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**Veblen's Lecture Notebooks on Foundations of  
Geometry in Report No. 2249b/2022, History of  
Mathematics through Collaboration: Toward a  
Composite Portrait of Oswald Veblen**

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foundations of mathematics, an interest of both men at this time, are the main feature of the early letters. Reference is also made to publications exchanged, and to the possibilities of visits in both directions.

Veblen's correspondence with Hardy takes a similar form, although here we begin to see features that are evident in his correspondence more generally: the gradual appearance, over the course of the 1930s, of references to aid for refugee mathematicians fleeing Europe, and a more general commentary on the European situation. For example, in a pair of letters that, based on their dates, must have crossed in November 1938, Hardy and Veblen both express the same view, and propose the same course of action (i.e., resignation), regarding Levi-Civita's recent dismissal as an editor of *Zentralblatt*.

The shift from mathematical discussions and exchange of publications is even more evident in the much more comprehensive surviving correspondence between Veblen and Weyl. At the end of the 1920s, Veblen assisted Weyl in his arrangements for a visit to the United States, and their correspondence of the time touches briefly upon geometry in relation to problems in mathematical physics – most notably, general relativity. But then, in the early 1930s, the problems encountered by mathematicians seeking to leave Germany are raised by Weyl, at first for others, and then for himself and his family. The correspondence continues throughout the decade, leading up to Weyl's move to Princeton, and becomes still warmer in the process: by 1940, they were addressing each other 'Dear Oswald' and 'Dear Hermann,' in contrast to the relative formality of their earlier correspondence.

In each of these three cases, the correspondence shifts away from scientific exchanges of a more standard type, as Veblen's network was co-opted for his efforts to aid refugees, and indeed also to help non-endangered individuals such as Russell, who sought a position in the United States. But, in line with our wider view of Veblen's scientific diplomacy, the network remained an informal one – Veblen continued to interact with his correspondents on an individual-to-individual basis, without involving any formal structures or organizations.

#### REFERENCES

- [1] O. Veblen, Oswald Veblen Papers, US Library of Congress, Box 3 Folder Communist charges against Veblen.
- [2] O. Veblen, Oswald Veblen Papers, US Library of Congress, Box 25, Folder Loyalty Cases: Struik, Dirk Jan, 1942-1952.
- [3] E. Luciano, Looking for a space of intellectual survival. The Jewish mathematical diaspora from fascist Italy (1938-1948), f.c
- [4] R. Siegmund-Schultze, *Mathematicians Fleeing from Nazi Germany: Individual Fates and Global Impact*, Princeton University Press, 2009.

#### **Veblen's Lecture Notebooks on Foundations of Geometry**

EMMYLOU HAFFNER, JEMMA LORENAT, NICOLAS MICHEL, DENIZ SARIKAYA

Emmylou Haffner, Jemma Lorenat, Nicolas Michel, Deniz Sarikaya

The starting point of our group is the study of Oswald Veblen’s notebook on E. H. Moore’s lectures on the foundations of geometry given in the Autumn of 1901 at the University of Chicago<sup>1</sup>, where Veblen was a graduate student. This notebook, which as far as we know has never been studied, is 45 pages long, and contains both notes from the lecture and additional notes showing how Veblen reappropriated the texts of the authors studied. The observation that some of Veblen’s notes are directly related to results stated in his dissertation prompted us to a close analysis of the contents. We propose a critical transcription of this notebook, along with an in-depth commentary, linked with original sources and with Veblen’s, E.H. Moore’s, N. J. Lennes’, and other participants’ later works.

Below, we describe four main themes that emerged through our group’s work and which we believe will be fruitful to explore further.

### 1. REWRITING AND DIGESTING MATHEMATICS, AN OVERVIEW

E. H. Moore’s seminar relied largely on reading five contemporary works: Hilbert’s 1899 *Grundlagen der Geometrie* [13], Pasch’s 1882 *Vorlesungen über neuere Geometrie* [17], Peano’s 1894 paper *Sui fondamenti della Geometria* [15], Ingrams’s 1899 *Elementi di Geometria per le scuole secondarie superiori* [18], and Schur’s 1901 paper *Ueber die Grundlagen der Geometrie* [16].<sup>2</sup>

Veblen wrote most of his lecture notes on the right-hand side of the notebook, keeping the left-hand side for later annotations. The amount of notes varies largely from one page to another. This organisation is consistent enough that the few exceptions should require special attention — and indeed seem to be additional reflections by Veblen.

