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# Topology generation of architectural meshes adapted to the support conditions

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**Abstract.** The performances of gridshells depend on the topology of the grid but also on the geometry of the underlying surface. Topology and geometry are linked: the shape of a hanging chain model depends on the pattern of the suspended fabric. In this paper, a method is proposed that generates topologies by varying continuous parameters. The obtained grids are adapted to the support conditions of the considered surface: elements converge towards the supports and arrive perpendicular to the edges otherwise. Grids are then lifted: the spatial mesh is the result of an optimisation to have a funicular surface and flat panels. The spatial mesh is then mechanically evaluated and its performance according to different criteria is represented in a Pareto front. A non-intuitive result shows that a large number of different topologies are situated on the Pareto front. These results show the strength of the method to explore different topologies.

**Keywords:** Topology finding · Gridshell · Form Finding.

## 1 Introduction

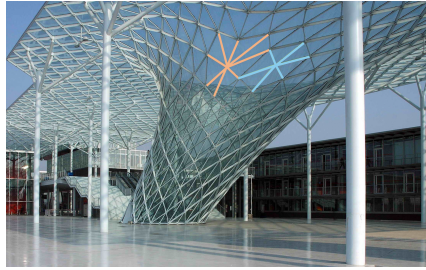
### 1.1 Necessity of topology generation

Meshes are everywhere in free form architecture, especially when it comes to constructing surfaces. On so-called discrete surfaces, or gridshells, made of beams and panels, the mesh is clearly visible: the beams are the edges of the mesh, the panels are the faces. The topology of the grid, i.e. the connectivity between the different elements, is thus apparent.

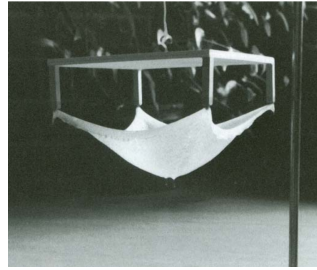
The construction of the surfaces must meet several requirements, of a geometrical nature and relative to manufacturing (flat panels are less expensive and easier to implement, for example), of a mechanical nature (the engineer seeks to reduce deformations, to direct the forces towards the supports) or even aesthetic (one prefers an equal distribution of the elements and panels of regular shape)

The geometry of the grid, its shape, is one of the ways to meet these objectives. But before the geometry, it is necessary to choose the topology of the grid.

The topology of the grid can be derived from the geometry to be discretised. In the case of the Milan Trade Fair (Figure 1), the abrupt change in curvature makes it necessary to introduce higher valence nodes (i.e. nodes that connect more elements, coloured on the image) in order to limit the distortion of the panels [16].



**Fig. 1.** Milan Trade Fair (Fuksas/SBP)



**Fig. 2.** Isler's hanging model [6]

But the topology of the grid can also control the resulting shape, in a process of form finding as illustrated by the image of a Heinz Isler model (Figure 2). This shape was obtained by suspending a net, whose topology (the textile orientation) influences the deformation under gravity loading. The influence of the topology is also illustrated in [5] where different fabric patterns are lifted, and in Figure 6 where a quad mesh with isolated singularities is submitted to gravity with a constraint on panel planarity.

In general, most form finding methods depend on the definition of an input grid topology, be it thrust network analysis [1], the Marionette method [10] or other historical methods developed by Frei Otto at the Institute for Lightweight Structures [12].

## 1.2 Existing methods of topology generation

Topological generation has not been widely studied for architectural structures, but is a subject of research in the computer graphics community (see surveys [2, 4] and [7, 9] which are closer to the method described hereafter).

Topological generation is not to be confused with topological optimisation, which optimises the distribution of material in an element.

R. Oval [13, 14] developed a method of topology generation based on the boundary of the surface to be meshed, as well as features such as points or curves chosen by the designer. The exploration of the different grids is then controlled by a grammar of elementary operations on strips.

