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Holonic Fuzzy Agents for Integrated CAD Product and Adaptive Manufacturing Cell Formation

A.-J. Fougères^{a*} and E. Ostrosi^b

^a *ECAM Rennes, Campus de Ker Lann - Bruz, Rennes, France*

^b *Université de Bourgogne Franche-Comté, UTBM, Pôle Industrie 4.0, Pôle ERgonomie et Conception des Systèmes ERCOS/ELLIADD EA4661, Belfort, France*

Abstract Cloud-based design and manufacturing is a dynamic service-oriented network where a set of CAD products is manufactured in virtual digital cells under certain constraints. This paper assumes that these CAD products can be modelled by a set of holonic fuzzy agents and defined from a set of fuzzy feature agents which themselves can be defined by a set of fuzzy face agents. Integrating the uncertainty in the modelling of the feature-product-workcenter-cell network by holonic fuzzy agents, this paper proposes an approach to address adaptive virtual manufacturing cell formation in cloud-based manufacturing. These cells are thus supported by digital technology. Intelligent manufacturing features, modelled as holonic fuzzy feature agents, are defined in CAD product models. Subsequently, the distributed capabilities of resources in cloud manufacturing are evaluated through holonic fuzzy agents, and the holonic structure of adaptive virtual manufacturing cells is formed by these agents. The software agent paradigm brings its reactivity, its dynamicity, its agility, and, thus, its adaptability to the holonic structure. The presentation of a scenario causing different changes in workcenters (e.g., unavailability of machines or addition of new products) illustrates the behaviour of holonic fuzzy agents during the reconfiguration of the virtual manufacturing cells holonic structure, thereby demonstrating the adaptive capacity of this kind of holonic structure.

Keywords: Intelligent manufacturing, adaptive virtual manufacturing cell, multiagent, holonic fuzzy agent

1 Introduction

Sustainable development should offer a holistic way of resolving many sharp objective conflicting goals in product design and manufacturing. Sustainability has also been defined as the opposite of crisis. If crisis is defined as the inability of a system to reproduce itself, then sustainability is the opposite: the long-term ability of a system to reproduce [Campbell, 1996]. This criterion is quite interesting because it is the essential criterion of life. It applies to natural ecosystems, economic and social systems, and also, artificial systems such as designs of products and manufacturing systems.

Products and manufacturing systems should be self-sustaining objects. The global model should also be self-sustaining. Therefore, the modelling of components needs to be self-sustaining. The ways in which these system components (product, manufacturing systems, and models) can be designed such that they are self-sustaining, self-standing, and suited for integration is an important subject of research on emergent behaviour [Valckenaers and Van Brussel, 2005]. If the models are self-sustaining then they should be capable of adapting themselves to new situations. This is an important characteristic of intelligent models [Meyer et al., 2009; Kang et al., 2016]. Adaptive models, influenced by factors of environment, are much more likely to emerge if they are composed of elements whose self-existence enhances the probability of success, survival, growth and diversification of their population in the future [Valckenaers and Van Brussel, 2005].

Cloud-based design and manufacturing has emerged as a new adaptive paradigm in networked digital manufacturing and design innovation [Schaefer, 2014; Zhang et al., 2014; Wu et al., 2015; Sudarsanam and Umasankar, 2019]. This paradigm transforms the design, simulation, computing, and

* Corresponding author: alain-jerome.fougeres@ecam-rennes.fr.

manufacturing resources and abilities into design and manufacturing services in order to form a design and manufacturing cloud and distribute it to the user on demand [Li et al., 2010]. New computing and service-oriented cloud-based models, such as materials design or product configuration, are also developed from existing advanced research models [Sallai et al., 2014; Issa et al., 2017]. In such a socio-technical process, large quantities of information and knowledge are widely distributed across multiple networked actors.

Inspired by the principles of cloud computing (e.g., no localization or proximity constraints, sharing of data and resources, etc.) [Xu, 2012; Ren et al., 2015], this paper proposes the integration of product design and cellular manufacturing in cloud-based design and manufacturing. A manufacturing cell executes a subset of well-defined production cycle operations belonging to a family of products on a restricted group of workcenters (i.e. set of machines) placed geographically together. This is known as the manufacturing cell identification problem. The main difficulty with the manufacturing cell identification problem is the dynamic of design and manufacturing data. Consequently, manufacturing cells have difficulties with the adjustment to the intensity or quality of design and manufacturing data. The concept of virtual manufacturing cells, a group of resources dedicated to the manufacturing of a part family but not reflected in the physical structure of the manufacturing system [McLean et al., 1982; Nomden et al., 2005], improves the adaptivity of the manufacturing systems. In addition, products, manufacturing systems as well as their models, should be designed holistically as adaptive objects. We can, then, design adaptive virtual manufacturing cells (*AVMC*).

In a cross-disciplinary design team, designers simultaneously select, create, modify, and evaluate design and manufacturing targets and real parameters of a product model [Choulier et al., 2015]. However, the concept of manufacturing features can integrate product data with manufacturing data for cell-formation problems. Thus, this research assumes that cloud-based design and manufacturing can be referred to as a dynamic service-oriented network [Duan et al., 2012; Zhang et al., 2014] where a set of *CAD* products, modelled by a set of features, can be manufactured in agile and adaptive virtual manufacturing cells under certain constraints. Holons is a very interesting concept for modelling adaptive systems [Leitão, 2004]. In its structural aspect, a holon is not an aggregation of elementary products and, in its functional aspect, is not a chain of elementary units of behaviour. It is a system that is composed of interrelated semi-autonomous subsystems, each of the latter being, in turn, hierarchic in structure, until we reach some lowest level of elementary subsystems. In relation to the holon, the concept of “attractor” is used to deal with the form and stability of the *AVMC*. An attractor is a stable product/workcenter or a stable group of products/workcenters toward which a manufacturing cell formation tends to evolve.

