



HAL
open science

Structure-preserving Observers for port-Hamiltonian systems via contraction analysis

Mario Spirito, Yann Le Gorrec, Bernhard Maschke

► **To cite this version:**

Mario Spirito, Yann Le Gorrec, Bernhard Maschke. Structure-preserving Observers for port-Hamiltonian systems via contraction analysis. 2024. hal-04344593v2

HAL Id: hal-04344593

<https://hal.science/hal-04344593v2>

Preprint submitted on 13 Mar 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Structure-preserving Observers for port-Hamiltonian systems via contraction analysis

Mario Spirito, Yann Le Gorrec, and Bernhard Maschke

Abstract—We address the design of structure-preserving nonlinear observers for port-Hamiltonian systems. Despite what is done in the literature on nonlinear observers for this class of systems, we consider a Luenberger-like identity observer dynamics with an affine output injection term. We provide some conditions on the observer gain that guarantee the passivity properties of the observer dynamics and of the system-observer cascade. We then consider the convergence properties of the observer scheme by exploiting the contraction theory (or convergent dynamics) approach, and we show the exponential convergence rate of the state reconstruction. We conclude the work by applying the proposed technique to a velocity observation of a PMSM system with only current measurements.

Index Terms—Structure-preserving, nonlinear functional observer, port-Hamiltonian systems, contraction theory, convergent dynamics, observer design.

I. INTRODUCTION

The port-Hamiltonian (pH) approach to modeling and controlling complex physical systems constitutes a well-established framework that originated with the seminal work by van der Schaft and Maschke [1], [2]. For a comprehensive overview on this field, including control techniques, one can refer to [3], [4], [5]. Port-Hamiltonian systems have the particular feature of describing the main physical properties of the system under consideration, such as energy dissipation, passivity, and power conservation laws. Because of this feature, they have been identified as a powerful framework for the modeling, simulation, and control of complex physical systems.

On the other hand, classical techniques to design nonlinear observer are high-gain observers [6], extended Kalman filter [7], KKL-observers [8], contractive observer [9], see the recent work [10] for an overview of the topic. Despite all the available approaches, the contraction theory approach is the only one that preserves the system structure. Thus this approach is the most suited for this paper. By exploiting the structure and properties of the port-Hamiltonian systems (such as the definition of a Hamiltonian function and its gradient) we provide a constructive way to design the observer dynamics.

The literature on pH systems contains numerous contributions to the design of controllers, see, e.g., [11] and [4]. In contrast, the design of observers for port-Hamiltonian systems has received rather limited attention.

In particular, for linear pH systems, state reconstruction can be naturally addressed with a standard Luenberger observer [12], thus making the error dynamics Hurwitz, see [13] for an application in the port-Hamiltonian formalism. However, for non-linear pH systems, one can exploit the Hamiltonian's gradient that is involved in the system structure. This changes the point of view of the design and gives additional insights into the observer's convergence and passivity properties, also for the case of linear pH system. For this latter class of systems, a compensator based on a dual observer has been proposed in [14], while in [15], the authors address the combined input-state reconstruction problem for linear port-Hamiltonian systems. The work [16] addresses the design of passive observers for linear port-Hamiltonian systems based on the LMI approach proposed in [17] and developed for stabilization purposes.

For nonlinear pH systems, there exist also several observer design methods. When referring to nonlinear pH systems, we can generally differentiate between two kinds of nonlinearities, viz. (a) nonlinearities in the interconnection structure and (b) nonlinearities in the gradient of the Hamiltonian/storage function. The former are characterized by state-dependent matrices of the pH systems; the latter are characterized by non-quadratic Hamiltonians. The combination of both nonlinearities can take place concurrently in the same dynamics, thus making the analysis more complicated.

The first work addressing the design of observers for nonlinear port-Hamiltonian systems was [18]. However, the proposed observer is shown to be asymptotically convergent only if the system reaches a steady state equilibrium. Later on, the authors in [19] present a passivity-based, globally exponentially convergent observer for pH systems with both nonlinear interconnection structure and Hamiltonian gradient. The proposed observer dynamics exploit additional measurements that are not conjugate to the system input and are available for feedback. Its design requires the solution of a set of algebraic equations and partial differential equations (PDEs).

The authors in [20] propose an observer design for pH systems with constant structure matrices and nonquadratic storages with an approach similar to the well-known interconnection and damping assignment passivity-based control (IDA-PBC), developed in [11]. Similarly, a passivity-based observer design was addressed in [21] and extended in [22]. It is worth noticing that the structure-preserving description in [20] is different from the one we consider in this work since we provide an analysis of the observer dynamics with an explicit port-Hamiltonian structure (involving an explicit Hamiltonian

M. Spirito and B. Maschke are with LAGEPP, University Claude Bernard Lyon 1, Villeurbanne, 69100 France, e-mail: spirito.mario@gmail.com (mario.spirito@univ-lyon1.fr).

Y. Le Gorrec is with FEMTO, Engineering School of Micromechanics and Microsystems (ENSMM), 25000 Besançon, France

Manuscript received ; revised .

function, an interconnection, and a resistance matrices), from which we deduce the passivity properties between the observer input signals and their conjugate outputs.

The observer structure for nonlinear pH system (with linear Hamiltonian gradient) has also been augmented with an integral action to improve the robustness to constant system perturbations of the proportional passivity-based observer in [23]. Recently, in [24] a Lyapunov-based observer design is described for a particular class of bilinear pH systems. In [25], instead, the authors proposed the design of a particular class of nonlinear port-Hamiltonian systems where the Hamiltonian function has a particular shape (a quadratic form with some nonlinearity depending on part of the system state). Along with this particular Hamiltonian shape, the authors consider exploiting, for the output injection term in the observer dynamics, some measurements that are not conjugated to the original system input. Among these measurements a part of the system state is directly available, that is the critical state part involved in the Hamiltonian nonquadratic form part, which simplifies a lot the scenario.

In [26], the authors propose an observer-based controller describing the passivity property of the system-observer cascade. They then couple the observer dynamics with a Model Predictive Control-like technique to obtain a controller design for port-Hamiltonian systems.

Despite all the proposed techniques, it can be seen that the observers in [18] and [19] are the only two approaches that apply to port-Hamiltonian systems with both nonlinearities in the interconnection structure and in the Hamiltonian gradient. However, the observer approach in [18] has not been proved to be asymptotically convergent in general, and the observer design in [19] is delicate (or weak) as it relies on the solution of a set of algebraic equations and PDEs.

In this paper, we propose a complete analysis regarding the passivity of the observer dynamics along with its subtleties in reconstructing the system state. In particular, it is not generally guaranteed that the observer states are a passive output for the overall observer dynamics. On the contrary, the gradient of the observer Hamiltonian is always a passive output with respect to the output injection term. This article is motivated by and concurrently paves the way for the design of an observer-based controller, which is currently under development and will be the topic of future work. In this work, we consider an observer design based on the convergence properties of nonlinear systems as introduced in [27], [28] and re-proposed in [29]. By properly exploiting this contraction/convergent¹ properties of nonlinear systems, we are able to find conditions on the observer gain that guarantees the exponential convergence of the observer and directly tune its convergence rate for some classes of port-Hamiltonian systems that are not restrictive in practical applications. This convergence property is achieved, for the class of systems under consideration, without involving any algebraic and/or partial differential equations.

The paper is structured as follows. In Section II, we introduce some preliminary concepts that are useful for the successive development. We then discuss the structure-preserving

functional observer dynamics in Section III and study the passivity of the corresponding error dynamics in Section IV. The theoretical part of the paper concludes with Section V, in which we describe the design techniques for two classes of port-Hamiltonian systems whose nonlinearities are first in the system dynamics matrices and then in the gradient of the associated Hamiltonian function. We then merge the two results into a summarizing theorem that applies to general nonlinear port-Hamiltonian systems, whose input matrix is constant. We then provide a numerical example to show the effectiveness of the proposed methods in Section VI, and we give some conclusions in Section VII.

Nomenclature

To apply a chain rule to a matrix $J(x)$ times a vector Qx multiplication, we consider the following notation

$$\frac{\partial (J(x)Qx)}{\partial x} = \nabla J(x, Qx) + J(x)Q$$

where $\nabla J(x, \cdot)$ represents the tensor (3D- matrix) obtained by involving the Jacobian of all the columns of $J(x)$, i.e., the i -th column of $\nabla J(x, Qx)$ is given by $\nabla J_i(x)Qx$, where $J_i(x)$ is the i -th column in $J(x)$.

II. PRELIMINARIES

In this section, we introduce the concepts of port-Hamiltonian systems and contractive dynamics, useful to the successive sections.