The method presented here is a continuation of this approach called "topology finding". It proposes a new paradigm where the exploration of the grids is controlled by continuous parameters defined at the boundary. It is therefore intuitive, and starts from the support conditions which are a crucial element in the behaviour of the structure but also its integration in an architectural context.

### 1.3 Importance of boundary conditions

The support conditions are the starting point for the method presented in this article. Beyond aesthetics, the main issue of gridshell design is to carry the load towards the supports where forces are transmitted to the ground.



**Fig. 3.** Smithsonian Institute, Washington, DC (Foster and Partners / Buro Happold - 2001) [8].



**Fig. 4.** Cabot Circus, Bristol (Chapman Taylor with Nayan Kulkarni / SKM - 2008) [17]

On both surfaces in Figures 3 and 4, the grid is formed of regular quadrilaterals. In the case of the Smithsonian Institute, the support conditions are punctual, and the topology of the grid is not changed at the columns. However, the thickness of the beams increases at this point where the forces are concentrated. In the case of Cabot Circus, the regular grid is supported by large edge beams, which interrupt the grid: at the boundary, the beams are of various lengths, and the connections may be unique.

The method, presented in the next section, generates topologies and grids based on parameters defined along the edge. The resulting grids have beams that converge at the supports, and beams that arrive perpendicular to the edge elsewhere. They are therefore optimal from a mechanical point of view.

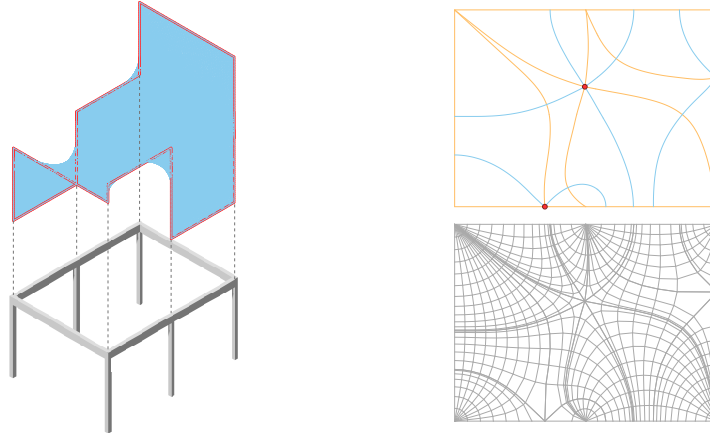
## 2 Topology and geometry generation method

### 2.1 Topology generation

The topology finding process is detailed in [3]. It relies on the computation of minimal surfaces that are only an intermediate tool in the topology finding process, but are not connected to the architectural shape. However, the boundary of



the minimal surface is chosen in relation to the supports of the real architectural surface, in order to find a topology and a grid that are adapted to the boundary conditions. One example is shown Figure 5, the minimal surface boundary is plotted in red, its vertical components are aligned with the columns shown underneath.



**Fig. 5.** Topology and grid generation process. A crenelated boundary (in red) is built over the real supports of the surface. This artificial boundary is controlled by height parameters. A minimal surface (in blue) is computed on this boundary. A topological skeleton is then retrieved (top right). It is made of singularities (red dots) of the height gradient field. Streamlines lines of the gradient are shown in blue, the level set is shown in orange. The skeleton defines patches that can be discretised regularly. The field is finally integrated to get patch subdivisions (bottom right).

The grid is the discretisation of streamlines of a field, namely the gradient field and its conjugate set the level set, on minimal surfaces which boundary is made of horizontal curves and vertical lines. This choice provides thus a proper alignment of the grid to the boundary, and smooth streamlines which intersect at 90 degrees. Moreover, level lines converge at the location of vertical boundary lines, which we choose to locate at the supports. By doing so forces are channelled to the supports. This phenomenon is illustrated Figure 5, where grey lines converge at six points on the boundary, where columns are, and arrive perpendicularly to the edge otherwise.

The boundary of the minimal surface can be modified: it is subdivided into slots whose heights can vary while keeping the same planar projection, and thus the resulting grid changes as well. This relation between the boundary and the grid is at the core of the generation process shown hereafter.