Introducing the concept of attractor, and integrating the uncertainty and adaptivity in a modelling based on holonic fuzzy agents, an *AVMC* in cloud-based design and manufacturing can be formed. Thus, the feature-product-workcenter components of the *AVMC* are agentified, meanings they are designed as entities capable of exhibiting intelligent behaviour (i.e., autonomous, reactive, adaptive, interacting with other agents, aware of their environment, etc. [Wooldridge, 2002]). Moreover, holonic fuzzy agent means that agent have fuzzy behaviour (i.e., fuzzy reactions, knowledge, reasoning, interpretations, etc.) and holonic structure.

To summarise: firstly, being aware of the geometrical and topological context in a *CAD* environment, feature agents interact to create a dynamic network between product agents and workcenter agents. The workcenter agents interact with the product agents to recognize each other and to form specific sub-communities, called *AVMC*, with holonic structures. Intelligent holonic cells are thus emerged networks of holonic fuzzy agents.

Secondly, intelligent holonic cells, in their turn, are adaptive, distributed, and dynamic objects. The software agent paradigm brings to the holonic structure its reactivity, its dynamicity, its flexibility, and, thus, its adaptability. So, in this paper we illustrate that self-structuring of *AVMC* can be possible.

Thirdly, in this context, the question about the stability and the consensus of models can be raised. The stability of a model depends on the interactions between the elementary models which constitute it and their respective forms. In the last analysis, the design can be seen as a qualitative discontinuity phenomenon in a continuous consensual background. Understanding and controlling the discontinuity is, therefore, an important subject of the design research.

The paper is organized as follows: in the second section, a literature review on some intelligent issues of cellular manufacturing systems is presented. In the third section, a holonic fuzzy agent modelling is presented. In the fourth section, the proposed holonic fuzzy agent model for *AVMC* formation in cloud-based design and manufacturing is outlined. In the fifth section, this proposed model is illustrated by a scenario showing the behaviour of holonic fuzzy agents during the reconfiguration of a virtual manufacturing cells' holonic structure, which demonstrates the adaptive capacity of this kind of holonic structure. *AVMC* formation, *AVMC* adaptation and *AVMC* consensus evaluation are presented. Finally, the conclusions and perspectives of this research study are presented.

2 Literature review

The problem of cell formation consists of grouping of machines into cells to process a single part family. The grouping depends on the set of characteristics of machines and the set of characteristics of parts to be manufactured. Many approaches have been developed to deal with the cell formation problem.

The approaches based on manufacturing part process plan use the part manufacturing plans as a criterion for the formation of manufacturing cells. These approaches identify either sequentially or simultaneously either part families or manufacturing cells. Production Flow Analysis with “nuclear synthesis” is the earliest approach proposed by Burbidge [Burbidge, 1975]. Cells formation based on the specification of seed machines [Wei and Gaither, 1990] or the initial assignment of one part to each cell [Sundaram and Lian, 1990] followed the principle of nucleus. Cluster analysis approaches are efficient for working with matrixes with a large number of data. Cells or part families are formed by clustering machines or parts according to their similarity using hierarchical [Tam, 1990; Gupta and Seifoddini, 1990] or non-hierarchical algorithms [Chandrasekharan and Rajagopalan, 1986]. Graph partitioning [Al-Qattan, 1990] and cross clustering [Kusiak and Chow, 1987] are also used to formalize the problem of cell formation. The cross-clustering approaches carry out the decomposition by simultaneously classifying the lines and the columns of the matrix in a way that the crossing of the set of parts and the set of machines should form a diagonal block correspondence. Mathematical programming is another formalism used in cell formation problems [Aalaei and Davoudpour, 2016; Ameli and Arkat, 2008]. However, the all-in-one objective function is often not practical due to computational and implementation complexities. Neural networks [Saidi-Mehrabad and Safaei, 2007], genetic algorithm [Mak et al., 2000; Yasuda et al., 2005], tabu search algorithm [Lei and Wu, 2006] are also used to identify part families and machine cells.

Though the manufacturing processes are interesting for the cell-formation problem, the majority of manufacturing companies more or less use the features of products for the implementation of cellular manufacturing systems because one cannot rely only on the manufacturing processes that often become obsolete with the evolution of products and workcenters. Parts are clustered together into part families based on their similar features. The types of manufacturing process plans are then defined for each part family. Workcenters are assigned according to the manufacturing operations to be done for each family. Thus the efficiency of the manufacturing cells is mainly dependent on the formation of part families. Fuzzy cluster analysis, fuzzy classification, and fuzzy equivalence in the process of part-family formation have been applied [Xu and Wang, 1989]. To deal with crisp and fuzzy part features and uncertainty in coding information the fuzzy logic is also used in cell formation problem [Ben-Arieh and Triantaphyllou, 1992; Ben-Arieh et al., 1996]. To integrate the product data with the manufacturing data, a fuzzy cell formation approach, which assumes that the product and manufacturing data exist jointly, was developed [Mutel and Ostrosi, 2002]. The product data are described by product features that also determine the manufacturing data.

An important issue in cellular manufacturing systems is how various distributed design and manufacturing data cooperate well within a networked digital manufacturing and design to form manufacturing cells. Holonic and multi-agent systems are among the most complex artefacts emanating from deliberate human design and development activities. From process and product points of view, the engineering design of cellular manufacturing can be assisted by holonic agent-based systems [Deen, 2013; Barbosa et al., 2015; Borangiu et al., 2016; Mařík et al., 2017].

