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# Queen Elisabeth Competition\*

Sébastien Courtin <sup>†</sup>, Marin Gohard <sup>‡</sup>, Annick Laruelle<sup>§</sup>

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## Abstract

The Queen Elisabeth International Music Competition of Belgium founded in 1937, is among the world's most prestigious and demanding music competitions. It alternates between cello, piano, violin, and voice and consists of three rounds in which each candidate gives a recital. Each round leads to the selection of a subset of candidates. This paper presents and analyses the rule used to select candidates and its specificities. It shows that the rule does not satisfy many of the classical properties deemed as desirable in the social choice literature.

KEYWORDS: Music competition; Social Choice properties; Voting rule

JEL: D71; D72; Z10

## 1 Introduction

At the theoretical level, the celebrated Arrow [1950] theorem points at the impossibility to find a perfect rule to aggregate preferences, while Gibbard [1973] and Satterthwaite [1975] theorems demonstrate that any voting rule offers the possibility of misrepresenting preferences to modify the collective outcome. In practice, sport and cultural competitions provide useful settings to examine voting mechanisms (Zitzewitz [2014]). They provide exogenous variations that one can otherwise only achieve in an experiment, but with the advantage of larger sample sizes and participants with strong interest in the outcome.

In most systems judges give an numerical evaluation within a given range of grades and candidates are then ranked on the basis of their averages. Evaluations may suffer a bias of favoritism. That is, submitted evaluations may differ from truthful ones in order to modify the global evaluation. Notorious examples of manipulation can be found in sports and music competition. Vote trading scandals marked the 1998 and 2002 Olympics Skating competitions (Zitzewitz [2006]). Favoritism was also detected in Ski jumping competitions (Krumer et al. [2022]) or figure skating competitions (Zitzewitz [2014]) for later periods. Yair [1995] finds in-group favoritism based on geographical

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proximity in Eurovision Song Contests. He identifies three blocs: the Nordic bloc, the Western bloc and the Mediterranean bloc. These results have since been repeatedly confirmed (see for instance Spierdijk and Vellekoop [2009]). Sosnowska and Zawislak [2019] and Kontek and Sosnowska [2020] suspect that some members of the jury of the XV Wieniawski International Violin Competition may have voted in a strategic way. Some competitions use trimmed averages (i.e. removing the largest and smallest values) or median evaluations with the objective of reducing the possible manipulations. The international skating union modified its rules in several occasions (see Bassett and Persky [1994], Truchon [2004], Zitzewitz [2014]). The reforms introduced in the evaluation and aggregation systems were inspired by the bias observed.

A second type of bias is more difficult to avoid: performing later in the competition seems to be an advantage. This is well-known by coaches who typically present their gymnasts from the poorest candidate to the best one in competitions (Plessner and Haar [2006]). Flôres Jr and Ginsburgh [1996], Glejser and Heyndels [2011] and Ginsburgh and Van Ours [2003] show that musicians who perform later on a given day obtain better classifications in the Queen Elisabeth competition. Data from the Idol series (Page and Page [2009]) or the "immortal song" contests (Kim et al. [2021]) confirm the serial position bias.

With a social choice approach, Truchon [2004] studies the properties of the rules that have been prescribed by the International Skating Union in the 1990's. Music competitions rules have also been analysed, as the Chopin International Piano competition (Sosnowska [2013, 2017]) and the Wieniawski International Violin Competition (Sosnowska [2017]). She shows that both rules violate some desirable social choice properties.

In this paper we focus on the rule used in the Queen Elisabeth International Music Competition of Belgium (in short QEC). This competition, founded in 1937, is among the world's most prestigious and demanding music competitions. Doing well at this competition helps musicians in their career (Ginsburgh and Van Ours [2003]). It alternates between cello, piano, violin, and voice. It consists of three rounds. At each round, candidates give a recital. A jury give them numerical evaluations. The first round leads to the selection of 24 semi-finalists; the second round makes a further selection of twelve finalists. At the third round, the finalists (referred to as laureates) play a contemporary concerto composed for the competition. They have one week to study it with a ban to communicate (Berehova and Volkov [2019]). This last round leads to the ranking of the twelve laureates. The ranking of the first six laureates is made public.

Following a social choice approach, we show that the QEC rule satisfies three basic properties: universal domain (all the final ranking are available), unanimity (if all judge are fervent supporters of a candidate, he must be selected) and non-dictatorship (only one judge does not determine the final ranking). Otherwise the rule violates many other properties, such as independence of irrelevant alternatives (the ranking of two candidates does not depend on a third candidate), participation (a candidate stay selected even after the removal of a friendly judge), reinforcement (a candidate selected by two disjoint sets of judges stay selected by their union), majority principle (a condorcet candidate must be selected (Condorcet [1785])) and monotonicity (increasing the grade of a candidate can not be harmful for him).

The rest of the paper is organized as follows. Section 2 describes the rule. Section 3 analyses the

properties and Section 4 concludes.

## 2 Description of the rule

Let  $\mathcal{J}$  be the universe of judges and  $\mathcal{C}$  be the universe of candidates. Here we study the rule for a given round. Each round is composed of a set of candidates  $C = \{c_p\}_{p \in \mathcal{C}}$ , a set of judges  $J = \{j^i\}_{i \in \mathcal{J}}$  and the numerical evaluations that the judges give to the candidates. The number of candidates is denoted by  $n_C$  and  $n^J$  denotes the number of judges.

Judges do not evaluate their former students. Before the competition starts, candidates give the list of the judges who have been their teachers. A procedure exists to punish any misreported relationship. First we construct the students/teachers profile. Let  $s_p^i = 0$  if judge  $j^i$  is a former teacher of candidate  $c_p$  (and thus judge  $j^i$  does not evaluate candidate  $c_p$ ) while  $s_p^i = 1$  otherwise. The students/teachers profile is denoted  $\mathbb{S}_C^J$ :

$$\mathbb{S}_C^J = (s_p^i)_{\substack{p \in \mathcal{C} \\ i \in \mathcal{J}}}$$

The number of judges who evaluate candidate  $c_p$  is denoted  $n_p$  while the number of candidates evaluated by judge  $j^i$  is denoted  $n^i$ . We have

$$n_p = \sum_{i \in \mathcal{J}} s_p^i \text{ and } n^i = \sum_{p \in \mathcal{C}} s_p^i.$$

Second we construct the grade profile, denoted  $\mathbb{U}_C^J$ . Let  $u_p^i$  be the numerical grade that judge  $j^i$  gives to candidate  $c_p$ . We set  $u_p^i = 0$  whenever  $s_p^i = 0$  (that is, if judge  $j^i$  is a former teacher of candidate  $c_p$ ). We have:

$$\mathbb{U}_C^J = (u_p^i)_{\substack{p \in \mathcal{C} \\ i \in \mathcal{J}}}$$

Note that the QEC restricts the range of grades  $u_p^i$  that can be given:  $u_p^i \in [0, 100]$  at the first round;  $u_p^i \in [50, 100]$  at the second round; and  $u_p^i \in [60, 100]$  at the third round.

A round can thus be summarized by  $\mathbb{B}_C^J = (\mathbb{S}_C^J, \mathbb{U}_C^J)$ , that we refer to as voting profile. Let  $\mathcal{B} = (\mathcal{S}, \mathcal{U})$  be the sets of all voting profiles that can be constructed from  $\mathcal{J}$  and  $\mathcal{C}$ .

**Definition 1.** A  $m$ -rule  $\mathcal{W}_m$  on  $\mathcal{B}$  associates to each voting profile  $\mathbb{B}_C^J$  a subset of  $m$  candidates from  $\mathcal{C}$ . The subset of selected candidates is denoted  $\mathcal{W}_m(\mathbb{B}_C^J)$ .

The QEC rule selects on the basis of averages. It starts by attributing a grade to candidates who have a former teacher within the jury. In this case the numerical grade is substituted by an average computed as follows.

Given a voting profile  $\mathbb{B}_C^J$ , consider student  $c_p$  who was a former student of judge  $j^i$  (that is,  $s_p^i = 0$ ). The student's average is computed. Denoting student  $c_p$ 's average by  $\bar{u}_p(\mathbb{B}_C^J)$  (or simply  $\bar{u}_p$  when there is no ambiguity), we have:

$$\bar{u}_p = \frac{1}{n_p} \sum_{i \in \mathcal{J}} u_p^i.$$

The teacher's ratio  $r^i$  is computed. The later is given by the teacher's average ( $\bar{u}^i$ ) divided by the other

judges' averages (denoted  $\bar{u}^{-i}$ ). That is,

$$r^i = \frac{\bar{u}^i}{\bar{u}^{-i}} \text{ where } \bar{u}^i = \frac{1}{n^i} \sum_{p \in C} u_p^i \text{ and } \bar{u}^{-i} = \frac{1}{n^J - 1} \sum_{j \in J, j \neq i} \bar{u}^j.$$

A ratio strictly larger (smaller) than 1 means that judge  $j^i$  is more (less) generous in her grades than her fellow judges are. The numerical grade that student  $c_p$  obtains from her teacher  $j^i$  is the student average multiplied by the teacher's ratio:  $r^i \bar{u}_p$ .

