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Mass transfer development over large rectangular wall region governed by secondary flows



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Mass transfer probes: measurement of wall shear stress

- Advantages:
 - can be used in a wide variety of flow;
 - no interference with the flow;
 - possibility of measuring time-varying flows;
 - in the case of electro-chemical probes: calibration is not necessary.
- Two basic spatial orientations between probe main edge and fluid flow:
 - perpendicular transfer surface, see Fig. 1b);
 - slanted transfer surface, see Fig. 1c).
- Governing equation of mass transport:

$$\frac{\partial c}{\partial t} + u_x \frac{\partial c}{\partial x} + u_y \frac{\partial c}{\partial y} + u_z \frac{\partial c}{\partial z} = D \frac{\partial^2 c}{\partial x^2} + D \frac{\partial^2 c}{\partial y^2} + D \frac{\partial^2 c}{\partial z^2} \quad (1)$$

- boundary conditions:

$$\begin{aligned} x \rightarrow -\infty \quad z > 0 \quad c &= c_\infty \\ x \in (0, L_x) \quad z = 0 \quad c &= c_w \\ x < 0 \cup x > L_x \quad z = 0 \quad \frac{\partial c}{\partial z} &= 0 \\ \forall x \quad z \rightarrow \infty \quad c &= c_\infty \end{aligned} \quad (2)$$

- Assumptions:

- steady or pseudo-steady state $\rightarrow \frac{\partial c}{\partial t} = 0$;
- the flow is homogeneous over the surface of the probe;
- convection is large enough that diffusion in the x direction can be neglected $\rightarrow D \frac{\partial^2 c}{\partial x^2} \approx 0$;
- concentration is changed negligibly in direction $y \rightarrow D \frac{\partial^2 c}{\partial y^2} \approx 0$;
- diffusion boundary layer thickness, δ_D , is smaller than the thickness of hydrodynamical boundary layer, $\delta_H \rightarrow u_x = S_x z$; $u_y = S_y z$, see Fig. 1a);
- a) for perpendicular transfer surface: $u_x \gg u_y, u_z \rightarrow u_y, u_z \approx 0$;
b) for slanted transfer surface: $u_x, u_y \gg u_z \rightarrow u_z \approx 0$.

