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Article **Two-phase flow modeling for bed erosion by a plane jet impingement**

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Abstract: This paper presents experimental and numerical studies on the erosion of a horizontal 1 granular bed by a two-dimensional plane vertical impinging jet to predict the eroded craters' size 2 scaling (depth and width). The simulations help understand the microscopic processes that govern 3 erosion in this complex flow. A modified jet-bed distance, accounting for the plane jet virtual origin, is successfully used to obtain a unique relationship between the crater size and a local Shields parameter. This work develops a two-phase flow numerical model to reproduce the experimental results. Different options for sediment's solid to liquid transition during erosion are proposed, tested, and discussed. One model is based on unified equations of continuum mechanics, others on modified 8 closure equations for viscosity or momentum transfer. A good agreement between the numerical 9 solutions and the experimental measurements is obtained. 10

Keywords: jet erosion test; sediment transport; numerical modeling; two-phase flow model; experiments; water injection dredging

1. Introduction

Vertical jet-induced scour erosion of soil has been studied for many industrial applications such as aerospace or hydraulic engineering [1–6]. As reported by Metzger et al. [1,2], in aerospace engineering, soil erosion and crater formation could generate problems for launching and landing spacecraft. In hydraulic engineering, we have several examples such as the work of Rouse [3] for testing criteria on erosion, the study of Hanson and Cook [4] assessing in situ the erodibility of soil material, and the research developed by Hanson and Hunt [5] in the so-called Jet Erosion Test (JET). We also have the work of Perng and Capart [6] for Water Injection Dredging (WID) and trenching in harbors by jet translation and possible jet inclination.

Experimental investigations of the jet normal impingement on plane surfaces have been conducted in various conditions. Although the jet nozzle section is circular in most cases [4,7–13], it can also be rectangular [14–17]. The circular and rectangular cases are called quasi-three-dimensional (3D) axisymmetric and two-dimensional (2D) plane jets, respectively. The jet can be either gas [1,2] or liquid [4,6] within unsubmerged [1,2] or submerged [8–17] conditions. Finally, the soil can be either cohesive [10] or non-cohesive [12,14].

Based on experiments of 3D axisymmetric jets impinging on a smooth non-erodible wall, Rajaratnam and Beltaos [8] and Phares et al. [16] divide the flow into three zones:

The free jet region. There is no influence on the boundary. It can be described by self-similar solutions [18,19].

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- The impinging region. It is characterized by a stagnation point on the jet axis at the wall with a deflexion of flow streamlines [20].
- The wall jet region. It is close to the wall but away from the impinging zone where the flow is mainly parallel to the wall [21–23].

Based on pressure and shear stress measurements on the bed, Rajaratnam and Beltaos [8] reported that the most extreme hydrodynamic action is located in the impinging region with maximum pressure at the stagnation point and maximum shear stress just away from it.

On erodible beds, two main crater shapes have been identified [1,2,7-9]. The first one consists of wide and shallow craters for high impingement heights *L* (distance from the jet nozzle to the initial flatbed). Here, the flow streamlines weakly deflect in the impinging region, and a smooth crater shape remains stable after the jet flow stop. For the second shape, narrower and deeper craters are obtained for small *L* and large flow rates. Here, the flow streamlines strongly deflect, and the crater shape changes after the jet stop with granular avalanches lowering the steep slopes.



Figure 1. Geometrical parameters of the eroded sediment bed for a flow (a) weakly and (b) strongly deflected. Here, *H* and *D* are the crater depth and width, respectively, and *L* is the distance above the granular bed .

Aderibigbe and Rajaratnam [9] state that these two crater regimes depend on the value of an adimensional erosion parameter. This parameter is defined for quasi-3D axisymmetric jets as

$$E = U_J \frac{b}{L} \left(\frac{1}{(s-1)gd} \right)^{1/2},$$
 (1)

where U_J is the mean velocity at the nozzle outlet, *b* is the nozzle diameter, *L* is the impingement height, *d* is the sediment grain size, and $s = \rho_p / \rho_f$ is the particle/fluid density ratio. This parameter is sometimes called a densimetric Froude number [15] and roughly corresponds to the Rouse number [3].

For the 3D axisymmetric experimental data [9], the erosion parameter *E* ranged up to 56 5. This parameter has been used by Sutherland and Dalziel [13] to investigate the crater 57 formation dynamics in granular beds by a 3D water jet. In the 2D turbulent plane jet case, 58 the erosion threshold is well governed by *E*, as shown in [15]. Recently, Badr et al. [17] 59 proposed that the modified Shields can control the erosion threshold for both turbulent 60 and laminar regimes. However, we should consider the correct decay law of self-similar 61 regimes and the precise position of the corresponding virtual origin. Moreover, in the 2D 62 plane jet test case of Badr et al. [17], the critical value of the local Shields number for erosion 63 is a constant $Sh_c \approx 1$ independently of the jet flow regime and the grain diameter of the 64 bed.