A. Port-Hamiltonian systems

Port-Hamiltonian systems can be formalized as nonlinear systems generated by an energy function H with related dynamics

$$\begin{aligned} \dot{x} &= (J(x) - R(x)) \nabla H(x) + g(x)u \\ y &= g(x)^\top \nabla H(x) \end{aligned} \quad (1)$$

where $x \in X \subset \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^m$ and $J, R \in \mathbb{R}^{n \times n}$ are matrix valued functions, $J(x)$ being skew-symmetric and $R(x)$ symmetric positive semidefinite, while $g(x) \in \mathbb{R}^{n \times m}$ plays the role of the input matrix. Moreover, having $R(x)$ positive semidefinite on X , guarantees the passivity property with storage function $H(x)$, whenever the Hamiltonian is lower bounded. Indeed, the energy balance equation reads:

$$\dot{H}(x) = y^\top u - \nabla H(x)^\top R(x) \nabla H(x) \leq y^\top u$$

thus, providing passivity with storage function H .

For the present developments, we consider the following Assumption.

Assumption 1. *The Hessian of the Hamiltonian is every bounded and positive definite, i.e., there exist positive real h_1 and h_2 such that for all $x \in X \subset \mathbb{R}^n$*

$$h_1 I \leq \nabla^2 H(x) \leq h_2 I.$$

The above assumption implies also that H is lower bounded and convex, and it can moreover play the role of a Lyapunov function.

¹From the stability point of view these notions are synonymous.

In the following, we consider the design of an observer dynamics that preserves the port-Hamiltonian structure (1) and we analyze the conditions to preserve the passivity of such an observer dynamics. In particular, when applying the identity observer approach to port-Hamiltonian systems, one has to consider an additional desired property along with guaranteeing the observer convergence/stability and its passivity characteristics, i.e., structure preservation. That is, in the port-Hamiltonian formalism, it is crucial to have explicit Hamiltonian/energy terms involved in the system dynamics (describing the effort of the system), then a symplectic structure for the energy conservation, a positive semidefinite matrix describing possible dissipative elements in the system, and a particular relationship between the input and the output matrices so to guarantee the passivity property of the system. Hence, the resulting *structure-preserving* observer must have all the features listed above in order to fit into the port-Hamiltonian formalism.

Remark 1. *Contrary to what has been addressed in [20], we highlight the difficulties of preserving the port-Hamiltonian structure for the observer dynamics, along with its passivity property, in Section III. The structure preservation addressed in [20] is then different to the one addressed here. Since they consider, as structure-preserving observer, a system whose dynamics is just stable, and no passivity property is taken into account. Their only focus is on the observer error origin stability. In the next section, we discuss the passivity properties of a functional observer in the port-Hamiltonian formalism and the related subtleties.*

B. Contraction theory

In Section V, we use the fundamental concept of contractive/convergent dynamics to prove the convergence of the observer. Contraction theory in the context of nonlinear control is nowadays a counterpart for the Lyapunov approach, allowing to guarantee the stability of the origin and convergence of the system trajectories.

This approach has been introduced in works such as [27], [28], for the case of constant metric, and re-proposed recently in [29], [30], [31], while their extension to the more general case of nonlinear Riemannian metrics can be found in [32] and [9]. See also [33] for the case of non-Euclidean L_1 and L_∞ metrics.

In the following, we will use the standard convergence result in [29] that exploits the standard metric $P \in \mathbb{R}^{n \times n}$ to describe a convergence dynamics property. In order to do so, we first introduce the definition of contractive systems.

Consider the system

$$\dot{x} = f(x, t) \quad (2)$$

where $x \in X \subseteq \mathbb{R}^n$ and $f : X \times \mathbb{R} \rightarrow \mathbb{R}^n$ is continuously differentiable in x and Lipschitz continuous in t . Let $\phi(x_0, t)$ denote the trajectory of (2) at time t originated at x_0 at time $t = 0$. We then have the following definition.

Definition II.1. *We say that system (2) defines a contraction if there exist real positive constants $\alpha, \kappa > 0$ such that*

$$\|\phi(x_1, t) - \phi(x_2, t)\| \leq \kappa \exp(-\alpha t) \|x_1 - x_2\|$$

for all initial conditions $x_1, x_2 \in X \subseteq \mathbb{R}^n$ and for all $t \geq 0$.

A sufficient condition to determine whether system (2) defines a contraction is the existence of a constant metric P for which the distance with this metric between any two system trajectories is monotonically decreasing in time. We thus have the following well-known result, also known as Demidovic condition in [29].

Lemma 1. *System (2) is a contraction on X if there exists a constant $P \in \mathbb{R}^{n \times n}$ symmetric and positive definite, such that*

$$P \frac{df}{dx}(x, t) + \frac{df^\top}{dx}(x, t) P < 0 \quad (3)$$

for all $x \in X \subseteq \mathbb{R}^n$.

III. STRUCTURE-PRESERVING OBSERVER FOR PORT-HAMILTONIAN SYSTEMS

We present in this section the passivity (structure-preserving) properties related to a general functional observer dynamics for a port-Hamiltonian system dynamics. We first introduce the following (passivity-related) assumption.

Assumption 2. *For any given functional of interest $q(x)$, $q : \mathbb{R}^n \rightarrow \mathbb{R}^p$, there exists a locally invertible matrix valued function $\psi : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times p}$ such that $q(x) = \psi^\top(x) \nabla H(x)$.*

Then a structure-preserving, and thus passive, port-Hamiltonian functional observer for the functional $q(x)$ reconstruction reads as

$$\begin{aligned} u_q &= \psi(\hat{x})^\dagger L(\hat{x})^\top y \\ \dot{\hat{x}} &= [(J(\hat{x}) - R(\hat{x})) - L(\hat{x})^\top g(\hat{x})^\top] \nabla H(\hat{x}) + \\ &\quad g(\hat{x})u + \psi(\hat{x})u_q \\ \hat{y} &= g(\hat{x})^\top \nabla H(\hat{x}) \\ q(\hat{x}) &= \psi(\hat{x})^\top \nabla H(\hat{x}) \end{aligned} \quad (4)$$

where $\psi(\hat{x})^\dagger$ is the local inverse of $\psi(x)$, existing by assumption, and the signals $(\hat{y}, q(\hat{x}))$ are the collocated outputs of the inputs (u, u_q) , that is, if $R(\hat{x}) + \text{sym}(g(\hat{x})L(\hat{x})) \geq 0$ for all $x \in X$, then the observer is passive with respect to the inputs (u, u_q) and their conjugated outputs $(\hat{y}, q(\hat{x}))$. It is also easy to notice that, if $\hat{x} \rightarrow x$ also $q(\hat{x}) \rightarrow q(x)$, thus the full order passive observer (4) can reconstruct the functional of interest if $\hat{x} - x \rightarrow 0$ for any trajectory $x(t)$ originated at $x_0 \in X_0 \subseteq X$ at time $t = 0$ of system (1).

However, from an implementation point of view, there is no need to consider the existence of the matrix values function ψ , because we can obtain an estimate of $q(x)$ by simply replacing \hat{x} into its argument, i.e., directly getting $q(\hat{x})$ from the available state evolution. Moreover, substituting the definition of u_q into the observer dynamics hides the presence of the local inverse of ψ .

The presence of such matrix ψ has been introduced only to preserve the structure of an explicit passive system, thus making the observer dynamics structure-preserving port-Hamiltonian with inputs (u, u_q) and outputs $(\hat{y}, q(\hat{x}))$. As a consequence, if such a matrix-valued function ψ exists, although possibly unknown and unemployable, the observer dynamics is intrinsically passive from the inputs (u, u_q) .

We now consider two particular cases, i.e., passive state and passive gradient reconstruction/observation.

A. Passive state observation

In this particular case, to obtain a *passive* state reconstruction, we need to assume a strong injectivity property of the Hamiltonian gradient $\nabla H(x)$, i.e., there exists a locally invertible ψ such that $x = \psi(x)\nabla H(x)$. Thus the observer dynamics reads as

$$\begin{aligned} u_q &= \psi(\hat{x})^\dagger L(\hat{x})^\top y \\ \dot{\hat{x}} &= [(J(\hat{x}) - R(\hat{x})) - L(\hat{x})^\top g(\hat{x})^\top] \nabla H(\hat{x}) + \\ &\quad g(\hat{x})u + \psi(\hat{x})u_q \\ \hat{y} &= g(\hat{x})^\top \nabla H(\hat{x}) \\ \hat{x} &= \psi(\hat{x})^\top \nabla H(\hat{x}). \end{aligned} \quad (5)$$

Again the knowledge of the matrix ψ and its locally invertibility property, are not necessary from the implementation point of view, but rather it plays an important role in the passivity analysis of the observer interconnections² when we use pure state-feedback techniques such as the IDA-PBC approach or any other control law of the form $\alpha(\hat{x})$. Obtaining $\psi(\hat{x})$ for the linear case, whenever $Q = Q^\top > 0$ in $\nabla H(x) = Qx$, is easy since $\psi(\hat{x})^\top = Q^{-1}$. In the nonlinear context, i.e., $\nabla H(x) \neq Qx$, we cannot provide any general solution for ψ , and the problem should be addressed on a case-by-case basis.