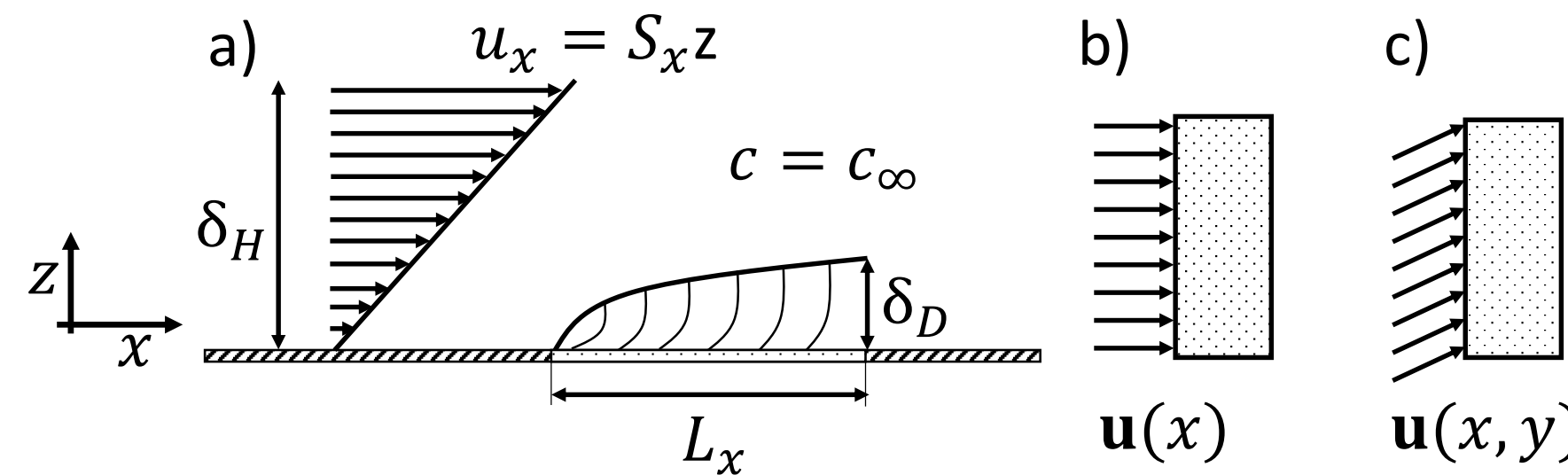


Figure 1: a) Description of mass transfer in the close of the probe; b) perpendicular transfer surface; c) slanted transfer surface.

Perpendicular transfer surface

- Simplification of eq. (1) according to the assumptions (1 - 6):

$$S_x z \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial z^2}; \quad (3)$$

- Analytical solution of eq. (3):

$$\frac{c - c_w}{c_\infty - c_w} = \frac{1}{\Gamma(4/3)} \int_0^\eta e^{-\eta^2} d\eta; \quad \eta = \frac{z}{L_x^{1/3}} \left[\frac{S_x}{9D} \right]^{1/3} \quad (4)$$

- Evaluation of mass transfer coefficient (Lévêque solution) on the base of eqs. (3) and (4):

$$\bar{k}_z = \frac{3}{2\Gamma(4/3)9^{1/3}} \left(\frac{D^2 S_x}{L_x} \right)^{1/3} = 0.8075 \left(\frac{D^2 S_x}{L_x} \right)^{1/3} \quad (5)$$

- Normalized Sherwood number:

$$\text{Sh}^* = \frac{\text{Sh}}{\text{Pe}_s^{1/3}} = \frac{\bar{k}_z L_x}{D} = \bar{k}_z \left(\frac{L_x}{D^2 S_x} \right)^{1/3} = 0.8075 \quad (6)$$

Slanted transfer surface

- Simplification of eq.(1) according to the assumptions (1 - 6):

$$S_x z \frac{\partial c}{\partial x} + S_y z \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial z^2} \quad (7)$$

- No analytical solution of eq. (7), but it is possible to find out:

- analytical solution of l_{eq} and Sh^* ;
- numerical solution of concentration profiles and Sh^* .