These kinds of systems are complex and adaptive [Miller and Page, 2009; Abbott and Hadžikadić, 2017; Morowitz, 2018]. Modelling and simulation of complex adaptive systems [Niazi, 2016] (and self-adaptive systems [Krupitzer et al., 2015; Hsieh and Lin, 2016; Zhang et al., 2017]) address a wide variety of characteristics: many heterogeneous interacting products, multiple scales, complicated transition laws, unpredicted emergence, sensitive dependence on initial conditions, path-dependent dynamics, ill-defined boundaries, interaction and self-organization of autonomous agents, combinatorial explosion, adaptivity to changing environments, co-evolving subsystems, and multilevel or non-equilibrium dynamics [San Miguel et al., 2012]. From this large set of complex adaptive system characteristics, we focused on three key properties:

- o *Self-organization property*. An organization is an integral arrangement according to a distribution of a set of elements in a hierarchical level; a self-organizing system has the ability to create and recreate this structure [Kauffman, 1993; Maguire et al., 2006; Brodu, 2008]; this property will be fundamental for adaptive virtual manufacturing cell formation.
- o *Nonlinearity property*. Behaviours and responses of a complex system are not deterministic and are influenced by the presence of nonlinear relationships and feedback loops. The implicit ability to exhibit linear or non-linear behaviours (order and/or chaos) is based on a response of the system to these stimuli [San Miguel et al., 2012]; this property will be fundamental for the reactivity of holonic fuzzy agents representing adaptive virtual manufacturing cells.
- o *Emerging behaviour property*. From nonlinear interactions in complex systems and their self-organized or chaotic structures, behaviours emerge [Kubik, 2003; Bar-Yam, 2004; Fromm, 2005; Kim, 2006]; this property will be fundamental for holonic fuzzy agents' generation and, finally, adaptive virtual manufacturing cell formation.

The idea of using the paradigm agent to design complex, adaptive, and interactive systems, either distributed or cooperative, is not a new one [Weiss, 1999]. Agent-based systems have been proposed in many industrial fields [Shen and Norrie, 1999; Marik and McFarlane, 2005; Monostri et al., 2006] because their distribution and decentralization allow industrial systems to be flexible [Leitão, 2009] and agents well suited to enabling cyber-physical systems [Vogel-Heuser et al., 2015]. An agent-based system can be fuzzy if agents that compose it are fuzzy. This means that agents have fuzzy knowledge and behaviours, their interactions, roles and resulting organizations are also fuzzy. Agents have their own level of knowledge and can share the knowledge or can communicate their knowledge [Fougères, 2003; Chira et al., 2006]. This knowledge can be fuzzy [Wang et al., 2010]. So, reactive and autonomous agents can update and follow the fuzzy information evolution coming from their environment and other agents [Ghasem-Aghaee and Ören, 2003]. By interpreting the fuzzy information that they receive or perceive, fuzzy agents interact within a multi-agent system; they can also interact in a fuzzy way. For instance, a fuzzy agent can discriminate a fuzzy value of interaction to evaluate its degree of affinity (or interest) with another fuzzy agent [Fougères and Ostrosi, 2013].

Multi-agent systems offer a major interest for the simulation and modelling of adaptive or self-adaptive phenomena, such as self-organization and emergence [van Aart, 2005; Valckenaers et al., 2003; Guessoum, 2004; Gleizes, 2012; De Lemos et al., 2013]. They are well adapted to manage the heterogeneity of some organizations [Nahm and Ishikawa, 2005]. Moreover, recent studies have shown that the development of multi-agent systems has become well controlled [Biswas, 2008].

The idea of the “holon” was introduced by Koestler [Koestler, 1967]. The “holon” represents a very interesting way to overcome the dichotomy between products and wholes, and to account for the integrative tendencies of design. Holons must be necessarily included in a typical vertical arrangement. With progressive accumulation and forming a nested hierarchical order, they are called a holarchy. Structural holarchies of the fractal type are used in engineering [Wang, 2001].

Fundamental insights into Holonic Systems Design, augmented with some research results on complex adaptive systems and with some implications for the design of Holonic Multi-Agent Systems, have also been discussed [Valckenaers and van Brussel, 2005]. The application of holonic concepts to manufacturing [Deen, 2013; Borangiu et al., 2016; Giret and Botti, 2009] was initially motivated by the inadequacy of existing manufacturing systems in the following two aspects: (a) dealing with the evolution of products within an existing production facility, and (b) maintaining a satisfactory performance outside of normal operating conditions [Valckenaers et al., 1998]. Modelling and reconfiguration in real-time of the holons' abstract behavioural specifications using internet-based

mobile agents has also been investigated. Issa et al. applied the concept of holonic agent in cloud-based design for configurations [Issa et al., 2017]. The interdependent properties of holons, inferred from these research works, are: *self-similarity, self-organization, self-preservation, self-adaptation, self-expansion, transcendence, utility, self-well-being, self-efficiency, emergence, dynamics, flexibility, and robustness.*

Initially formed to respond to a stable and robust demand, cells have been evolved dynamically to respond to an unstable and flexible demand. However, due to the lack of intelligent technologies, the dynamic cells have found difficulties in real implementation. It is the concept of virtual cells, initially introduced by McLean et al. [McLean et al., 1982], that allows exploiting the processing of product families into virtually grouped workcenters. The concept of virtual cells is also innovative in intelligent cloud based design and manufacturing. Holonic and fuzzy agents can be used to create dynamic service-oriented networked digital design and manufacturing, from CAD products to virtual intelligent cells' recognition. This issue is further developed in this paper.

3 Holonic fuzzy agent modelling

Why should we be interested in holonic fuzzy agents for manufacturing cell formation requires, we believe, a much better understanding of some key properties of such systems. Manufacturing cells should be dynamic, adaptive, and evolving systems. Manufacturing cells also are fuzzy interaction-based organization. Interactions between product design and product manufacturing are necessary. The manufacturing cell formation is also a fuzzy information and knowledge-based process. The adequacy of the holons' paradigm and fuzzy agents' paradigms for intelligent virtual cell formation is, thus, complete. Holonic fuzzy agents can overcome the lack of adaptivity of manufacturing cells. Holonic fuzzy agents can also be modelled to integrate the behaviour of both fuzzy agents and holonic agents.

