



**HAL**  
open science

## Modelling of the child's lower urinary system

Lisa Grandjean, Nadia Boudaoud, Guillaume Dolle, Stéphanie Salmon,  
Marie-Laurence Poli-Merol

► **To cite this version:**

Lisa Grandjean, Nadia Boudaoud, Guillaume Dolle, Stéphanie Salmon, Marie-Laurence Poli-Merol. Modelling of the child's lower urinary system. ENUMATH 2023, Sep 2023, Lisbonne, Portugal. hal-04350535

**HAL Id: hal-04350535**

**<https://hal.science/hal-04350535>**

Submitted on 18 Dec 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



# MODELING OF THE CHILD'S LOWER URINARY SYSTEM

Lisa Grandjean<sup>1</sup>, Nadia Boudaoud<sup>2</sup>, Guillaume Dollé<sup>1</sup>,

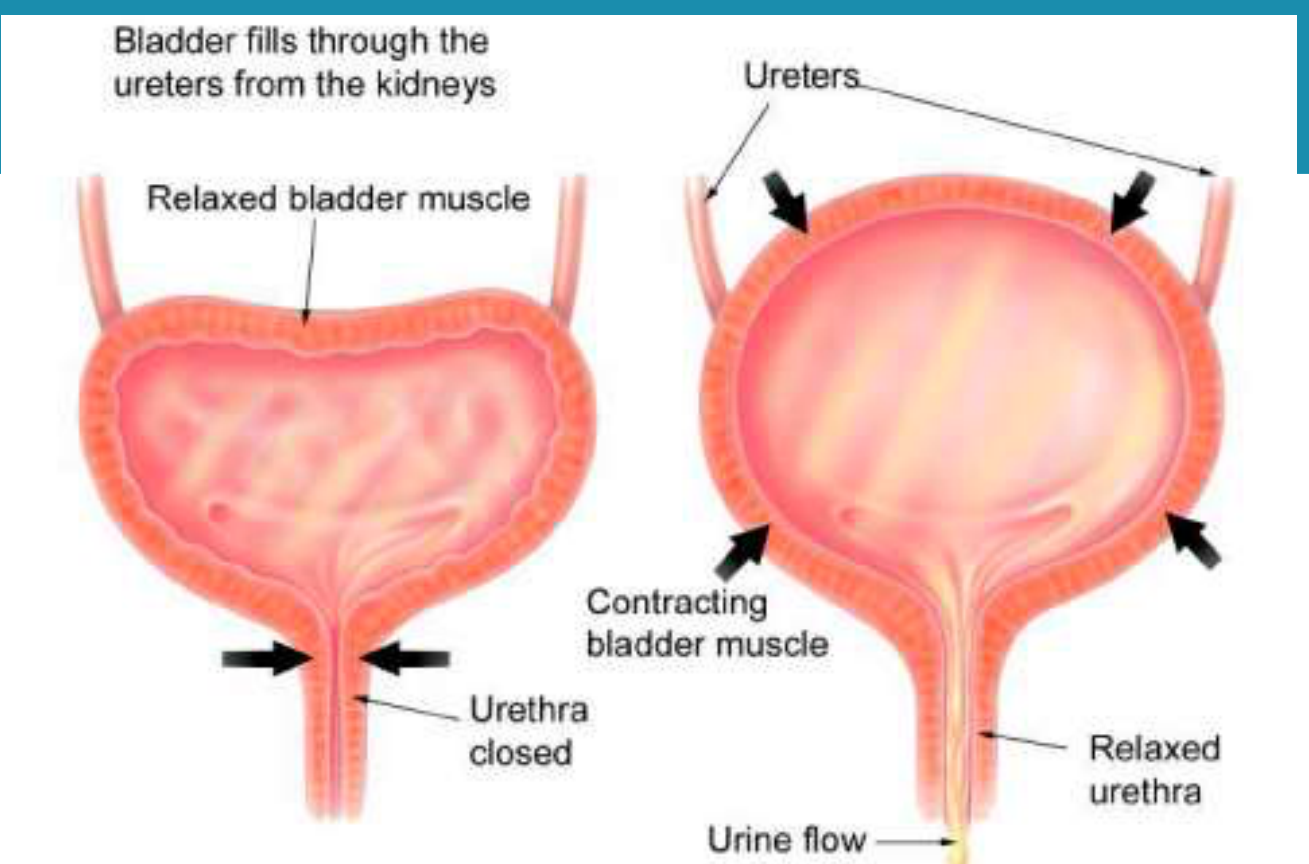
Stéphanie Salmon<sup>1</sup> and Marie-Laurence Poli-Merol<sup>2</sup>

<sup>1</sup>LMR UMR CNRS 9008 University of Reims, <sup>2</sup>CHU of Reims, pediatric surgery



## Motivation & Research Goals

The bladder has the function of receiving the urine produced by the kidneys, keep it and then evacuate. In order to study the mechanism/movement during **filling**<sup>(ii)</sup> and **voiding**<sup>(i)</sup>, we build with paediatric surgeons a simple **numerical model** of the 5-10 years old child's bladder based on **real data** such as volumes, urodynamic data or medical images like cystography.