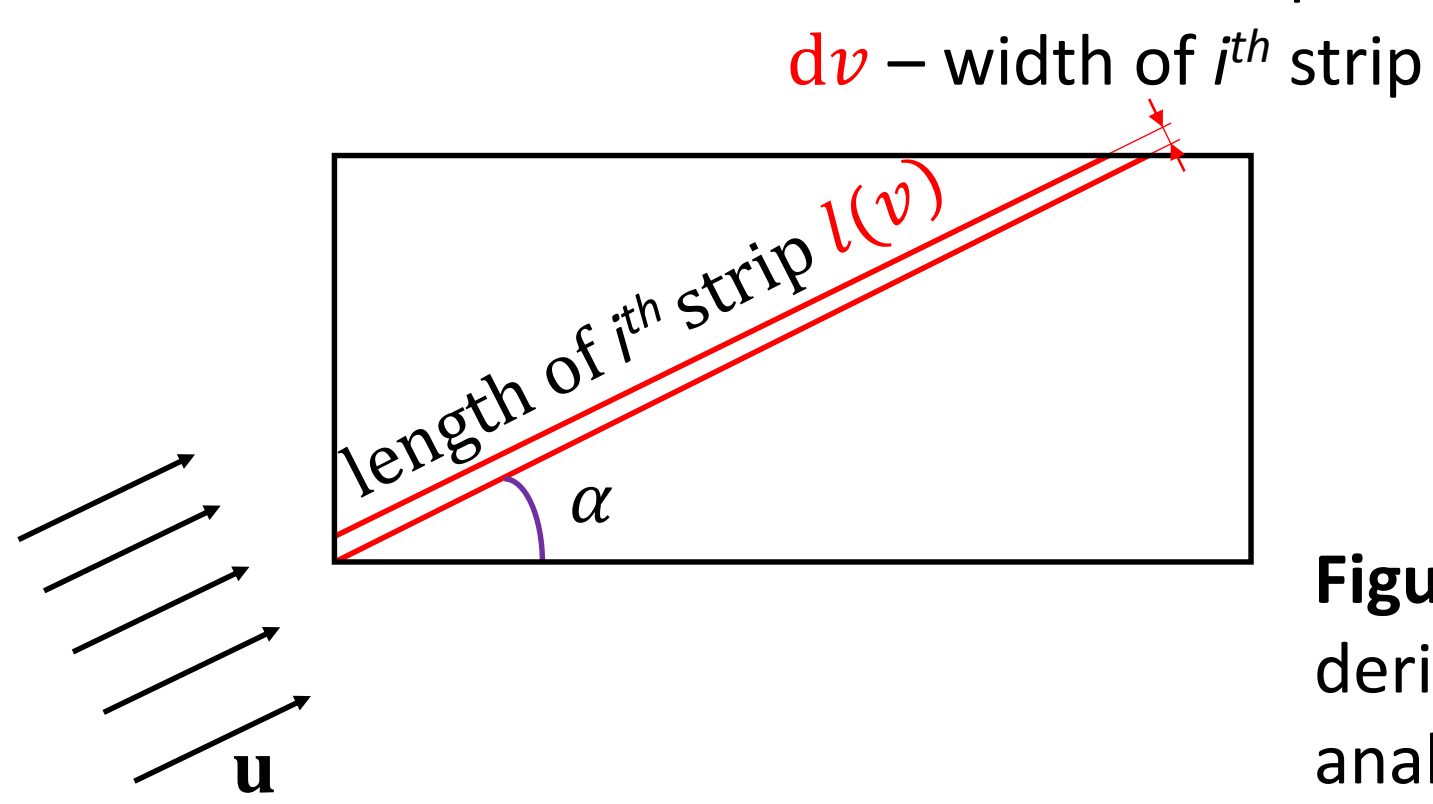


Figure 2: Scheme for derivation of analytical l_{eq} and Sh^*

Analytical solution of equivalent length and normalized Sherwood number

- Mass transfer coefficient for i^{th} strip is defined as:

$$k_z = 0.8075 \left(\frac{D^2 \|S\|}{l(v)} \right)^{1/3} \quad (8)$$

- The mass transfer coefficient for the whole probe is evaluation on the base of summation (integral) of all stripes over the entire surface of the probe. :

$$\bar{k}_z = \frac{1}{L_x L_y} \int k_z l(v) dv = 0.8075 \frac{(D^2 \|S\|)^{1/3}}{L_x L_y} \int l(v)^{2/3} dv = 0.8075 \left(\frac{D^2 \|S\|}{l_{eq}} \right)^{1/3} \quad (9)$$

- Evaluation of equivalent length l_{eq} :

$$l_{eq}^{1/3} = \frac{L_x L_y}{\int l(v)^{2/3} dv} = \begin{cases} \frac{L_x^{1/3}}{(1-\beta)^{1/6} \left(1 + \frac{R\sqrt{\beta}}{5\sqrt{1-\beta}} \right)} & \text{for } \beta \leq \beta_c \\ \frac{L_x^{1/3}}{\beta^{1/6} R^{1/3} \left(1 + \frac{\sqrt{1-\beta}}{5R\sqrt{\beta}} \right)} & \text{for } \beta > \beta_c \end{cases} \quad \text{where } \beta_c = \frac{1}{1+R^2} \quad (10)$$

- The normalized Sherwood number Sh^* is defined as:

$$\text{Sh}^* = \frac{\text{Sh}}{\text{Pe}_s^{1/3}} = \bar{k}_z \left(\frac{l}{D^2 \|S\|} \right)^{1/3} = 0.8075 \left(\frac{D^2 \|S\|}{l_{eq}} \right)^{1/3} \left(\frac{l}{D^2 \|S\|} \right)^{1/3} = 0.8075 \left(\frac{l}{l_{eq}} \right)^{1/3} \quad (11)$$