B. Passive gradient observation

In this particular case, we are interested in the reconstruction of the system Hamiltonian's gradient, and thus the matrix-valued function ψ is the identity matrix, thus the functional observer trivially reads as

$$\begin{aligned} u_q &= L(\hat{x})^\top y \\ \dot{\hat{x}} &= [(J(\hat{x}) - R(\hat{x})) - L(\hat{x})^\top g(\hat{x})^\top] \nabla H(\hat{x}) + \\ &\quad g(\hat{x})u + Iu_q \\ \hat{y} &= g(\hat{x})^\top \nabla H(\hat{x}) \\ q(\hat{x}) &= I\nabla H(\hat{x}). \end{aligned} \quad (6)$$

Reconstructing/observing the Hamiltonian gradient is by far the simplest case of *passive* functional observation because the functional $q(\hat{x}) = \nabla H(\hat{x})$ is intrinsically passive with respect to the injection term $u_q = L(\hat{x})^\top y$, without the need to involve any matrix-valued function $\psi(\hat{x})$.

Remark 2. In standard port-Hamiltonian nomenclature, $\nabla H(\hat{x})$ is usually referred to as the effort variable.

Remark 3. It is clear from the above analysis that getting a *passive observer* might be a hard task, although it might not be needed to construct a passivity-preserving controller. See also Fig. 1 to have a schematic point of view of the passivity part of the observer dynamics.

As already touched before, any functional q can be reconstructed by the full order observer, whenever the states of the

observer \hat{x} converge to the system state evolution x . Hence, from a convergence point of view, it is enough to study the deviation evolution between the observer's and the system's state, as proposed in the following section.

For the sake of exposition, from the following subsection on, we use the notation $M = M(x)$, $\hat{M} = M(\hat{x})$, and $\tilde{M} = \hat{M} - M$, for $M \in \{J, R, g, H\}$, while $\tilde{\nabla}H = \nabla\hat{H} - \nabla H = \nabla H(\hat{x}) - \nabla H(x)$.

C. System-observer cascade passivity

We consider now the augmented system given by the cascade of the system and the observer dynamics. For this augmented system, due to the dynamics interconnection, the passivity conditions has to be considered only between the input signals (u, u) and the conjugated outputs (y, \hat{y}) . In particular, we have that the augmented system has again a port-Hamiltonian structure, i.e., defining $H_a(x, \hat{x}) = H(x) + H(\hat{x})$ we have

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} &= [J_a(x, \hat{x}) - R_a(x, \hat{x})] \nabla H_a(x, \hat{x}) + g_a(x, \hat{x})u_a \\ &= \begin{bmatrix} J - R & 0 \\ \hat{L}^\top \hat{g}^\top & \hat{J} - \hat{R} - \hat{L}^\top \hat{g}^\top \end{bmatrix} \begin{pmatrix} \nabla_x H_a(x, \hat{x}) \\ \nabla_{\hat{x}} H_a(x, \hat{x}) \end{pmatrix} \\ &\quad + \begin{bmatrix} g & 0 \\ 0 & \hat{g} \end{bmatrix} \begin{pmatrix} u \\ u \end{pmatrix} \\ \begin{pmatrix} y \\ \hat{y} \end{pmatrix} &= g_a(x, \hat{x})^\top \nabla H_a(x, \hat{x}) \\ &= \begin{bmatrix} g & 0 \\ 0 & \hat{g} \end{bmatrix}^\top \begin{pmatrix} \nabla_x H_a(x, \hat{x}) \\ \nabla_{\hat{x}} H_a(x, \hat{x}) \end{pmatrix}. \end{aligned} \quad (7)$$

And thus, for storage function H_a , the passivity condition is satisfied if the cascade resistive matrix $R_a(x, \hat{x})$ is positive semidefinite, i.e.,

$$R_a(x, \hat{x}) = \begin{bmatrix} R & -\frac{1}{2}\hat{g}\hat{L} \\ -\frac{1}{2}\hat{L}^\top \hat{g}^\top & \hat{R} + \text{sym}(\hat{L}^\top \hat{g}^\top) \end{bmatrix} \geq 0 \quad (8)$$

and this condition can be satisfied, in general, under strong assumptions³ on L , despite what is reported in the analysis by [26, eq. (6)]. Thus in general, the cascade system-observer dynamics is not passive from the inputs (u, u) to outputs (y, \hat{y}) . A reason to conclude this non-passivity property of the system-observer cascade can be found in the high-gain-like approach to obtain a robust observer convergence. In other words, the usual choice of the output injection gain $L(\hat{x})$ employs 'high-gain' terms to provide a fast convergence of the observer dynamics, especially when $L(\hat{x})$ is taken as a constant matrix. Thus, the design of a fast convergent observer goes, in principle, against some possibly desired passivity property of the system-observer cascade.

However, it should be noted that the passivity property can be useful when exploiting the observer dynamics into a dynamical controller for the system. Thus, closing the loop

²For example the design can be interested in some passivity properties of the system-observer cascade or closed loop.

³Take for example a scalar system case with R positive real. In order to satisfy the passivity condition, one should take a sufficiently small L , and thus a robust high-gain approach will go against the passivity requirements.

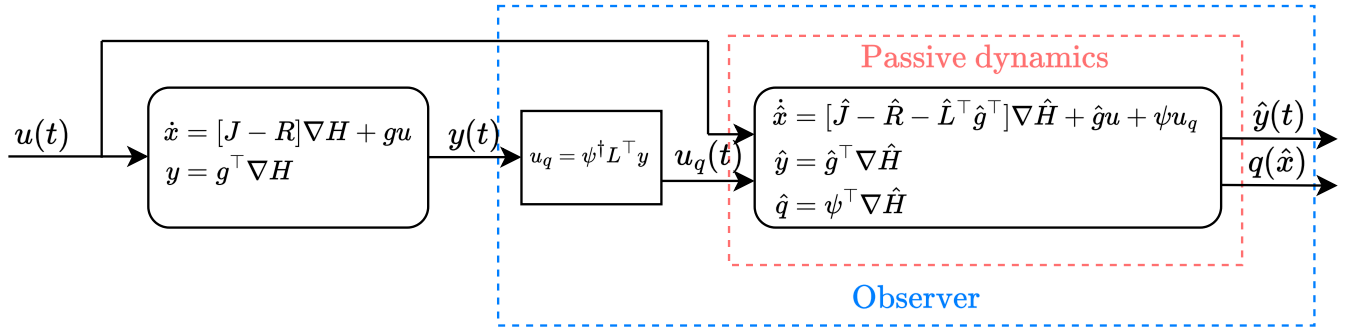


Fig. 1. System-Observer cascade configuration highlighting the passive part of the observer dynamics.

with an appropriate choice of the controller gain matrix, such as the symplectic feedback

$$u = -L^\top(\hat{x})\nabla H(\hat{x}) + v,$$

provides the desired passivity property, this time with respect to the input (v, v) and the output (y, \hat{y}) of the closed-loop dynamics.

For the sake of exposition, in the remaining part of the paper, we consider the observer state dynamics as

$$\dot{\hat{x}} = (\hat{J} - \hat{R} - \hat{L}^\top \hat{g}^\top)\nabla \hat{H} + \hat{g}u(t) + \hat{L}^\top y(t). \quad (9)$$

IV. OBSERVER ERROR PASSIVITY ANALYSIS

In order to analyze the convergence of the observer state \hat{x} to the system state trajectory x , the standard approach is to introduce the error coordinate $\tilde{x} = \hat{x} - x$. By considering the evolution in the observer dynamics (9) and in the system dynamics (1), we can obtain again a port-Hamiltonian \tilde{x} dynamics, as reported in the following Lemma

Lemma 2. *Given the observer dynamics (9) with state \hat{x} and in the system dynamics (1) with state x , the error dynamics reads as*

$$\begin{aligned} \dot{\tilde{x}} &= (\hat{J} - \hat{R} - \hat{L}^\top \hat{g}^\top)\tilde{\nabla}H + \tilde{g}u + (\tilde{J} - \tilde{R} - \hat{L}^\top \tilde{g}^\top)\nabla H \\ \tilde{y} &= \tilde{g}^\top \tilde{\nabla}H \end{aligned} \quad (10)$$

where \tilde{y} are the conjugate outputs to the inputs u .