We start by defining the model of fuzzy agent. Table 1 describes the different elements of this model: (1) the fuzzy agent-based system \tilde{M}_a ; (2) the behaviour of a fuzzy agent $\tilde{\alpha}_i$, inspired by the feedback loops *perceive-decide-act* [Brun et al., 2009]; (3) the behavioural functions of a fuzzy agent; and (4) the fuzzy interactions \tilde{i}_i between two fuzzy agents.

The model of fuzzy agent having been proposed, a model of holonic fuzzy agent can be defined. This new model, integrating agent, fuzzy and holonic concepts, respects the interdependent holonic properties presented in section 2:

- self-adaptation, self-organization, transcendence, self-efficiency, dynamics, and emergence are intrinsic properties of the multiagent paradigm;
- self-similarity, self-preservation, self-expansion, and robustness are intrinsic properties of the holonic agent paradigm;
- utility, self-well-being, and flexibility are intrinsic properties of the fuzzy agent paradigm.

Thus a set \tilde{H} of holonic fuzzy agents, forming a holonic fuzzy agent-based system respecting the equation 1 (Table 1), is defined recursively as follows:

- each fuzzy agent $\tilde{\alpha}_i \in \tilde{A}$, forms an atomic holonic fuzzy agent \tilde{h}_i , $\tilde{h}_i = (\{\tilde{\alpha}_i\}, \{\tilde{\alpha}_i\}, \emptyset) \in \tilde{H}$, and
- $\tilde{h}_i = (\tilde{H}_0, \tilde{H}', \tilde{R}) \in \tilde{H}$, where $\tilde{H}' \subseteq \tilde{H}$ is the set of holonic fuzzy agents that participate in \tilde{h}_i , and $\tilde{H}_0 \subseteq \tilde{H}'$ is the non-empty set of holonic fuzzy agents that represent \tilde{h}_i to the environment and is responsible for coordinating the actions inside \tilde{h}_i . \tilde{R} defines a fuzzy relationship inside \tilde{h}_i (*i.e.* \tilde{R} specifies the holonic fuzzy agent organization). Each \tilde{h}_i respects the equation 2 (Table 1) where $K_{\tilde{h}_i} = K_{\tilde{\alpha}_i} \cup (\tilde{H}_0, \tilde{H}', \tilde{R})$.

Table 1. Model of fuzzy agent [Fougères and Ostrosi, 2013]

Model	Meaning	
$\tilde{M}_\alpha = \langle \tilde{A}, \tilde{I}, \tilde{P}, \tilde{O}, \Phi_{\tilde{A}} \rangle$ (1)	\tilde{A} : is a set of fuzzy agents \tilde{I} : is a set of fuzzy interactions between fuzzy agents \tilde{P} : is a set of fuzzy roles that fuzzy agents can perform \tilde{O} : is a set of fuzzy organizations defined for fuzzy agents (subsets of strongly linked fuzzy agents or holonic fuzzy agents) $\Phi_{\tilde{A}}$: is a set of functions of fuzzy agents' generation defined as follows: $\varphi : \tilde{A}^n \rightarrow \tilde{A}^m \quad \varphi(\tilde{\alpha}_i, \tilde{\alpha}_j) \Rightarrow \tilde{A}' = \tilde{A} + \{ \tilde{\alpha}_j \} .$	
	$\tilde{\alpha}_i = \langle \Phi_{\Pi(\tilde{\alpha}_i)}, \Phi_{\Delta(\tilde{\alpha}_i)}, \Phi_{\Gamma(\tilde{\alpha}_i)}, K_{\tilde{\alpha}_i} \rangle$ (2)	$\Phi_{\Pi(\tilde{\alpha}_i)}$: is the function of observation of the fuzzy agent $\tilde{\alpha}_i$ $\Phi_{\Delta(\tilde{\alpha}_i)}$: is the function of decision of the fuzzy agent $\tilde{\alpha}_i$ $\Phi_{\Gamma(\tilde{\alpha}_i)}$: is the function of action of the fuzzy agent $\tilde{\alpha}_i$ $K_{\tilde{\alpha}_i}$: is the set of knowledge of the fuzzy agent $\tilde{\alpha}_i$ (decision rules, characteristics of the domain, acquaintances, and dynamic knowledge).
	$\Phi_{\Pi(\tilde{\alpha}_i)} : (E_{\tilde{\alpha}_i} \cup I_{\tilde{\alpha}_i}) \times \Sigma_{\tilde{\alpha}_i} \rightarrow \Pi_{\tilde{\alpha}_i}$ $\Phi_{\Delta(\tilde{\alpha}_i)} : \Pi_{\tilde{\alpha}_i} \times \Sigma_{\tilde{\alpha}_i} \rightarrow \Delta_{\tilde{\alpha}_i}$ $\Phi_{\Gamma(\tilde{\alpha}_i)} : \Delta_{\tilde{\alpha}_i} \times \Sigma \rightarrow \Gamma_{\tilde{\alpha}_i}$ (3)	$E_{\tilde{\alpha}_i}$: is the set of fuzzy observed events of the fuzzy agent $\tilde{\alpha}_i$ $I_{\tilde{\alpha}_i}$: is the set of fuzzy interactions of the fuzzy agent $\tilde{\alpha}_i$ $\Sigma_{\tilde{\alpha}_i}$: is the set of fuzzy state of the fuzzy agent $\tilde{\alpha}_i$ $\Pi_{\tilde{\alpha}_i}$: is the set of fuzzy perceptions of the fuzzy agent $\tilde{\alpha}_i$ $\Delta_{\tilde{\alpha}_i}$: is the set of fuzzy decisions of the fuzzy agent $\tilde{\alpha}_i$ $\Gamma_{\tilde{\alpha}_i}$: is the set of fuzzy actions of the fuzzy agent $\tilde{\alpha}_i$ Σ is the state of the fuzzy agent-based system \tilde{M}_α
		$\tilde{I}_i = \langle \tilde{\alpha}_s, \tilde{\alpha}_r, \tilde{\gamma}_c \rangle$ (4)