- The resulting value of the Sh^* is dependent on the choice of length l in the eq. (11), see Fig. 3:

$$\text{Sh}^* = \begin{cases} 0.8075 (1-\beta)^{1/6} \left(1 + \frac{R\sqrt{\beta}}{5\sqrt{1-\beta}} \right) & \text{for } l = L_x; \beta \leq \beta_c \\ 0.8075 \beta^{1/6} R^{1/3} \left(1 + \frac{\sqrt{1-\beta}}{5R\sqrt{\beta}} \right) & \text{for } l = L_x; \beta > \beta_c \\ 0.8075 & \text{for } l = l_{eq}; \forall \beta \end{cases} \quad (12)$$

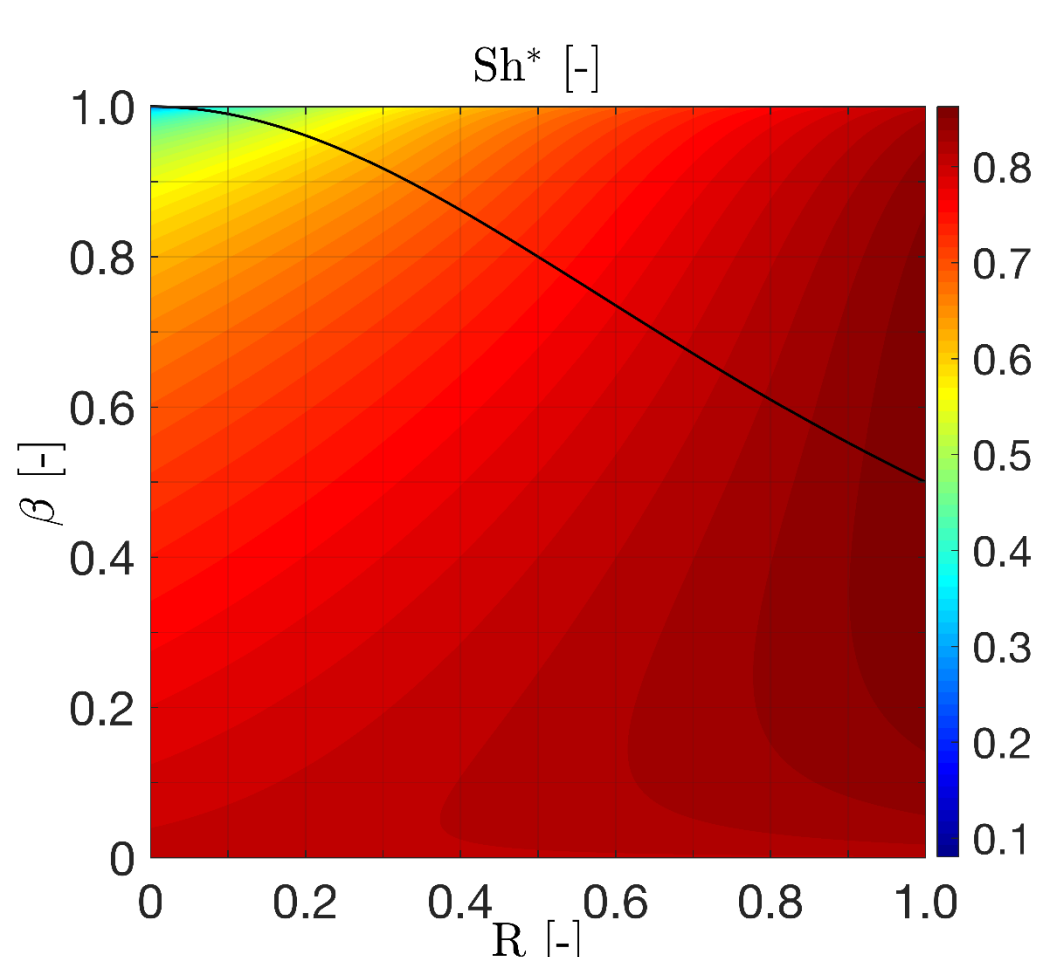


Figure 3: Normalized Sherwood number evaluated for length $l = L_x$ in dependence on the parameters β and ratio R . Black curve describes the value of critical β_c .