On the other hand, several numerical studies have been proposed to study the crater formation by an incident jet. We can clearly identify two types of numerical modeling frameworks. In the first type, numerical models apply Computational Fluid Dynamics (CFD) software for computing the fluid motion only above the bed [1,24-26]. The second 69 modeling framework is based on two-phase approaches, in which the motion equations 70 solve both solid and fluid phases [27–34]. 71

The common point of models based on CFD software is that the sediment bed is 72 considered as impermeable wall (no-slip conditions). But their modeling approaches 73 mainly differ in treating the two-phase interaction between the fluid and the sediment bed. 74 For example, Metzger et al. [1] use CFD++ software to study the steady-state flow above 75 different stationary boundary conditions (or crater geometries) but calculate the porous 76 flow inside the sediment bed by Darcy's law and pressure values. Weidner et al. [24] and 77 Mercier et al. [25] use FLUENT to dynamically adapt the computational grid into the bed's 78 evolving geometry; however, they do not consider the sediment below the bed interface. 79

For the two-phase formulation, the Eulerian approach usually models the fluid phase; 80 but, Eulerian or Lagrangian techniques can be used for the solid phase. A Lagrangian model 81 for the solid phase enables a microscopic description of the particle motion. The flowing 82 (eroded) and jammed (non-eroded) regions in the sediment bed are directly simulated 83 grain by grain. However, this framework suffers from high computational costs due to the 84 number of solid particles. For examples of this formulation, we refer the work of Kuang et al. [27], and more recently the studies of Benseghier et al. [28,29]. 86

On the other hand, in the Eulerian-Eulerian framework, clear fluid and solid particles are treated as two interpenetrating fluids. The fluid-particles and particle-particle 88 interactions are described at the scale of the continuum media [31–34]. However, if only 89 a liquid-like behavior of solid particles is modeled to handle the motion of the granular 90 phase for the jet problem, the sediment motion becomes unrealistic, particularly in the 91 jammed region [31,32]. 92

Therefore, solid-like and liquid-like behaviors of sediment are essential and need to 93 be considered for more accurate models. In recent years, few efforts has been performed 94 to include this property in the Eulerian approach; for example, Yuan et al. [32] apply an 95 incipient motion theory in their numerical simulations, Uh Zapata et al.[33] proposed a 96 modified inter-phase momentum transfer formulation, and Wang et al. [34] use a model 97 for the solid-fluid mixture combined with a viscoplastic description of soil. Consequently, 98 this paper proposes a unified formulation for continuum mechanics to simulate vertical-jet 99 induced erosion with a unique local parameter governing the solid/liquid transition for 100 the sediment. This model is based on a technique initially developed by Greenshields and 101 Weller [35] to handle fluid-structure interactions of flexible and impermeable tubes under 102 fluid flows. 103

This paper's originality relies on the experimental results and new numerical model 104 for submerged vertical plane jet-induced craters on horizontal non-cohesive sediment. The 105 remaining document organization is as follows. Section 2 presents the experimental setup to generate craters, measuring their sizes for different impingement distances and jet velocities. 107 The presentation of the two-phase model, using the two-phase Euler-Euler approach, is 108 proposed in Section 3. We give a complete description of the model improvements, which 109 produce both solid-like and liquid-like behaviors of the sediment. Next, Section 4 shows 110 that the correct local Shields number choice for this governing parameter leads to very close 111 agreement with the experimental results. Finally, we present the discussion, conclusions, 112 and future work. 113

2. Laboratory Experiments

In this section, the experimental results for the crater depth and width are analyzed 115 following Badr et al. [17] in terms of the self-similar jet model. We pay particular attention 116 to the corresponding virtual origin. Simple scaling for the crater sizes is found in terms of a 117 local erosion parameter. 118

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2.1. Experimental Setup

The experimental set-up consists of a vertical plane water jet impinging a horizontal granular bed immersed in water, as sketched in Figure 2. The grains are monodisperse glass beads with diameter $d = 350 \ \mu$ m with the density $\rho_p = 2.5 \times 10^3 \ \text{kg/m}^3$. The granular bed is 10 cm in height (about 300-grain diameters) with a solid volume fraction of about 0.6. It lies on the bottom of a 3D rectangular container of dimensions $50 \times 20 \times 3.2 \ \text{cm}^3$ filled with water (density $\rho_f = 10^3 \ \text{kg/m}^3$ and kinematic viscosity $\nu_f = 10^{-6} \ \text{m}^2/\text{s}$), see Figure 2(a). The container walls are in PolyMethyl MethAcrylate (PMMA), allowing visualization.

A vertical plane water jet is generated at the exit of an injector of an inner rectangular cross-section of dimensions $b \times w_J = 0.4 \times 2.6 \text{ cm}^2$. The outside cross-section of the injector is $1 \times 3.2 \text{ cm}^2$ to fit within the container gap. The injector is at a distance *L* above the granular bed, see Figure 2(b).

The mean velocity of the jet is given by $U_J = Q/(bw_J)$. A gear pump controls the steady water flow Q with very low negligible fluctuations in a closed circuit. Here, water is sucked through the bed from the porous bottom of the container and re-injected by the pump through the injector.



Figure 2. Sketch of the device to study the erosion of a granular bed by a water jet: (a) hydraulic circuit to prepare the initial bed by fluidization, and (b) water jet injector dimensions.

In this paper, the jet Reynolds number is defined as

$$Re = \frac{U_J b}{v_f}.$$
 (2)

The injector is long enough (20 cm) to ensure a laminar regime inside the injector. It also has a parabolic Poiseuille velocity profile across the thickness *b* at the nozzle in the range of jet Reynolds numbers $Re < 2 \times 10^3$.

