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Unfolding engineering metamaterials design: relaxed micromorphic modeling of large-scale acoustic meta-structures [1]

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Introduction

Metamaterial (from μετά, "beyond"): Architectured material showing exotic mechanical characteristics that cannot be found in nature.

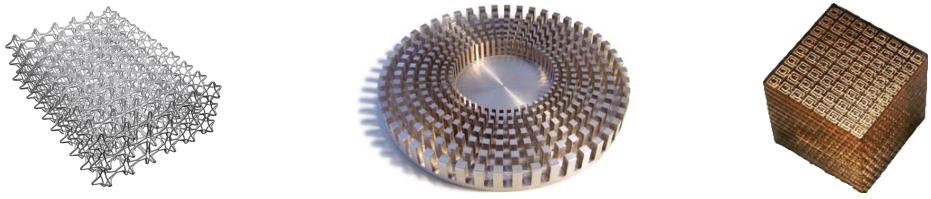


Figure 1: (*left to right*) (a) Foam with negative Poisson ratio [Lakes 1988]. (b) Seismic cloack [Guenneau 2014]. (c) Ultrasound Lense [Fang 2006].

Band-gap metamaterials inhibit wave propagation in particular frequency ranges.

Such effect can be achieved by:

- Bragg scattering
- local resonance

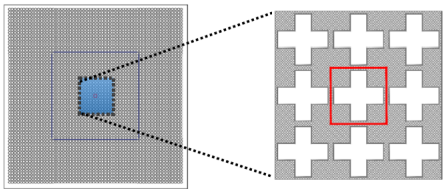
Modeling metamaterials: the relaxed micromorphic model

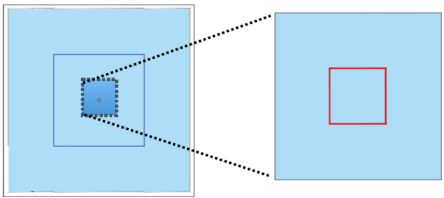
$\longrightarrow$ Such materials could be advantageously used in mechanical engineering

$\times$ Classical models are not suited for such structures

A homogenized model is required:

$\longrightarrow$ The relaxed micromorphic model allows such simplification

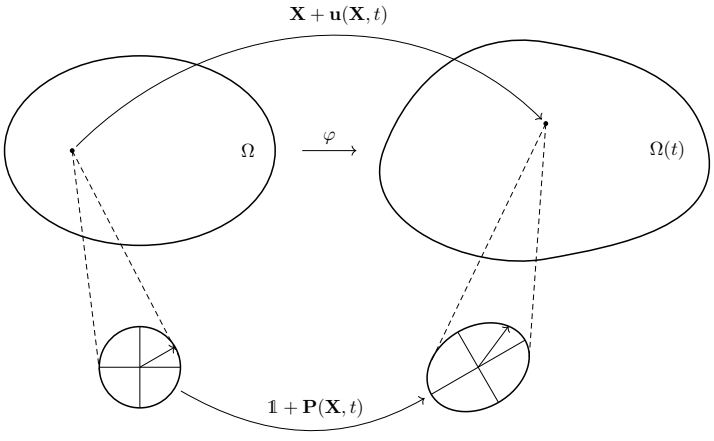




We present here:

- Design and manufacturing of such metastructures
- Experimental characterization and numerical modelling

Enriching the kinematic description of our media



Displacement: $\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ Microdistorsion: $\mathbf{P} = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix}$

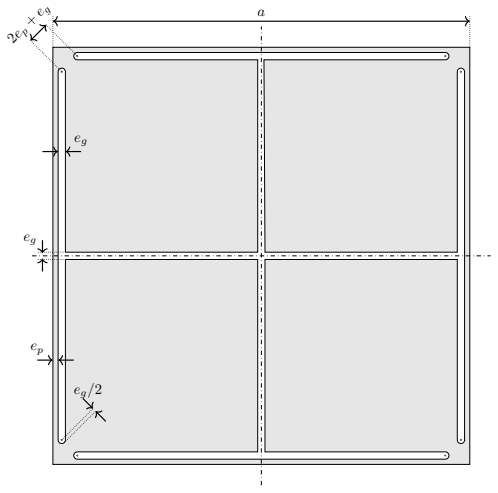
Lagrangian density of the relaxed micromorphic model