With this substitution we obtain a numerical grade for each candidate, denoted  $v_p^i$ . Let

$$\mathbb{V}_C^J = (v_p^i)_{\substack{p \in C \\ i \in J}} \text{ with } v_p^i = \begin{cases} u_p^i & \text{if } s_p^i = 1 \\ r^i \bar{u}_p & \text{if } s_p^i = 0. \end{cases}$$

Before computing the candidates' average, outlier grades are eliminated. A grade which lies much (20 percent) above (or below) the average is substituted by the average plus (minus) 20 percent. Formally, for each  $p \in C$ , candidate  $c_p$ 's average grade is computed. We obtain

$$(\bar{v}_p)_{p \in C} \text{ with } \bar{v}_p = \frac{1}{n^J} \sum_{i \in J} v_p^i.$$

We eliminate the outlier grades. This final grade that judge  $j^i$  gives to candidate  $c_p$  is denoted by  $w_p^i$ . We have:

$$\mathbb{W}_C^J = (w_p^i)_{\substack{p \in C \\ i \in J}} \text{ with } w_p^i = \begin{cases} 0.8 \bar{v}_p & \text{if } v_p^i < 0.8 \bar{v}_p \\ 1.2 \bar{v}_p & \text{if } v_p^i > 1.2 \bar{v}_p \\ v_p^i & \text{otherwise.} \end{cases}$$

The final grades are the trimmed averages over the different judges.

$$(\bar{w}_p)_{p \in C} \text{ with } \bar{w}_p = \frac{1}{n^J} \sum_{i \in J} w_p^i.$$

**Definition 2.** The QEC  $m$ -rule  $\mathcal{W}_m^*$  associates to each voting profiles  $\mathbb{B}_C^J$  the subset  $\mathcal{W}_m^*(\mathbb{B}_C^J)$  composed of the  $m$  candidates with the largest  $\bar{w}_p$ .

The QEC uses three different  $m$ -rules:  $\mathcal{W}_{24}^*$  at the first round,  $\mathcal{W}_{12}^*$  at the second round and  $\mathcal{W}_6^*$  at the third round. Note that at the third stage the six laureates are ranked.

**Example 1.** Consider a round with 10 candidates  $\{c_1, c_2, \dots, c_{10}\}$  and 5 judges  $\{j^1, j^2, \dots, j^5\}$ . Four candidates ( $m = 4$ ) will be selected at this round. The round is represented by  $\mathbb{B}_C^J = (\mathbb{S}_C^J, \mathbb{U}_C^J)$  where in matrix  $\mathbb{S}_C^J$  and  $\mathbb{U}_C^J$  the value of  $s_p^i$  and  $u_p^i$  appears in the  $i^{\text{th}}$  row and the  $p^{\text{th}}$  column (similarly for the other matrices).

$$S_C^J = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & \mathbf{0} & 1 & 1 & 1 \\ 1 & 1 & 1 & \mathbf{0} & 1 & 1 & 1 & 1 & 1 & \mathbf{0} \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & \mathbf{0} & 1 & 1 \end{pmatrix}, \mathbb{U}_C^J = \begin{pmatrix} 90 & 85 & 60 & 40 & 33 & 50 & 75 & 15 & 95 & 30 \\ 95 & 12 & 32 & 58 & 60 & 61 & \mathbf{0} & 20 & 32 & 10 \\ 75 & 22 & 81 & \mathbf{0} & 52 & 65 & 25 & 64 & 72 & \mathbf{0} \\ 20 & 50 & 34 & 48 & 45 & 85 & 22 & 23 & 64 & 40 \\ 47 & 59 & 51 & 52 & 60 & 42 & 95 & \mathbf{0} & 62 & 37 \end{pmatrix}$$

We have  $s_4^3 = u_4^3 = 0$ , which means that candidate  $c_4$  is a former student of judge  $j^3$ . Candidates  $c_7$ ,  $c_8$  and  $c_{10}$  have also a former teacher within the jury. For these students and their teachers (judges  $j^2$ ,  $j^3$ , and  $j^5$ ) we compute the students averages and their teacher ratio to obtain their grades:

$$\begin{aligned} \bar{u}_4 &= 49.5 & \bar{u}^3 &= 57 & \bar{u}^{-3} &= 49.7 & r^3 &= 1.15 & v_4^3 &= 56.8 \\ \bar{u}_7 &= 54.2 & \bar{u}^2 &= 42.2 & \bar{u}^{-2} &= 53.4 & r^2 &= 0.79 & v_7^2 &= 42.9 \\ \bar{u}_8 &= 30.5 & \bar{u}^5 &= 56.1 & \bar{u}^{-5} &= 49.9 & r^5 &= 1.12 & v_8^5 &= 34.3 \\ \bar{u}_{10} &= 29.2 & \bar{u}^3 &= 57 & \bar{u}^{-3} &= 49.7 & r^3 &= 1.15 & v_{10}^3 &= 33.6 \end{aligned}$$

We then have :

$$V_C^J = \begin{pmatrix} 90 & 85 & 60 & 40 & 33 & 50 & 75 & 15 & 95 & 30 \\ 95 & 12 & 32 & 58 & 60 & 61 & \mathbf{42.9} & 20 & 32 & 10 \\ 75 & 22 & 81 & \mathbf{56.8} & 52 & 65 & 25 & 64 & 72 & \mathbf{33.6} \\ 20 & 50 & 34 & 48 & 45 & 85 & 22 & 23 & 64 & 40 \\ 47 & 59 & 51 & 52 & 60 & 42 & 95 & \mathbf{34.3} & 62 & 37 \end{pmatrix}$$

The candidates' averages are then computed, together with the lower and upper bounds:

$$\begin{aligned} (\bar{v}_p)_{p \in C} &= (65.4 & 45.6 & 51.6 & 51 & 50 & 60.6 & 52 & 31.3 & 65 & 30.1) \\ (0.8\bar{v}_p)_{p \in C} &= (52.3 & 36.5 & 41.3 & 40.8 & 40 & 48.5 & 41.6 & 25 & 52 & 24.1) \\ (1.2\bar{v}_p)_{p \in C} &= (78.5 & 54.7 & 61.9 & 61.1 & 60 & 72.7 & 62.4 & 37.5 & 78 & 36.1) \end{aligned}$$

The following table gives the final grades of the candidates: it may be the initial value ( $u_p^i$ ), the substituted value ( $v_p^i$ ), the lower bound of the average ( $0.8\bar{v}_p$ ) or the upper bound of the average ( $1.2\bar{v}_p$ ). We obtain

$$W_C^J = \begin{pmatrix} 78.5 & 54.7 & 60 & 40.8 & 40 & 50 & 62.4 & 25.1 & 78 & 30 \\ 78.5 & 36.5 & 41.3 & 58 & 60 & 61 & 42.9 & 25.1 & 52 & 24.1 \\ 75 & 36.5 & 61.9 & 56.8 & 52 & 65 & 41.6 & 37.5 & 72 & 33.6 \\ 52.3 & 50 & 41.3 & 48 & 45 & 72.7 & 41.6 & 25 & 64 & 36.1 \\ 52.3 & 54.7 & 51 & 52 & 60 & 48.5 & 62.4 & 34.3 & 62 & 36.1 \end{pmatrix}$$

The final averages are then computed.

$$(\bar{w}_p)_{p \in C} = (\mathbf{67.3} & 46.5 & 51.1 & 51.1 & \mathbf{51.4} & \mathbf{59.4} & 50.2 & 29.4 & \mathbf{65.6} & 32)$$

The selected candidates are the four ones with the largest averages. We obtain  $\mathcal{W}_4^*(\mathbb{B}_C^J) = \{c_1, c_9, c_6, c_5\}$ .

### 3 Properties of the rules

In this section we check the properties that the QEC rule satisfies. First we start by the minimal properties. Obviously any possible subset can be selected (universal domain) and there is no dictator. Another property that the rule satisfies is the unanimity. Then we show that the rule displays various "voting paradoxes", that is the rule violates properties deemed as desirable.

#### 3.1 Unanimity property

The unanimity property (also known as the Pareto property, see Arrow [1951] and Fishburn [1974a]) can be stated as follows in our context. If all grades of candidate  $c_p$  are higher than those of candidate  $c_q$  then the selection of candidate  $c_q$  implies the selection of candidate  $c_p$ . The following proposition shows that the unanimity condition is satisfied.

**Definition 3.** Let  $\mathbb{B}_C^J$  be a voting profile. Candidate  $c_p$  dominates candidate  $c_q$  in  $\mathbb{B}_C^J$  if and only if  $s_p^i = s_q^i$  and  $u_p^i \geq u_q^i$  for all  $i \in J$ .

**Definition 4.** A  $m$ -rule  $\mathcal{W}_m$  on  $\mathcal{B}$  satisfies Unanimity if for each  $\mathbb{B}_C^J \in \mathcal{B}$  such that candidate  $c_p$  dominates candidates  $c_q$  in  $\mathbb{B}_C^J$  the following condition holds:  $c_q \in \mathcal{W}_m(\mathbb{B}_C^J)$  implies that  $c_p \in \mathcal{W}_m(\mathbb{B}_C^J)$ .

**Proposition 1.** The  $m$ -rule  $\mathcal{W}_m^*$  satisfies Unanimity.

*Proof.* Consider a profile  $\mathbb{B}_C^J \in \mathcal{B}$  with  $c_p$  and  $c_q$  two candidates belonging to  $C$ .

i) Firstly we assume that  $s_p^i = 1$  for all  $p \in C$  and all  $i \in J$  which implies that  $v_p^i = u_p^i$ . Let  $c_p$  get grades  $u_p^1, \dots, u_p^{n_j}$  and candidate  $c_q$  grades  $u_q^1, \dots, u_q^{n_j}$  such that  $u_p^i > u_q^i$  for all  $i \in J$ . Assume finally that  $c_q \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ . We must consider two cases:

i) there is no outlier grade, i.e.  $w_p^i = u_p^i$  and  $w_q^i = u_q^i$ . This implies that  $\bar{w}_p = \frac{1}{n^j} \sum_{i \in J} w_p^i > \frac{1}{n^j} \sum_{i \in J} w_q^i = \bar{w}_q$ .