2.2. Measure of the Crater Dimensions

The erosion process is observed until the resulting crater reaches a quasi-stationary state in a few seconds. The size of the jet-induced scour hole is characterized by the two measured parameters: crater depth and width. The depth, *H*, is defined as the distance from the bottom of the crater to the initial bed surface. The width, *D*, is also determined at the level of the initial bed surface and not at the level of the crater crests, as shown in Figure 1.

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The measurements are extracted from the images taken by a camera placed in front of the container with backlighting. When the crater stops evolving, we measure the results from the difference between the quasi-stationary image and the initial flatbed image. This procedure allows an easy and accurate determination of H and D. It is important to remark that the measured crater has dynamical sizes, which results from the dynamical crater equilibrium sustained by the steady jet flow. In that case, the dynamic angle may be larger than the avalanche angle.

2.3. Experimental Results

The crater size measurements are shown in Table 1 and Figure 3. We use six different 154 jet velocities in the range of $0.16 < U_J < 0.47 \text{ m/s}$ (630 < Re < 1900) and different jet-bed distances ($2 \le L/b \le 40$). Figure 3 shows the crater depth *H* and width *D* normalized by the jet thickness *b*.

Table 1. Experimental results using different experimental parameters of U_J , L and jet thickness b = 0.004 m.

Mean velocity <i>U_J</i> (m/s)	Reynolds number Re	Impingemen distance L (m)	t Crater width D (m)	Crater depth <i>H</i> (m)	Mean velocity <i>U_J</i> (m/s)	Reynolds number Re	Impingemen distance L (m)	t Crater width D (m)	Crater depth <i>H</i> (m)
0.4712	1885	0.1454	0.0152	0.0914	0.3140	1256	0.1117	0.0078	0.0446
0.4712	1885	0.1305	0.0209	0.0953	0.3140	1256	0.1039	0.0086	0.0429
0.4712	1885	0.0848	0.0223	0.0672	0.3140	1256	0.0885	0.0114	0.0407
0.4712	1885	0.0596	0.0254	0.0570	0.3140	1256	0.0753	0.0130	0.0433
0.4712	1885	0.0360	0.0252	0.0340	0.3140	1256	0.0626	0.0172	0.0408
0.4712	1885	0.1086	0.0253	0.0906	0.3140	1256	0.0514	0.0159	0.0457
0.4314	1726	0.1590	0.0055	0.0871	0.3140	1256	0.0419	0.0214	0.0509
0.4314	1726	0.1259	0.0129	0.0864	0.2356	942	0.0681	0.0021	0.0254
0.4314	1726	0.0796	0.0209	0.0699	0.2356	942	0.0644	0.0051	0.0281
0.4314	1726	0.0630	0.0204	0.0475	0.2356	942	0.0521	0.0088	0.0275
0.4314	1726	0.0429	0.0252	0.0502	0.2356	942	0.0397	0.0119	0.0243
0.3926	1570	0.1063	0.0146	0.0780	0.2356	942	0.0326	0.0110	0.0198
0.3926	1570	0.1018	0.0231	0.0711	0.2356	942	0.0207	0.0123	0.0120
0.3926	1570	0.0766	0.0218	0.0615	0.2356	942	0.0098	0.0205	0.0190
0.3926	1570	0.0561	0.0216	0.0499	0.1570	628	0.0534	0.0042	0.0184
0.3926	1570	0.0341	0.0228	0.0324	0.1570	628	0.0412	0.0061	0.0167
0.3926	1570	0.0200	0.0248	0.0359	0.1570	628	0.0291	0.0072	0.0160

Figure 3(a) shows that for a given mean jet velocity U_I , the crater depth *H* is higher 158 at lower *L* values. Furthermore, *H* decreases towards zero as *L* increases. Above a critical 159 distance, L_c , there is no erosion and thus no crater formation is seen anymore. Note that the 160 critical distance L_c also increases with U_I . For a given value of L (see for instance L/b = 10), 161 *H* increases with the jet velocity U_I . At low enough U_I , here $U_I < 0.1$ m/s, no erosion 162 can be seen whatever the value of the jet-bed distance. On the other hand, Figure 3(a) 163 shows that the crater width D increases with L and U_I for a non-zero value, D_0 , close to 164 the erosion threshold. 165



Figure 3. Experimental results of the crater (a) depth and (b) width normalized by the jet width *b* as function of the normalized jet-bed distance L/b for different jet Reynolds numbers.

Previous behavior can be understood when considering the jet spatial evolution downstream of the injector nozzle. As the jet Reynolds number is larger than 200, the jet regime is turbulent with a local velocity u_0 at the jet axis. This velocity decreases according to the distance *L* with a law that can be approximated by the following self-similar model for free jets [36]:

$$\frac{u_0}{U_I} = \frac{1}{K\sqrt{L/b - \lambda/b}},\tag{3}$$

where $K \approx 1/2$ is the decay rate and λ is the distance of the virtual origin of the jet from the nozzle exit. Here $\lambda > 0$ for a virtual origin downstream of the jet nozzle. In the proposed experimental conditions, λ is equal to 10*b*. From jet momentum conservation, the jet width δ increases in the meantime with *L* by the following law:

$$\frac{\delta}{b} = K^2 (L/b - \lambda/b), \tag{4}$$

where $K^2 \approx 1/4$ corresponds to the jet opening angle $\theta = \tan^{-1}(K^2) \approx 15^{\circ}$. We remark that the jet velocity spatial profile (3)-(4) is only valid downstream the potential core. By contrast, in the injector, the jet velocity u_0 remains constant and equal to $u_0 = (3/2)U_J$ for a laminar parabolic Poiseuille profile.