Now, we can apply this holonic fuzzy agent model to the specific problem of intelligent *AVMC* formation. Firstly, features, products, workcenters, and cells, composing the *AVMC*, are fuzzy agentified (7). Thus, each feature of the set $x = \{x_1, x_2, \dots, x_n\}$ is transformed into a fuzzy agent $\tilde{x}_{[1..n]} \in \tilde{A}_x$; each product of the set $p = \{p_1, p_2, \dots, p_o\}$ is transformed into a fuzzy agent $\tilde{p}_{[1..o]} \in \tilde{A}_p$; each workcenter of the set $w = \{w_1, w_2, \dots, w_p\}$ is transformed into a fuzzy agent $\tilde{w}_{[1..p]}$; and each generated cell of the set $c = \{c_1, c_2, \dots, c_q\}$ (q being dependent of the dynamic of the *AVMC*) is transformed into a fuzzy agent $\tilde{c}_{[1..q]}$ (7):

$$\text{Fuzzy Agentification: } X, P, W, C \rightarrow \tilde{A}_x, \tilde{A}_p, \tilde{A}_w, \tilde{A}_c \quad (7)$$

Secondly, the *AVMC* is modelled as a fuzzy holonic agent-based system. From the fuzzy agent's product, workcenter and cell, holonic fuzzy agents are generated (8):

$$\text{Holonisation: } \tilde{A}_p, \tilde{A}_w, \tilde{A}_c \rightarrow \tilde{H}_p, \tilde{H}_w, \tilde{H}_c \quad (8)$$

where $P_j^i \in \tilde{H}_p$, with $P_j^0 \in \tilde{A}_p$ and $P_j^{i>1} = \{P_1^0, P_2^0, \dots, P_1^i, \dots, P_o^i\}$; $W_j^i \in \tilde{H}_w$ with $W_j^0 \in \tilde{A}_w$ and $W_j^{i>1} = \{W_1^0, W_2^0, \dots, W_1^i, \dots, W_p^i\}$; $C_j^i \in \tilde{A}_c$, $C_j^i = \langle W_j^i, P_j^i, R_j^i \rangle$ with $W_j^i \subset H_w$, $P_j^i \subset H_p$, and $R_j^i: W_j^i \leftrightarrow P_j^i$.

Thus, a fuzzy holon agent cell C_j^i is defined by the relationship R_j^i between two fuzzy holon agents workcenter W_j^i and product P_j^i that will satisfy the properties mentioned above, especially those of self-organization and self-adaptation.

4 Holonic fuzzy agent modelling for adaptive virtual cell formation

4.1 Modelling of fuzzy agents

CAD products are modelled by a set of fuzzy agents and defined from a set of fuzzy feature agents which themselves are defined by a set of fuzzy face agents. Any manufacturing feature $x_i \in X$ can be characterized by a set of faces $F_{X_i} = \{f_1, f_2, \dots, f_j, \dots, f_m\}$ that satisfy a set of topological and geometric relations. Each face $f_i \in F_{X_i}$ is transformed into a fuzzy face agent $\tilde{f}_i \in \tilde{A}_F$. Then a graph of fuzzy face agents can form a feature [Fougères and Ostrosi, 2018]. All elements of the model are fuzzy agentified and have specific knowledge (Table 2).

Table 2. Fuzzy agents' knowledge.

Fuzzy agents	Knowledge
<i>Fuzzy feature agent</i> $\tilde{x}_i \in \tilde{A}_X$	<ul style="list-style-type: none"> - A graph $G_{\tilde{x}_i} = (\tilde{A}_{F_{\tilde{x}_i}}, R)$, where $\tilde{A}_{F_{\tilde{x}_i}}$ is the set of fuzzy face agents making up the fuzzy feature agent \tilde{x}_i, and R is the set of relationships between fuzzy face agents. - The set of fuzzy product agents $\tilde{A}'_P \subset \tilde{A}_P$ including the fuzzy feature agent \tilde{x}_i. - A set of fuzzy agent feature-specific decision rules $\Delta_{\tilde{x}_i}$. - The ability to generate a holonic fuzzy feature agent $(\varphi(\tilde{x}_i, \tilde{h}_j \in \tilde{H}_X))$.
<i>Fuzzy product agent</i> $\tilde{p}_i \in \tilde{A}_P$	<ul style="list-style-type: none"> - The set of fuzzy feature agents $\tilde{A}'_X \subset \tilde{A}_X$ including the fuzzy product agent \tilde{p}_i. - A set of tuples <form, dimension, tolerance> characterized features of the fuzzy product agent \tilde{p}_i. - The set of fuzzy workcenter agents $\tilde{A}'_W \subset \tilde{A}_W$ which have the capability to realize features of the product agent \tilde{p}_i. - A set of fuzzy agent product-specific decision rules $\Delta_{\tilde{p}_i}$. - The ability to generate a holonic fuzzy product agent $(\varphi(\tilde{p}_i, \tilde{h}_j \in \tilde{H}_P))$.
<i>Fuzzy workcenter agent</i> $\tilde{w}_i \in \tilde{A}_W$	<ul style="list-style-type: none"> - The set of fuzzy feature agents $\tilde{A}'_X \subset \tilde{A}_X$ that the workcenter agent \tilde{w}_i can realize. - The set of fuzzy product agents $\tilde{A}'_P \subset \tilde{A}_P$ that the workcenter agent \tilde{w}_i can realize. - A set of fuzzy agent workcenter-specific decision rules $\Delta_{\tilde{w}_i}$. - The ability to generate a holonic fuzzy workcenter agent $(\varphi(\tilde{w}_i, \tilde{h}_j \in \tilde{H}_W))$.
<i>Fuzzy cell agent</i> $\tilde{c}_i \in \tilde{A}_C$	<ul style="list-style-type: none"> - The set $\tilde{A}'_W \subset \tilde{A}_W$ of fuzzy workcenter agents including the fuzzy cell agent \tilde{c}_i - The set $\tilde{A}'_P \subset \tilde{A}_P$ of fuzzy product agents that the fuzzy cell agent \tilde{c}_i can manufacture. - A set \tilde{R} of relationships between fuzzy product agents and fuzzy workcenter agents ($\tilde{R}: \tilde{A}'_P \rightarrow \tilde{A}'_W$). - A set of fuzzy agent cell-specific decision rules $\Delta_{\tilde{c}_i}$. - The ability to generate a holonic fuzzy cell agent $(\varphi(\tilde{c}_i, \tilde{h}_j \in \tilde{H}_C))$.