We observed that the notes constitute a first assimilation of the sources studied, likely mediated by Moore. Many of these notes are translations of the original sources, which were easy to trace. However, some of the translations are slightly inaccurate, sometimes leading to annotations by Veblen showing his confusion. For example, the translation of Hilbert’s Axiom IV<sub>1</sub> is:

If  $A, B$  are 2 pts on a line  $a$  and further  $A'$  is a pt on the same or /an/other<sup>3</sup> line  $a'$ , /then/ on a given side\* of the pt  $A'$  on the line  $a'$ , there is always one and only one pt  $B'$  so that the segment  $AB$  (or  $BA$ ) is congruent to the segment  $A'B'$ , in not[atio]n

$$AB \equiv A'B'.$$

Each segment is congruent to itself i.e.

$$AB \equiv AB.$$

(p. 14)

<sup>1</sup>O. Veblen, MSS 44016, Library of Congress.

<sup>2</sup>Veblen wrote as a reference for Peano the first volume of his 1889 *I Principii Di Geometria Logicamente Esposti* [14] but our analysis of the notes showed that this reference does not correspond to the notes in question, which are mostly literal translations of the axioms in [15].

<sup>3</sup>Words written /like this/ were added later by Veblen.

The \* added after the word “side” is written in pencil and refers to a note added on the left-hand side page, which simply reads “side?”. In fact, Hilbert’s original German is:

*Wenn  $A, B$  zwei Punkte auf einer Geraden  $a$  und ferner  $A'$  ein Punkt auf derselben oder einer anderen Geraden  $a'$  ist, so **kann man auf einer gegebenen Seite der Geraden  $a'$  von  $A'$  stets einen und nur einen Punkt  $B'$  finden**, so dass die Strecke  $AB$  (oder  $BA$ ) der Strecke  $A'B'$  congruent ist. [13, 10]*

The bold part of that axiom, which caused Veblen’s confusion, states that from a given side of the line  $a'$  of  $A'$ , that is, the side of  $a'$  that  $A'$  lies in, one can always find one and only one point  $B'$ , which certainly makes the idea of a “side” much more understandable.

Other similar examples can be found. An exhaustive comparison of the original sources with the notes is thus needed. In addition, notational and conceptual changes are noticeable in a number of cases, suggesting a deeper examination of the sources, which should be investigated further. The first goal of our commentary will be to map the influences and references exhaustively — including Moore’s 1902 paper *On the Projective Axioms of Geometry* in which he refers to this seminar (see Section [4] of this report). We will also consider other archival sources in Veblen’s and in Moore’s archives if we find any relevant ones.<sup>4</sup>

On the other hand, the notebook also exhibits many notes showing Veblen’s original reflection and digesting of the sources, some of which can be traced to his 1904 dissertation and to later works (see the commentary on “Thm 5” in the section below). Our second goal will be to correlate the contents of the notebook to Veblen’s dissertation, as well as to his later works in the foundations of geometry and topology and to his pedagogical reflections. We will propose a general overview, which will map the contents of his notebook into the contents of his dissertation, so as to better understand its conceptual and textual genesis. We are, here again, hoping to find additional sources in his archive at the Library of Congress, in particular relating to the writing of his dissertation.

## 2. FROM SOURCES TO LECTURE NOTES TO PUBLICATIONS

To illustrate the dynamic evolution between Veblen’s sources, lecture notes, and publications, we here present two complementary representative cases. These capture what could be further examined on the basis of the notebook — how Veblen selected and rewrote results from published sources and how he continued to reorder, reduce, and refine them.