In such a self-similar model, the reduced jet-bed distance, $L - \lambda$, appears as the pertinent length scale that may govern the crater size. Thus, taking into account the virtual origin of the jet, a relevant dimensionless parameter for the jet erosion strength is the local erosion parameter *E* or Rouse number. In the present inertial regime of high particle Reynolds number, this parameter is given by

$$E = U_J \left(\frac{b}{L-\lambda}\right)^{1/2} \left(\frac{1}{(s-1)gd}\right)^{1/2},\tag{5}$$

which corresponds to the ratio of the local jet velocity to the grain settling velocity. Equation (5) is similar to expression (1) proposed in [9]. However, besides considering the effective jet-sediment distance, the model differs in the exponent -1/2 (instead of -1) for the *L* scaling. This exponent is coming from the self-similar free jet models for a 2D plane (instead of 3D axisymmetric) flow geometry.

Figure 4 shows the crater size data normalized by the reduced distance $L - \lambda$ as a function of the dimensionless erosion parameter *E* given by equation (5). We compute *E* using s - 1 = 1.5, g = 9.8 m/s² and $d = 350 \mu$ m. Note that the different curves collapse 191

into a master curve of simple trend. The dimensionless crater depth $H/(L - \lambda)$ increases linearly with *E* above a critical value E_c corresponding to erosion threshold, as follows

$$H/(L-\lambda) = m_H(E-E_c), \tag{6}$$

where $m_H = 0.7 \pm 0.05$ and $E_c = 1 \pm 0.1$. Similarly, the dimensionless crater size $D/(L-\lambda)$ increases linearly with *E* above the critical value E_c ; however, it has a non-zero value D_0 at threshold. Thus we get

$$D/(L-\lambda) = m_D(E-E_c) + D_0^*,$$
(7)

where $m_D = 1.4 \pm 0.1$ and $D_0^* = D_0 / (L - \lambda) = 0.55 \pm 0.05$.



Figure 4. Experimental results of the crater (a) depth and (b) width normalized by the reduced jet-bed distance $L - \lambda$ as a function of *E* for different jet Reynolds numbers. The solid line corresponds to the linear fit model through the data.

The critical value $E_c \approx 1$ found here for the erosion threshold at vanishing crater depth agrees very well with the critical value one reported by Badr et al. [17] with just a visual criterion of the few first moving grains in a similar setup. It is worth noting that taking into account the distance λ of the virtual origin of the jet from the nozzle exit is crucial to obtain a good rescaling of the data and such simple scaling laws. Indeed, the critical value $E_c \approx 1$ at the threshold is very satisfying, meaning that *E* is a pertinent parameter for describing vertical jet-induced erosion.

Finally, considering the evolution of crater sizes H and D with the jet flow action, the shape of the crater evolves with a typical slope H/(D/2) that tends toward the value $2m_H/m_D \approx 1$ at large erosive strength E ($E \gg 1$). This value corresponds to a global slope angle of about 45°, thus much larger than the usual critical avalanche angle 25° without any jet flow [37]. This shows that at large E, the crater slope is enhanced by an upward-back flow of the jet, as happens in homogeneous parallel shear flow [38].

3. Mathematical Model

The present two-phase flow model comes from a strict application of the mathematical formulation of Drew and Lahey [39] using an Eulerian-Eulerian description for the fluidparticle system. In this section, the governing equations for the two phases corresponding to clear fluid (water) and disperse medium (sediment particles in a fluid) are briefly recalled with special closure laws.

The present two-phase flow model is initially based on the two-fluid model presented in Barbry et al. [40] and Nguyen et al. [41,42]. This model only assumes a liquid-like behavior for the disperse phase (solid grains). We refer to this formulation as the standard or classical model. However, it cannot account for a possible solid-like behavior of the granular bed. In the present study on scouring erosion, we propose an improved model considering mobile and immobile parts of the granular media by a simple switch from fluid mechanics to soil mechanics equations. The benefit of this new treatment for the sediment

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particles is illustrated and justified in the numerical results when the method is compared with the initial two-fluid model. 224

3.1. Governing Equations

Before specifying the novel feature introduced to the model in the present paper, let us first recall the initial two-fluid model. For a constant width-integrated 2D x/z two-phase flow model, the governing equations in an Eulerian formulation for the *k*-phase read:

$$\frac{\partial(\alpha_k \rho_k)}{\partial t} + \nabla \cdot (\alpha_k \rho_k \mathbf{u}_k) = 0, \qquad (8)$$

$$\frac{\partial(\alpha_k \rho_k \mathbf{u}_k)}{\partial t} + \nabla \cdot (\alpha_k \rho_k \mathbf{u}_k \mathbf{u}_k) = \nabla \cdot (\alpha_k \bar{\sigma}_k) + \alpha_k \rho_k \mathbf{g} + \mathcal{M}_k, \tag{9}$$

where k = f stands for the clear fluid (water) and k = s for the solid particles, respectively. The volume fraction is represented by values α_k with $\alpha_f + \alpha_s = 1$. Here, ρ_k is the density, $\mathbf{u}_k = (u_k, w_k)$ is the velocity vector, and \mathbf{g} stands for gravity. The symbol \mathcal{M}_k refers to the inter-phase momentum transfer, and $\overline{\sigma}_k$ is the total stress tensor of phase k. In this paper, the double bar over a symbol means a tensor of order two.