Numerical solution of concentrations profile and normalized Sherwood number

- Definition of hydrodynamical parameter β and non-dimensional characteristics of the system:

$$\beta = \frac{S_y^2}{S_x^2 + S_y^2} = \frac{S_y^2}{\|S\|^2} \Rightarrow S_x = \sqrt{1-\beta} \|S\|; S_y = \sqrt{\beta} \|S\| \quad (13)$$

$$c^+ = \frac{c - c_w}{c_\infty - c_w}; x^+ = \frac{x}{L_x}; y^+ = \frac{y}{L_y}; z^+ = \frac{z}{\delta_D}; \delta_D = \left(\frac{Dl}{\|S\|} \right)^{1/3}; R = \frac{L_x}{L_y} \quad (14)$$

- Modification of eq. (7):

$$\frac{\partial c^+}{\partial x^+} + \frac{\sqrt{\beta} R}{\sqrt{1-\beta}} \frac{\partial c^+}{\partial y^+} = \frac{1}{z^+ \sqrt{1-\beta}} \frac{\partial^2 c^+}{(\partial z^+)^2} \quad (15)$$

- Numerical solution of eq. (7), see Fig. 4. The shape of the concentration profiles is dependent on the parameters β and ratio R .

- Sh^* is evaluated from derivative of concentration in the z direction on the probe surface using a relation, see Fig. 5:

$$\text{Sh}^* = \int_0^1 \int_0^1 - \frac{\partial c^+}{\partial z^+} dx^+ dy^+ \quad (16)$$

- Final values of Sh^* is dependent on the choice of length l . For example, the length l can be replaced by the width of the electrode L_x or by the equivalent length:

- $l = L_x$ and for $\forall \beta$:

$$\frac{\partial c^+}{\partial x^+} + \frac{\sqrt{\beta} R}{\sqrt{1-\beta}} \frac{\partial c^+}{\partial y^+} = \frac{1}{z^+ \sqrt{1-\beta}} \frac{\partial^2 c^+}{(\partial z^+)^2} \Rightarrow \text{Sh}^*(R, \beta), \text{ see Fig. 3} \quad (17)$$

- $l = l_{eq}$ and for $\beta \leq \beta_c$:

$$\frac{\partial c^+}{\partial x^+} + \frac{\sqrt{\beta} R}{\sqrt{1-\beta}} \frac{\partial c^+}{\partial y^+} = \frac{1}{z^+ \sqrt{1-\beta}} \sqrt{1-\beta} \left(1 + \frac{R\sqrt{\beta}}{5\sqrt{1-\beta}} \right)^3 \frac{\partial^2 c^+}{(\partial z^+)^2} \Rightarrow \text{Sh}^* = 0.8075 \quad (18)$$

- $l = l_{eq}$ and for $\beta > \beta_c$:

$$\frac{\partial c^+}{\partial x^+} + \frac{\sqrt{\beta} R}{\sqrt{1-\beta}} \frac{\partial c^+}{\partial y^+} = \frac{1}{z^+ \sqrt{1-\beta}} \sqrt{\beta} R \left(1 + \frac{\sqrt{1-\beta}}{5R\sqrt{\beta}} \right)^3 \frac{\partial^2 c^+}{(\partial z^+)^2} \Rightarrow \text{Sh}^* = 0.8075 \quad (19)$$