**Generation of grid skeleton** In this study, grids with quad panels are generated. Some triangular panels appear at the boundary, and the boundary point

they connect is called a pole. Other irregular points, located on the boundary or in the shape, are called singularities. At these points on the minimal surface, the gradient is null: singularities are saddle points. The topology finding process falls out into few parts: the determination of singularities (shown in red in Figure 5) and the computation of a skeleton. The skeleton, represented with blue and orange curves Figure 5, is made of streamlines that connect singularities between them and to the boundary. It defines patches: quadrilateral or triangular zones that can be discretised regularly.

## 2.2 Determination of the geometry

**Direct method for determining the strip subdivisions** The number of patch subdivisions is the same for all patches of a strip (a strip is a series of patches that are connected along their opposite edges) and is computed to get a target average edge length (of about 1.2 meters in our example).

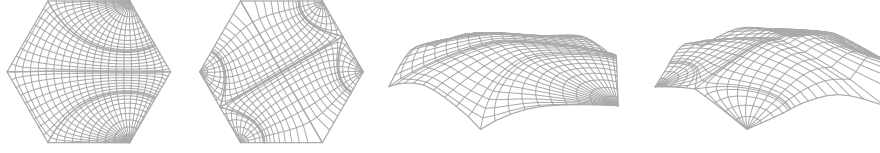
The resulting grid is made of quads panels (except at poles) that are not distorted because they come from the discretisation of lines that meet at right angle. There are overall distributed nicely in the mesh. However, concentration of lines at poles, and large faces at singularities can be noted. This could easily be remeshed locally, and the former grids are kept for the evaluation, despite this constructive remark.

**Direct method for computing an adapted shape** In this study, in order to compare topologies, only one shape is computed for each topology. This shape depends on the topology, but is not the only one that is adapted to this topology. It is the result an optimisation under two major constraints: flat panels and a funicular shape. This optimisation is run with Kangaroo [15]. The funicular load is proportional to the tributary area in plane, therefore all grids, that have the same planar projection, are submitted to the same loading. Other goals are applied for achieving a smooth shape: mesh edge lengths are controlled, and a stiffness is provided at the ridge beams to smooth curvature. The optimised shape is rescaled to get a maximum target height that is the same for all grids (and corresponds to a ratio height/span of 0.2). Grid heights could also be normalised to get a constant enclosed volume. This lifting process is illustrated on two grids with different topologies in Figure 6: because the shape depends on the topology, the resulting geometries vary.

As mentioned in the introduction, this method for generating topologies can also be used as a form finding method, as the resulting geometry is directly related to the topology of the grid. Therefore, each geometry is also evaluated as an isotropic shell, in order to compare their performance.

## 3 Case study: generation of grids on a hexagonal boundary

The method is illustrated on a regular shape, that can be used to cover an interior space or as a building (like the restaurant of Felix Candela in Xochimilco

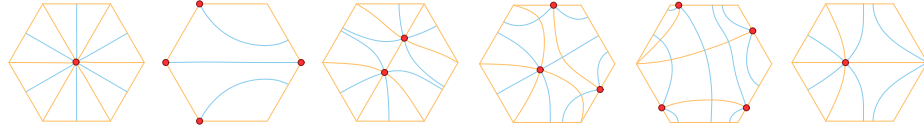


**Fig. 6.** Two grid examples that are lifted with a gravity load and constrained to have planar panels. The two lifted shapes differ, because grid topologies are different.

[11]). We consider a point supported structure, whose boundary projection is an hexagon, where supports are located at the discontinuities of the boundary. The crenellated contour is thus made of six slots, and five parameter heights are considered (when removing the rigid body motion). Some topologies that are found are shown Figure 7.

### 3.1 Topologies that can be reached with this method

Even if this contour is very symmetrical, many topologies can appear. The types of topologies that can occur are controlled by the Poincaré Hopf theorem: the sum of the indices of the vertices must be equal to the Euler characteristic of the surface (equal to 1 in this case).