First, how did Veblen import material from mathematical sources and lectures into his notebook? The case of one theorem, simply referred to by Veblen as “Thm 5”, is particularly illuminating. This theorem is included twice in the notebook, both on the left-hand side, indicating a later addition. The first iteration in pencil on a small piece of paper folded and glued into the notebook asserts that “Any 2

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<sup>4</sup>Moore, Eliakim Hastings. Papers, Hanna Holborn Gray Special Collections Research Center, University of Chicago Library.

inside pts can be joined by a st line not intersecting  $\Delta$  [this is Veblen’s notation for triangle in his notebooks] for if [...] not we should have  $BABC$  and  $B$  would be out.”

Thm 5 is more formally presented (now as “Theorem 5”) a few pages earlier in the text. This time, however, it is in ink with the annotation that it is now “stated in language sug. by Prof. Moores [sic] remarks,” which leads us to conclude that this is a later version. The page includes a diagram representing the line  $a$  and a “broken line”  $A'AB$  such that  $AB$  intersects  $a$ . The numeration and the diagram are slight modifications of Hilbert’s *Satz 5* (pp. 7–8) in his *Folgerungen aus den Axiomen der Verknüpfung und der Anordnung*. Notably, Hilbert does not include a proof for the theorems in this section with the claim that they follow *ohne Mühe* [without difficulty] from the axioms. Hilbert’s figure (Fig. 5) is nearly identical to Veblen’s except that, in the latter, regions above and below  $a$  are respectively labelled  $+\alpha$  and the region below with  $-\alpha$ .

In Veblen’s dissertation [9], this result becomes Theorem 26: “Any line decomposes a plane in which it lies into two regions.” The figure is gone, the proof is formalized to align with earlier definitions and axioms. Still, the proof invokes “observation” — a surprising choice in a paper on axiomatic foundations.

Veblen builds from this result in his work on Analysis situs, beginning with “Theory on Plane Curves in non-Metrical Analysis Situs” in which he references it as a “special case of a simple polygon” for the Jordan Curve Theorem [1]. The result is also cited in the dissertation of his classmate N. J. Lennes, who was also in Moore’s 1901 seminar [8]. Both publications refer explicitly to Moore’s seminar. These iterations of “the same” theorem prompt further investigations of figures, definitions, and symbols as they circulated from Hilbert’s publications to classrooms in Chicago.

The second case concerns Veblen’s “Projective Axioms of Dec 14. 1901” that appear toward the end of the notebook. Their physical and temporal proximity directly links these course notes to what would become Veblen’s 1904 dissertation. On the right-hand side of the page are nine axioms labeled 1 through 9. On the left-hand page, Veblen writes “Rearrange” and then uses Roman numerals to reorder and reduce this list. Below are further notes, dated April 9th (probably 1902), where Veblen further modifies the language.

A detailed cross-text comparison of these axioms demonstrates Veblen’s keen attention to language, experimentation with reducing or expanding axioms (both within each axiom but also as a system), and sensitivity to audience expectations. The almost trivial proofs immediately derived from these axioms that appear in his notebooks are condensed in his dissertation but then expanded in his pedagogical writings, for instance [10]. Components of Veblen’s formulation are first cited in [3]. This study complements that of “Thm 5” as here we have an instance of Veblen’s original mathematics that cemented his reputation in axiomatic foundations of geometries.

### 3. VEBLEN'S FOUNDATIONAL VIEWS IN GLOBAL AND INTERDISCIPLINARY CONTEXT

Around the time of Oswald Veblen's first forays into mathematics, new questions and approaches pertaining to the foundations of geometry were increasingly gaining traction, especially in Germany, Italy, and the United States. Veblen's notebook demonstrates close attention to many of the iconic texts of turn-of-the-century foundations. On a subtler level, Veblen's position at the University of Chicago). We shall determine to what extent Veblen adopted or modified the philosophical interpretations of his own mathematical work Hilbert proposed, as well as the role played by Moore in mediating these ideas (comparing f.i. [13] with [9] and with the work of Moore).