The total stress tensor of phase *k* is given by

$$\bar{\bar{\tau}}_k = \left(-p_k \bar{\bar{I}} + \bar{\bar{\tau}}_k + \bar{\bar{\tau}}_k^{Re}\right). \tag{10}$$

where \bar{I} is the identity tensor. Equation (10) is the sum of three contributions: the pressure (p_k) of phase k, the deviatoric viscous ($\bar{\tau}_k$), and Reynolds stresses ($\bar{\tau}_k^{Re}$). Following Lundgren [43] for the slow motion of two-fluids, the non-turbulent stresses are expressed by: 238

$$\alpha_f \bar{\bar{\sigma}}_f = -\alpha_f p_f \bar{\bar{I}} + 2\mu_f \left(\alpha_s \bar{\bar{D}}_s + \alpha_f \bar{\bar{D}}_f \right), \tag{11}$$

$$\alpha_s \bar{\bar{\sigma}}_s = -\alpha_s p_s \bar{\bar{I}} + 2\mu_f \alpha_s \beta \left(\alpha_s \bar{\bar{D}}_s + \alpha_f \bar{\bar{D}}_f \right), \tag{12}$$

where $\overline{D}_k = \frac{1}{2} (\nabla \mathbf{u}_k + \nabla \mathbf{u}_k^T)$ is the shear rate tensor of phase *k*. In equations (11) and (12), the parameter μ_f is the dynamic viscosity of the clear fluid (water) and β is an increasing function of the solid volume fraction of grains, for which we use the expression of Graham [44]. We refer to Nguyen et al. [41,42] for more details.

To compute the Reynolds stresses $\bar{\tau}_k^{Re}$ for the two-phase model, the standard $k_f - \epsilon_f$ model is used for the fluid phase. For the turbulent model of the solid phase, we use the closure equations $k_s - k_{fs}$ (turbulent kinetic energy of particulate phase, k_s , with covariance of fluid and solid velocity fluctuations, k_{fp}). This model is considered a two-way coupling because it can evaluate the effects of solid particles on fluid turbulence and vice-versa [41,45].

The momentum exchange between phases, \mathcal{M}_k , is decomposed into jump conditions (p_{ki} and $\overline{\tau}_{ki}$) and different forces (\mathcal{M}_k^*) acting on phase k, as follows 250

$$\mathcal{M}_f = \left[p_{fi} \bar{I} - \bar{\bar{\tau}}_{fi} \right] \nabla \alpha_f + \mathcal{M}_f^*, \tag{13}$$

$$\mathcal{M}_s = \left[p_{si} \bar{\bar{I}} - \bar{\bar{\tau}}_{si} \right] \nabla \alpha_s + \mathcal{M}_s^*. \tag{14}$$

Here, the subscript *i* specifies the solid/fluid interface. As the interfacial tension is zero, there is no discontinuity of the pressure $(p_{si} = p_{fi} = p_f - \rho_f ||\mathbf{u}_f - \mathbf{u}_s||^2/4$, where $\|\cdot\|$ means Euclidean norm) and the shear stress $(\bar{\tau}_{si} = \bar{\tau}_{fi} = \beta \bar{\tau}_f)$ across the fluid/sediment interface (Drew and Lahey [39]).

In equations (13) and (14), the momentum exchanged between phases ($\mathcal{M}_{f}^{*} = -\mathcal{M}_{s}^{*}$) ²⁵⁵ is the sum of the drag and lift forces exerted on the solid particles, the added mass effect, ²⁵⁶

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the Faxén correction and the Basset historical term. In the present formulation, we only 257 retain the drag force as it is dominant, as follows 258

$$\mathcal{M}_{s}^{*} = F_{D} \mathbf{u}_{r}, \qquad F_{D} = \alpha_{s} \rho_{s} / \tau_{fs}, \tag{15}$$

where $\mathbf{u}_r = \mathbf{u}_f - \mathbf{u}_s - \mathbf{u}_d$ is the relative velocity of fluid-particles. Here \mathbf{u}_d is the drift 250 velocity which represents the correlation between the fluctuating velocity of the fluid and 260 the instantaneous spatial distribution of particles. The drag force is also expressed as a 261 function of the solid concentration (α_s) and the particle relaxation time τ_{fs} given by 262

$$\tau_{fs} = 4d\rho_s / \left(3\rho_f C_D ||\mathbf{u}_r||\right), \qquad C_D = \frac{24}{Re_p} f(Re_p). \tag{16}$$

For turbulent two-phase flows, the drift velocity and particle relaxation time are very important, as discussed in Nguyen et al. [41]. For the present study, the drag coefficient 264 (C_D) is computed from the formula of Haider and Levenspiel [46], where Re_v is the particle Reynolds number. 266

3.2. A Unified Momentum Equation for the Solid Phase

The set of equations (8)-(16) roughly describes the fluid-sediment system as a two-268 fluid system. In particular, equations (11) and (12) for the viscous stress correspond to a 269 rheological law for complex fluids. Such a model is only valid below the packing fraction when the granular material has a liquid-like suspension behavior. However, we expect a 271 solid-like behavior for the particulate phase close to the packing fraction at the bed surface 272 or deeper in bulk. 273

