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MODELING OF THE CHILD'S LOWER URINARY SYSTEM

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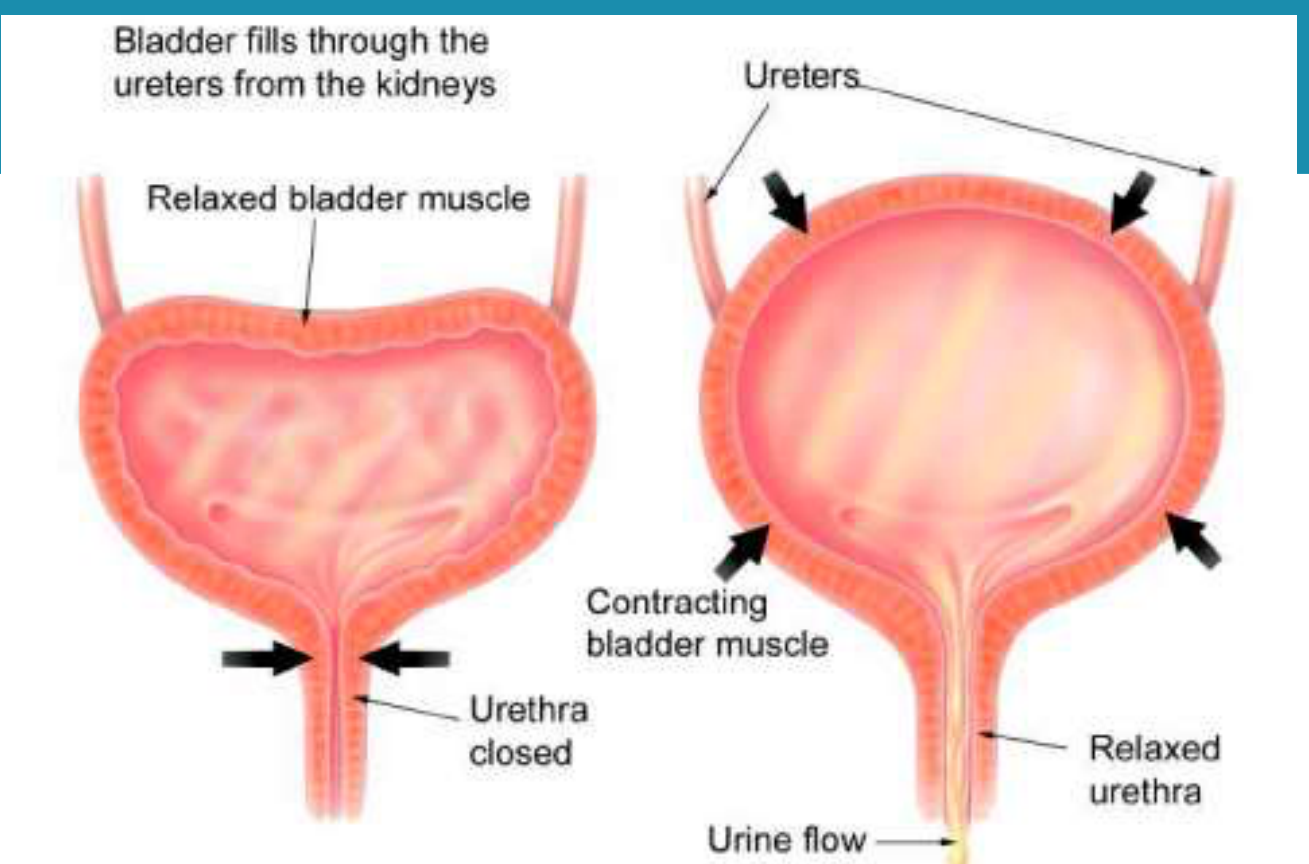
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Motivation & Research Goals

The bladder has the function of receiving the urine produced by the kidneys, keep it and then evacuate. In order to study the mechanism/movement during **filling**⁽ⁱⁱ⁾ and **voiding**⁽ⁱ⁾, we build with paediatric surgeons a simple **numerical model** of the 5-10 years old child's bladder based on **real data** such as volumes, urodynamic data or medical images like cystography.