Generalized strain w_m and kinetic k_m energy density:

$$
\begin{aligned}
k_m(\dot{\mathbf{u}}, \boldsymbol{\nabla}\dot{\mathbf{u}}, \dot{\mathbf{P}}) &= \frac{1}{2}\langle \dot{\mathbf{u}}, \rho_m \dot{\mathbf{u}} \rangle \text{ (Cauchy inertia)} \\
&+ \frac{1}{2}\langle \operatorname{sym} \dot{\mathbf{P}}, \mathbb{J}_m \operatorname{sym} \dot{\mathbf{P}} \rangle + \frac{1}{2}\langle \operatorname{skew} \dot{\mathbf{P}}, \mathbb{J}_c \operatorname{skew} \dot{\mathbf{P}} \rangle \text{ (free micro-inertia)} \\
&+ \frac{1}{2}\langle \operatorname{sym} \boldsymbol{\nabla}\dot{\mathbf{u}}, \mathbb{T}_e \operatorname{sym} \boldsymbol{\nabla}\dot{\mathbf{u}} \rangle + \frac{1}{2}\langle \operatorname{skew} \boldsymbol{\nabla}\dot{\mathbf{u}}, \mathbb{T}_c \operatorname{skew} \boldsymbol{\nabla}\dot{\mathbf{u}} \rangle \text{ (gradient micro-inertia)} \\
w_m(\boldsymbol{\nabla}\mathbf{u}, \mathbf{P}) &= \frac{1}{2}\langle \operatorname{sym}(\boldsymbol{\nabla}\mathbf{u} - \mathbf{P}), \mathbb{C}_e \operatorname{sym}(\boldsymbol{\nabla}\mathbf{u} - \mathbf{P}) \rangle \text{ (elastic energy)} \\
&+ \frac{1}{2}\langle \operatorname{sym} \mathbf{P}, \mathbb{C}_m \operatorname{sym} \mathbf{P} \rangle \text{ (micro self energy)} \\
&+ \frac{1}{2}\langle \operatorname{skew}(\boldsymbol{\nabla}\mathbf{u} - \mathbf{P}), \mathbb{C}_c \operatorname{skew}(\boldsymbol{\nabla}\mathbf{u} - \mathbf{P}) \rangle \text{ (local rotational elastic coupling)}
\end{aligned}
$$

$\longrightarrow$ Systems are modeled by the introduction of an appropriate Action

The metamaterial unit cell



a	20	[mm]
e_g	0.35	[mm]
e_p	0.25	[mm]
ρ_{Ti}	4400	[kg/m^3]
E_{Ti}	112	[GPa]
ν_{Ti}	0.34	-

Figure 2: (*left*) Geometry of the architectured cell with its parametrization and (*right*) its mechanical properties (titanium alloy TA6V).