Proof. The proof comes from substituting the observer's and system's dynamics into the time derivative of \tilde{x} , i.e., $\dot{\tilde{x}} = \dot{\hat{x}} - \dot{x}$. Using the notation introduced in the Lemma, we have

$$\begin{aligned} \dot{\tilde{x}} &= (\hat{J} - \hat{R})\nabla \hat{H} + \hat{L}^\top (g^\top \nabla H - \hat{g}^\top \nabla H + \hat{g}^\top \nabla H - \hat{g} \nabla \hat{H}) \\ &\quad + (\hat{g} - g)u - (J - R)\nabla H \\ &= (\hat{J} - \hat{R})\tilde{\nabla}H - (J - R - (\hat{J} - \hat{R}))\nabla H + \tilde{g} \\ &\quad - \hat{L}^\top (\hat{g}^\top \tilde{\nabla}H + \tilde{g}^\top \nabla H) \\ &= (\hat{J} - \hat{R} - \hat{L}^\top \hat{g}^\top)\tilde{\nabla}H + \tilde{g}u + (\tilde{J} - \tilde{R} - \hat{L}^\top \tilde{g}^\top)\nabla H \end{aligned}$$

and thus we obtained an error dynamics that has a port-Hamiltonian structure, although the error Hamiltonian function \tilde{H} is not explicitly defined. \square

In general, $\tilde{\nabla}H$ is not only a function of \tilde{x} and thus the corresponding Hamiltonian function \tilde{H} , obtained by integration

in the \tilde{x} usually depends also on the system state variable x . In particular, by involving the multivariable mean value theorem, as usually done in contraction analysis [29], we can write $\tilde{\nabla}H$ explicitly in form of a symmetric matrix-valued function and the error state \tilde{x} , i.e.,

$$\begin{aligned} \tilde{\nabla}H &= \nabla H(\tilde{x} + x) - \nabla H(x) \\ &= \int_0^1 \nabla^2 H(x(t) + s(\hat{x}(t) - x(t)))ds \cdot (\hat{x} - x) \\ &= \int_0^1 \nabla^2 H(x(t) + s\tilde{x}(t))ds \tilde{x} = Q(x, \tilde{x})\tilde{x} \end{aligned}$$

where

$$Q(x, \tilde{x}) := \int_0^1 \nabla^2 H(x(t) + s\tilde{x}(t))ds.$$

However, integrating $Q(x, \tilde{x})\tilde{x}$ to get a time dependent⁴ Hamiltonian functional \tilde{H} for the observer error dynamics is in general not an easy task but, by introducing the so-called Availability function

$$\mathcal{A}(x, \tilde{x}) = H(x + \tilde{x}) - H(x) - \tilde{x}^\top \nabla H(x) \quad (11)$$

one can easily see that the partial derivative of $\mathcal{A}(x, \tilde{x})$ with respect to \tilde{x} gives

$$\nabla_{\tilde{x}}\mathcal{A}(x, \tilde{x}) = \nabla H(x + \tilde{x}) - \nabla H(x) = \tilde{\nabla}H$$

Unfortunately, the partial derivative of \mathcal{A} with respect to x does not provide the gradient of the original system Hamiltonian $\nabla H(x)$. Thus the system-observer error cascade has a port-Hamiltonian structure, but in general, it is not port-Hamiltonian because there is no natural expression of passivity, i.e., the cascade dynamics reads as

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{\tilde{x}} \end{pmatrix} &= \begin{bmatrix} J - R & 0 \\ \tilde{J} - \tilde{R} - \hat{L}^\top \tilde{g}^\top & \hat{J} - \hat{R} - \hat{L}^\top \hat{g}^\top \end{bmatrix} \begin{pmatrix} \nabla_x H(x) \\ \nabla_{\tilde{x}} \mathcal{A}(x, \tilde{x}) \end{pmatrix} \\ &\quad + \begin{bmatrix} g & 0 \\ 0 & \tilde{g} \end{bmatrix} \begin{pmatrix} u \\ u \end{pmatrix} \\ \begin{pmatrix} y \\ \tilde{y} \end{pmatrix} &= \begin{bmatrix} g & 0 \\ 0 & \tilde{g} \end{bmatrix}^\top \begin{pmatrix} \nabla_x H(x) \\ \nabla_{\tilde{x}} \mathcal{A}(x, \tilde{x}) \end{pmatrix}, \end{aligned} \quad (12)$$

with no explicit Hamiltonian/storage function through which we can describe the passivity property of the cascade.

⁴Due to the state dependence $x(t)$.

If we additionally assume that there exists a function $\mathcal{H}(\tilde{x}, x)$ such that

$$\frac{\partial \mathcal{H}(\tilde{x}, x)}{\partial \tilde{x}} = \frac{\partial H}{\partial x}(\tilde{x} + x) - \frac{\partial H}{\partial x}(x), \quad \frac{\partial \mathcal{H}(\tilde{x}, x)}{\partial x} = \frac{\partial H}{\partial x}(x), \quad (13)$$

then passivity conditions simplifies for the system cascade, as detailed in the following corollary.

Lemma 3. *Assume there exists a Hamiltonian function $\mathcal{H}(x, \tilde{x})$ with property (13). Then the cascade (1)-(10), is passive with respect to the inputs (u, \tilde{u}) and outputs (y, \tilde{y}) , with storage function $\mathcal{H}(x, \tilde{x})$, if the following passivity condition holds*

$$\begin{bmatrix} -2R & (\tilde{J} - \tilde{R} - \hat{L}^\top \tilde{g}^\top)^\top \\ (\tilde{J} - \tilde{R} - \hat{L}^\top \tilde{g}^\top) & -2(\hat{R} + \text{sym}(\hat{L}^\top \hat{g}^\top)) \end{bmatrix} \leq 0. \quad (14)$$

Proof. The proof of the Lemma simply comes by noticing the structure of the System-Observer error cascade (12) is indeed port-Hamiltonian with a well-defined Hamiltonian function $\mathcal{H}(x, \tilde{x})$, i.e.,

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{\tilde{x}} \end{pmatrix} &= \begin{bmatrix} J - R & 0 \\ \tilde{J} - \tilde{R} - \hat{L}^\top \tilde{g}^\top & \hat{J} - \hat{R} - \hat{L}^\top \hat{g}^\top \end{bmatrix} \begin{pmatrix} \nabla_x \mathcal{H}(x, \tilde{x}) \\ \nabla_{\tilde{x}} \mathcal{H}(x, \tilde{x}) \end{pmatrix} \\ &\quad + \begin{bmatrix} g & 0 \\ 0 & \tilde{g} \end{bmatrix} \begin{pmatrix} u \\ \tilde{u} \end{pmatrix} \\ \begin{pmatrix} y \\ \tilde{y} \end{pmatrix} &= \begin{bmatrix} g & 0 \\ 0 & \tilde{g} \end{bmatrix}^\top \begin{pmatrix} \nabla_x \mathcal{H}(x, \tilde{x}) \\ \nabla_{\tilde{x}} \mathcal{H}(x, \tilde{x}) \end{pmatrix} \end{aligned}$$

where the matrix in the LMI (14) is two times the symmetric part of the cascade structure matrices. Thus computing the time derivative of $\mathcal{H}(x, \tilde{x})$ proves the Lemma, i.e., $\dot{\mathcal{H}}(x, \tilde{x}) \leq y^\top u + \tilde{y}^\top \tilde{u}$. \square

A particular case happens, when the Hamiltonian function of the system $H(x)$ is a quadratic form, i.e., $H(x) = x^\top Qx$, with associate positive definite matrix, $Q = Q^\top > 0$. In particular, in this situation the Availability function $\mathcal{A}(x, \tilde{x}) = H(\tilde{x})$, the cascade Hamiltonian function $\mathcal{H}(x, \tilde{x})$ can be taken can the sum of the two dynamics Hamiltonian, i.e., $\mathcal{H}(x, \tilde{x}) = H(x) + H(\tilde{x})$.

This case is also particularly relevant for the convergence analysis of the observer dynamics, as considered in the following section.

V. OBSERVER DESIGN VIA CONTRACTION ANALYSIS

In this section, we present some results on the design of a convergent⁵ observer dynamics (9), reported here for completeness,

$$\dot{\hat{x}} = (\hat{J} - \hat{R} - \hat{L}^\top \hat{g}^\top) \nabla \hat{H} + \hat{g}u(t) + \hat{L}^\top y(t),$$

for two classes of port-Hamiltonian systems with constant input matrix $g(x) = g$, i.e., for systems with quadratic form Hamiltonian function H and for systems with constant parameters J and R , but with non quadratic form Hamiltonian.

The key idea behind the following results is that the observer dynamics is the system dynamics initialized on a different initial condition. In order to proceed with the analysis, we first

introduce a modification of the system dynamics (1), which however does not change its state trajectory. In particular, we consider adding the terms $L^\top(y - y) = 0$ into the x dynamics, so that the state dynamics of (1) now reads as

$$\dot{x} = (J - R - L^\top g^\top) \nabla H + gu(t) + L^\top y(t) \quad (15)$$

where we consider the system output y as an additional ‘fake input’. With this reformulation, we see that both the observer and system dynamics have the same structure with augmented input signals (u, y) . Moreover, it is clear in this context that the observer dynamics is the system dynamics initialized at a different initial condition. This reformulation allows us to exploit the contraction property, i.e., Lemma 1, on the original system (15), in order to obtain the observer convergence on a certain domain of interest.