(i) ALE Navier-Stokes equations with free surface^[2]

Navier-Stokes (NS) equations in Arbitrary Lagrangian Eulerian (ALE) formulation

$$\begin{cases} \frac{\partial \vec{u}}{\partial t} + (\vec{u} - \vec{c}) \cdot \nabla \vec{u} - \nu \Delta \vec{u} + \nabla p = 0 & \text{in } \Omega_t \\ \nabla \cdot \vec{u} = 0 & \text{in } \Omega_t, t \in [0, T_v], T_v < 30s \end{cases} \quad (1)$$

\vec{u}, p : fluid velocity and pressure

\vec{c} : domain velocity (harmonic extension)

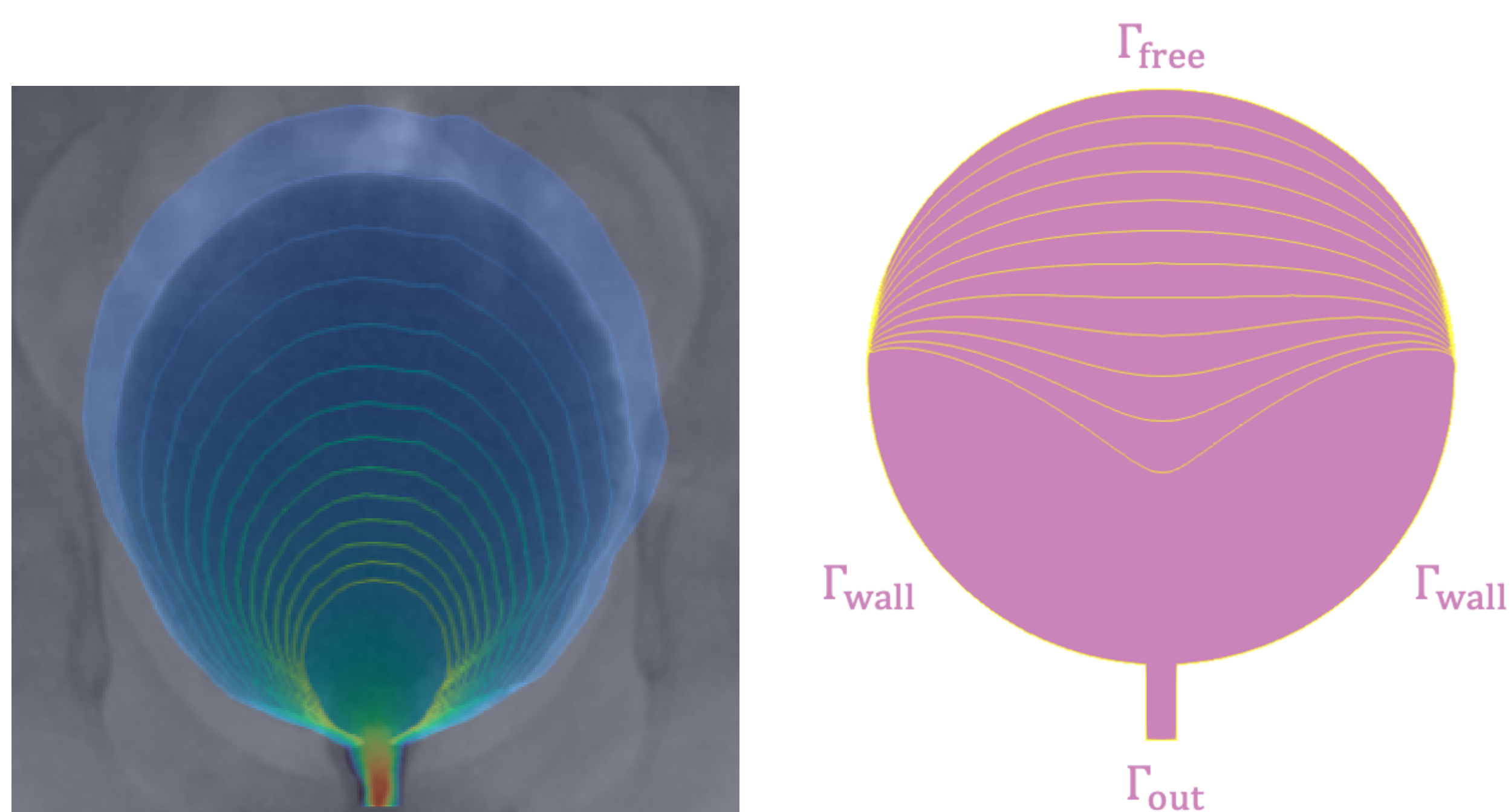
Voiding boundary conditions with free surface (Γ_t^{free}) :