### (i) ALE Navier-Stokes equations with free surface<sup>[2]</sup>

**Navier-Stokes (NS) equations in Arbitrary Lagrangian Eulerian (ALE) formulation**

$$\begin{cases} \frac{\partial \vec{u}}{\partial t} + (\vec{u} - \vec{c}) \cdot \nabla \vec{u} - \nu \Delta \vec{u} + \nabla p = 0 & \text{in } \Omega_t \\ \nabla \cdot \vec{u} = 0 & \text{in } \Omega_t, t \in [0, T_v], T_v < 30s \end{cases} \quad (1)$$

$\vec{u}, p$  : fluid velocity and pressure

$\vec{c}$  : domain velocity (harmonic extension)

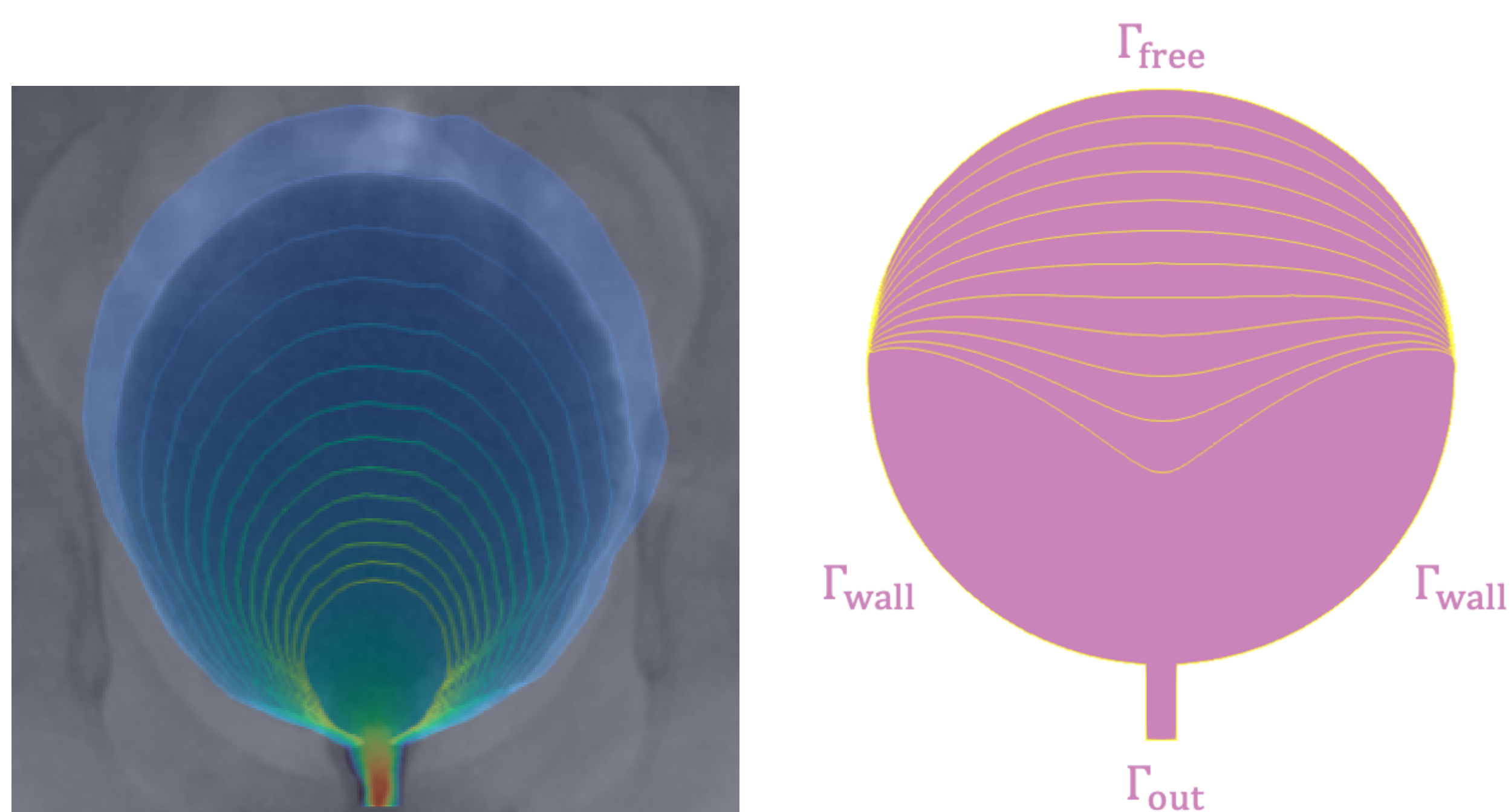
**Voiding** boundary conditions with free surface ( $\Gamma_t^{free}$ ) :