The numerical solution given by equations (8)-(16) are unphysical and erroneous in 274 high solid fraction regions: the sediment's *liquid-like* behavior is clearly visualized by wavy 275 iso-contour lines in the sediment bed, see Section 4.1. We remark that similar results are 276 obtained using the Eulerian model proposed by Qian et al. [31] and Yuan et al. [32] for this 277 problem. To overcome this issue, this paper adapts a unified formulation, mainly inspired 278 by Greenshields and Weller [35], to describe both the *liquid-like* and *solid-like* behavior of 279 the sediment phase. 280

3.2.1. Solid-like model

For the solid-like model, we assume a Hookean material that undergoes small defor-282 mations. In this case, a linear elastic, isotropic model is used. Thus, the strain tensor $(\bar{\varepsilon}_s)$ 283 corresponds to the time integration of the strain rate tensor (\overline{D}_s) as follows: 284

$$\sigma_s = -p_s \bar{\bar{I}} + \operatorname{dev}(\bar{\bar{\sigma}}_s) = -p_s \bar{\bar{I}} + \int_{t_0}^{t_n} \operatorname{dev}(\dot{\bar{\sigma}}_s) dt + \operatorname{dev}(\bar{\bar{\sigma}}_0),$$
(17)

where the deviatoric component of $\bar{\sigma}$ is defined as $dev(\bar{\sigma}) \equiv \bar{\sigma} - \frac{1}{2}tr(\bar{\sigma})\bar{l}$, and $dev(\bar{\sigma}_0)$ is the deviatoric solid stress at the initial time. Equation (17) can be approximated by a 286 quadrature formula for the integral, as follows 287

$$\int_{t_0}^{t_n} \operatorname{dev}(\sigma_s) dt = \omega_n \operatorname{dev}(\dot{\sigma}_s)_n + \sum_{i=1}^{n-1} \omega_i \operatorname{dev}(\dot{\sigma}_s)_i,$$
(18)

where ω_i represents the quadrature weighting functions. Here, $(\cdot)_n$ indicates the current 288 time (t_n) . Note that the sum is taken over previous stages t_i (i = 1, ..., n - 1). 289

Next, we can substitute (18) into (17). Now, using the relationship between ε_s and D_s , 290 we get 291

$$\bar{\sigma}_s = -p_s \bar{I} + 2G\omega_n \operatorname{dev}(\bar{D}_s) + \Sigma, \tag{19}$$

267

where the shear modulus is given in terms of the elastic Young modulus (E) and the 292 Poisson's ratio (ν) by $G = E/2(1 + \nu)$. In equation (19), Σ is given by 203

$$\Sigma = \sum_{i=1}^{n-1} \omega_i \operatorname{dev}(\dot{\bar{\sigma}}_s)_i + \operatorname{dev}(\bar{\sigma}_0),$$
(20)

representing the accumulation of elastic stress from the beginning of the loading. Thus, the 294 solid-like behavior of the granular phase can be model as 295

$$\alpha_s \bar{\bar{\sigma}}_s = -\alpha_s p_s \bar{\bar{I}} + \alpha_s 2G\omega_n \operatorname{dev}(\bar{\bar{D}}_s) + \alpha_s \Sigma.$$
⁽²¹⁾

Finally, we obtain a unified momentum equation for the sediment incorporating (22)296 into the two-phase flow model (8)-(15). Thus, to unify liquid-like model (12) and solid-like 297 model (21), we introduce a smooth transition function (F), as follows 298

$$\alpha_{s}\bar{\sigma}_{s} = (1-F)\left[-\alpha_{s}p_{s}\bar{I} + \alpha_{s}\beta\mu\left(\alpha_{s}\bar{D}_{s} + \alpha_{f}\bar{D}_{f}\right)\right] +F\left[-\alpha_{s}p_{s}\bar{I} + \alpha_{s}2G\omega_{n}\operatorname{dev}(\bar{D}_{s}) + \alpha_{s}\Sigma\right],$$
(22)

where $0 \le F \le 1$. We remark that if F = 1 then equation (22) reduces to the *solid-like* 299 model (21). If F = 0 then a *liquid-like* motion is obtained by equation (12). In other terms, the improvement proposed in this paper has non-intrusive character since the original 301 two-fluid model is recovered. For the fluid phase, equations remain unchanged.

3.3. Solid-Liquid Transition of the Granular Phase

In the original development of Greenshields and Weller [35], F is a step function that 304 takes either the value 0 or 1 according to the fluid or solid regions, respectively, since there is no interpenetration between them. In our modeling, F varies continuously from 1 to 0 306 using a smooth solid-liquid transition for the sediment. 307

From a physical viewpoint, function *F* encodes the erosion or fluidization criterion 308 because it determines at any point of the computational domain if the disperse phase 309 is flowing (eroded/fluidized bed for F = 0) or not (static bed for F = 1). Therefore, 310 this function's formulation is as complex as the physics of the erosion threshold. From a 311 numerical point of view, function F should be monotone and sufficiently smooth with a 312 continuous first derivative to avoid any numerical instability. 313

This paper considers for *F* a S-shape function with the following analytical expression 314

$$F = \frac{1}{2}(1 - \text{erf}(\xi)),$$
(23)

where erf is the Gaussian error function. We remark that equation (23) has a unique 315 parameter ξ governing the liquid-solid transition. In the numerical simulations, we select 316

$$\xi = Sh - Sh_c, \qquad Sh = \frac{u_f^2 + w_f^2}{(s-1)gd}.$$
 (24)

corresponding to the deviation of the local Shields number *Sh* from its critical value for 317 erosion Sh_c . This choice is consistent with the erosion criteria of Badr et al. [17] for the 318 plane jet impinging into granular beds. This selection also agrees with the dimensionless 319 crater depth and width formulas given in the experimental results, as presented in the 320 following section. 321