$$\begin{cases} \vec{u} = 0 & \text{on } \Gamma^{wall} \\ \vec{u} = \vec{u}_{out} & \text{on } \Gamma^{out} \\ \nu \frac{\partial \vec{u}}{\partial \vec{n}} - p \vec{n} = -p_e \vec{n} - \sigma(\kappa) \vec{n} & \text{on } \Gamma_t^{free}, t \in [0, T_v], T_v < 30s \end{cases}$$

p_e : external (here abdominal) pressure

κ, σ : curvature and surface tension coefficient of Γ_t^{free}

Problem solved with FreeFem++^[1], using the FEM.



First steps of voiding based on medical images (left) and results with a simplified geometry

Model limits :

- Filling : preserve the waving shape and not inflate as expected.
- Fluid velocity drives the system.
- Detrusor action not taken into account.

(ii) Fluid-Structure Interaction (FSI)

FSI equations • NS + ALE (1) for the fluid in Ω_t^F ($t \in [0, T_f], T_f \approx 3h$)

- Despite large displacements, linear elasticity to test FSI

$$\rho^S \partial_{tt} \vec{d}^S - \nabla \cdot (\sigma^S(\vec{d}^S)) = f^S \quad \text{in } \Omega_t^S \quad (2)$$

- with the coupling conditions on the interface Σ_t :

$$\begin{cases} \vec{u}^F = \partial_t \vec{d}^S = \vec{u}^S & \text{on } \Sigma_t \\ \sigma^F(\vec{u}^F, p) \vec{n}^F = -\sigma^S(\vec{d}^S) \vec{n}^S & \text{on } \Sigma_t \end{cases} \quad (3)$$

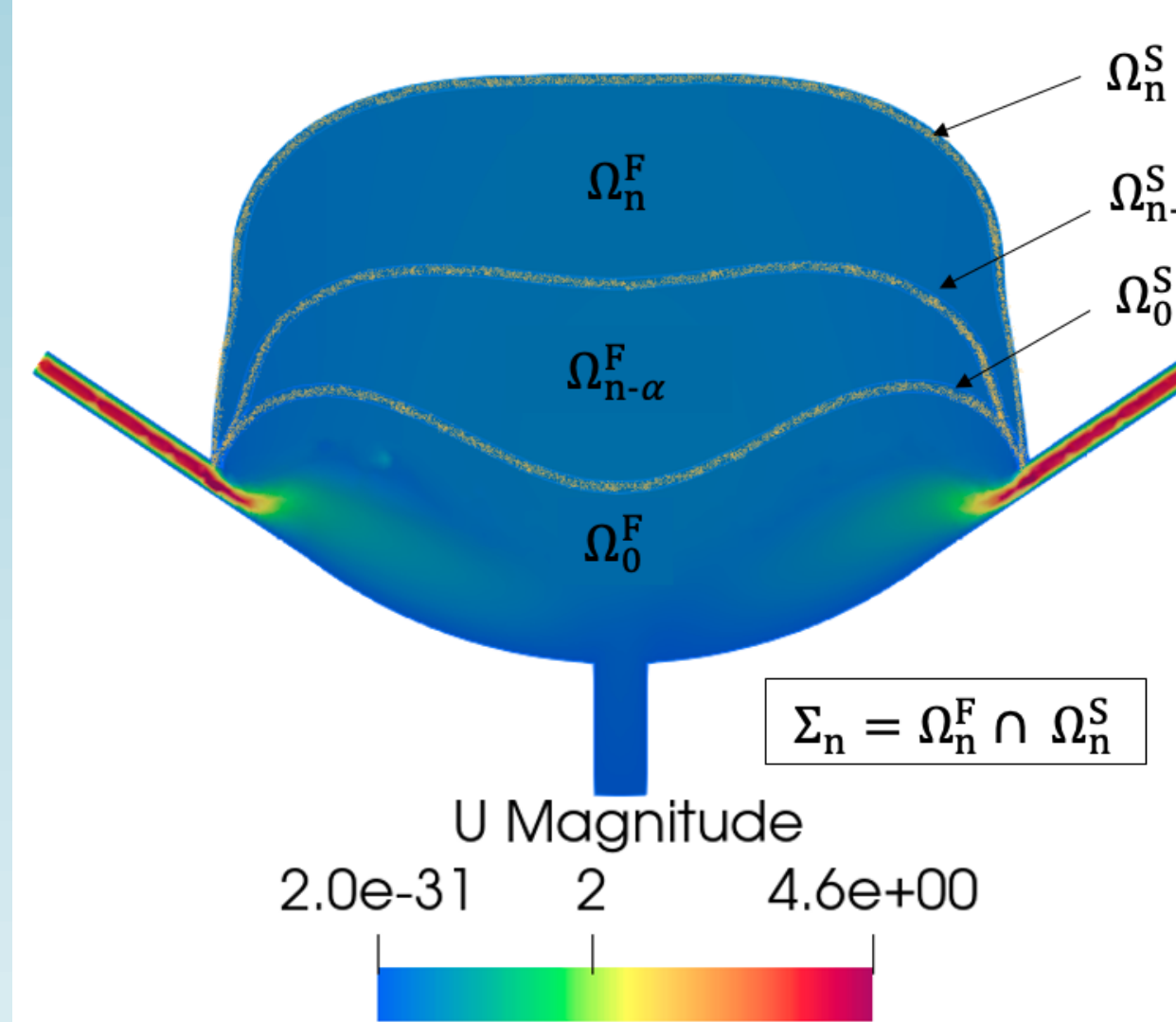
Monolithic formulation for FSI^[3]