$$\begin{cases} \vec{u} = 0 & \text{on } \Gamma^{wall} \\ \vec{u} = \vec{u}_{out} & \text{on } \Gamma^{out} \\ \nu \frac{\partial \vec{u}}{\partial \vec{n}} - p \vec{n} = -p_e \vec{n} - \sigma(\kappa) \vec{n} & \text{on } \Gamma_t^{free}, t \in [0, T_v], T_v < 30s \end{cases}$$

$p_e$  : external (here abdominal) pressure

$\kappa, \sigma$  : curvature and surface tension coefficient of  $\Gamma_t^{free}$

Problem solved with FreeFem++<sup>[1]</sup>, using the FEM.



First steps of voiding based on medical images (left) and results with a simplified geometry

#### Model limits :

- Filling : preserve the waving shape and not inflate as expected.
- Fluid velocity drives the system.
- Detrusor action not taken into account.

### (ii) Fluid-Structure Interaction (FSI)

**FSI equations** • NS + ALE (1) for the fluid in  $\Omega_t^F$  ( $t \in [0, T_f], T_f \approx 3h$ )

- Despite large displacements, linear elasticity to test FSI

$$\rho^S \partial_{tt} \vec{d}^S - \nabla \cdot (\sigma^S(\vec{d}^S)) = f^S \quad \text{in } \Omega_t^S \quad (2)$$

- with the coupling conditions on the interface  $\Sigma_t$ :

$$\begin{cases} \vec{u}^F = \partial_t \vec{d}^S = \vec{u}^S & \text{on } \Sigma_t \\ \sigma^F(\vec{u}^F, p) \vec{n}^F = -\sigma^S(\vec{d}^S) \vec{n}^S & \text{on } \Sigma_t \end{cases} \quad (3)$$

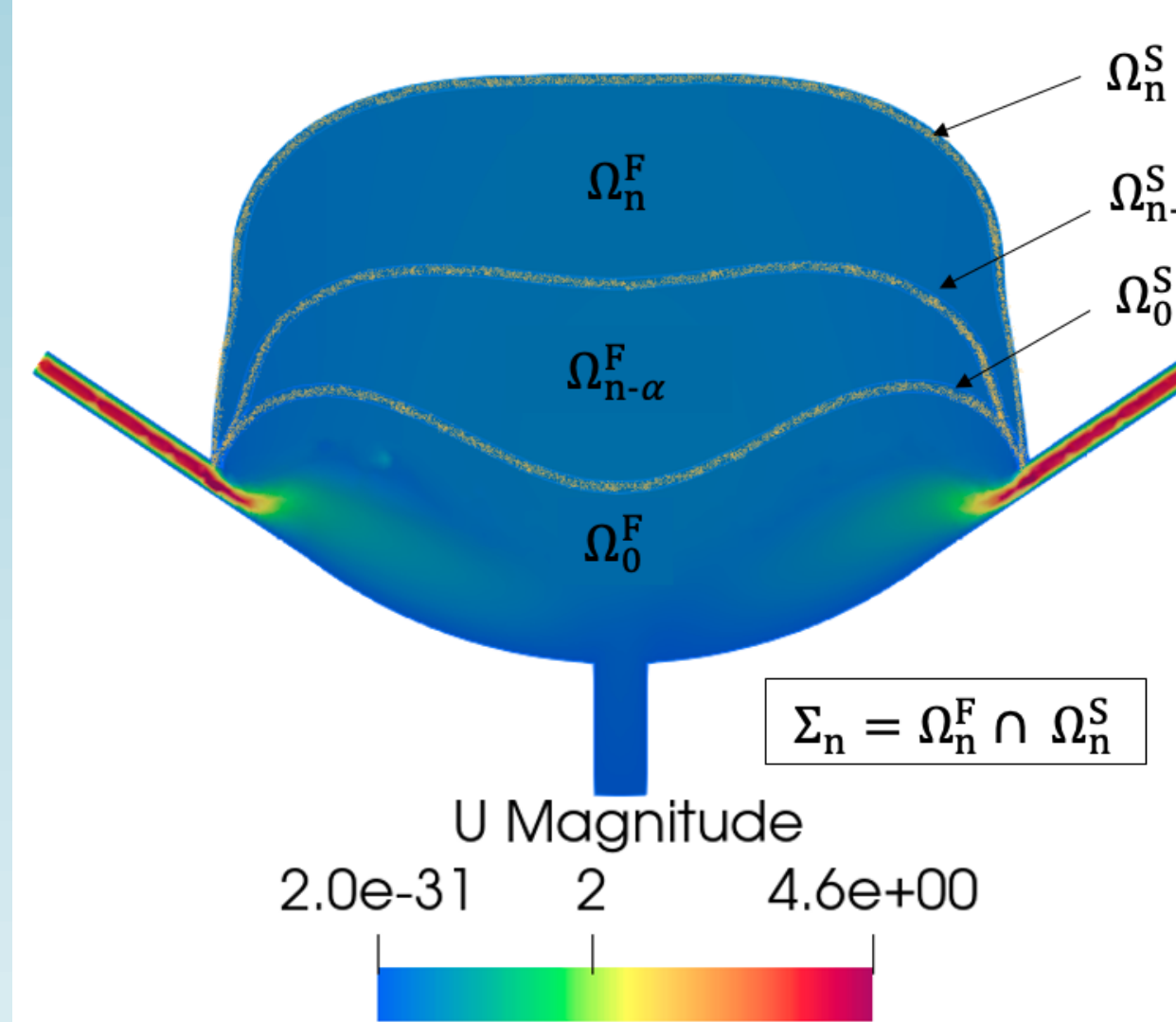
#### Monolithic formulation for FSI<sup>[3]</sup>