**Fig. 7.** Selection of topologies that can be achieved with the presented method. Various combinations of vertex indices can be created.

### 3.2 Mechanical model

**Structural behaviour** Three structural types are tested: a shell (isotropic), a braced gridshell (anisotropic but with a membrane stiffness) and a gridshell without bracing (anisotropic). The gridshell without bracing serves as a reference for equivalent mechanical properties. The bracing section for the diagonals of the braced gridshell is taken as a tenth of the regular beams section. The shell properties are chosen in order to get a similar rigidity than the braced gridshell.

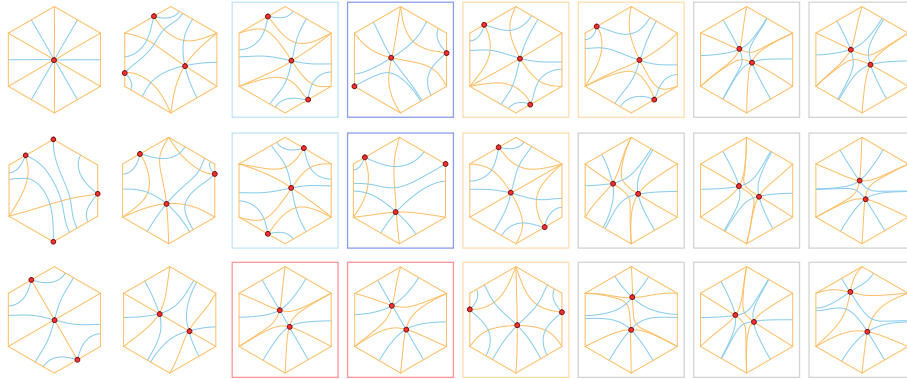
**Applied loads and boundary conditions** A symmetric gravity load is applied to mesh points and proportional to the tributary area. Hinged supports are set at the six corners.

**Initialisation of the model** Some parameters are chosen at the first iteration and kept constant for the rest of the generation process (same structure mass and section inertia in all grids). The only variables of the generation are those changing the topology, i.e the slot heights of the boundary of the minimal surface.

**Evaluation of the grids** The grid geometry is thus given, and its properties can be evaluated. Among all the criteria that can be chosen, dimensionless ratios are built: the thrust divided by the vertical reaction, which can be a design criteria for structures on top of existing building for example, the ratio between axial and bending strain energies, which characterises deformations and forces in the structure, and deflection, which is a design factor for relatively low structures (deflection is kept as it is, since the height and span of the different structures are the same). Since several criteria are involved, the Pareto front of the generated grids is evaluated.

### 3.3 Results

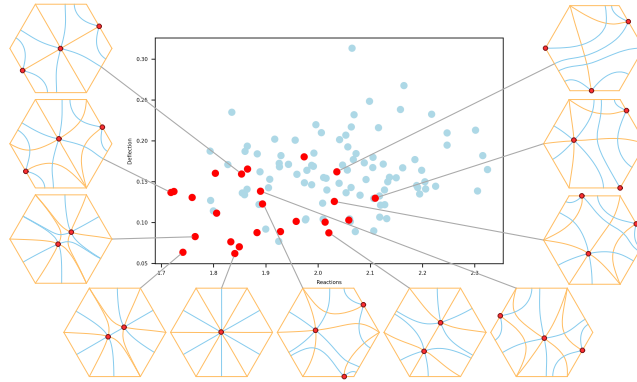
In total, three structural topologies are tested. More than a hundred of grids are generated, but one cannot prove that it constitutes an exhaustive scan of the space of grids achievable with this method. Nevertheless, this sample allows to see the diversity of possible designs, and to compare grids between them (see Figure 8).



**Fig. 8.** Grids that belong to the Pareto front when evaluated for a symmetric loading. Different topologies appear, but also geometric variations of the same topology, which are framed with colours.