In the following cases, we focus our attention on the output injection matrix L by considering it constant, along with the input matrix g . Because the observer dynamics can suffer from the peaking problem, the state of the observer could in principle ‘exit’ the original system state space X . In order to guarantee the uniform convergence property of the observer we need to study the contractivity of the observer on a larger state space \hat{X} encompassing the one of the original system X , i.e., $X \subseteq \hat{X}$.

A. Observer design with constant Hessian

In this subsection, we consider the case in which the system (15) has a quadratic form Hamiltonian, i.e., $H(x) = \frac{1}{2}x^\top Qx$, with $Q = Q^\top$ positive definite, so that $\nabla H(x) = Qx$, and we have the following

Theorem V.1. *Given system (15), with $x \in X$, with $g(x) = g$ constant, and $\nabla H(x) = Qx$, with Q positive definite. The observer dynamics (9), with $\hat{x} \in \hat{X}$, converges to the system state, i.e., $\hat{x} \xrightarrow[t \rightarrow \infty]{} x$, if there exists a constant L such that for any $x \in \hat{X}$*

$$R'(x) + \text{sym}(gL) \geq \beta I > 0, \quad (16)$$

where

$$R'(x) = R - \text{sym}(\nabla J(x, Qx)Q^{-1} - \nabla R(x, Qx)Q^{-1}). \quad (17)$$

Moreover, for α satisfying $\beta I \geq \alpha Q^{-1}$, the convergence rate of the observer error is of the form

$$|\tilde{x}(t)| \leq \mu(Q) \exp(-\alpha t) |\tilde{x}(0)|. \quad (18)$$

with α being a positive constant.

Proof. To have a convergent observer, due to $X \subseteq \hat{X}$, we need to guarantee the convergence property of (15) for any $x \in \hat{X}$. That is, according to Lemma 1 and considering the dynamics in (15) as $\dot{x} = F(x, t)$, we analyze its gradient

$$\begin{aligned} \frac{\partial F}{\partial x}(x, t) &= \left(\frac{\partial J(x)}{\partial x} - \frac{\partial R(x)}{\partial x} \right) Qx \\ &\quad + (J(x) - R(x) - L^\top g^\top) Q \\ &= [\nabla J(x, Qx)Q^{-1} - \nabla R(x, Qx)Q^{-1} \\ &\quad + (J(x) - R(x) - L^\top g^\top)] Q \\ &= (J'(x) - R'(x) - L^\top g^\top) Q. \end{aligned}$$

⁵By convergent observer dynamics we mean that $\hat{x} \rightarrow x$ as $t \rightarrow \infty$.

Then the convergence property can be studied with respect to the metric Q , i.e., defining $V = \tilde{x}^\top Q \tilde{x} = 2\tilde{H}$ and taking its time derivative, we have by the mean value theorem

$$\begin{aligned}\dot{V} &= 2\tilde{x}^\top Q(F(\hat{x}, t) - F(x, t)) = \\ &= 2\tilde{x}^\top Q \int_0^1 \frac{\partial F}{\partial x}(\bar{x}(t, s), t) ds \tilde{x} \\ &= 2\tilde{x}^\top Q \int_0^1 (J'(\bar{x}(t, s)) - R'(\bar{x}(t, s)) - L^\top g^\top) ds Q \tilde{x}\end{aligned}$$

where $\bar{x}(t, s) = x + s(\hat{x} - x)$ for $s \in [0, 1]$. We can hence write

$$\dot{V} = -\tilde{x}^\top Q \int_0^1 (R'(\bar{x}(t, s)) + \text{sym}(gL)) ds Q \tilde{x} < 0$$

Then, by standard arguments, since $R' + \text{sym}(gL)$ is uniformly positive, there exists a positive α such that for every $x \in \hat{X}$

$$-R'(x) - \text{sym}(gL) \leq -\alpha Q^{-1},$$

so that we have (18) by applying the comparison lemma [34, Lemma 3.4] to the Lyapunov function. \square

Remark 4. This result resembles the approach proposed in [21] and it is the dual of the one proposed in [20] in which the authors assume that the system matrices are constant and the only nonlinearity is due to the gradient of the Hamiltonian $\nabla H(\hat{x})$.

B. Observer design with constant structural matrices

In this subsection, we consider a system dynamics (15) whose matrices J, R, g are constant and that the only nonlinearity comes from the gradient of the Hamiltonian function $\nabla H(x)$. We additionally assume that the Hessian of H is diagonalizable by means of a constant orthogonal matrix T_H , i.e., $\nabla^2 H(x) = T_H^\top \Lambda(\nabla^2 H(x)) T_H$, where $\Lambda(\nabla^2 H)$ is the matrix with the eigenvalues of $\nabla^2 H(x)$ on the main diagonal. For this class of systems, we have the following result

Theorem V.2. Consider (15) with constant matrices J, R, g , and consider $Q = \nabla^2 H(x_{\min})$ where x_{\min} is such that, for all $x \in \hat{X}$, $\nabla^2 H(x) - \nabla^2 H(x_{\min}) \geq 0$. Moreover, assume that there exists a constant orthogonal matrix T_H such that $\nabla^2 H(x) = T_H^\top \Lambda(\nabla^2 H)(x) T_H$ for all $x \in \hat{X}$. Then the observer dynamics (9) is convergent on \hat{X} with metric Q if there exists a constant L such that

$$R(x) + \text{sym}(gL) \geq \beta I > 0, \quad (19)$$

for all $x \in \hat{X}$. Moreover, for α satisfying $\beta I \geq \alpha Q^{-1}$ we can explicitly determine the exponential convergence of the observation error as

$$|\tilde{x}(t)| \leq \mu(Q) \exp(-\alpha t) |\tilde{x}(0)|. \quad (20)$$

Proof. The proof of this theorem leverages on the orthogonal transformation T_H . In particular, by definition of Q , we first introduce $\Delta Q(x) := \nabla^2 H(x) - Q \geq 0 \forall x \in \hat{X}$, that is the deviation matrix between $\nabla^2 H(x)$ and Q and is always positive semidefinite on the domain of interest \hat{X} . Then, since

T_H is a constant transformation, it diagonalizes the matrix Q ($:= \nabla^2 H(x_{\min})$) and, hence, also the matrix $\Delta Q(x)$.

We now show by contradiction that the matrix

$$Q(J - R - L^\top g^\top) \Delta Q(x)$$

is negative semidefinite for all $x \in \hat{X}$. Thus, assume it is positive semidefinite, then we have

$$\begin{aligned}Q(J - R - L^\top g^\top) \Delta Q(x) &= \\ &= T_H^\top \Lambda(Q) T_H (J - R - L^\top g^\top) T_H^\top \Lambda(\Delta Q(x)) T_H \\ &\leq \lambda_{\max}(Q) (J - R - L^\top g^\top) \lambda_{\max}(\Delta Q(x)) \leq 0\end{aligned}$$

by the property (19) of L . Hence, we have a contradiction. In particular, we have

$$\begin{aligned}T_H^\top \Lambda(Q) T_H (J - R - L^\top g^\top) T_H^\top \Lambda(\Delta Q(x)) T_H \\ \leq \lambda_{\min}(Q) (J - R - L^\top g^\top) \lambda_{\min}(\Delta Q(x)) = 0\end{aligned}$$

because $\Delta Q(x)$ is positive semidefinite for all $x \in \hat{X}$.

We can now proceed with the convergence proof considering as Lyapunov function $V = \tilde{x}^\top Q \tilde{x}$. Taking the time derivative of V we have

$$\begin{aligned}\dot{V} &= 2\tilde{x}^\top Q(F(\hat{x}, t) - F(x, t)) = 2\tilde{x}^\top Q \int_0^1 \frac{\partial F}{\partial x}(\bar{x}(t, s), t) ds \tilde{x} \\ &= 2\tilde{x}^\top \int_0^1 Q [(J - R - L^\top g^\top) \nabla^2 H(\bar{x}(t, s))] ds \tilde{x} \\ &= 2\tilde{x}^\top \int_0^1 Q [(J - R - L^\top g^\top) (Q + \Delta Q(\bar{x}(t, s)))] ds \tilde{x} \\ &\leq 2\tilde{x}^\top Q [(J - R - L^\top g^\top) Q] \tilde{x} < 0, \quad \forall \tilde{x} \neq 0.\end{aligned}$$

Then, by standard arguments, since $R + \text{sym}(gL)$ is uniformly positive, there exists a positive α such that

$$-R - \text{sym}(gL) \leq -\alpha Q^{-1},$$

so that we have (20) by applying the comparison lemma [34, Lemma 3.4] to the Lyapunov function. \square

Remark 5. A particular case in which Theorem V.2 can be applied is when H is separable, so that its Hessian has a natural diagonal structure and thus T_H is the identity matrix.

Note that our result, compared to [20], is more powerful since it does not involve finding the solution of a Algebraic Riccati Inequality (or ARE with a arbitrary small ϵ , as presented in their paper). But on the other hand, it assumes the existence of a constant transformation matrix T_H to diagonalize the Hessian of H .