In the first pages of the notebook (cf. O. Veblen, MSS 44016, Library of Congress, 6 RHS), Veblen remarks that "[t]he idea of the proof of the existence of a science is probably due to Hilbert, (Paris congress)" and raises the question of studying "whether the axioms are independent what is the simplest set of axioms." Those themes are picked up in later publications, like in the published version of his thesis ([9]). Similarly, (cf. [12] stresses that we need "to distinguish between geometry as a branch of mathematics and geometry as a branch of physics".<sup>5</sup>

Moreover, Veblen displayed an active interest in theoretical physics, especially in relativity theory, to which he himself contributed via his *geometry of paths program* developed during his time at Princeton (cf. [7]). These activities further deepened his aforementioned understanding of geometry as both an axiomatic science and a branch of physics (cf. [11]). How do these views relate the *Laboratory Method* advanced by Moore [4]? Veblen was in particular interested to foster exchanges between mathematicians and physicist. Those efforts were in part unsuccessful due to disinterest within the scientific community of American physicists and in part due to the failure of Veblen's efforts to attract funding from the Rockefeller foundation.

### 4. COLLABORATIVE DYNAMICS AND THE MAKING OF VEBLEN'S GEOMETRY

Perhaps more than for his own scientific achievements, Veblen is oft remembered for his active leadership within the American mathematical community [19, pp. 63-101]. This leadership, as other groups in this workshop have demonstrated, involved the creation, strengthening, and administration of spaces and institutions specifically designed to grow this community. But Veblen's mathematical production also consists to a rather unusual degree (for the times) in co-authored articles and textbooks. In this section, we wish to focus on other kinds of collaborative and mentoring activities, namely the hands-on tutoring and fostering of promising students, whether it be as a supervisor of master's and doctoral theses or as the organizer of graduate seminars.

<sup>5</sup>This idea is further elaborated: "When you try to make a point you really make a spot, and when you try to draw a line you really make a strip. But the more accurately you succeed in indicating the points and drawing the lines, the more beautifully will the result of the experiment be in accord with the theorem stated." (p. 211)

These latter activities were ones which Veblen himself had directly experienced under E. H. Moore. The notebook under study, as well as its traces in later published articles, all bear witness to how formative these experiences were. In Veblen's famous 1905 paper on Jordan's closed curve theorem, for instance, Moore's seminar on the foundations of geometry is explicitly credited for provided the impetus and scientific environment in which novel ideas and approaches flourished – not only Veblen's own, but also those of Lennes, another student of Moore at the University of Chicago between 1898 and 1907:

This case was under discussion at the University of Chicago in 1901-02 in connection with Professor Moore's seminar on Foundations of Geometry. Mr. N. J. Lennes gave a proof in his master's thesis (1903), . . . Another proof appears as theorem 28 in the writer's dissertation . . . The present paper owes much to the discussions of the subject that have taken place under the leadership of Professor Moore.<sup>6</sup>

Conversely, while staying in Chicago after his graduation until 1905, Veblen followed in Moore's footsteps by organizing his own seminars and taking a proactive stance toward the mentoring of other graduate students, who sometimes were barely younger than him. This practice finds echo for instance in the first scientific publication of fellow Chicago graduate R. L. Moore, wherein Veblen is credited not only for orienting Moore towards certain topics but for having "given [him] much help in the way of actual contributions."<sup>7</sup> And it is in one such seminar, the site of constant interaction with other graduate students such as Lennes but also William Henry Bussey (a student of Leonard Dickson), that Veblen first worked out the content of his axiomatic theory of finite projective geometry.<sup>8</sup>

Thus, seminar teaching and research were so interwoven as to be inseparable in Veblen's mathematical life. This part of the project seeks to examine the modalities of such collaborative dynamics and how they contributed to shape Veblen's geometry, culminating with his 1910 textbook on projective geometry. It will do so using the Moore notebook and other lecture notes preserved at the University of Chicago, as well as a comparative analysis of the scientific publications on geometry written by the members of the Chicago seminars between 1900 and 1905.