$$\begin{cases} \int_{\Omega_n^F} \rho^F \frac{u^{n+1}}{\Delta t} \cdot v dx + \int_{\Omega_n^F} \rho^F ((u^n - c^n) \cdot \nabla) u^{n+1} \cdot v dx \\ - \int_{\Omega_n^F} (\nabla \cdot v) p^{F,n+1} dx + \int_{\Omega_n^F} 2\mu^F \epsilon(u^{n+1}) : \epsilon(v) dx \\ + \int_{\Omega_n^F} (\nabla \cdot u^{n+1}) q dx + \int_{\Omega_n^S} \rho^S \frac{u^{n+1}}{\Delta t} \cdot v dx + \int_{\Omega_n^S} L(u^{n+1}) : \nabla v dx \\ = \int_{\Omega_n^F} \rho^F \frac{u^n}{\Delta t} \cdot v dx + \int_{\Omega_n^S} \rho^S \frac{u^n}{\Delta t} \cdot v dx + \int_{\Omega_n^{S,F}} \rho^{S,F} g \cdot v dx \end{cases} \quad (4)$$

with

$$L(u^{n+1}) = (\Delta t)(\lambda^S (\nabla \cdot u^{n+1}) I + \mu^S (\nabla u^{n+1} + (\nabla u^{n+1})^T)) + \lambda^S (\nabla \cdot d^n) I + \mu^S (\nabla d^n + (\nabla d^n)^T)$$

λ^S, μ^S : Lamé's coefficients



Algorithm While $V < Vmax$ do :

1. Solve the linear system 4
2. Compute the fluid mesh velocity \vec{c}^{n+1} by harmonic extension of \vec{u}^{n+1} on Σ_t
3. Define the map by $T_n(\hat{x}) = \hat{x} + (\Delta t) \vec{c}^{n+1} 1_{\Omega_n^F}(\hat{x}) + (\Delta t) \vec{u}^{n+1} 1_{\Omega_n^S}(\hat{x})$
4. Set $\Omega_{n+1} = T_n(\Omega_n)$

Model limits :

- Change structure model to obtain more physiological behavior.
- ALE for large displacements have meshing difficulties : remesh the domain every 10 iterations to prevent the triangles from reversing.

References

- [1] F. Hecht, New development in FreeFem++, Journal of Numerical Mathematics, 2012, vol. 20, no. 3-4, p. 251-265
- [2] B. MAURY (1996) Characteristics ALE Method for the Unsteady 3D Navier-Stokes Equations with a Free Surface, International Journal of Computational Fluid Dynamics, 6:3, 175-188, DOI: 10.1080/10618569608940780
- [3] Murea, C.M. Three-Dimensional Simulation of Fluid-Structure Interaction Problems Using Monolithic Semi-Implicit Algorithm. Fluids 2019, 4, 94, DOI: 10.3390/fluids4020094

Conclusion and perspectives

- First promising results consistent with some physiological data.
- Explore other models to be able to compute a complete cycle (filling and voiding) in 3D considering different time scales.

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