4.2 Modelling of fuzzy agents' relationships

A relationship can be defined between the universal set of CAD products P and the universal set of manufacturing features X (cf. Figure 1 for an example of CAD product composed by basic features). In the case of feature interaction, because of the loss of topological information, such, for instance, as the adjacency between two faces, the manufacturing feature membership to a product implies an uncertainty. This uncertainty means the existence of a fuzzy relationship $\tilde{R}_3 = (P, X)$

between the set of products P and the set of manufacturing features X . This fuzzy relationship is a subset of the Cartesian product $P \times X$, with the membership function $\mu_{\tilde{R}_3}(p, x) \in [0, 1]$.

The manufacturability of a product is evaluated in terms of the capabilities of available workcenters to manufacture intermediate features called the operational features x_{ij} , taking into account the following manufacturing properties: form, dimensions, and tolerances. In this way, there is a relationship between the set of operational features X and the set of workcenters W , which indicates the capability of workcenters to realize these operational features. However, the expert knowledge on workcenters is expressed in natural language, and it is introduced by subjective descriptions. In addition, the nature of knowledge, on the capability of a workcenter to realize an operational feature set, is gradual. Therefore, a fuzzy relationship $\tilde{R}_5 = (X, W)$, characterized by the membership function $\mu_{\tilde{R}_5}(x, w) \in [0, 1]$ can be defined by softening the definition of the classic characteristic function. This membership indicates in what degree a feature can be manufactured in a workcenter.

From CAD modelling, a product is composed of features which, in turn, should be manufactured in a set of workcenters. Therefore, the composed relationship $\tilde{R}_7 = (P, W)$ between the universal set of products and the universal set of workcenters is a fuzzy one. The max-min composition between the fuzzy relationship $\tilde{R}_3 = (P, X)$ and the fuzzy relationship $\tilde{R}_5 = (X, W)$ is the fuzzy set $\tilde{R}_7 = (P, W)$. The membership function is, then, defined as follows (9):

$$\mu_{\tilde{R}_7}(p, w) = \mu_{\tilde{R}_3} \circ \mu_{\tilde{R}_5}(p, w) = \max_x \min [\mu(p, x), \mu(x, w)], \quad \forall p \in P, \forall w \in W \quad (9)$$

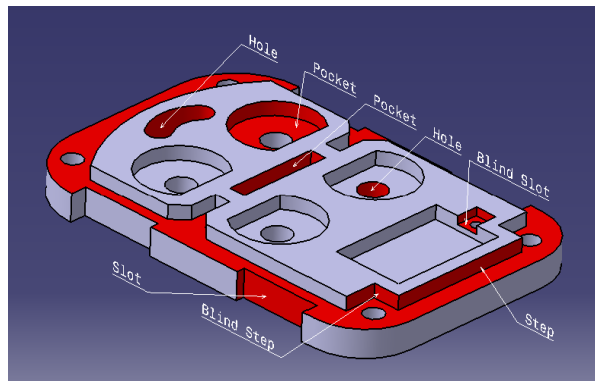


Fig. 1. Example of CAD product composed by basic features.

4.3 Formation of holonic fuzzy agents' virtual cells

The architecture of holons shows *horizontal* and *vertical* self-similarity. Horizontal self-similarity relates to self-similarity across different specializations on one level of aggregation. Vertical self-similarity refers to self-similarity across different levels of aggregation: higher-level resource holons work similarly to lower-level resource holons. Therefore, the first proposition of our work postulates that the intelligent AVMC should have the structure of the holon. That means creating “*intelligent virtual manufacturing cells within an intelligent virtual manufacturing cell.*” The second proposition of our work postulates that intelligent virtual manufacturing holons tend to evolve toward some particular physical elements: products or resources. These are called *attractors*. An attractor is a structurally stable element, product, or resource in cloud-based design and manufacturing. Properties relating to fuzzy cell formation are shown in Table 3.

Following the *Preferences of designer* property, in order to increase the flexibility of the solutions, the products or the workcenters can be weighted by weight w_i (respectively w_j). In addition, the limits of acceptability for variable values, whether communicated formally or established informally by experience, are familiar to engineers in industry. The acceptable intervals correspond to the intervals over which the designer preference is greater than zero. For considering the degrees of preferences of

the designer for the elements in a fuzzy set, the acceptable intervals α -levels, over which the designer preference is equal or greater than a preferred threshold value, can be used.

Table 3. Properties of fuzzy cell formation

Properties	Meaning
<i>Holon</i>	AVMC is a fuzzy holon.
<i>Coupling</i>	A holonic cell is defined through two basic coupled elements: a holonic product and a holonic workcenter.
<i>Attractor</i>	An attractor is a structurally stable element, product or workcenter toward which a cell tends dynamically to evolve.
<i>Force of attraction</i>	An attractor exerts a force of attraction on either of the other elements.
<i>Condition of attraction</i>	The contribution of element to a cell must be greater than or equal to an acceptable value of the force of attraction, called Acceptable Force of Attraction (AFA). The AFA varies between 0 and 1.
<i>Competition of fuzzy design cells and Condition of transfer</i>	The contribution of element (product or workcenter) to its own cell must be greater than its contribution to all the other cells.
<i>Equivalent cells</i>	The construction of equivalent cells reposes on the possibility that an element should be able to contribute in an equal way to several cells.
<i>Preferences of designer</i>	This property stipulates that a designer can build a preference relationship in the set of products or workcenters.