C. Observer design: a general case

This result can be easily extended and merged with the results presented in the previous subsection (Sec.V-A), under the existence assumption of such a transformation matrix T_H , if (16) can be satisfied for all $x \in \hat{X}$ by a constant L .

Theorem V.3. Consider (15) with constant matrix g , and consider Q and T_H defined as in Theorem V.2. Then the observer dynamics (9) is convergent on \hat{X} with metric Q if there exists a constant L such that (16) is satisfied for all

$x \in \hat{X}$. Moreover, for some positive α , we can explicitly determine the exponential convergence of the observation error as in (20)

The proof is simply an application of the previous Theorems' proofs and thus it is omitted.

Remark 6. *The results shown in this section hold for any value of u and y , and allow us to determine an exponential convergence of the observed dynamics with convergence rate α . Such a value only depends on the choice of L and, because Q is a fixed metric independent of L , we can possibly make α arbitrarily large by properly defining L . This property paves the way for the design of an observer-based controller that leverages the Separation Principle.*

D. Discussion on the choice of L

The existence of the matrix L satisfying (16) and/or (19), introduced for the observer design, can be determined through the sufficient condition introduced in [25, Lemma 4]. In particular, the authors show that there exists a matrix L , such that for all $x \in \hat{X}$, (16) holds with $R'(x) \geq 0$, if and only if (16) can be 'already' satisfied by $L = g^\top$. Thus, for $R'(x) \geq 0$, this 'observability' condition can be easily checked by first setting $L = g^\top$. We can then set $L = \gamma g^\top$, with positive scalar γ chosen sufficiently high, to possibly obtain a desired convergence rate α .

For $R'(x)$ which is not sign defined, but bounded from below we can still get a similar property as detailed in the following generalization of Lemma 4 in [25].

Lemma 4. *Define \underline{R}' as the minimum value for R' in (17), i.e., for all $x \in \hat{X}$, $R'(x) \geq \underline{R}'$, and assume it to take finite values on \hat{X} . If there exists a $\Gamma = \Gamma^\top$ such that $\underline{R}' + g\Gamma g^\top \geq 0$, then there exists an L such that (16) if and only if (16) is satisfied by $L = (\Gamma + I)g^\top$.*

The proof reads as in [25] when considering $R(x) = \underline{R}' + g\Gamma g^\top \geq 0$, and thus it is omitted.

In this case, to obtain the observer convergence rate and satisfy (16), we can define $L = (\Gamma + \gamma I)g^\top$, with Γ as defined above, and set $\gamma > 0$ according to the desired observer convergence rate.

Note that however, condition (16) is only sufficient and can be restrictive for proving the convergence of the observer dynamics. In particular, it is still possible to obtain the attractiveness of the origin for \tilde{x} with $R'(x) + \text{sym}(gL) \geq 0$ for all $x \in \hat{X}$, if some additional condition on the error dynamics is fulfilled. For linear systems, this condition is easy to find, as shown in [17, Proposition 6] and exploited in the linear case below, and it is related to kernel space of $R + \text{sym}(gL)$, see Proposition 1 below. The extension to the nonlinear case is left for future work and it is currently under development. Moreover, we believe that this type of approach can also lead to a separation principle in the nonlinear context for the class of port-Hamiltonian systems with a constant input matrix because the input signal does not affect the exponential rate of convergence of the observer dynamics.

E. The linear case

A particular case of the scenarios considered above is the linear case, for which we can give a larger characterization of the observer convergence. In particular, it is well-known that for linear systems, the observation error dynamics is autonomous, i.e., (10) is independent from the $u(t)$ and ∇H terms. Thus in this scenario, a sufficient condition for the observer convergence is that (19) is satisfied. However, we can think of the matrix L to be parameterized into a symplectic J_L , positive definite⁶ R_L and the g matrices, i.e., we write

$$L = (J_L + R_L)g^\top.$$

With this parameterization of L , the observer error dynamics (10) reads as

$$\dot{\tilde{x}} = (J + gJ_Lg^\top - (R + gR_Lg^\top))Q\tilde{x} = (J_o - R_o)Q\tilde{x} \quad (21)$$

and if $R_o \geq 0$ we can still have asymptotic stability of the origin for \tilde{x} , as described in the following corollary.

Proposition 1. *Denote by k the dimension of the kernel of $R_o \geq 0$, and assume it is spanned by $\{r_1, \dots, r_k\}$. Then (21) is asymptotically stable if and only if the matrix*

$$(sI - QJ_o) [r_1 | r_2 | \dots | r_k]$$

has rank k , for every $s = j\omega$, $\omega \in \mathbb{R}$.

The proof of this proposition follows the same steps as that for [17, Proposition 6], and it is thus omitted.

We can then characterize the stability of an autonomous linear system, rather than placing its eigenvalues on the open left hand side, by exploiting the positive semidefiniteness of its equivalent resistive part along with some geometrical conditions related to its kernel space.

It is worth noticing that the proposed approach is more general than the one described in [13], since in that work the authors assume to have a Hurwitz $(J_o - R_o)$ matrix, and this might not always be true even in the case of full system observability.

VI. EXAMPLE

Linear gradient: sensorless velocity measurement of PMSM system [26]

We consider here the case of a 3-phase Permanent Magnetic Synchronous Motor (PMSM) in the $d - q$ coordinates, with unmeasured shaft velocity. Its dynamics reads as

$$\begin{aligned} \begin{pmatrix} \dot{\varphi}_d \\ \dot{\varphi}_q \\ \dot{p} \end{pmatrix} &= \left(\begin{bmatrix} 0 & 0 & \varphi_q \\ 0 & 0 & -\varphi_d - \Phi \\ -\varphi_q & \varphi_d + \Phi & 0 \end{bmatrix} - \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & \beta \end{bmatrix} \right) \begin{pmatrix} \frac{\varphi_d}{L} \\ \frac{\varphi_q}{L} \\ \frac{p}{J_m} \end{pmatrix} \\ &+ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} u = [J(x) - R]\nabla H(x) + gu \\ y &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} \frac{\varphi_d}{L} \\ \frac{\varphi_q}{L} \\ \frac{p}{J_m} \end{pmatrix} = g^\top \nabla H(x) \end{aligned}$$

⁶Previously, we considered $R_L = \Gamma + \gamma I$.

where φ_d, φ_q are the stator magnetic fluxes along the d and q axes, respectively, while Φ is the constant rotor magnetic flux, L is the phase inductance, r the phase resistance, p is the shaft momentum, J_m its inertia, and β is the viscous friction coefficient. The control signals u are the two stator phase voltages v_d, v_q , and the available outputs the corresponding phase currents i_d, i_q . The associated Hamiltonian function is then

$$H = \frac{\varphi_d^2}{2L} + \frac{\varphi_q^2}{2L} + \frac{p^2}{2J_m}$$

This model has constant input matrix and linear Hamiltonian gradient, with a diagonal Q matrix, thus it fits precisely into the framework described in Section V-A. In order to design the observer dynamics (9), according to the case analysed in Section V-A, we have to investigate the partial derivative of $J(x)Qx$ with respect to x , i.e.,

$$\frac{\partial (J(x)Qx)}{\partial x} = \frac{\partial}{\partial x} \begin{pmatrix} \varphi_q \frac{p}{J_m} \\ -(\varphi_d + \Phi) \frac{p}{J_m} \\ \frac{\varphi_q}{L} \\ \frac{\varphi_d + \Phi}{L} \end{pmatrix} = \begin{bmatrix} 0 & \frac{p}{J_m} & \frac{\varphi_q}{J_m} \\ -\frac{p}{J_m} & 0 & -\frac{\varphi_d + \Phi}{J_m} \\ \frac{\varphi_q}{L} & 0 & 0 \\ \frac{\varphi_d + \Phi}{L} & 0 & 0 \end{bmatrix}$$

We are then interested in the symmetric part of $\frac{\partial (J(x)Qx)}{\partial x} Q^{-1}$, that is

$$\text{sym} \left(\frac{\partial (J(x)Qx)}{\partial x} Q^{-1} \right) = \begin{bmatrix} 0 & 0 & \frac{1}{2}\varphi_q \\ 0 & 0 & -\frac{1}{2}\varphi_d \\ \frac{1}{2}\varphi_q & -\frac{1}{2}\varphi_d & 0 \end{bmatrix}$$

so that the resulting equivalent resistive matrix $R'(x)$ in (17) is then given by

$$R - \text{sym} \left(\frac{\partial (J(x)Qx)}{\partial x} Q^{-1} \right) = \begin{bmatrix} r & 0 & \frac{1}{2}\varphi_q \\ 0 & r & -\frac{1}{2}\varphi_d \\ \frac{1}{2}\varphi_q & -\frac{1}{2}\varphi_d & \beta \end{bmatrix}.$$

In general, this $R'(x)$ is not sign definite, since its principal minors have no positive sign, i.e.,

$$r > 0; \quad r^2 > 0; \quad r^2\beta - r\frac{1}{4}(\varphi_d^2 + \varphi_q^2) > 0.$$

The last inequality can be analyzed by considering the fact that the supplied current norm $\sqrt{i_d^2 + i_q^2}$ saturates in any practical application⁷. Then, we can write an upper bound for $\varphi_d^2 + \varphi_q^2$, that is valid for any working configuration of the motor, i.e.,

$$\varphi_d^2 + \varphi_q^2 = (i_d^2 + i_q^2)L^2 = i_{\max}^2 L^2. \quad (22)$$

Although a lower bound for the determinant of $R'(x)$ can be found, there is no guarantee that the resistance r is sufficiently large to overcome the norm of the magnetic flux, $\varphi_d^2 + \varphi_q^2$, in any configuration and for any friction coefficient β . We thus consider as output injection matrix $L^\top = \gamma g$, where γ is a positive parameter to be tuned. With this choice the equivalent resistance matrix is

$$R'(x) + \gamma G G^\top = \begin{bmatrix} r + \gamma & 0 & \frac{1}{2}\varphi_q \\ 0 & r + \gamma & -\frac{1}{2}\varphi_d \\ \frac{1}{2}\varphi_q & -\frac{1}{2}\varphi_d & \beta \end{bmatrix}, \quad (23)$$

⁷Moreover, the magnetic fluxes usually saturate even at lower current values.