#### REFERENCES

- [1] Oswald Veblen, Theory on Plane Curves in Non-Metrical Analysis Situs, Transactions of the American Mathematical Society, **6**:1, pp. 83-98, 1905.
- [2] Robert Lee Moore, Sets of Metrical Hypotheses for Geometry, Transactions of the American Mathematical Society, **9**:4, pp. 487-512, 1908.
- [3] Eliakim Hastings Moore, Sets of Metrical Hypotheses for Geometry, Transactions of the American Mathematical Society, **3**:1, pp. 142-158, 1902.
- [4] Eliakim Hastings Moore, On the Foundations of Mathematics, Science, **17**, pp.401-416, 1903.

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<sup>6</sup>[1, p. 83].

<sup>7</sup>[2, p. 488].

<sup>8</sup>[5, p. 245].

- [5] Oswald Veblen and William Henry Bussey, *Finite Projective Geometry*, Transactions of the American Mathematical Society, **7**:2, pp. 241-259, 1906.
- [6] John Corcoran, *Categoricity, History and Philosophy of Logic*, History and Philosophy of Logic, **1**:1-2, pp. 187-207, 1980, doi 10.1080/01445348008837010.
- [7] Jim Ritter, *Geometry as Physics: Oswald Veblen and the Princeton School*, Mathematics meets physics, ed. Karl-Heinz Schlote and Martina Schneider, Verlag Harri Deutsch, pp. 148-179, 2011.
- [8] Nels Johann Lennes, *Theorems on the Polygon and Polyhedron*, PhD Thesis, University of Chicago, 1904.
- [9] Oswald Veblen, *A System of Axioms for Geometry*, Transactions of the American Mathematical Society, **5**:3, pp. 343-384, 1904.
- [10] Oswald Veblen, *The Foundations of Geometry*, Monographs on Modern Mathematics Relevant to the Elementary Field, ed. J.W.A. Young, pp. 3-51, Longmans, Green and Co., 1911.
- [11] Oswald Veblen, *Geometry and Physics*, Science, **57**, pp. 129-139, 1923.
- [12] Oswald Veblen, *Certain Aspects of Modern Geometry. A course of three lectures delivered at the Rice Institute on Jan. 8, 11, 12, 1932*, Rice Institute Pamphlets, **31**, 1934.
- [13] David Hilbert, *Grundlagen der Geometrie*, Teubner, Leipzig, 1899.
- [14] Giuseppe Peano, *I principii di geometria logicamente esposti*, Fratelli Bocca, Torino, 1889.
- [15] Giuseppe Peano, *Sui fondamenti della Geometria*, Rivista di Matematica, **4**, pp. 51-90, 1894.
- [16] Friedrich Schur, *Ueber die Grundlagen der Geometrie*, Mathematische Annalen, **55**, pp. 265-292, 1901.
- [17] Moritz Pasch, *Vorlesungen über die neuere Geometrie*, Teubner, Leipzig, 1882.
- [18] Giuseppe Inghiri, *Elementi di Geometria per le scuole secondarie superiori*, Tip. Cenerelli, Bologna, 1899.
- [19] Karen Hunger Parshall, *The New Era in American Mathematics, 1920-1950*, Princeton University Press, Princeton & Oxford, 2022.

## **Veblen at the Institute for Advanced Study: Building for Excellence**

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This collaborative project explores mathematician Oswald Veblen and his early work at the Institute for Advanced Study (IAS).

### 1. THE INSTITUTE FOR ADVANCED STUDY

In early 1929, Louis Bamberger and his sister, Caroline Bamberger Fuld, sold their department stores, Bamberger & Co., to R.H. Macy Company and began to focus on how to invest their sizable fortune. Initially, they hoped to found a medical school for Jewish students. They enlisted the help of their longtime accountant, Samuel Leidesdorf, and attorney, Herbert Maass, to help determine the feasibility of transforming this idea into reality. When Leidesdorf and Maass began to explore this possibility, they were referred to Abraham Flexner, an educator who had completed a review of Medical Schools in North America and published his findings in 1910 in his *Medical Education in the United States and Canada* [1], otherwise known as “The Flexner Report.”

Rather than endorse Bamberger and Fuld’s idea for a new medical school, Flexner proposed an educational initiative with an entirely different focus. This series of events led Flexner to meet with Bamberger and Fuld in early 1930. This