$$\begin{cases} \int_{\Omega_n^F} \rho^F \frac{u^{n+1}}{\Delta t} \cdot v dx + \int_{\Omega_n^F} \rho^F ((u^n - c^n) \cdot \nabla) u^{n+1} \cdot v dx \\ - \int_{\Omega_n^F} (\nabla \cdot v) p^{F,n+1} dx + \int_{\Omega_n^F} 2\mu^F \epsilon(u^{n+1}) : \epsilon(v) dx \\ + \int_{\Omega_n^F} (\nabla \cdot u^{n+1}) q dx + \int_{\Omega_n^S} \rho^S \frac{u^{n+1}}{\Delta t} \cdot v dx + \int_{\Omega_n^S} L(u^{n+1}) : \nabla v dx \\ = \int_{\Omega_n^F} \rho^F \frac{u^n}{\Delta t} \cdot v dx + \int_{\Omega_n^S} \rho^S \frac{u^n}{\Delta t} \cdot v dx + \int_{\Omega_n^{S,F}} \rho^{S,F} g \cdot v dx \end{cases} \quad (4)$$

with

$$L(u^{n+1}) = (\Delta t)(\lambda^S (\nabla \cdot u^{n+1}) I + \mu^S (\nabla u^{n+1} + (\nabla u^{n+1})^T)) + \lambda^S (\nabla \cdot d^n) I + \mu^S (\nabla d^n + (\nabla d^n)^T)$$

$\lambda^S, \mu^S$  : Lamé's coefficients



Filling of the bladder

**Algorithm** While  $V < Vmax$  do :

1. Solve the linear system 4
2. Compute the fluid mesh velocity  $\vec{c}^{n+1}$  by harmonic extension of  $\vec{u}^{n+1}$  on  $\Sigma_t$
3. Define the map by  $T_n(\hat{x}) = \hat{x} + (\Delta t) \vec{c}^{n+1} 1_{\Omega_n^F}(\hat{x}) + (\Delta t) \vec{u}^{n+1} 1_{\Omega_n^S}(\hat{x})$
4. Set  $\Omega_{n+1} = T_n(\Omega_n)$

#### Model limits :

- Change structure model to obtain more physiological behavior.
- ALE for large displacements have meshing difficulties : remesh the domain every 10 iterations to prevent the triangles from reversing.

## References

- [1] F. Hecht, New development in FreeFem++, Journal of Numerical Mathematics, 2012, vol. 20, no. 3-4, p. 251-265
- [2] B. MAURY (1996) Characteristics ALE Method for the Unsteady 3D Navier-Stokes Equations with a Free Surface, International Journal of Computational Fluid Dynamics, 6:3, 175-188, DOI: 10.1080/10618569608940780
- [3] Murea, C.M. Three-Dimensional Simulation of Fluid-Structure Interaction Problems Using Monolithic Semi-Implicit Algorithm. Fluids 2019, 4, 94, DOI: 10.3390/fluids4020094

## Conclusion and perspectives

- First promising results consistent with some physiological data.
- Explore other models to be able to compute a complete cycle (filling and voiding) in 3D considering different time scales.

Grant: American Memorial Hospital Foundation