TABLE I
PMSM SYSTEM PARAMETERS

J_m	0.012	$[Kg m^2]$	L	$3.8 \cdot 10^{-3}$	$[H]$
β	0.0026	$[Nms/rad]$	r	0.225	$[\Omega]$
Φ	0.17	$[Wb]$	i_{\max}	15	$[A]$

whose principal minors are given by

$$r + \gamma > 0; \quad (r + \gamma)^2 > 0; \quad (r + \gamma)^2\beta - (r + \gamma)\frac{1}{4}(\varphi_d^2 + \varphi_q^2).$$

To guarantee that (23) is positive definite, it is easy to see that

$$\gamma \geq \max \left\{ \frac{i_{\max}^2}{4\beta} L^2 - r, 0 \right\}.$$

This allows to guarantee an exponential convergence of the observation error, independently of the applied input voltages.

For the numerical results we set $\gamma = 10$, and we initialize

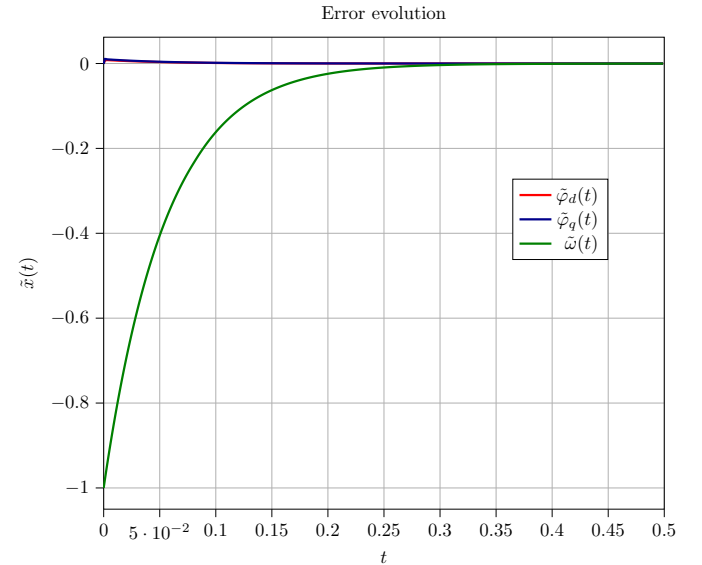


Fig. 2. Error evolution with $u = (v_d, v_q)$ constant $(-4V, 3V)$.

system dynamics at $(\varphi_d(0), \varphi_q(0), \omega(0)) = (0, 0.001, 1)$, and the observer at the origin, $\hat{x}(0) = 0$. We then test the observer dynamics in two different scenarios. We first set the input voltage to a constant value $(v_d, v_q) = (-4V, 3V)$ and the corresponding states evolution is depicted in Fig. 3, while the observer error evolution in Fig. 2. Then we consider the case in which the input voltage has a cosinusoidal evolution, i.e., $(v_d(t), v_q(t)) = (-4 \cos(10t)V, 3 \cos(3t)V)$ as depicted in Fig. 4, and the corresponding states evolution is depicted in Fig. 5, while the observer error evolution in Fig. 6. It is worth noticing, that the observer convergence is independent from the input signal as a consequence of the constant input matrix g .

VII. CONCLUSIONS

In this work, we presented a general overview of the structure-preserving observer for port-Hamiltonian systems.

In particular, we analyzed the observer structure that preserves the passivity for this class of systems and we then

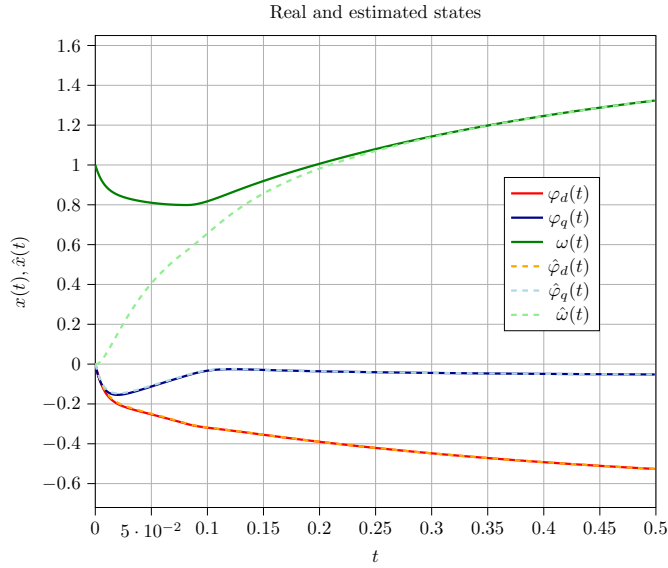


Fig. 3. System and observer states evolution with $u = (v_d, v_q)$ constant $(-4V, 3V)$.

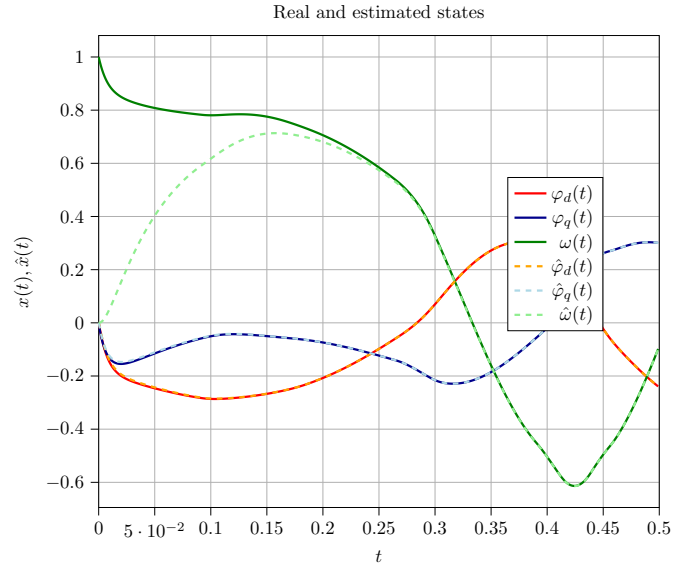


Fig. 5. System and observer states evolution with cosinusoidal inputs $(v_d, v_q) = (-4 \cos(10t)V, 3 \cos(3t)V)$, depicted in Fig. 4.

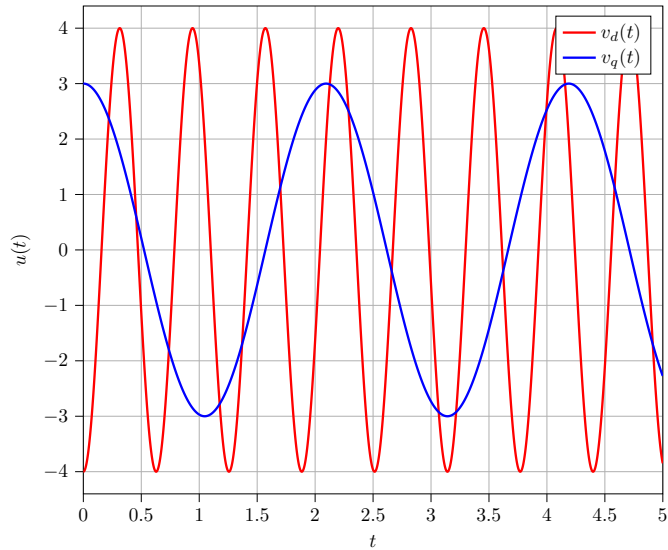


Fig. 4. Cosinusoidal inputs $(v_d, v_q) = (-4 \cos(10t)V, 3 \cos(3t)V)$.