In Table 4 we present a distributed algorithm of virtual cell formation executed by each holonic fuzzy product agent \tilde{P}_i^x . The main ten steps of this algorithm are identified by the indices (S0) to (S9): **(S0)** *initial conditions and preferences of designer*; **(S1)** *coefficient of attraction force*; **(S2)** *vertical self-similarity*; **(S3)** *fuzzy attractor identification*; **(S4)** *attraction of fuzzy workcenter holons*; **(S5)** *attraction of fuzzy product holons*; **(S6)** *equivalent fuzzy cell holons and transfer*; **(S7)** *new fuzzy cell holon*; **(S8)** *fuzzy cells within a fuzzy cell*; and **(S9)** *evaluation of the end of fuzzy cell holons' formation*.

The problem of consensus-searching has been firstly modelled as the problem of searching for the best overlap between conjectures [Ostrosi et al., 2010]. To measure consensus, two key concepts have been proposed: the *intra consensus design coefficient*, noted k_1 , and the *inter consensus design coefficient*, noted k_2 (Table 5). Higher values for the *consensus design coefficients* lead to an increase in consensus. With these definitions, the problem of consensus-seeking has been formulated in terms of the *intra consensus design coefficient* and the *inter consensus design coefficient*. The function to be maximized defined as *global consensus design coefficient* and noted k (Table 5), is the following (10):

$$k = \alpha k_1 + (1 - \alpha) k_2 \quad (10)$$

As the conjectures are defined by cells and corresponding product families, the degree of overlap between conjectures can be seen as the degree of overlap between cells and product families belonging to these conjectures. Consequently, two problems can be formulated. The first problem can be formulated as follows: if a set of conjectures is found to satisfy a set of measurable criteria, then what is the consensus conjecture which globally satisfies the set of criteria? Indeed, at least if there is a conjecture from a criterion which fully overlaps with a conjecture from every criterion, the conjectures are consensual for the set of considered criteria. In this case, we can say that all the criteria are consensual toward a conjecture without dilemma. This is the ideal consensus. The second problem can be formulated as follows: if a set of conjectures is found to satisfy a set of measurable criteria, what is the best overlap between these conjectures? The best overlap between all conjectures allows us to distinguish the consensual share of the set of conjectures. Indeed, the consensual share will define workcenters and products which can always be grouped together. The most consensual workcenters and products will represent the nucleus of the cellular manufacturing system.

Table 4. Virtual cells formation, based on holonic product attractors

Algorithm	Meaning
(S0) Given a set of p fuzzy product holons $\{\tilde{P}_1^0, \dots, \tilde{P}_p^0\} \in \tilde{H}_p$; // initial conditions Given a set of m fuzzy workcenter holons $\{\tilde{W}_1^0, \dots, \tilde{W}_m^0\} \in \tilde{H}_w$; // initial conditions Given a set of c fuzzy cell holons $\{\tilde{C}_1^0, \dots, \tilde{C}_c^0\} \in \tilde{H}_c$; // initial conditions Given the 2 sets of weights $\{w_1, \dots, w_i\}$ and $\{w_1, \dots, w_j\}$ // initial conditions	
EACH part holon agent \tilde{P}_i^x DO {	
(S1) $CAF \leftarrow 1$ $\tilde{X}_i^x \subseteq \tilde{A}_x$ $\tilde{W}_i^x \subseteq \tilde{A}_w$ $_h \leftarrow false$	// current CAF = 1.0 // set of features that compose the product holon \tilde{P}_i^x // set of workcenters that manufacture \tilde{P}_i^x // \tilde{P}_i^x is not included in a product holon of higher level
(S2) WHILE ($_h == false$) { FOR j in $1..Card(F^x)$	// while \tilde{P}_i^x is not included in a holon of higher level // loop to know how many features compose \tilde{P}_i^x
$nbX_i^x[j] \leftarrow Exchange_1(\tilde{P}_i^x, \tilde{P}_j^x)$	
(S3) IF ($\max(Card(\tilde{X}_i^x), nbX_i^x)$) THEN	// \tilde{P}_i^x is composed by the greatest number of features
$A_i \leftarrow \tilde{P}_i^x$	// \tilde{P}_i^x is an attractor of product holons
(S4) generate(\tilde{P}_i^{x+1}) generate(\tilde{W}_i^{x+1})	// generation of a new product holon $\tilde{P}_i^{x+1} \leftarrow \tilde{P}_i^x$ // generation of a new workcenter holon $\tilde{W}_i^{x+1} \leftarrow \tilde{W}_i^x$
(S5) calculate($CAF_i, \tilde{W}_i^x, \tilde{W}_i^{x+1}$) FOR k in $1..Card(\tilde{W}_i^x)$	// calculation of $CAF_i \leftarrow Card(\tilde{W}_i^{x+1})/Card(\tilde{W}_i^x)$ // loop to get the list of families manufactured by the
$F_m \leftarrow Exchange_2(\tilde{P}_i^x, \tilde{W}_k^x)$	// workcenter holons that manufacture \tilde{P}_i^x
(S6) FOR l in $1..Card(F_m)$ $CAF_i.[\tilde{P}_i^x] \leftarrow Exchange_3(\tilde{P}_i^x, \tilde{P}_l^x, \tilde{W}_i^{x+1})$ IF ($CAF_i.[\tilde{P}_i^x] \geq CAF_{encours}$) THEN $\tilde{P}_i^{x+1} \leftarrow \tilde{P}_i^x$ inform($\tilde{P}_i^x, \tilde{P}_l^x, \tilde{P}_i^{x+1}$) ELSE inform($\tilde{P}_i^x, \tilde{P}_l^x, !\tilde{P}_i^{x+1}$)	// loop for each family manufactured by workcenters of \tilde{W}_i^x // exchange to get the CAF of \tilde{P}_i^x // \tilde{P}_i^x is an attractor for the product holon \tilde{P}_i^x // \tilde{P}_i^x is included in the new product holon \tilde{P}_i^{x+1} // \tilde{P}_i^x is informed of inclusion in the product holon \tilde{P}_i^{x+1} // \tilde{P}_i^x is informed of not inclusion in the holon \tilde{P}_i^{x+1}
(S7) generate($\tilde{C}_i^{x+1}, A_i, \tilde{P}_i^{x+1}, \tilde{W}_i^{x+1}$) ELSE	// generation of the new cell holon \tilde{C}_i^{x+1} // \tilde{P}_i^x not composed by the greatest number of features
wait(inform($\tilde{P}_j^x, \tilde{P}_i^x, a$)) IF ($a == \tilde{P}_i^{x+1}$) THEN $_h \leftarrow true$	// \tilde{P}_i^x is awaiting information of possible attraction // \tilde{P}_i^x is attracted by the product holon \tilde{P}_j^x // \tilde{P}_i^x is included in a product holon of higher level
(S6') $A_i \leftarrow \tilde{P}_j^x$ ELSE $_h \leftarrow false$	// attractor of \tilde{P}_i^x is the product holon \tilde{P}_j^x // \tilde{P}_i^x is always not included in a holon of higher level
(S8) lower(CAF)	// lower the current CAF (for instance $CAF = CAF - 0,1$)
(S9) } END WHILE } END DO	