Therefore  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

ii) there is an outlier grade:

a) Assume that  $u_p^i = v_p^i < 0.8 \bar{v}_p$  for some  $i \in J$ . Then  $w_p^i = 0.8 \bar{v}_p > u_p^i > u_q^i$  for some  $i \in J$ . This implies that  $\bar{w}_p = \frac{1}{n} \sum_{i \in J} w_p^i > \frac{1}{n} \sum_{i \in J} w_q^i = \bar{w}_q$  and  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

b) Suppose that  $u_q^i = v_q^i > 1.2 \bar{v}_q$  for some  $i \in J$ . Therefore  $w_q^i = 1.2 \bar{v}_q < u_q^i < u_p^i$  for some  $i \in J$ . Then  $\bar{w}_q < \bar{w}_p$  and  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

c) Assume that  $u_p^i = v_p^i > 1.2 \bar{v}_p$  for some  $i \in J$ . Then  $w_p^i = 1.2 \bar{v}_p < u_p^i$  for some  $i \in J$ . Since  $u_p^i > u_q^i$  for all  $i \in J$ , we have  $1.2 \bar{v}_p > 1.2 \bar{v}_q$ . And since  $u_q^i = v_q^i \leq 1.2 \bar{v}_q$  for all  $i \in J$ , this implies that  $w_p^i > w_q^i$  for all  $i \in J$ . We then have  $\bar{w}_p > \bar{w}_q$  and  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

d) Suppose that  $u_q^i = v_q^i < 0.8 \bar{v}_q$  for some  $i \in J$ . Then  $w_q^i = 0.8 \bar{v}_q > u_q^i$  for some  $i \in J$ . Since  $u_p^i > u_q^i$  for all  $i \in J$ , we then have  $0.8 \bar{v}_p > 0.8 \bar{v}_q$ . And since  $u_p^i = v_p^i \geq 0.8 \bar{v}_p$  for all  $i \in J$ , this implies that  $w_q^i < w_p^i$  for all  $i \in J$ . Therefore  $\bar{w}_p > \bar{w}_q$  and  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

e) Consider  $u_p^i = v_p^i < 0.8 \bar{v}_p$  for some  $i \in J$  and  $u_q^i = v_q^i > 1.2 \bar{v}_q$  for some  $i \in J$ . According to Case a and Case b, it is obvious that  $\bar{w}_p > \bar{w}_q$  and  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

f) Let  $u_p^i = v_p^i > 1.2\bar{v}_p$  for some  $i \in J$  and  $u_q^i = v_q^i < 0.8\bar{v}_q$  for some  $i \in J$ . According to Case c and Case d, we have  $\bar{w}_p > \bar{w}_q$  and  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

g) Let  $u_p^i = v_p^i > 1.2\bar{v}_p$  for some  $i \in J$  and  $u_q^i = v_q^i > 1.2\bar{v}_q$  for some  $i \in J$ . According to Case b and Case c, we have  $\bar{w}_p > \bar{w}_q$  and  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

h) Finally suppose that  $u_p^i = v_p^i < 0.8\bar{v}_p$  for some  $i \in J$  and  $u_q^i = v_q^i < 0.8\bar{v}_q$  for some  $i \in J$ . According to Case a and Case d, we have  $\bar{w}_p > \bar{w}_q$  and  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

II) Now assume that  $s_p^i = s_q^i = 0$  for  $i \in J$ ;  $s_p^j = s_q^j = 1$  for  $j \neq i$  and  $s_r^j = 1$  for all  $r \neq p, q$  and all  $j \neq i$ . WLOG assume that  $i = 1$ . Let  $c_p$  get grades  $u_p^2, \dots, u_p^{n_j}$  and candidate  $c_q$  grades  $u_q^2, \dots, u_q^{n_j}$  such that  $u_p^j > u_q^j$  for all  $j \neq i = 1$ . We then have  $\bar{u}_p = \frac{1}{n^j} \sum_{j \neq i=1} u_p^j > \frac{1}{n^j} \sum_{j \neq i=1} u_q^j = \bar{u}_q$ . Therefore  $v_p^1 = \bar{u}_p \cdot r^1 > v_q^1 = \bar{u}_q \cdot r^1$ . Similarly to Case I, if  $c_q \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ , then  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ . □

Note that the property of unanimity holds whenever we require the condition  $s_p^i = s_q^i$  for candidate  $c_p$  and candidate  $c_q$ . For a general discussion on the violation of unanimity in peer ratings system, see Yew-Kwang and Guang-Zhen [2003].

### 3.2 Condorcet properties

The Condorcet properties give conditions that guarantee the selection (or non selection) of a candidate. The classical properties concern the Condorcet winner (a candidate who obtains a higher grade than any other candidate for a majority of judges) or Condorcet loser (a candidate who obtains a lower grade than any other candidate for a majority of judges). The QEC  $m$ -rule does not necessarily select the Condorcet winner (if there is one) and may select the Condorcet loser (if there is one). Here we show that the QEC  $m$ -rule may even violate a weaker property: the majority criterion. The majority criterion states that (i) a *strong* Condorcet winner (if there is one) should be selected and (ii) a *strong* Condorcet loser (if there is one) should not be selected.

Let  $\alpha_p(\mathbb{B}_C^J)$  (resp.  $\beta_p(\mathbb{B}_C^J)$ ) be the number of times that candidate  $c_p$  is ranked first (resp. last) in  $(\mathbb{B}_C^J)$ . We have:

$$\alpha_p^i(\mathbb{B}_C^J) = \begin{cases} 1 & \text{if } u_p^i = \text{Max}_{p \in C} u_p^i \\ 0 & \text{otherwise} \end{cases} \quad \text{and } \alpha_p(\mathbb{B}_C^J) = \sum_{i \in J} \alpha_p^i(\mathbb{B}_C^J)$$

$$\beta_p^i(\mathbb{B}_C^J) = \begin{cases} 1 & \text{if } u_p^i = \text{Min}_{p \in C} u_p^i \\ 0 & \text{otherwise} \end{cases} \quad \text{and } \beta_p(\mathbb{B}_C^J) = \sum_{i \in J} \beta_p^i(\mathbb{B}_C^J)$$

**Definition 5.** Let  $\mathbb{B}_C^J$  be a voting profile. (i) Candidate  $c_p$  is a *strong Condorcet winner* in  $\mathbb{B}_C^J$  if and only if  $\alpha_p(\mathbb{B}_C^J) > \frac{n^j}{2}$ ; (ii) candidate  $c_p$  is a *strong Condorcet loser* in  $\mathbb{B}_C^J$  if and only if  $\beta_p(\mathbb{B}_C^J) > \frac{n^j}{2}$ .

**Definition 6.** A  $m$ -rule  $\mathcal{W}_m$  on  $\mathcal{B}$  satisfies the *Majority criterion* if: (i)  $c_p \in \mathcal{W}_m(\mathbb{B}_C^J)$  whenever  $c_p$  is a *strong Condorcet winner* in  $\mathbb{B}_C^J$ ; (ii)  $c_p \notin \mathcal{W}_m(\mathbb{B}_C^J)$  whenever  $c_p$  is a *strong Condorcet loser* in  $\mathbb{B}_C^J$ .

The following proposition shows that a *strong* Condorcet loser may be selected and a *strong* Condorcet winner may be excluded from the selection.



**Proposition 2.** *The  $m$ -rule  $\mathcal{W}_m^*$  does not satisfy the Majority criterion.*

*Proof.* Consider a profile  $\hat{\mathbb{B}}_C^J = (\hat{\mathbb{S}}_C^J, \hat{\mathbb{U}}_C^J)$  with 4 candidates,  $C = \{c_1, \dots, c_4\}$  and 3 judges,  $J = \{j^1, \dots, j^3\}$ . Two candidates will be selected. We have

$$\hat{\mathbb{S}}_C^J = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \text{ and } \hat{\mathbb{U}}_C^J = \begin{pmatrix} 70 & 68 & 42 & 40 \\ 3 & 17 & 65 & 78 \\ 50 & 48 & 41 & 40 \end{pmatrix}$$

We can observe that  $j^1$  and  $j^3$  rank  $c_1$  first and  $c_4$  last. Candidate  $c_1$  is thus a strong Condorcet winner and Candidate  $c_4$  is thus a strong Condorcet loser. The candidates averages and the bounds for the outliers give

$$\begin{aligned} (\bar{v}_p)_{c_p \in C} &= (41 \quad 44.3 \quad 49.3 \quad 52.7) \\ (0.8\bar{v}_p)_{c_p \in C} &= (32.8 \quad 35.5 \quad 39.5 \quad 42.1) \\ (1.2\bar{v}_p)_{c_p \in C} &= (49.2 \quad 53.2 \quad 59.2 \quad 63.2) \end{aligned}$$

Outliers grades are then substituted by  $0.8\bar{v}_p$  or  $1.2\bar{v}_p$  and the final averages are computed:

$$(\bar{w}_p)_{c_p \in C} = (43.7 \quad 45.6 \quad 47.4 \quad 49.2)$$

We obtain  $\mathcal{W}_2^*(\hat{\mathbb{B}}_C^J) = \{c_4, c_3\}$ . The *strong* Condorcet loser  $c_4$  is selected while the *strong* Condorcet winner  $c_1$  is not selected.  $\square$

### 3.3 Independence of irrelevant alternatives property

The property of the independence of irrelevant alternatives (Arrow [1951], Fishburn [1974b,c, 1977]) can be stated in our context as follows: if we eliminate a candidate who was not selected, the selection of the candidates should not be modified.

**Definition 7.** *A  $m$ -rule  $\mathcal{W}_m$  on  $\mathcal{B}$  satisfies Independence of irrelevant alternatives if  $\mathcal{W}_m(\mathbb{B}_C^J) = \mathcal{W}_m(\mathbb{B}_{C \setminus p}^J)$  whenever candidate  $c_p \notin \mathcal{W}_m(\mathbb{B}_C^J)$ .*

The following proposition tells us that the property can be violated. That is, the selected candidates may depend on the presence or absence of a non selected candidate.