**Pareto Front** The Pareto front is calculated for the gridshell without bracing. Over the 138 tested grid, 24 of them belong to the Pareto front, and are represented Figure 8. These grids can be grouped into topological families. Indeed,

the generation method controls both the topology of the grid and the position of the elements in space. Some topologies are also equivalent (with one rotation) and are grouped together. Finally, eleven different topologies are present on the Pareto front.



**Fig. 9.** Projection of the grids without bracing, in the plane of the support reactions and deflection criteria. The grids belonging to the Pareto front are shown in red, one representative of each topology family of the Pareto front is displayed on the side.

These grids are plotted in the objective space, a three dimensional space whose axis are the supports reactions, the ratio of strain energies and deflection. The grids belonging to the Pareto front are represented in red among all other generated grids represented in blue in Figure 9. The results are represented as a projection in the plane of two criteria (in this case the deflection on the ordinate and the ratio of horizontal to vertical reactions on the abscissa). A representative of each family is also displayed. This visualisation tool shows how the grids are positioned in relation to each other. It also allows the designer to make choices according to the criteria to be met.

**Comparison of the structure typologies** Three structural typologies are tested and compared in Table 1. Values are computed for two topologies, the symmetrical one with one singularity in the center, and another topology with a singularity in the surface, two in corners, and no symmetry in the grid (these topologies are shown in Figure 11 and the resulting 3D shapes are shown Figure 10).

As expected, adding bracing improves drastically the behaviour of the grid, in particular with regard to strain energies. The relative performance depends on the type of structure considered: with bracing, the symmetrical topology works better than the asymmetrical topology. Deflection is much larger for the shell models than for the grids.

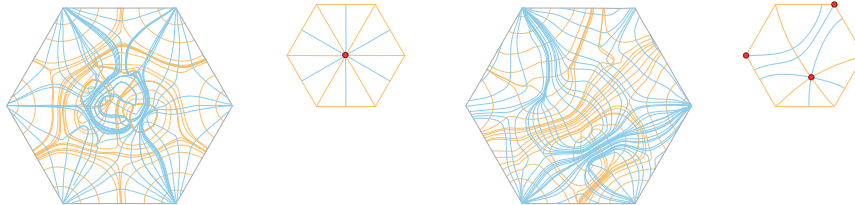
**Table 1.** Comparison of the performance of two grids, for three structural types (R. for reactions, E. for strain energy and d for deflection).

	3-symmetrical topology			asymmetrical topology		
	R.	E.	d (mm)	R.	E.	d (mm)
Gridshell	1.84	4.70	0.62	2.13	5.16	1.0
Braced gridshell	1.79	1.44	0.49	2.11	1.29	0.75
Shell	1.83	0.47	5.7	2.19	0.59	18



**Fig. 10.** Perspective views of the grid whose performances are compared for the different structural typologies: a symmetric topology with a central singularity (left) and an asymmetric topology with one singularity in the surface and two at corners (right).

The shell model serves as an evaluation of the geometry obtained with the grid topology. Stresslines can be computed on this shell, and compared to the grid that was used to create the geometry. Two examples are shown Figure 11. The stress lines of the two funicular forms are completely different. However, the shape of the lines is close to the underlying grid topology. The deviations may be due to the added planarity objective, or to the method of determining these lines on imperfect meshes. This topic deserves to be explored in more detail.



**Fig. 11.** Stress lines under gravity loading computed on the symmetric grid and on a grid with no symmetries and singularities located in the surface and on the boundary, with their associated topology.

## 4 Conclusion and perspectives

The presented method allows the generation of a variety of grid topologies, all adapted to the support conditions, by varying parameters defined along the edge. This variety is reflected in the Pareto front of the case study: a real choice is

offered to the designer according to the chosen criteria. The topology generation tool can also be used upstream of many formfinding methods, for which the topology is a given. Local remeshing techniques and local optimisations could be added to improve grid performance. The link between topology and isotropic shells deserves further investigation.

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