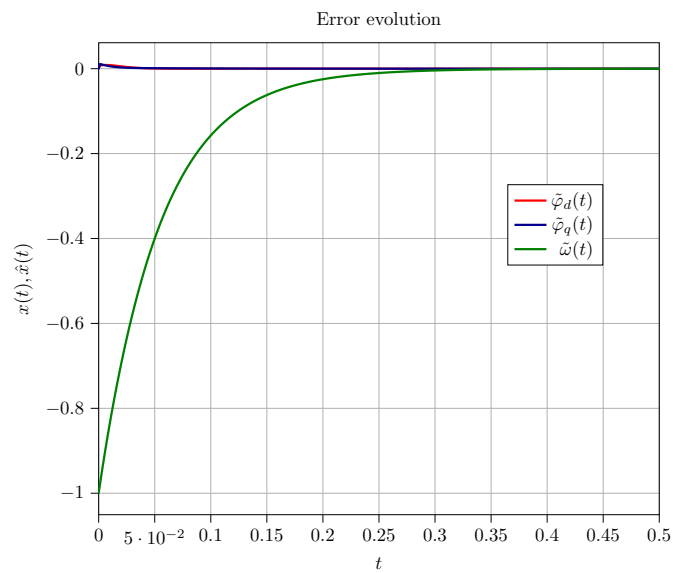


Fig. 6. Error evolution with cosinusoidal inputs $(v_d, v_q) = (-4 \cos(10t)V, 3 \cos(3t)V)$, depicted in Fig. 4.

analyzed the conditions for passivity of the system-error dynamics cascade. We then exploit the convergence/contractive systems approach to determine the convergence of the observer dynamics onto the system dynamics for a class of port-Hamiltonian systems whose input matrix is constant in time.

We conclude the work with an example of industrial interest in which we apply the proposed design paradigm to a PMSM system in order to reconstruct the system state under two different classes of input signals, thus showing the effectiveness of the presented results.

ACKNOWLEDGMENT

This work was supported by the ANR IMPACTS project ANR-21-CE48-0018.

REFERENCES

- [1] B. M. Maschke and A. J. van der Schaft, "Port-controlled Hamiltonian systems: modelling origins and systemtheoretic properties," in *Nonlinear Control Systems Design 1992*. Elsevier, 1993, pp. 359–365.
- [2] A. van der Schaft and B. M. Maschke, "The Hamiltonian formulation of energy conserving physical systems with external ports," *AEÜ International journal of electronics and communications*, vol. 49, no. 5/6, pp. 362–371, 1995.
- [3] V. Duindam, A. Macchelli, S. Stramigioli, and H. Bruyninckx, *Modeling and control of complex physical systems: the port-Hamiltonian approach*. Springer Science & Business Media, 2009.
- [4] A. Van der Schaft, *L₂-gain and passivity techniques in nonlinear control*. Springer, 2000.
- [5] A. Van Der Schaft, D. Jeltsema *et al.*, "Port-Hamiltonian systems theory: An introductory overview," *Foundations and Trends® in Systems and Control*, vol. 1, no. 2-3, pp. 173–378, 2014.
- [6] H. K. Khalil and L. Praly, "High-gain observers in nonlinear feedback

- control,” *International Journal of Robust and Nonlinear Control*, vol. 24, no. 6, pp. 993–1015, 2014.
- [7] B. Anderson and J. Moore, “Optimal filtering,” 1979.
- [8] N. Kazantzis and C. Kravaris, “Nonlinear observer design using Lyapunov’s auxiliary theorem,” *Systems & Control Letters*, vol. 34, no. 5, pp. 241–247, 1998.
- [9] V. Andrieu, B. Jayawardhana, and L. Praly, “Transverse exponential stability and applications,” *IEEE Transactions on Automatic Control*, vol. 61, no. 11, pp. 3396–3411, 2016.
- [10] P. Bernard, V. Andrieu, and D. Astolfi, “Observer design for continuous-time dynamical systems,” *Annual Reviews in Control*, vol. 53, pp. 224–248, 2022.
- [11] R. Ortega, A. Van Der Schaft, B. Maschke, and G. Escobar, “Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems,” *Automatica*, vol. 38, no. 4, pp. 585–596, 2002.
- [12] D. Luenberger, “Observers for multivariable systems,” *IEEE transactions on automatic control*, vol. 11, no. 2, pp. 190–197, 1966.
- [13] S. Zenfari, M. Laabissi, and M. E. Achhab, “Proportional observer design for port Hamiltonian systems using the contraction analysis approach,” *International Journal of Dynamics and Control*, vol. 10, no. 2, pp. 403–408, 2022.
- [14] P. Kotyczka and M. Wang, “Dual observer-based compensator design for linear port-hamiltonian systems,” in *2015 European Control Conference (ECC)*. IEEE, 2015, pp. 2908–2913.
- [15] M. Atitallah, R. El Harabi, and M. N. Abdelkrim, “Fault detection and estimation based on full order unknown input hamiltonian observers,” in *2015 IEEE 12th International Multi-Conference on Systems, Signals & Devices (SSD15)*. IEEE, 2015, pp. 1–7.
- [16] J. Toledo, H. R. Estay, Y. Wu, and Y. Le Gorrec, “Linear matrix inequality design of observer-based controllers for port-hamiltonian systems,” *IEEE Transactions on Automatic Control*, vol. 68, no. 10, pp. 6184–6191, 2023.
- [17] S. Prajna, A. van der Schaft, and G. Meinsma, “An lmi approach to stabilization of linear port-controlled hamiltonian systems,” *Systems & control letters*, vol. 45, no. 5, pp. 371–385, 2002.
- [18] Y. Wang, S. S. Ge, and D. Cheng, “Observer and observer-based h_∞ control of generalized hamiltonian systems,” *Science in China Series F: Information Sciences*, vol. 48, pp. 211–224, 2005.
- [19] A. Venkatraman and A. Van der Schaft, “Full-order observer design for a class of port-Hamiltonian systems,” *Automatica*, vol. 46, no. 3, pp. 555–561, 2010.
- [20] A. Yaghmaei and M. J. Yazdanpanah, “Structure preserving observer design for port-Hamiltonian systems,” *IEEE Transactions on Automatic Control*, vol. 64, no. 3, pp. 1214–1220, 2018.
- [21] B. Biedermann, P. Rosenzweig, and T. Meurer, “Passivity-based observer design for state affine systems using interconnection and damping assignment,” in *2018 IEEE Conference on Decision and Control (CDC)*. IEEE, 2018, pp. 4662–4667.
- [22] B. Biedermann and T. Meurer, “Observer design for a class of nonlinear systems combining dissipativity with interconnection and damping assignment,” *International Journal of Robust and Nonlinear Control*, vol. 31, no. 9, pp. 4064–4080, 2021.
- [23] B. Vincent, N. Hudon, L. Lefèvre, and D. Dochain, “Port-hamiltonian observer design for plasma profile estimation in tokamaks,” *Ifac-PapersOnline*, vol. 49, no. 24, pp. 93–98, 2016.
- [24] M. Rojas, C. Granados-Salazar, and G. Espinosa-Pérez, “Observer design for a class of nonlinear Hamiltonian systems,” *IFAC-PapersOnLine*, vol. 54, no. 19, pp. 125–130, 2021.
- [25] M. Pfeifer, S. Caspart, F. Strehle, and S. Hohmann, “Full-order observer design for a class of nonlinear port-hamiltonian systems,” *IFAC-PapersOnLine*, vol. 54, no. 19, pp. 149–154, 2021.
- [26] N. Vu, T. Pham, I. Prodan, and L. Lefèvre, “Port-hamiltonian observer for state-feedback control design,” in *2023 European Control Conference (ECC)*. IEEE, 2023, pp. 1–6.
- [27] B. P. Demidovich, *Lectures on stability theory*. Nauka, Moscow, 1967, (in Russian).
- [28] J. L. Willems, *Stability theory of dynamical systems*, 1970.
- [29] A. Pavlov, A. Pogromsky, N. van de Wouw, and H. Nijmeijer, “Convergent dynamics, a tribute to boris pavlovich demidovich,” *Systems & Control Letters*, vol. 52, no. 3, pp. 257–261, 2004.
- [30] A. Pavlov, N. Van De Wouw, and H. Nijmeijer, “Convergent systems: analysis and synthesis,” *Control and observer design for nonlinear finite and infinite dimensional systems*, pp. 131–146, 2005.
- [31] —, *Uniform output regulation of nonlinear systems: a convergent dynamics approach*. Springer, 2006, vol. 205.
- [32] W. Lohmiller and J.-J. E. Slotine, “On contraction analysis for non-linear systems,” *Automatica*, vol. 34, no. 6, pp. 683–696, 1998.
- [33] A. Davydov, S. Jafarpour, and F. Bullo, “Non-euclidean contraction theory via semi-inner products,” *arXiv preprint arXiv:2103.12263*, 2021.
- [34] H. K. Khalil, “Nonlinear systems,” *Patience Hall*, vol. 115, 2002.