**Proposition 3.** *The  $m$ -rule  $\mathcal{W}_m^*$  does not satisfy the Independence of irrelevant alternatives.*

*Proof.* Consider  $\mathbb{B}_{C \setminus 7}^J = (\mathbb{S}_{C \setminus 7}^J, \mathbb{U}_{C \setminus 7}^J)$  obtained by eliminating  $c_7$  in  $\mathbb{B}_C^J$  of Example 1. This does not modify the candidates' averages but modify the judges' averages and ratios, which in turn modifies the grade of the candidates who had a former teacher in the jury

$$\begin{aligned} \bar{u}^3 &= 61.6 & \bar{u}^{-3} &= 48.6 & r^3 &= 1.27 & v_4^3 &= 62.8 \\ \bar{u}^5 &= 51.2 & \bar{u}^{-5} &= 50.1 & r^5 &= 1.02 & v_8^5 &= 30.6 \\ \bar{u}^3 &= 61.6 & \bar{u}^{-3} &= 48.6 & r^3 &= 1.27 & v_{10}^3 &= 37.1 \end{aligned}$$

The average of candidates  $c_4$  increases

$$\begin{aligned} (\bar{v}_p)_{c_p \in C} &= (65.4 \quad 45.6 \quad 51.6 \quad 52.2 \quad 50 \quad 60.6 \quad 30.5 \quad 65 \quad 30.8) \\ (0.8\bar{v}_p)_{c_p \in C} &= (52.3 \quad 36.5 \quad 41.3 \quad 41.7 \quad 40 \quad 48.5 \quad 24.4 \quad 52 \quad 24.6) \\ (1.2\bar{v}_p)_{c_p \in C} &= (78.5 \quad 54.7 \quad 61.9 \quad 62.6 \quad 60 \quad 72.7 \quad 36.6 \quad 78 \quad 37) \end{aligned}$$

and

$$(\bar{w}_p)_{c_p \in C} = (67.3 \quad 46.5 \quad 51.1 \quad 52.5 \quad 51.4 \quad 59.4 \quad 28.1 \quad 65.6 \quad 33.1)$$

We obtain  $\mathcal{W}_4^*(\mathbb{B}_{C \setminus 7}^J) = \{c_1, c_9, c_6, c_4\} \neq \{c_1, c_9, c_6, c_5\} = \mathcal{W}_4^*(\mathbb{B}_C^J)$ .  $\square$

### 3.4 Participation properties

The participation properties have been introduced by Fishburn and Brams [1983], Ray [1986] and Moulin [1988b] and can be divided in four different requirements: (i) positive abstention: If a candidate is not selected, he cannot be selected either if a judge who gives him the highest grades is removed from the jury. (ii) positive participation: If a candidate is selected, he remains selected if a judge who gives him the highest grade is added to the jury. (iii) negative abstention: If a candidate is selected, he remains selected if a judge who gives him the smallest grade is removed from the jury. (iv) negative participation: If a candidate is not selected, he cannot be selected either if a judge who gives him the smallest grade is added to the jury.

**Definition 8.** A  $m$ -rule  $\mathcal{W}_m$  on  $\mathcal{B}$  satisfies:

- (i) Positive abstention if  $c_p \notin \mathcal{W}_m(\mathbb{B}_C^J)$  and  $\alpha_p^i(\mathbb{B}_C^J) = 1$  imply that  $c_p \notin \mathcal{W}_m(\mathbb{B}_C^{J^i})$ ;
- (ii) Positive participation if  $c_p \in \mathcal{W}_m(\mathbb{B}_C^J)$  and  $\alpha_p^i(\mathbb{B}_C^J) = 1$  imply that  $c_p \in \mathcal{W}_m(\mathbb{B}_C^{J \cup i})$ ;
- (iii) Negative abstention if  $c_p \in \mathcal{W}_m(\mathbb{B}_C^J)$  and  $\beta_p^i(\mathbb{B}_C^J) = 1$  imply that  $c_p \in \mathcal{W}_m(\mathbb{B}_C^{J^i})$ ;
- (iv) Negative participation if  $c_p \notin \mathcal{W}_m(\mathbb{B}_C^J)$  and  $\beta_p^i(\mathbb{B}_C^J) = 1$  imply that  $c_p \notin \mathcal{W}_m(\mathbb{B}_C^{J \cup i})$ .

**Proposition 4.** The  $m$ -rule  $\mathcal{W}_m^*$  does not satisfy : (i) Positive abstention; (ii) Positive participation; (iii) Negative abstention; (iv) Negative participation.

*Proof.* (i) Consider a profile  $\check{\mathbb{B}}_C^J = (\mathbb{S}_C^J, \check{\mathbb{U}}_C^J)$  with 10 candidates,  $C = \{c_1, \dots, c_{10}\}$  and 5 judges,  $J = \{j^1, \dots, j^5\}$ . Four candidates will be selected for the next round.  $\mathbb{S}_C^J$  is identical to Example 1 and

$$\check{\mathbb{U}}_C^J = \begin{pmatrix} 90 & 85 & 60 & 40 & 64 & 47 & 75 & 15 & 95 & 30 \\ 95 & 46 & 50 & 58 & 60 & 61 & 0 & 20 & 32 & 10 \\ 74 & 75 & 74 & 0 & 77 & 76 & 74 & 74 & 74 & 0 \\ 20 & 50 & 44 & 48 & 45 & 85 & 22 & 23 & 64 & 40 \\ 47 & 59 & 51 & 52 & 60 & 42 & 95 & 0 & 62 & 37 \end{pmatrix}$$

The former students' averages and the teachers ratios permit to complete all candidates grades

$$\begin{aligned}
\bar{u}_4 &= 49.5 & \bar{u}^3 &= 74.7 & \bar{u}^{-3} &= 52.0 & r^3 &= 1.43 & v_4^3 &= 71 \\
\bar{u}_7 &= 66.5 & \bar{u}^2 &= 48 & \bar{u}^{-2} &= 58.8 & r^2 &= 0.82 & v_7^2 &= 54.3 \\
\bar{u}_8 &= 33 & \bar{u}^5 &= 56.1 & \bar{u}^{-5} &= 56.7 & r^5 &= 0.99 & v_8^5 &= 32.6 \\
\bar{u}_{10} &= 29.2 & \bar{u}^3 &= 74.7 & \bar{u}^{-3} &= 52.0 & r^3 &= 1.43 & v_{10}^3 &= 42
\end{aligned}$$

The averages and the bounds for outliers are computed

$$\begin{aligned}
(\bar{v}_p)_{c_p \in C} &= (65.2 & 63 & 55.8 & 53.8 & 61.2 & 62.2 & 64.1 & 32.9 & 65.4 & 31.8) \\
(0.8\bar{v}_p)_{c_p \in C} &= (52.2 & 50.4 & 44.6 & 43 & 49 & 49.8 & 51.2 & 26.3 & 52.3 & 25.4) \\
(1.2\bar{v}_p)_{c_p \in C} &= (78.2 & 75.5 & 67 & 64.6 & 73.4 & 74.6 & 76.9 & 39.5 & 78.5 & 38.2)
\end{aligned}$$

This gives the final averages

$$(\bar{w}_p)_{c_p \in C} = (\mathbf{67} \quad \mathbf{62.1} \quad 54.5 \quad 53.1 \quad 61.3 \quad 62 \quad \mathbf{66.3} \quad 30.2 \quad \mathbf{66.2} \quad 33.7)$$

We have  $\mathcal{W}_4^*(\check{B}_C^J) = \{c_1, c_7, c_9, c_2\}$

Now consider the profile  $\check{B}_C^{J\setminus 3} = (S_C^{J\setminus 3}, \check{U}_C^{J\setminus 3})$  obtained from  $\check{B}_C^J$  when  $j^3$  does not take part to the jury. The new grade profile becomes

$$\check{U}_C^{J\setminus 3} = \begin{pmatrix} 90 & 85 & 60 & 40 & 64 & 47 & 75 & 15 & 95 & 30 \\ 95 & 46 & 50 & 58 & 60 & 61 & 0 & 20 & 32 & 10 \\ 20 & 50 & 44 & 48 & 45 & 85 & 22 & 23 & 64 & 40 \\ 47 & 59 & 51 & 52 & 60 & 42 & 95 & 0 & 62 & 37 \end{pmatrix}$$

The former students' averages and the teachers ratios permit to complete all candidates grades

$$\begin{aligned}
\bar{u}_7 &= 64 & \bar{u}^2 &= 48 & \bar{u}^{-2} &= 53.4 & r^2 &= 0.9 & v_7^2 &= 57.5 \\
\bar{u}_8 &= 19.3 & \bar{u}^5 &= 56.1 & \bar{u}^{-5} &= 50.7 & r^5 &= 1.1 & v_8^5 &= 21.4
\end{aligned}$$

The averages and the bounds for outliers are computed

$$\begin{aligned}
(\bar{v}_p)_{c_p \in C} &= (63 & 60 & 51.3 & 49.5 & 57.3 & 58.8 & 62.4 & 19.8 & 63.2 & 29.3) \\
(0.8\bar{v}_p)_{c_p \in C} &= (50.4 & 48 & 41 & 39.6 & 45.8 & 47 & 49.9 & 15.9 & 50.6 & 23.4) \\
(1.2\bar{v}_p)_{c_p \in C} &= (75.6 & 72 & 61.5 & 59.4 & 68.7 & 70.5 & 74.8 & 23.8 & 75.9 & 35.1)
\end{aligned}$$

This gives the final averages

$$(\bar{w}_p)_{c_p \in C} = (\mathbf{63} \quad 57.3 \quad 51.3 \quad 49.5 \quad \mathbf{57.5} \quad 56.4 \quad \mathbf{64.3} \quad 20.1 \quad \mathbf{63.1} \quad 30.9)$$

We have  $W_4^*(\check{B}_C^{J\setminus 3}) = \{c_7, c_9, c_1, c_5\}$ .

Therefore starting from competition  $\check{B}_C^{J\setminus 3}$  the deletion of judge  $j^3$ , whose highest grade is for candidate  $c_5$ , leads to the selection of candidate  $c_5$ .