Fitting of the dispersion curves

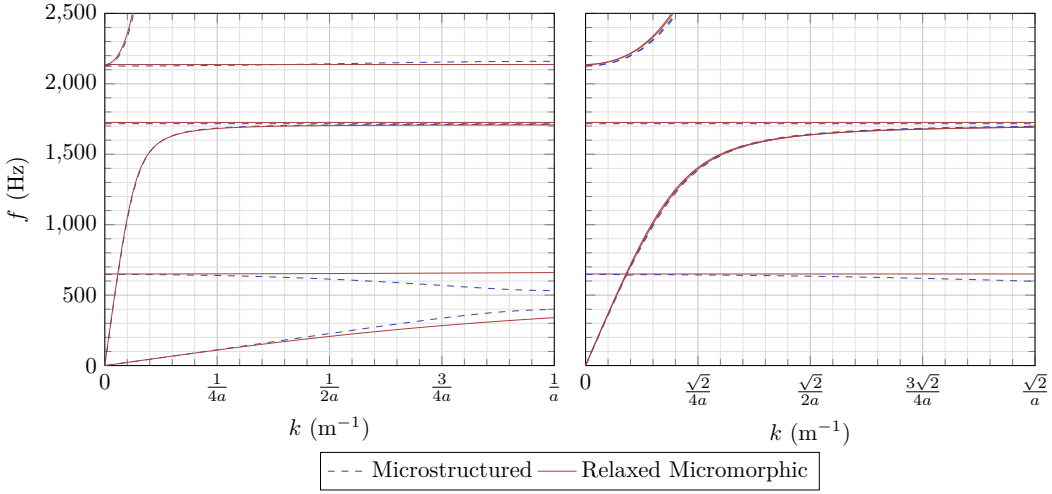
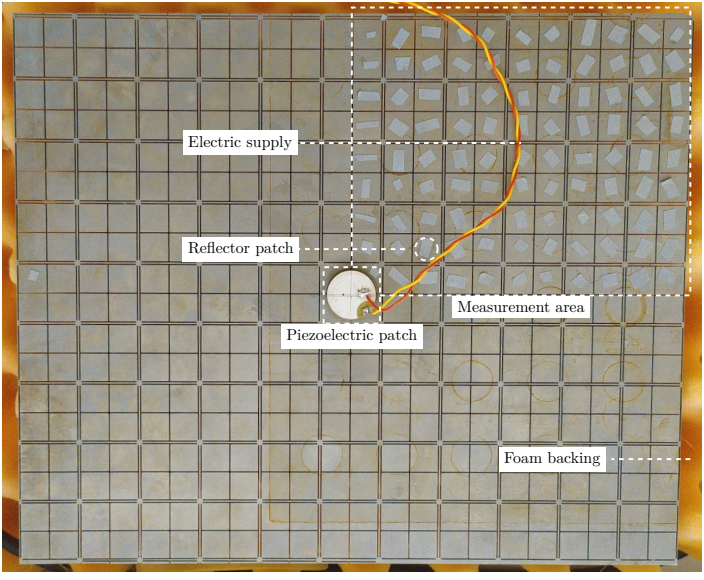
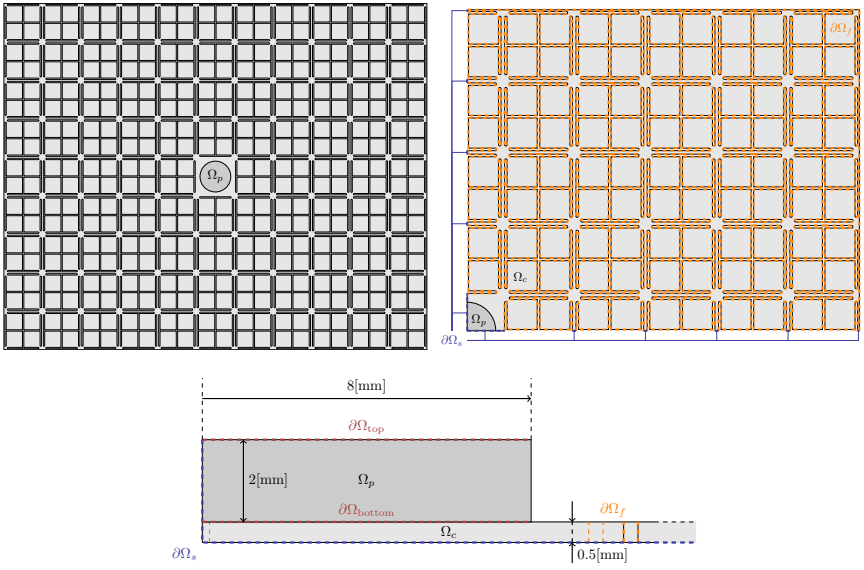


Figure 3: Dispersion curves of the microstructured and the relaxed micromorphic systems along ΓX (*left*, propagation at 0°) and ΓM (*right*, propagation at 45°).

Experimental set-up of the *proof-of-concept*



Comparison with the experiment: the microstructured plate



Modelling the microstructured plate

Action functional of the system:

$$\mathcal{A}_{\text{int}}\left[\mathbf{u}, V\right] = \int_{t_1}^{t_2} \left[\int_{\Omega_p} (k_p - w_p - q) \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 + \frac{e}{2} \int_{\Omega_c} (k_c - w_c) \mathrm{d}x_1 \mathrm{d}x_2 \right] \mathrm{d}t$$

Plain strain hypothesis in the microstructured plate:

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ 0 \end{pmatrix} \text{ in } \Omega_c, \ \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \text{ in } \Omega_p \text{ and } u_3 = 0 \text{ on } \Omega_{\text{bottom}}$$

