Temporal Hyperbolic Graphs as Null Models for Brain Dynamics

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1 Introduction

Null models are crucial for determining the degree of significance when testing hypotheses about brain dynamics modeled as a temporal complex network [7, 12]. The comparison between the hypothesis being tested on empirical data and on the null model enables us to assess the extent to which an apparently remarkable feature of the former can be attributed to randomness.

In this work, we consider networks generated from resting-state functional magnetic resonance imaging (fMRI) signals from the Human Connectome Project [1] using standard processing techniques [9]. The network nodes correspond to brain regions while the edges represent the presence of a correlation greater than a certain value between the signals of two regions. Over time, edges may appear and disappear, indicating that the associated correlation may jump above and below a certain value.

While null models for static networks have been studied extensively [12, 4, 2, 6], there is a lack of attention to temporal networks. Therefore, our investigation focuses on the study of temporal null models. In doing so, we focus on the ability of the null models to reproduce the temporal small-worldness present in the empirical data, a property associated with the efficient exchange of information at local and global scales over time.

Specifically, we compare two models taken from the literature [11], which we will refer to as the Random Temporal Edge Graph Model and Random Temporal Permuted Times Graph Model, and three models that have not been previously considered in the context of brain dynamics which are the Random Temporal Square Graph Model, the Random Temporal Torus Graph Model [8] and the Random Temporal Hyperbolic Graph Model.

The latter is a temporal version of the Random Hyperbolic Graph Model which has gained prominence in the complex network research community for its ability to capture some important properties of real-world networks, such as small-worldness and a high-tail degree distribution [5]. In addition, a recent paper shows that brain networks are best represented in the hyperbolic disk, which is a projection of hyperbolic space, showing relevance to its anatomical counterpart [14]. Our analysis indicates that it is also a particularly suitable null model for reproducing the temporal small-worldness observed in the empirical data.

2 Methods

Models We consider five random temporal graph models. The Random Temporal Permuted Times Graph (RTPT) Model is created by randomizing the timing of each contact in the real data network [11]. Similarly, the Random Temporal Edge Graph (RTE) Model is a permutation of the real data network where, in each snapshot, one end of an edge is reassigned to a different node [11].

The Random Temporal Square Graph (RTS) Model and the Random Temporal Torus Graph (RTT) Model are created by randomly placing nodes within a square and a torus, respectively and then connecting them if they are within a certain distance [8]. To introduce a temporal dependency to the models, the node positions are updated at each time step by applying a displacement vector with a uniformly chosen direction in $[0, 2\pi)$ and length in $(0, v)$, where v is the speed parameter of the models, and recomputing the edges accordingly.

The Random Temporal Hyperbolic Graph (RTH) Model is a temporal variant of Krioukov's Random Hyperbolic Graph (RH) model [5]. Geometrically, nodes are quasi-uniformly distributed within a disk with a radius of R, which is centered on the upper half of a hyperboloid with a specified negative curvature of $K = -\zeta^2$. Formally, writing the node positions in polar coordinates (r, θ), where $r \in [0, R]$ and $\theta \in [0, 2\pi]$, we have that the angular density is $\rho(\theta) = \frac{1}{2\pi}$ and the radial coordinate density is $\rho(r) = \alpha \frac{\sinh(\alpha r)}{\cosh(\alpha R)-1}$. The α parameter controls the spread of the point positions. If $\alpha = \zeta$, the points follow a uniform distribution. If $\alpha > \zeta$, the points are more likely to be near the border of the disk, otherwise, they are more likely to be near the center. Two nodes, x and y, are connected if they are within hyperbolic distance $d_H < R$, where, considering the polar coordinate (r_x, θ_x) of x and (r_y, θ_y) of y, d_H is defined as

$$d_H(x,y) = \frac{1}{\zeta} \operatorname{acosh}(\cosh \zeta r_x \cosh \zeta r_y - \sinh \zeta r_x \sinh \zeta r_y \cos(\pi - |\pi - |\theta_x - \theta_y||)).$$

To obtain the temporal version, we update the point position for as many time steps as there are in the real data networks we want to compare to. The point position update is chosen to ensure that the marginal distribution at each time step is the same as the initial distribution. Specifically, the polar coordinate is updated by adding to the previous value a number chosen according to a uniform distribution over $(0, v)$, and then computing the modulo 2π. For the radial coordinate, we add to the previous value a number chosen according to a uniform distribution over $(-v, v)$, and we reflect the result in the interval $[0, 1]$ to keep the distribution uniform in $[0, 1]$.

As for the curvature parameter ζ, we set it to $\zeta = 1$, since (Lemma 1.1 of [3]) if two RH graphs have the same ratio $\frac{\zeta}{\alpha}$ and the other parameters are equal, then they produce the same distribution on graphs. The parameters of the model are v and α.

Temporal Small-Worldness The first static definition of small-worldness of a graph G was introduced by Watts and Strogatz [13] as the combination of a high clustering coefficient C_G (as in lattice graphs) and a short average path length L_G (as in Erdős-Rènyi graphs). The previous measures are combined into the ratio $\frac{C_G}{L_G}$. If the ratio is greater than 1, it indicates a significant small-world property of the network. To apply

this to temporal networks, it is necessary to modify the previous measures to take into account the time evolution. We thus define the temporal small-worldness of a temporal graph G_T

$$S_{G_T} = \frac{TC_{G_T}}{TL_{G_T}}$$

where TC is the average of the global clustering coefficient over time and TL is defined as the average of the shortest path length between all pairs of nodes over time.

Optimization of the parameters The model parameters are determined using an optimization method that minimizes the area between the temporal small-worldness curves of the empirical data and the random temporal model over all different average degrees (see Fig. 1). The curves are created by interpolating the temporal small-worldness values at different average degrees. The average degrees differ based on the correlation threshold variation in the cases of empirical networks and the RTPT and RTE models, as well as the variation of the connection radius in the RTS, RTT, and RTH models.

3 Results

In Fig. 1, the blue lines are the values of the empirical network of 1042 subjects, created with the Schaefer atlas of 302 regions [10]. For all the models, not depending on the real data, the illustrated lines are the median of the temporal small-worldness values over 10 random model realizations, instead, in the case of the real data, the RTE and RTPT models, the lines correspond to the median over 1042 subjects. The shadows correspond to the values between the first and third quartiles. Note that for RTT, RTS and RTH graph models the shadows are hardly visible given the minimal variance. Looking at the random models, the median of the RTPT Graph Model has higher values with respect to those of the RTE Graph Model. We can observe that the RTT and RTS models perform well on high-average degree regimes. However, the RTH graph model not only exhibits the closest temporal small-worldness values but also follows the same trend as the empirical data.

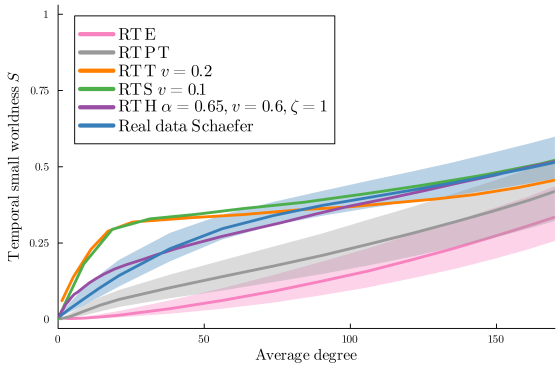

Fig. 1.

Acknowledgments

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