(ii) Starting from profile  $\check{\mathbb{B}}_C^{\wedge 3}$  the addition of judge  $j^3$ , whose highest grade is for candidate  $c_5$ , leads to the non selection of candidate  $c_5$ .

(iii) Consider a profile  $\check{\mathbb{B}}_C^J = (\mathcal{S}_C^J, \check{\mathbb{U}}_C^J)$  with 10 candidates,  $C = \{c_1, \dots, c_{10}\}$  and 5 judges,  $J = \{j^1, \dots, j^5\}$ . Four candidates will be selected for the next round. Once again  $\mathcal{S}_C^J$  is identical to Example 1 and

$$\check{\mathbb{U}}_C^J = \begin{pmatrix} 90 & 85 & 60 & 40 & 64 & 47 & 75 & 15 & 95 & 30 \\ 95 & 46 & 50 & 58 & 60 & 61 & 0 & 20 & 32 & 10 \\ 74 & 75 & 74 & 0 & 79 & 76 & 74 & 74 & 74 & 0 \\ 44 & 44 & 44 & 48 & 43 & 45 & 44 & 45 & 64 & 46 \\ 47 & 59 & 51 & 52 & 60 & 42 & 95 & 0 & 62 & 37 \end{pmatrix}$$

The former students' averages and the teachers ratios permit to complete all candidates grades

$$\begin{array}{lllll} \bar{u}_4 = 49.5 & \bar{u}^3 = 75 & \bar{u}^{-3} = 52.7 & r^3 = 1.42 & v_4^3 = 70,4 \\ \bar{u}_7 = 72 & \bar{u}^2 = 48 & \bar{u}^{-2} = 59.5 & r^2 = 0.81 & v_7^2 = 58.1 \\ \bar{u}_8 = 38.5 & \bar{u}^5 = 56.1 & \bar{u}^{-5} = 57.4 & r^5 = 0.98 & v_8^5 = 37.6 \\ \bar{u}_{10} = 30.7 & \bar{u}^3 = 75 & \bar{u}^{-3} = 52.7 & r^3 = 1.42 & v_{10}^3 = 43.7 \end{array}$$

The averages and the bounds for outliers are computed

$$\begin{array}{ll} (\bar{v}_p)_{c_p \in C} = & (70 \quad 61.8 \quad 55.8 \quad 53.7 \quad 61.2 \quad 54.2 \quad 69.2 \quad 38.3 \quad 65.4 \quad 33.3) \\ (0.8\bar{v}_p)_{c_p \in C} = & (56 \quad 49.4 \quad 44.6 \quad 42.9 \quad 49 \quad 43.4 \quad 55.4 \quad 30.7 \quad 57.3 \quad 26.7) \\ (1.2\bar{v}_p)_{c_p \in C} = & (84 \quad 74.2 \quad 67 \quad 64.4 \quad 73.4 \quad 65 \quad 83.1 \quad 46 \quad 78.5 \quad 40) \end{array}$$

This gives the final averages

$$(\bar{w}_p)_{c_p \in C} = (70.8 \quad 61.2 \quad 54.5 \quad 53.1 \quad 61.3 \quad 52.3 \quad 69.1 \quad 38 \quad 66.2 \quad 34.7)$$

We have  $W_4^*(\check{\mathbb{B}}_C^J) = \{c_1, c_7, c_9, c_5\}$

Now consider the profile  $\check{\mathbb{B}}_C^{\wedge 4} = (\mathcal{S}_C^{\wedge 4}, \check{\mathbb{U}}_C^{\wedge 4})$  obtained from  $\check{\mathbb{B}}_C^J$  when  $j^4$  does not take part to the jury. The new grade profile is

$$\check{\mathbb{U}}_C^{\wedge 4} = \begin{pmatrix} 90 & 85 & 60 & 40 & 64 & 47 & 75 & 15 & 95 & 30 \\ 95 & 46 & 50 & 58 & 60 & 61 & 0 & 20 & 32 & 10 \\ 74 & 75 & 74 & 0 & 79 & 76 & 74 & 74 & 74 & 0 \\ 47 & 59 & 51 & 52 & 60 & 42 & 95 & 0 & 62 & 37 \end{pmatrix}$$

The former students' averages and the teachers ratios permit to complete all candidates grades

$$\begin{array}{lllll} \bar{u}_4 = 50 & \bar{u}^3 = 75 & \bar{u}^{-3} = 54.7 & r^3 = 1.37 & v_4^3 = 68,5 \\ \bar{u}_7 = 81.3 & \bar{u}^2 = 48 & \bar{u}^{-2} = 63.7 & r^2 = 0.75 & v_7^2 = 61.3 \\ \bar{u}_8 = 36.3 & \bar{u}^5 = 56.1 & \bar{u}^{-5} = 61.0 & r^5 = 0.92 & v_8^5 = 33.4 \\ \bar{u}_{10} = 25.7 & \bar{u}^3 = 75 & \bar{u}^{-3} = 54.7 & r^3 = 1.37 & v_{10}^3 = 35.2 \end{array}$$

The averages and the bounds for outliers are computed

$$\begin{aligned} (\bar{v}_p)_{c_p \in C} &= (76.5 \quad 66.3 \quad 58.8 \quad 54.6 \quad 65.8 \quad 56.5 \quad 76.3 \quad 35.6 \quad 65.8 \quad 28) \\ (0.8\bar{v}_p)_{c_p \in C} &= (61.2 \quad 53 \quad 47 \quad 43.7 \quad 52.6 \quad 45.2 \quad 61 \quad 28.5 \quad 52.6 \quad 23.4) \\ (1.2\bar{v}_p)_{c_p \in C} &= (91.8 \quad 79.5 \quad 70.5 \quad 65.6 \quad 78.9 \quad 67.8 \quad 91.6 \quad 42.7 \quad 78.9 \quad 33.7) \end{aligned}$$

This gives the final averages

$$(\bar{w}_p)_{c_p \in C} = (79.3 \quad 66.6 \quad 57.9 \quad 54.8 \quad 65.7 \quad 55.3 \quad 75.5 \quad 33.3 \quad 66.9 \quad 29.9)$$

We obtain  $W_4^*(\mathbb{B}_C^{\wedge 4}) = \{c_1, c_7, c_9, c_2\}$ .

Therefore starting from profile  $\mathbb{B}_C^J$  the deletion of judge  $j^4$ , whose lowest grade is for candidate  $c_5$ , leads to the non selection of candidate  $c_5$ .

(iv) Starting from profile  $\mathbb{B}_C^{\wedge 4}$  the addition of judge  $j^4$ , whose lowest grade is for candidate  $c_5$ , leads to the selection of candidate  $c_5$ .  $\square$

### 3.5 Reinforcement property

The reinforcement property is also known as separability property or consistency property (see Smith [1973], Young [1974], Moulin [1988a]). In our context it can be stated as follows: consider two voting profiles with the same set of candidates and different sets of judges. If a candidate is selected in the two profiles, Reinforcement requires that the candidate is also selected in a new profile composed of the same set of candidates and the union of the sets of judges.

**Definition 9.** Let  $\mathbb{B}_C^J$  and  $\mathbb{B}_C^{J'}$  two voting profiles, with  $J \cap J' = \emptyset$ . A  $m$ -rule  $\mathcal{W}_m$  on  $\mathcal{B}$  satisfies Reinforcement if  $c_p \in \mathcal{W}_m(\mathbb{B}_C^J)$  and  $c_p \in \mathcal{W}_m(\mathbb{B}_C^{J'})$  imply  $c_p \in \mathcal{W}_m(\mathbb{B}_C^{J \cup J'})$ .

**Proposition 5.** The  $m$ -rule  $\mathcal{W}_m^*$  does not satisfy Reinforcement.

*Proof.* Consider a profile  $\mathbb{B}_C^J$  with 10 candidates,  $C = \{c_1, \dots, c_{10}\}$  and 5 judges,  $J' = \{j^6, \dots, j^{10}\}$ . Four candidates will be selected.  $\mathbb{S}_C^{J'}$  is identical to Example 1 and

$$\mathbb{W}_C^{J'} = \begin{pmatrix} 90 & 85 & 60 & 60 & 33 & 44 & 75 & 15 & 95 & 30 \\ 95 & 12 & 32 & 58 & 60 & 55 & 0 & 20 & 32 & 10 \\ 75 & 22 & 81 & 0 & 52 & 60 & 25 & 64 & 72 & 0 \\ 20 & 50 & 34 & 48 & 45 & 55 & 22 & 23 & 64 & 40 \\ 47 & 59 & 51 & 52 & 60 & 42 & 95 & 0 & 62 & 37 \end{pmatrix}$$

The former students' averages and the teachers ratios permit to complete all candidates grades

$$\begin{aligned} \bar{u}_4 &= 54.5 & \bar{u}^8 &= 56.3 & \bar{u}^{-8} &= 49.1 & r^8 &= 1.15 & v_4^8 &= 62.6 \\ \bar{u}_7 &= 54.3 & \bar{u}^7 &= 41.6 & \bar{u}^{-7} &= 52.8 & r^7 &= 0.79 & v_7^7 &= 42.7 \\ \bar{u}_8 &= 30.5 & \bar{u}^{10} &= 56.1 & \bar{u}^{-10} &= 49.2 & r^{10} &= 0.75 & v_8^{10} &= 34.8 \\ \bar{u}_{10} &= 29.3 & \bar{u}^8 &= 56.3 & \bar{u}^{-8} &= 49.1 & r^8 &= 1.15 & v_{10}^8 &= 33.6 \end{aligned}$$