3.4. Numerical Technique

To solve the system of equations (8)-(17), we use the techniques initially developed 323 by Guillou et al. [47] and later improved by Uh Zapata et al. [48]. The σ -transformation is 324 applied to the vertical coordinates to fit the computing mesh to the free surface evolution. 325

303

A finite volume formulation approximates spatial derivatives. A projection technique 326 [49] is applied to calculate the pressure and velocity for each phase. We use a staggered 327 grid to avoid the spurious oscillations induced by the projection technique [50]. Advection 328 terms in Equation (9) are handled by a Total Variation Diminishing (TVD) scheme. The 329 time scheme is implicit in both vertical and horizontal directions. 330

A Successive Order Relaxation (SOR) iterative solver is used for the resulting linear 331 equation system. Furthermore, the code has been fully parallelized through Message 332 Passing Interface (MPI) and Graphics Processing Unit (GPU), which significantly optimize 333 the performance of the two-phase flow program. 334

4. Numerical Results

In this section, the numerical results are obtained using a 2D rectangular domain with 336 0.25 m of height and 0.2 m of width. We impose a Poiseuille profile of velocity at the nozzle 337 outlet of thickness *b*, which is fixed at a distance *L* from the bed. 338

In the simulations, we consider a rectangular mesh grid. The highest resolution is 339 given by a uniform mesh of 401 points along the horizontal and 501 points in the vertical 340 direction. Thus, the spatial steps are given by $\Delta x = \Delta z = 0.5$ mm. This choice corresponds 341 to a fine grid resolution close to one-grain diameter. Thus, it would not be pertinent to 342 consider finer meshes. For this fine space resolution, small time steps are reduced to 343 $\Delta t = 2 \times 10^{-6}$ sec to have stable simulations. In agreement with the experiments, each run 344 lasts a few seconds of simulated time. 345

4.1. Original and Unified Model

To illustrate the benefit of the proposed treatment for the solid-liquid transition, Figure 5 presents the numerical results using the original two-phase flow model (transition 348 function F = 0). For these simulations, we use a coarse grid ($\Delta x = 1 \text{ mm}$, $\Delta z = 2 \text{ mm}$), and 349 parameters b = 4 mm, $U_I = 0.4$ m/s, and L = 2 cm). The initial flatbed interface is located 350 at z = -0.15 m. Although we have a crater at the early stages, the bed shows unrealistic 351 wavy modulation of the bed. 352



Figure 5. Numerical simulations at different stages for the standard two-phase flow model: the solid concentration with fluid field (top), and velocity magnitude of the fluid (bottom).

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On the other hand, Figure 6 shows the numerical results using the proposed unified two-phase Euler-Euler model. For these simulations, we use the same parameters as the standard model. Note that the qualitative behavior is now close to the experimental observations: the bed does not present a wavy behavior but a localized crater. Furthermore, zero particle velocity is predicted within the sediment bed. In other words, our numerical treatment for the solid-liquid transition in the model mainly corrects the identified numerical inaccuracies associated with the original two-fluid formulation.



Figure 6. Solid concentration (top) and fluid velocity magnitude (bottom) for the proposed two-phase flow model using a solid-liquid transition model of the sediment phase at different stages.

As previously pointed out, the profile function plays an essential role in modeling correctly the solid phase. Figure 7 shows the profiles for F given by equation (23) and the numerical results of previous simulations. Note how this function correctly varies continuously from 1 to 0 dividing the computational domain into *liquid-like* and *solid-like* regions. We remark the correlation between the scour dimensions and the values of F.



Figure 7. Numerical results of the liquid-solid transition function *F* at different stages. The fluid velocity field is also plotted as a reference.

Although previous results shows the correct implementation of the model, the mesh resolution does not allow to fully observe the complex structure of the water jet and crater 366 formation. Figures 8 and 9 present the velocity field and solid volume fraction results at 367 t = 1.5 seconds using b = 8 mm, $U_I = 0.323$ m/s, and L = 4.6 cm. In these simulations, we 368 use a fine ($\Delta x = \Delta z = 0.5$ mm), a medium ($\Delta x = \Delta z = 1$ mm), and a coarse ($\Delta x = \Delta z = 2$ 369 mm) grid resolutions. As illustrated in Figure 8, the plane jet flow shows a complex 370 structure with some fluctuation and vortices along the jet. However, the jet flow structure is 371 out of the scope of the present paper, so we only focus on the cratering. Note that the region 372 of solid-like behavior of the sediment is characterized by low fluid velocity, in contrast to 373 the high velocity just above this region. 374



Figure 8. Numerical results of the fluid velocity (a) field, and (b) magnitude with streamlines at t = 1.5 s using b = 8 mm, $U_I = 0.323$ m/s, and L = 4.6 cm for different grid resolutions.



Figure 9. Solid volume fraction at t = 1.5 s using b = 8 mm, $U_J = 0.323$ m/s, and L = 4.6 cm for different grid resolutions.

The abrupt variation in the velocity magnitude indicates that a continuous interface line can model the solid-liquid transition zone. This interface cane be verified in Figures 9 showing 100 contour lines of the solid volume fraction distributed uniformly in the interval [0, 0.55]. Note that the transition zone between the sediment bed and the liquid is less diffusive as the mesh resolution increases, as expected. This interface is used to measure the crater size (H, D).