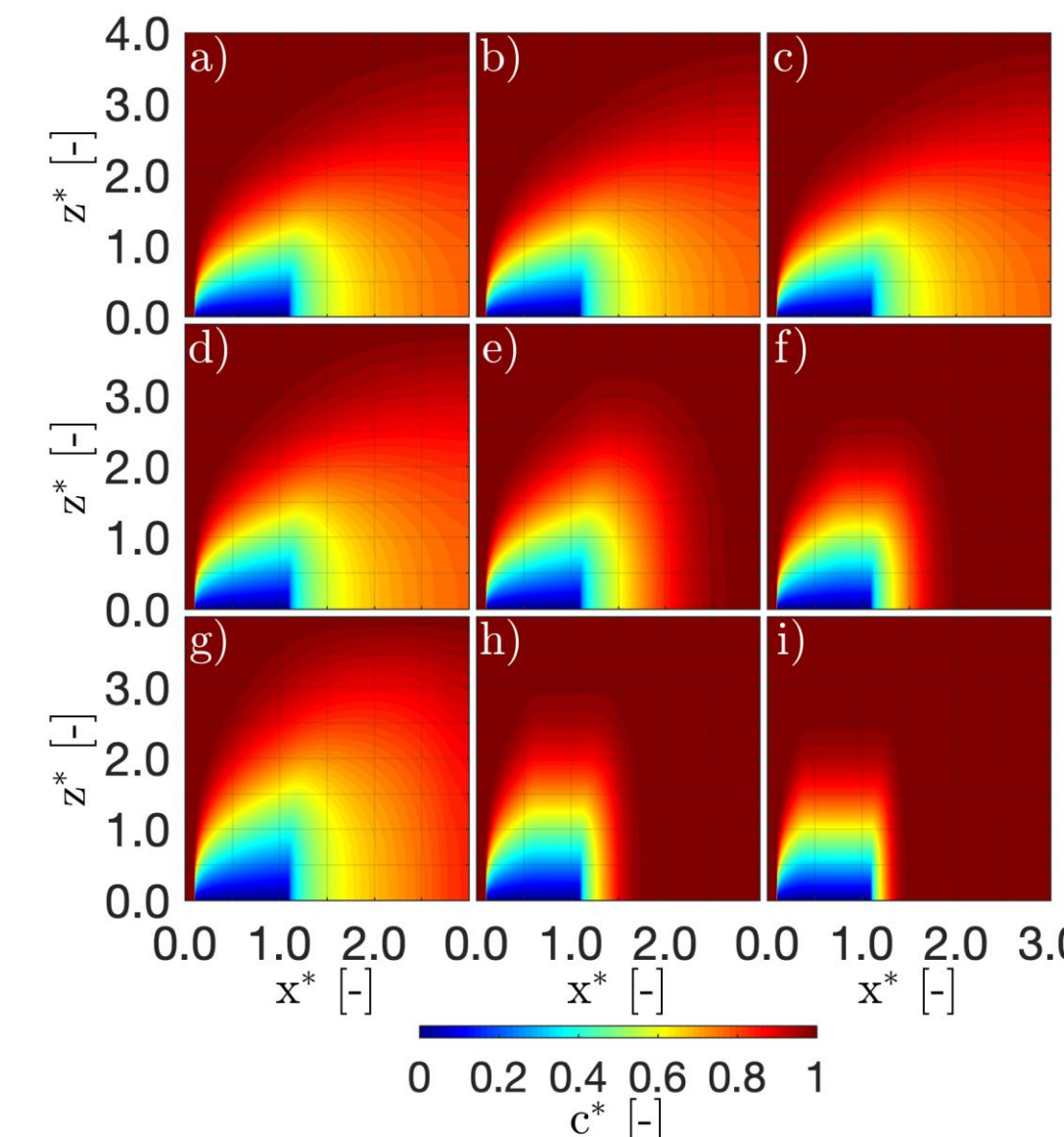


Figure 4: Concentration profile in the plane x^+z^+ for $y^+ = L_y/2$: 1st row - $\beta = 0$; 2nd row - $\beta = 0.4$; 3rd row - $\beta = 0.8$; 1st column - $R = 0.1$; 2nd column - $R = 0.4$; 3rd column - $R = 0.8$.