The averages and the bounds for outliers are computed

$$\begin{aligned}(\bar{v}_p)_{c_p \in C} &= ( 65.4 \quad 45.6 \quad 51.6 \quad 56.1 \quad 50 \quad 51.2 \quad 51.9 \quad 31.4 \quad 65 \quad 30.1 ) \\(0.8\bar{v}_p)_{c_p \in C} &= ( 52.3 \quad 36.5 \quad 41.3 \quad 44.9 \quad 40 \quad 41 \quad 41.5 \quad 25.1 \quad 52 \quad 24.1 ) \\(1.2\bar{v}_p)_{c_p \in C} &= ( 78.5 \quad 54.7 \quad 62 \quad 67.3 \quad 60 \quad 61.4 \quad 62.3 \quad 37.6 \quad 78 \quad 36.1 )\end{aligned}$$

This gives the final averages

$$(\bar{w}_p)_{c_p \in C} = ( \mathbf{67.3} \quad 46.5 \quad 51.1 \quad \mathbf{56.1} \quad \mathbf{51.4} \quad 51.2 \quad 50.1 \quad 29.5 \quad \mathbf{65.6} \quad 32 )$$

and  $\mathcal{W}_4^*(\mathbb{B}'_C) = \{c_1, c_9, c_4, c_5\}$

Now consider a profile  $\mathbb{B}^{J \cup J'}$  where we combine profile  $\mathbb{B}'_C$  and profile  $\mathbb{B}^J_C$  defined in Example 1. The 10 candidates are the same in  $\mathbb{B}'_C$  and in  $\mathbb{B}^J_C$  and there are now 10 judges :  $j^1, j^2, j^3, j^4, j^5, j^6, j^7, j^8, j^9$  and  $j^{10}$ . The grades of the judges in this combined profile are similar to their respective grades in round  $\mathbb{B}'_C$  and  $\mathbb{B}^J_C$ . Likewise the former students/teachers matrix is a combination of  $\mathbb{S}^{J'}$  and  $\mathbb{S}^J$ .

$$\mathbb{U}^{J \cup J'}_C = \begin{pmatrix} 90 & 85 & 60 & 40 & 33 & 50 & 75 & 15 & 95 & 30 \\ 95 & 12 & 32 & 58 & 60 & 61 & 0 & 20 & 32 & 10 \\ 75 & 22 & 81 & 0 & 52 & 65 & 25 & 64 & 72 & 0 \\ 20 & 50 & 34 & 48 & 45 & 85 & 22 & 23 & 64 & 40 \\ 47 & 59 & 51 & 52 & 60 & 42 & 95 & 0 & 62 & 37 \\ 90 & 85 & 60 & 60 & 33 & 44 & 75 & 15 & 95 & 30 \\ 95 & 12 & 32 & 58 & 60 & 55 & 0 & 20 & 32 & 10 \\ 75 & 22 & 81 & 0 & 52 & 60 & 25 & 64 & 72 & 0 \\ 20 & 50 & 34 & 48 & 45 & 55 & 22 & 23 & 64 & 40 \\ 47 & 59 & 51 & 52 & 60 & 42 & 95 & 0 & 62 & 37 \end{pmatrix}$$

The former students' averages and the teachers ratios permit to complete all candidates grades

$$\begin{aligned}\bar{u}_4 &= 52 & \bar{u}^3 &= 57 & \bar{u}^{-3} &= 50.2 & r^3 &= 1.14 & v^3_4 &= 59.1 \\ \bar{u}_7 &= 54.3 & \bar{u}^2 &= 42.2 & \bar{u}^{-2} &= 51.8 & r^2 &= 0.81 & v^2_7 &= 44.2 \\ \bar{u}_8 &= 30.5 & \bar{u}^5 &= 56.1 & \bar{u}^{-5} &= 50.3 & r^5 &= 1.12 & v^5_8 &= 34 \\ \bar{u}_{10} &= 29.3 & \bar{u}^3 &= 57 & \bar{u}^{-3} &= 50.2 & r^3 &= 1.14 & v^3_{10} &= 33.2 \\ \bar{u}_4 &= 52 & \bar{u}^8 &= 56.4 & \bar{u}^{-8} &= 50.2 & r^8 &= 1.12 & v^8_4 &= 58.3 \\ \bar{u}_7 &= 54.3 & \bar{u}^7 &= 41.6 & \bar{u}^{-7} &= 51.9 & r^7 &= 0.80 & v^7_7 &= 43.4 \\ \bar{u}_8 &= 30.5 & \bar{u}^{10} &= 56.1 & \bar{u}^{-10} &= 50.3 & r^{10} &= 1.12 & v^{10}_8 &= 34 \\ \bar{u}_{10} &= 29.3 & \bar{u}^8 &= 56.4 & \bar{u}^{-8} &= 50.2 & r^8 &= 1.12 & v^8_{10} &= 32.8\end{aligned}$$

The averages and the bounds for outliers are computed

$$\begin{aligned}(\bar{v}_p)_{c_p \in C} &= ( 65.4 \quad 45.6 \quad 51.6 \quad 53.3 \quad 50 \quad 55.9 \quad 52.2 \quad 31.2 \quad 65 \quad 30 ) \\(0.8\bar{v}_p)_{c_p \in C} &= ( 52.3 \quad 36.5 \quad 41.3 \quad 42.7 \quad 40 \quad 44.7 \quad 41.7 \quad 25 \quad 52 \quad 24 ) \\(1.2\bar{v}_p)_{c_p \in C} &= ( 78.5 \quad 54.7 \quad 61.9 \quad 64 \quad 60 \quad 67.1 \quad 62.6 \quad 37.4 \quad 78 \quad 36 )\end{aligned}$$

This gives the final averages

$$(\bar{w}_p(\mathbb{B}_C^{J \cup J'}))_{c_p \in C} = (67.3 \quad 46.5 \quad 51.1 \quad 53.6 \quad 51.4 \quad 54.7 \quad 50.5 \quad 29.3 \quad 65.6 \quad 31.8)$$

and  $\mathcal{W}_4^*(\mathbb{B}_C^{J \cup J'}) = \{c_1, c_9, c_6, c_4\}$ .

To conclude we have  $c_5 \in \mathcal{W}_4^*(\mathbb{B}_C^J)$ ,  $c_5 \in \mathcal{W}_4^*(\mathbb{B}_C^{J'})$  and  $c_5 \notin \mathcal{W}_4^*(\mathbb{B}_C^{J \cup J'})$ .  $\square$

### 3.6 Monotonicity property

The monotonicity property (Smith [1973]) requires that if some judge increases the grade of a selected candidate (with no other modification), this candidate remains selected.

**Definition 10.** Consider a candidate  $c_p$ , a judge  $j^i$  and two voting profiles  $\mathbb{B}_C^J = (\mathbb{S}_C^J, \mathbb{U}_C^J)$  and  $\tilde{\mathbb{B}}_C^J = (\mathbb{S}_C^J, \tilde{\mathbb{U}}_C^J)$  with  $\tilde{u}_i^p > u_i^p$ ,  $\tilde{u}_i^q = u_i^q$  for all  $q \neq p$ ,  $\tilde{u}_j^p = u_j^p$  for all  $j \neq i$ ,  $\tilde{u}_j^q = u_j^q$  for all  $j \neq i$  and  $q \neq p$ .

A rule  $\mathcal{W}_m$  on  $\mathcal{B}$  satisfies Monotonicity if  $c_p \in \mathcal{W}_m(\mathbb{B}_C^J)$  implies that  $c_p \in \mathcal{W}_m(\tilde{\mathbb{B}}_C^J)$ .

**Proposition 6.** The  $m$ -rule  $\mathcal{W}_m^*$  does not satisfy monotonicity.

*Proof.* Consider a profile  $\mathbb{B}_C^J$  with 2 candidates,  $C = \{c_1, c_2\}$  and 4 judges,  $J = \{j^1, \dots, j^4\}$ . One candidate will be selected.

$$\mathbb{S}_C^J = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \text{ and } \mathbb{U}_C^J = \begin{pmatrix} 0 & 50 \\ 39 & 51 \\ 61 & 50 \\ 50 & 50 \end{pmatrix}$$

The former students' averages and the teachers ratios permit to complete all candidates grades

$$\bar{u}_1 = 50 \quad \bar{u}^1 = 50 \quad \bar{u}^{-1} = 50.2 \quad r^1 = 0.99 \quad v_1^1 = 49.8$$

The averages and the bounds for outliers are then

$$\begin{aligned} (\bar{v}_p)_{c_p \in C} &= (50 \quad 50.3) \\ (0.8\bar{v}_p)_{c_p \in C} &= (40 \quad 40.2) \\ (1.2\bar{v}_p)_{c_p \in C} &= (60 \quad 60.3) \end{aligned}$$

This gives the final averages

$$(\bar{w}_p)_{c_p \in C} = (49.9 \quad 50.3)$$

and  $\mathcal{W}_1^*(\mathbb{B}_C^J) = \{c_2\}$

Consider now a profile  $\tilde{\mathbb{B}}_C^J$  with 2 candidates,  $C = \{c_1, c_2\}$  and 4 judges,  $J = \{j^1, \dots, j^4\}$ . One candidate will be selected.  $\tilde{\mathbb{S}}_C^J$  is identical to  $\mathbb{S}_C^J$

$$\tilde{\mathbb{U}}_C^J = \begin{pmatrix} 0 & 53 \\ 39 & 51 \\ 61 & 50 \\ 50 & 50 \end{pmatrix}$$

The former students' averages and the teachers ratios permit to complete all candidates grades

$$\bar{u}_1 = 53 \quad \bar{u}^1 = 50.2 \quad \bar{u}^{-1} = 50.2 \quad r^1 = 1.06 \quad v_1^1 = 52.8$$

We have

$$\begin{aligned} (\bar{v}_p)_{c_p \in C} &= (50.7 \quad 51) \\ (0.8\bar{v}_p)_{c_p \in C} &= (40.6 \quad 40.8) \\ (1.2\bar{v}_p)_{c_p \in C} &= (60.8 \quad 61.2) \end{aligned}$$

This gives the final averages

$$(\bar{w}_p)_{c_p \in C} = (51.1 \quad 51)$$

and  $\mathcal{W}_1^*(\tilde{\mathbb{B}}_C^J) = \{c_1\}$

We have  $c_2 \in \mathcal{W}_1^*(\mathbb{B}_C^J)$  and  $c_2 \notin \mathcal{W}_1^*(\tilde{\mathbb{B}}_C^J)$  where  $j^1$  increases the grade of  $c_2$ , so the monotonicity is not respected. □

Although monotonicity is not fulfilled, a candidate  $c_p$  will be non selected after the increase of his grade only in a restrictive scenario, that is when the average of  $c_q$ , the last non selected candidate, is closed to those of  $c_p$ , and when  $c_q$  is a former student of the judge who increases the grade of  $c_p$ .