4.2. Crater Size Predictions

(a)

 $U_1 = 0.28 \ m/s$

As discussed in Section 3.3, we use the local parameter $\xi = Sh - Sh_c$ for governing the sediment solid-liquid transition, where Sh is the local Shields number Sh and Sh_c stands for its critical value. Notice that this formulation considers the total fluid velocity instead of only its tangential component. Indeed, we systematically obtain a zero value of the local Shields parameter at the stagnation point in front of the jet by considering only the horizontal component of the fluid velocity. As a result, the scour hole geometry would present a non-eroded zone at the jet axis with a whiskers-like shape.

Numerical results have shown to have an excellent agreement with experimental data 389 using $Sh_c = 0.9$. This value also agrees with the critical Shields number near one reported 390 by Badr et al. [17]. Figure 10 presents the fluid velocity field for b = 4 mm with different 391 values of the impingement distance (*L*) and mean jet velocity (U_I) using the same $Sh_c = 0.9$. 392 Results demonstrate that the crater is wider and shallower as L increases, as expected. We 393 also notice that the velocity field for the fluid phase presents more oscillations for higher jet 394 velocities. Experiments also show these jet oscillations. Moreover, the proposed two-phase 395 model generates both strongly and weakly deflected cases with recirculation zones inside 396 or outside the crater, respectively (see Figures 10(c) and (d)). 397

(b)

 $U_{I}=0.28 \ m/s$



Figure 10. Velocity field and crater shapes for different jet configurations using b = 4 mm and fine mesh ($\Delta x = \Delta z = 0.5$ mm).

To verify the assumptions made for the velocity based on the Shield parameter and $_{398}$ its critical value, we test a series of three numerical simulations for different jet widths b_{399}

equal to 4 mm, 6 mm, and 8 mm (Table 2). The critical value $Sh_c = 0.9$ is kept constant for all numerical simulations. Numerical results confirm that D is larger and H is smaller as L_{401} increases. We note that the crater dimensions also increases as the mean jet velocity U_J is larger, as expected.

Mean velocity	Impingement distance	Jet width	Jet-bed distance	Reynolds number	Erosion parameter	Crater width	Crater depth
U_J (m/s)	<i>L</i> (mm)	<i>b</i> (mm)	L/b	Re	E	<i>D</i> (mm)	<i>H</i> (mm)
0.18	40	4	10	720	2.24	12	3
0.18	50	4	12.5	720	1.29	15	5
0.28	50	4	12.5	1120	2.01	32	10
0.28	60	4	15	1120	1.56	35	6
0.30	40	4	10	1200	3.74	23	10
0.30	60	4	15	1200	1.67	40	10
0.37	46	4	11.5	1480	3.11	40	15
0.37	80	4	20	1480	1.54	50	10
0.28	70	6	11.66	1680	2.28	38	15
0.28	80	6	13.33	1680	1.82	42	13
0.18	80	8	10	1440	2.24	25	10
0.18	90	8	11.25	1440	1.58	30	10

Finally, Figure 11 compares the numerical and experimental results of the crater dimensions *D* and *H*. The experimental data is the same as the one presented in Figure 4. The straight lines correspond to the linear models from experimental data given by equations (6) and (7). The numerical results are taking according to the information given in Table 2. The comparison shows a very good agreement for all values. These findings show the capacity of the proposed two-phase model to reproduce complex flows with sediment.



Figure 11. Comparison between experiments and numerical results for (a) the crater depth and (b) width normalized by the reduced jet-bed distance $(L - \lambda)$. Solid lines correspond to linear models.

5. Discussion and Conclusions

In this paper, we study experimentally and numerically the crater sizes generated on 411 horizontal non-cohesive sediment by a submerged vertical 2D plane jet. We have shown 412 that the crater depth and width, measured under various impinging distances and jet 413 velocities conditions, are governed by the effective impinging distance $L - \lambda$. This distance 414 is corrected from the virtual jet origin and by a dimensionless erosion parameter E. The 415 proposed erosion parameter, inspired by the one proposed by Aderibigbe and Rajaratnam 416 [9], also accounts for the virtual origin of the jet, as proposed by Sutherland and Dalziel 417 [13] and Badr et al. [17]. The dimensionless crater sizes evolve approximately linearly with 418

A two-phase numerical model has been used successfully to simulate the crater 421 formation. We suppress numerical inaccuracies from a standard two-fluid model by 422 considering a new formulation for the sediment. It consists of a unified equation for the 423 solid phase, which employs a smooth function governing the transition between the liquid-424 like and solid-like regimes. This function is based on the deviation of a local Shields number 425 from a critical value $Sh_c \approx 1$. Numerical results agree accurately to the experimental data 426 for all the proposed tests. Further studies are focused on testing the proposed approach 427 with different configurations. For instance, we are interested in scour holes generated by 428 inclined or even horizontal jet flows. 429

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Abbreviations

The following abbreviations are used in this manuscript:

- 2D Two-dimensional
- 3D Three-dimensional
- JET Jet Erosion Test
- WID Water Injection Dredging
- CFD Computational Fluid Dynamics
- MPI Message Passing Interface
- SOR Succesive-Over Relaxation
- GPU Graphics Processing Unit
- TVD Total Variation Diminishing
- PMMA PolyMethyl MethAcrylate

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