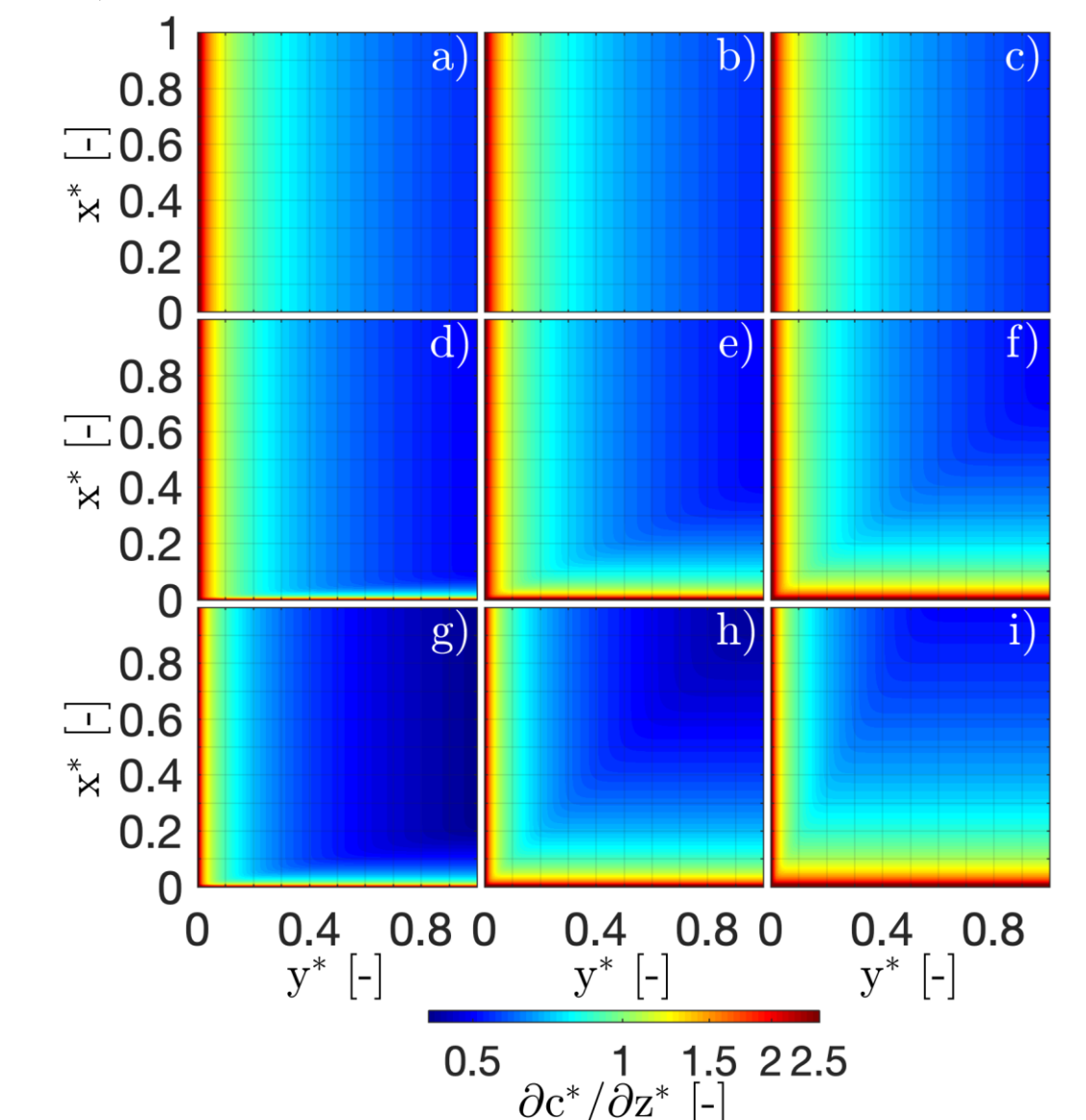


Figure 5: Derivative of concentration in the z direction on the probe surface (logarithmic colorbar): 1st row - $\beta = 0$; 2nd row - $\beta = 0.4$; 3rd row - $\beta = 0.8$; 1st column - $R = 0.1$; 2nd column - $R = 0.4$; 3rd column - $R = 0.8$.

Conclusion:

We have derived an equation describing the equivalent length and normalized Sherwood number as a function of the fluid flow angle (characterized by parameter β) and the aspect ratio of the rectangular mass transfer probe ($R = L_x/L_y$). The given equations could serve for better interpretation of the measurement of wall shear stress (ED or hot-film method) especially for fluid flow associated with secondary flows.

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