Let a voting profile  $\mathbb{B}_C^J$ , for a candidate  $c_p$ , we set  $n_p^L(\mathbb{B}_C^J)$  (resp.  $n_p^H(\mathbb{B}_C^J)$ ) as the number of grades lower (resp. higher) than  $0.8\bar{v}_p$  ( $1.2\bar{v}_p$ ). For  $\mathbb{B}_C^J$  and  $\tilde{\mathbb{B}}_C^J$ , we denote  $\Delta n_p^L = n_p^L(\tilde{\mathbb{B}}_C^J) - n_p^L(\mathbb{B}_C^J)$  and  $\Delta n_p^H = n_p^H(\tilde{\mathbb{B}}_C^J) - n_p^H(\mathbb{B}_C^J)$ .

**Proposition 7.** Let  $c_p$  and  $c_q$  two candidates, a judge  $j^1$  and two profiles  $\mathbb{B}_C^J = (\mathcal{S}_C^J, \mathcal{U}_C^J)$  and  $\tilde{\mathbb{B}}_C^J = (\mathcal{S}_C^J, \tilde{\mathcal{U}}_C^J)$  with  $\bar{u}_1^p = u_1^p + x$ ,  $\bar{u}_1^k = u_1^k$  for all  $k \neq p$ ,  $\bar{u}_i^p = u_i^p$  for all  $i \neq 1$  and all  $p$ . Assume that  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$  and that  $c_q \notin \mathcal{W}_m^*(\mathbb{B}_C^J)$  with  $\bar{w}_p(\mathbb{B}_C^J) - \bar{w}_q(\mathbb{B}_C^J) = \epsilon$ .

The  $m$ -rule  $\mathcal{W}_m^*$  does not satisfy Monotonicity if and only if  $c_q$  is a former student of  $j^1$  and  $\epsilon < \beta$ , with

$$\beta = \begin{cases} \frac{0.2(\Delta n_q^H(\mathbb{B}_C^J) - \Delta n_q^L(\mathbb{B}_C^J))\bar{v}_q(\mathbb{B}_C^J)}{n^J} + \frac{(n^J - 0.2n_q^L(\mathbb{B}_C^J) + 0.2n_q^H(\mathbb{B}_C^J))}{n^J} \cdot \frac{\bar{u}_q(\mathbb{B}_C^J)}{n^J} \cdot \frac{x}{\bar{u}^{-1}(\mathbb{B}_C^J)} \\ - \frac{0.2(\Delta n_p^H - \Delta n_p^L)\bar{v}_p(\mathbb{B}_C^J)}{n^J} - \frac{(n^J - 0.2n_p^L(\mathbb{B}_C^J) + 0.2n_p^H(\mathbb{B}_C^J))}{n^J} \cdot \frac{x}{n^J} \end{cases}$$

*Proof.* Consider a candidate  $c_p$ , a judge  $j^1$  and two voting profiles  $\mathbb{B}_C^J = (\mathcal{S}_C^J, \mathcal{U}_C^J)$  and  $\tilde{\mathbb{B}}_C^J = (\mathcal{S}_C^J, \tilde{\mathcal{U}}_C^J)$  with  $\bar{u}_1^p = u_1^p + x$ ,  $\bar{u}_1^q = u_1^q$  for all  $q \neq p$ ,  $\bar{u}_i^p = u_i^p$  for all  $i \neq 1$  and all  $p$ . Assume that  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$  and that a candidate  $c_q \notin \mathcal{W}_m^*(\mathbb{B}_C^J)$  with  $\bar{w}_p(\mathbb{B}_C^J) - \bar{w}_q(\mathbb{B}_C^J) = \epsilon$ .  $c_q$  can be seen as the candidate with the highest grade of the non selected candidates. Assume that candidates different from  $c_p$  and  $c_q$  are not former students of a judges, i.e.  $s_r^i = 1$  for all  $r \neq p, q$  and all  $i \in J$ . For candidates  $c_p$  and  $c_q$  we must consider differents cases.

I)  $c_p$  and  $c_q$  are not former students, which implies that both for  $\mathbb{B}_C^J$  and  $\tilde{\mathbb{B}}_C^J$  we have  $v_p^i = u_p^i$  for all  $i$  in  $J$  and all  $c_p$  in  $C$ . Note that for  $c_q$ , the presence of outlier grades or not have no incidence between  $\mathbb{B}_C^J$  and  $\tilde{\mathbb{B}}_C^J$ , then  $\bar{w}_q(\mathbb{B}_C^J) = \bar{w}_q(\tilde{\mathbb{B}}_C^J)$ . We must distinguish between the case with outlier grades and the case without for candidate  $c_p$ .

I.i) there is no outlier grades for  $c_p$  both for  $\mathbb{B}_C^J$  and  $\tilde{\mathbb{B}}_C^J$ , i.e.  $w_p^i = v_p^i = u_p^i$ . This implies that  $\bar{w}_p(\tilde{\mathbb{B}}_C^J) > \bar{w}_p(\mathbb{B}_C^J) > \bar{w}_q(\mathbb{B}_C^J) = \bar{w}_q(\tilde{\mathbb{B}}_C^J)$ , then  $c_p \in \mathcal{W}_m^*(\tilde{\mathbb{B}}_C^J)$ .



I.ii) there is outlier grades for  $c_p$  in  $\mathbb{B}_C^J$ . If  $v_p^i(\mathbb{B}_C^J) < 0.8\bar{v}_p(\mathbb{B}_C^J)$  for some  $i \in J$ , then  $w_p^i(\mathbb{B}_C^J) = 0.8\bar{v}_p(\mathbb{B}_C^J)$  for some  $i \in J$  and  $0.8\bar{v}_p(\mathbb{B}_C^J) > 0.8\bar{v}_p(\mathbb{B}_C^J)$  for some  $i \in J$ . Likewise if  $v_p^i(\mathbb{B}_C^J) > 1.2\bar{v}_p(\mathbb{B}_C^J)$  for some  $i \in J$ , then  $w_p^i(\mathbb{B}_C^J) = 1.2\bar{v}_p(\mathbb{B}_C^J)$  for some  $i \in J$  and  $1.2\bar{v}_p(\mathbb{B}_C^J) > 1.2\bar{v}_p(\mathbb{B}_C^J)$  for some  $i \in J$ . Therefore this implies that  $\bar{w}_p(\mathbb{B}_C^J) > \bar{w}_p(\mathbb{B}_C^J) > \bar{w}_q(\mathbb{B}_C^J) = \bar{w}_q(\mathbb{B}_C^J)$ , and then  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

II)  $c_p$  is a former students of judges different from  $j^1$  and there is no former students for  $c_q$ . WLOG, we assume that he is only the former students of  $j^2$  which implies that both for  $\mathbb{B}_C^J$  and  $\mathbb{B}_C^J$ ,  $v_p^2 \neq u_p^2 = 0$  and  $v_p^i = u_p^i$  for all  $i \neq 2$ . The increase of  $c_p$ 's grade by  $j^1$  implies that  $\bar{u}_p(\mathbb{B}_C^J) > \bar{u}_p(\mathbb{B}_C^J)$ ,  $\bar{u}^1(\mathbb{B}_C^J) > \bar{u}^1(\mathbb{B}_C^J)$  and that  $\bar{u}^{-i}(\mathbb{B}_C^J) > \bar{u}^{-i}(\mathbb{B}_C^J)$  for all  $i \neq 1$ . Therefore  $r^1(\mathbb{B}_C^J) = \frac{\bar{u}^1(\mathbb{B}_C^J)}{\bar{u}^{-1}(\mathbb{B}_C^J)} > r^1(\mathbb{B}_C^J)$  and  $r^2(\mathbb{B}_C^J) = \frac{\bar{u}^2(\mathbb{B}_C^J)}{\bar{u}^{-2}(\mathbb{B}_C^J)} < r^2(\mathbb{B}_C^J)$ .

We study now the sign of  $v_p^2(\mathbb{B}_C^J) - v_p^2(\mathbb{B}_C^J)$ . Since  $v_p^2(\mathbb{B}_C^J) = \bar{u}_p(\mathbb{B}_C^J) * r^2(\mathbb{B}_C^J) = (\bar{u}_p(\mathbb{B}_C^J) + \frac{x}{n^J-1}) * (\frac{\bar{u}^2(\mathbb{B}_C^J)}{\bar{u}^{-2}(\mathbb{B}_C^J) + \frac{x}{n^J-1}})$ , we have  $v_p^2(\mathbb{B}_C^J) - v_p^2(\mathbb{B}_C^J) = r^2(\mathbb{B}_C^J) * \frac{x}{n^J-1} * \frac{n_C * \bar{u}^{-2}(\mathbb{B}_C^J) - \bar{u}_p(\mathbb{B}_C^J)}{n_C * \bar{u}^{-2}(\mathbb{B}_C^J) + \frac{x}{n^J-1}}$ . Let  $\min \bar{u}^{-2}(\mathbb{B}_C^J)$  the possible minimum of  $\bar{u}^{-2}(\mathbb{B}_C^J)$  and  $\min \bar{u}^i(\mathbb{B}_C^J)$  the possible minimum of  $\bar{u}^i(\mathbb{B}_C^J)$  for all  $i \neq 2$ . Assume that  $\min \bar{u}^i(\mathbb{B}_C^J) = \frac{u_p^i(\mathbb{B}_C^J)}{n_C}$  for all  $i \neq 2$ . Then  $\min \bar{u}^{-2}(\mathbb{B}_C^J) = \frac{1}{n^J-1} \sum_{\substack{j^i \in J \\ i \neq 2}} \min \bar{u}^i(\mathbb{B}_C^J) = \frac{1}{n^J-1} \sum_{\substack{j^i \in J \\ i \neq 2}} \frac{u_p^i(\mathbb{B}_C^J)}{n_C}$ . Therefore  $n_C * \min \bar{u}^{-2}(\mathbb{B}_C^J) = \frac{1}{n^J-1} \sum_{\substack{j^i \in J \\ i \neq 2}} u_p^i(\mathbb{B}_C^J) = \bar{u}_p(\mathbb{B}_C^J)$  which implies that  $v_p^2(\mathbb{B}_C^J) - v_p^2(\mathbb{B}_C^J) \geq 0$ .

