns his experience of seeing a hydrodynamic phenomenon: “... *rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height.*”

To proceed with our arguments, let us agree on the fact that, what Russell observed is a change of state of a physical system, namely, the water in the canal. This change of state is what we may call a hydrodynamic wave phenomenon. Of course, there are many types of them and Russell's was one particular type. According to **PR**, if we consider an equation to represent such a wave, which we may call the wave equation, then the wave equation should remain unaffected under the coordinate transformation that relates any two inertial frames which are in uniform translatory motion with respect to each other. This is because the wave can be observed from both these frames. In Russell's case, one frame is the rest frame of the bank of the channel where he stood at first and the other frame is the horseback on which he rode so as to follow the wave. **PC** demands that Russell's observation of wave velocity and its dependence on how fast he rode on the horseback must reflect in the wave equation as well. Combining these two point of views we may demand that observation of a wave from two different inertial frames, with velocities corresponding to the respective frames, must be reflected in the wave equation in a way such that

- the wave velocity manifests the frame dependence to take into account the physical reality of observing different wave velocities from different inertial frames,
- the wave equation remains invariant under the coordinate transformation that connects these two inertial frames so as to manifest the observation of the wave from both frames.

In a nutshell, we aim to extract the computational content of the verbal statements of physics which we primarily construct to express our experience. That is, such a line of inquiry is actually concerned with the refinement of the language of physics in relation to what we conceive of as physical or real, through our experience and experimental observations. The significance of our demand and the potential consequences of such a line of inquiry can be understood further by consulting refs. [56–60].

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