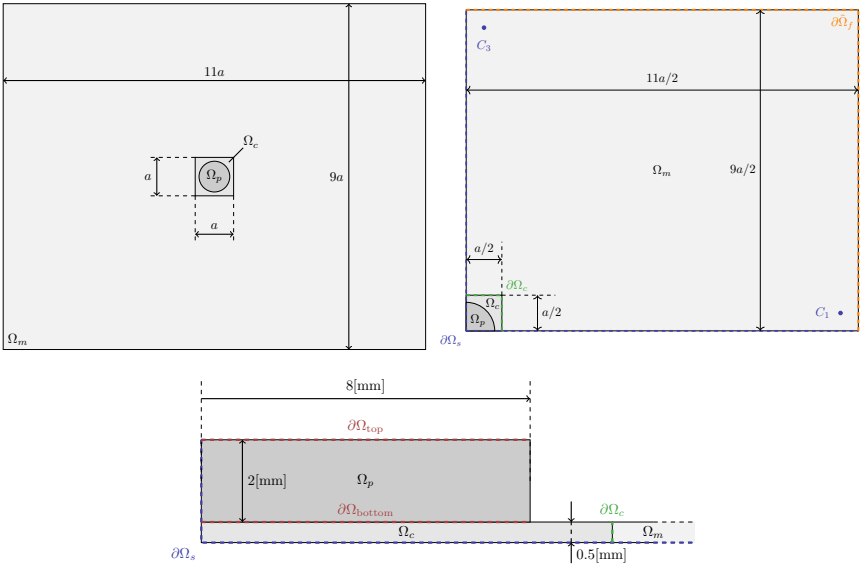
Piezoelectric excitation and symmetry conditions:

$$\begin{cases} V = 0 & \text{on } \partial\Omega_{\text{bottom}} \\ V = V_0 & \text{on } \partial\Omega_{\text{top}} \end{cases} \quad \text{and} \quad \begin{cases} \langle \mathbf{u}, \mathbf{n} \rangle = 0 \\ \langle \mathbf{D}, \mathbf{n} \rangle = 0 \end{cases} \quad \text{on } \partial\Omega_s$$

Computation of the FRF under Comsol:

$$FRF(\mathbf{x}) = \frac{|\dot{\mathbf{u}}|}{V_0} \qquad \text{on} \quad \mathbf{x} = C_1 = (0.105, 0.005) \text{ [m]}$$

Modelling for the relaxed micromorphic medium



Action functional of the relaxed micromorphic model

Action functional of the system:

$$\mathcal{A}_{\text{int}}[\mathbf{u}, \mathbf{P}, V] = \int_{t_1}^{t_2} \left[\int_{\Omega_p} (k_p - w_p - q) \mathrm{d}\Omega + \frac{e}{2} \int_{\Omega_c} (k_c - w_c) \mathrm{d}\Gamma \right] \mathrm{d}t$$
$$+ \frac{e}{2} \int_{t_1}^{t_2} \int_{\Omega_m} (k_m - w_m) \mathrm{d}\Gamma \mathrm{d}t$$

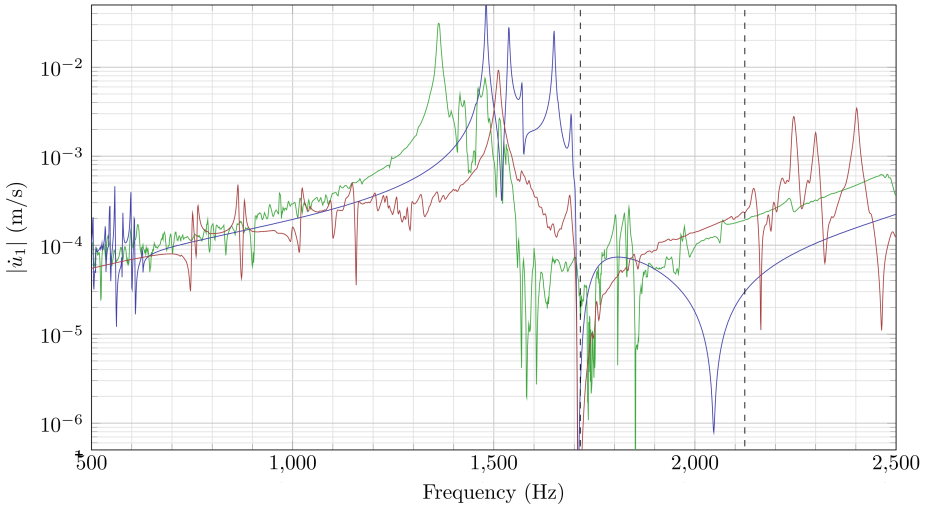
Plane strain hypothesis in the plate:

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \text{ in } \Omega_p, \ \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ 0 \end{pmatrix} \text{ in } \Omega_c, \ \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ 0 \end{pmatrix} \text{ and } \mathbf{P} = \begin{pmatrix} P_{11} & P_{12} & 0 \\ P_{21} & P_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ in } \Omega_m$$

Traction-free and symmetry boundary conditions:

$$\begin{cases} (\widehat{\sigma}_{ij} + \widetilde{\sigma}_{ij}) n_j = 0 & \text{on } \partial\tilde{\Omega}_f \\ (\widehat{\sigma}_{ij} + \widetilde{\sigma}_{ij}) n_j = \sigma^c_{ij} n_j & \text{on } \partial\Omega_c \end{cases} \text{ and } \begin{cases} u_i n_i = 0 \\ (\delta_{ki} - n_k n_i)(P_{ij} n_j) = 0 \end{cases} \text{ on } \partial\Omega_s$$

Comparison with the experiment in C_1



Recalibration and perturbation of the model

× Shift between the theoretical FRFs and the experimental one
× Theoretical band gap "deeper" than the experimental one

⟶ Reconsider the hypotheses made in the framework of our modelling:

- linear hypothesis
- idealized geometry
- plane strain hypothesis
- idealized constitutive laws
- idealized boundary conditions

⟶Recalibration of the parameters:

$$E^{\text{recalibration}} = (1+\kappa_w)E_{\text{Ti}} \qquad \text{and} \qquad \rho_c^{\text{recalibration}} = (1+\kappa_k)\rho_{\text{Ti}}$$

⟶Perturbation (θ_ρ and θ_E follow a zero-mean probability law):

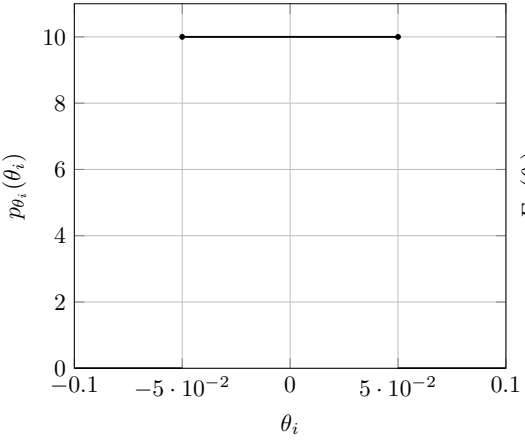
$$\begin{cases} \rho_c(\mathbf{x}) = (1+\theta_\rho(\mathbf{x}))\rho_c^{\text{recalibration}} = (1+\theta_\rho(\mathbf{x}))(1+\kappa_k)\rho_{\text{Ti}} \\ E(\mathbf{x}) = (1+\theta_E(\mathbf{x}))E^{\text{recalibration}} = (1+\theta_E(\mathbf{x}))(1+\kappa_w)E_{\text{Ti}} \end{cases}$$

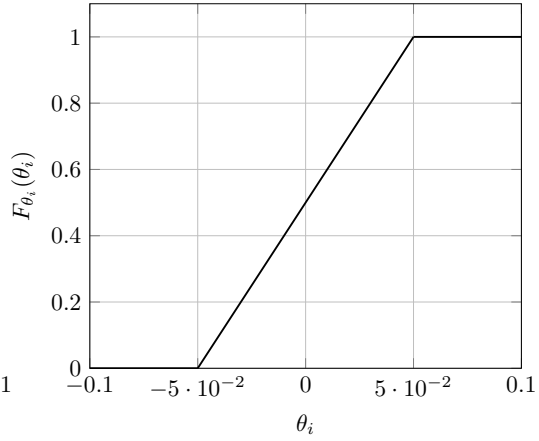
Recalibration and perturbations chosen

$\longrightarrow$ Optimal values of recalibration parameters:

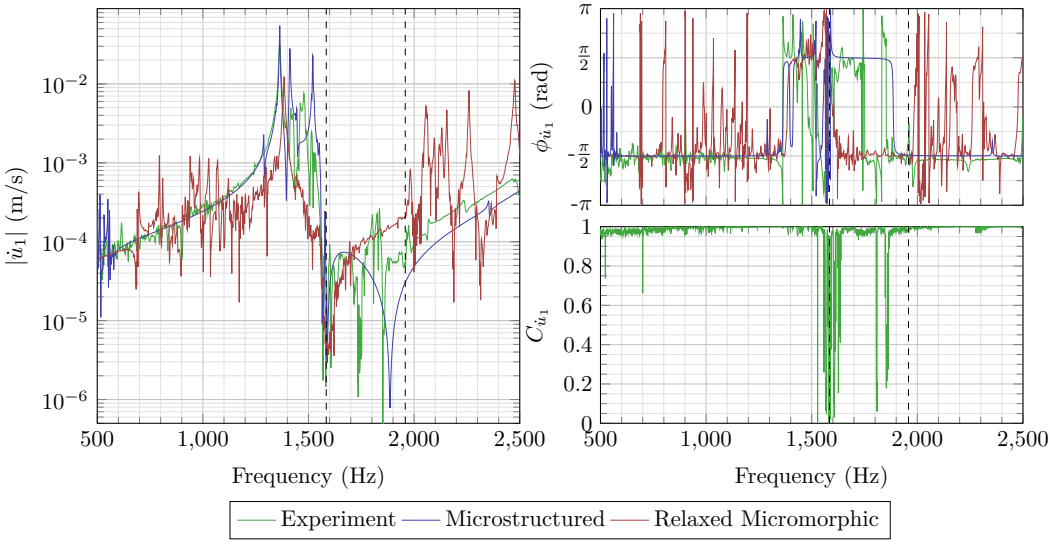
$$\kappa_k = +0.05 \qquad \text{and} \qquad \kappa_w = -0.10875 \tag{1}$$

$\longrightarrow$ Continuous uniform distribution used for both models:





Test/computation comparison of $\dot{u}_1$ at C_1



Response of the plate at 1589 Hz

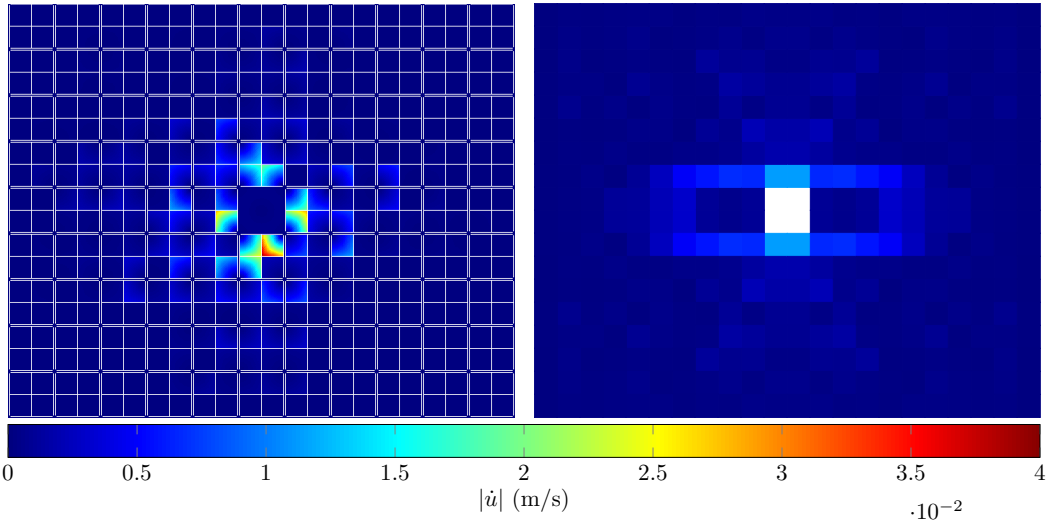


Figure 4: $|\dot{\mathbf{u}}|$ for the microstructured model (*left*) and for the symmetrized experiment (*right*).

Conclusions and perspectives

Achievements:

- Design and characterization of a metastructure with band-gap
- Numerical validation of the relaxed micromorphic model
- Integration of metamaterials in new structures
- Successful comparison with the experiment:
  Manufacturing defects taken into account

Perspectives:

- Manufacturing of energy focusing metastructures
- Take into account mechanical damping
- Improvement of the perturbation procedure:
  Limited to the band-gap frequencies

Thank you for your attention!

Questions?