Therefore  $\bar{v}_p(\mathbb{B}_C^J) \geq \bar{v}_p(\mathbb{B}_C^J)$  and whatever there is outlier grades or not (see Case I), we have  $\bar{w}_p(\mathbb{B}_C^J) > \bar{w}_p(\mathbb{B}_C^J) > \bar{w}_q(\mathbb{B}_C^J) = \bar{w}_q(\mathbb{B}_C^J)$ , and then  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

III)  $c_q$  is a former students of judges different from judge 1. WLOG, we assume that he is only the former students of  $j^2$  which implies that both for  $\mathbb{B}_C^J$  and  $\mathbb{B}_C^J$ ,  $v_q^2 \neq u_q^2 = 0$  and  $v_q^i = u_q^i$  for all  $i \neq 2$ . As already mentioned in Case II, if judge 1 increases the grade of candidate  $c_p$ ,  $r^2(\mathbb{B}_C^J) < r^2(\mathbb{B}_C^J)$  and then  $v_q^2(\mathbb{B}_C^J) < v_q^2(\mathbb{B}_C^J)$ . Therefore the grade of  $c_q$  does not increase. And whatever  $c_p$  is a former student or not (see Case I and Case II) we deduce that  $\bar{w}_p(\mathbb{B}_C^J) > \bar{w}_p(\mathbb{B}_C^J) > \bar{w}_q(\mathbb{B}_C^J) > \bar{w}_q(\mathbb{B}_C^J)$ , and then  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ .

IV)  $c_q$  is a former students of judge 1. WLOG assume he is the former student of only judge 1 which implies that both for  $\mathbb{B}_C^J$  and  $\mathbb{B}_C^J$ ,  $v_q^1 \neq u_q^1 = 0$  and  $v_q^i = u_q^i$  for all  $j^i \neq j^1$ . As already mentioned if judge 1 increases the grade of  $c_p$  this implies that  $\bar{v}_p(\mathbb{B}_C^J) = \bar{v}_p(\mathbb{B}_C^J) + \frac{x}{n^J}$ . And whatever  $c_p$  is a former student or not we have  $\bar{w}_p(\mathbb{B}_C^J) = \frac{(n^J - n_p^L(\mathbb{B}_C^J) - n_p^H(\mathbb{B}_C^J))\bar{v}_p(\mathbb{B}_C^J) + 0.8n_p^L(\mathbb{B}_C^J)\bar{v}_p(\mathbb{B}_C^J) + 1.2n_p^H(\mathbb{B}_C^J)\bar{v}_p(\mathbb{B}_C^J)}{n^J} = \bar{w}_p(\mathbb{B}_C^J) + \frac{0.2(\Delta n_p^H - \Delta n_p^L)\bar{v}_p(\mathbb{B}_C^J)}{n^J} + \frac{(n^J - 0.2n_p^L(\mathbb{B}_C^J) + 0.2n_p^H(\mathbb{B}_C^J))\frac{x}{n^J}}{n^J}$ . For  $c_q$  we have  $\bar{v}_q(\mathbb{B}_C^J) = \frac{\bar{v}_q(\mathbb{B}_C^J)n^J + \bar{u}_q(\mathbb{B}_C^J)\frac{\bar{u}^1(\mathbb{B}_C^J) - \bar{u}^1(\mathbb{B}_C^J)}{\bar{u}^{-1}(\mathbb{B}_C^J)}}{n^J} = \bar{v}_q(\mathbb{B}_C^J) + \frac{\bar{u}_q(\mathbb{B}_C^J)}{n^J} \cdot \frac{\frac{x}{n^J}}{\bar{u}^{-1}(\mathbb{B}_C^J)}$ . And therefore  $\bar{w}_q(\mathbb{B}_C^J) = \frac{(n^J - n_q^L(\mathbb{B}_C^J) - n_q^H(\mathbb{B}_C^J))\bar{v}_q(\mathbb{B}_C^J) + n_q^L(\mathbb{B}_C^J)0.8\bar{v}_q(\mathbb{B}_C^J) + n_q^H(\mathbb{B}_C^J)1.2\bar{v}_q(\mathbb{B}_C^J)}{n^J} = \bar{w}_q(\mathbb{B}_C^J) + \frac{0.2(\Delta n_q^H - \Delta n_q^L)\bar{v}_q(\mathbb{B}_C^J)}{n^J} + \frac{(n^J - 0.2n_q^L(\mathbb{B}_C^J) + 0.2n_q^H(\mathbb{B}_C^J))\bar{u}_q(\mathbb{B}_C^J)}{n^J} \cdot \frac{\frac{x}{n^J}}{\bar{u}^{-1}(\mathbb{B}_C^J)}$ .

Assume first that  $\bar{w}_p(\mathbb{B}_C^J) - \bar{w}_q(\mathbb{B}_C^J) > \frac{0.2(\Delta n_q^H(\mathbb{B}_C^J) - \Delta n_q^L(\mathbb{B}_C^J))\bar{v}_q(\mathbb{B}_C^J)}{n^J} + \frac{(n^J - 0.2n_q^L(\mathbb{B}_C^J) + 0.2n_q^H(\mathbb{B}_C^J))\bar{u}_q(\mathbb{B}_C^J)}{n^J} \cdot \frac{\frac{x}{n^J}}{\bar{u}^{-1}(\mathbb{B}_C^J)} -$

$\frac{0.2(\Delta n_p^H - \Delta n_p^L) \bar{v}_p(\mathbb{B}_C^J)}{n^J} - \frac{(n^J - 0.2n_p^L(\mathbb{B}_C^J) + 0.2n_p^H(\mathbb{B}_C^J)) \frac{x}{n^J}}{n^J} = \beta$ . This implies that  $\bar{w}_p(\mathbb{B}_C^J) - \bar{w}_q(\mathbb{B}_C^J) > (\bar{w}_q(\mathbb{B}_C^J) - \bar{w}_q(\mathbb{B}_C^J)) - (\bar{w}_p(\mathbb{B}_C^J) - \bar{w}_p(\mathbb{B}_C^J))$  and then  $\bar{w}_p(\mathbb{B}_C^J) > \bar{w}_q(\mathbb{B}_C^J)$ , therefore  $c_p \in \mathcal{W}_m^*(\mathbb{B}_C^J)$ . Now consider the converse,  $\bar{w}_p(\mathbb{B}_C^J) - \bar{w}_q(\mathbb{B}_C^J) < \beta$ , this implies that  $\bar{w}_p(\mathbb{B}_C^J) < \bar{w}_q(\mathbb{B}_C^J)$  and then  $c_p \notin \mathcal{W}_m^*(\mathbb{B}_C^J)$  and monotonicity is not fulfilled when  $\epsilon < \beta$ . □

## 4 Conclusion

We can summarize the different results following a classification given by Felsenthal [2010]. He distinguishes two types of paradoxes: the "straightforward" ones and the "conditional" ones. Straightforward paradoxes are paradoxes where the relevant data (the number of judges, the number of candidates, the number of candidates that must be selected, the preference of every judges among the competing candidates, the amount of information judges have regarding all other judges preference,...) leads to a "surprising" and arguably undesirable outcome if the property is not satisfied. The conditional paradoxes are those where changing one relevant datum while holding constant all other relevant data leads to a undesirable outcome. In Table 1, a "√" means that the property is satisfied by  $\mathcal{W}_m^*$ , while "×" means that the property is not satisfied. A "×<sup>1</sup>" means that the property is not satisfied due to the attribution of grade for candidates that have teachers within the jury and a "×<sup>1,2</sup>" means that the attribution of the missing grades and/or the outliers play a role in the violation of the property.

Table 1: Properties of the  $m$ -rule  $\mathcal{W}_m$

Straightforward paradox			
Universal Domain	Unanimity	Dictator	Majority criterion
√	√	√	×
Conditional paradox			
Independence	Participation	Reinforcement	Monotonicity
× <sup>1</sup>	× <sup>1,2</sup>	×	× <sup>1</sup>

From this table we can conclude that the QEC  $m$ -rule violates many desirable properties. Among the "straightforward" paradox, the rule violates the majority criterion. This illustrates the tension between rankings and intensities of preferences: The criterion is based on the rankings while the rule makes averages of grades, and thus includes the intensities of the preferences. The rule also violates all conditional properties. One source is how a student with a former teacher within the jury obtains his evaluation. The evaluation depends on his average and on the teacher's average. Modifying the set of candidates or the set of judges modifies the teacher's average and thus the student's grade. This explains how the paradoxes may appear. The elimination of the outliers is the second source of the violation of the conditional properties. The elimination of the outliers has been introduced to reduce the possible manipulation of the notes by the jury. This is done at the cost of possible violation of conditional properties. Note that in the case of the monotonicity Proposition 7 puts a strict limit to this violation.

The violation of many properties raises the question of whether more classical rules of social choice may substitute the rule used in the QEC competition with similar results.

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