In [Fougères and Ostrosi, 2013] we have proposed a distributed algorithm by which fuzzy agents seek to build consensus: fuzzy agents of a first partition seek to regroup with fuzzy agents of a second partition by maximizing the weights of their mutual relations. The consensus corresponds to new fuzzy agents, called consensus cluster agents. The different consensus cluster agents thus formed seek, in turn, to establish higher levels of consensus cluster agents. In the present approach of AVMC

formation, we adapted these algorithms (see Table 5) for checking the association stability of workcenters or products in the same cells in different configurations.

Table 5. Consensus seeking by fuzzy product and workcenter agents

Algorithms	Description
<pre> EACH agent \tilde{p}_i DO $\tilde{P}'_r \leftarrow \tilde{P}_r$ IF (receiveMsg ($T_{\tilde{W}_r}$)) THEN update (\tilde{W}_r); $r \leftarrow \text{Card} (\tilde{W}_r)$ ENDIF FOR $p = 1$ to r $k_{1,i,p} = \sum_{\tilde{w}_j \in \tilde{W}_p} \mu(\tilde{p}_i, \tilde{w}_j) / \text{Card} (\tilde{W}_p)$; $k_{2,i,p} = \sum_{\tilde{w}_j \in \tilde{W}_p} (1 - \mu(\tilde{p}_i, \tilde{w}_j)) / \text{Card} (\tilde{W} - \tilde{W}_p)$ $k_{i,p} = \alpha k_{1,i,p} + (1 - \alpha) k_{2,i,p}$ ENDFOR $k_{i,p*} = \max (k_{i,1}, \dots, k_{i,r})$ $\tilde{P}_{p*} \leftarrow \tilde{P}_{p*} + \{\tilde{p}_i\}$ IF ($\tilde{P}_{p*} \neq \tilde{P}'_{p*}$) THEN diffuse ($\tilde{p}_i, \tilde{W}, T_{\tilde{P}_r}, (\tilde{p}_i, \tilde{P}_{p*})$); diffuse ($\tilde{w}_j, \tilde{C}_{p*}, T_{\tilde{W}_r}, (\tilde{w}_j, \tilde{W}_{p*})$) ENDIF WHILE ((receiveMsg ($T_{\tilde{W}_r}$)) </pre>	<pre> // P agents search for a partition of fuzzy workcenter agents // save the current partition P_r // if reception of a new partition W_r // update the groups of fuzzy workcenter agents // iteration on r groups of fuzzy workcenter agents // compute k1 (cf. equation 10) // compute k2 (cf. equation 10) // compute k (cf. equation 10), $\alpha \in [0..1]$ // end of iteration on r groups of fuzzy workcenter agents // max k founded in the group P^* // assign P_i to the product of fuzzy product P_{p*} // inform only if modification // diffuse assigning to fuzzy workcenter agents // diffuse assigning to the fuzzy consensus cluster agents C_{p*} // while information of modifications is sent by agents of W </pre>
<pre> EACH agent \tilde{w}_j DO $\tilde{W}'_r \leftarrow \tilde{W}_r$ IF (receiveMsg ($T_{\tilde{S}_r}$)) THEN update (\tilde{P}_r); $r \leftarrow \text{Card} (\tilde{P}_r)$ ENDIF FOR $p = 1$ to r $k_{1,j,p} = \sum_{\tilde{p}_i \in \tilde{P}_p} \mu(\tilde{p}_i, \tilde{w}_j) / \text{Card} (\tilde{P}_p)$ $k_{2,j,p} = \sum_{\tilde{p}_i \in \tilde{P}_p} (1 - \mu(\tilde{p}_i, \tilde{w}_j)) / \text{Card} (\tilde{P} - \tilde{P}_p)$ $k_{j,p} = \alpha k_{1,j,p} + (1 - \alpha) k_{2,j,p}$ ENDFOR $k_{j,p*} = \max (k_{j,1}, \dots, k_{j,r})$ $\tilde{W}_{p*} \leftarrow \tilde{W}_p + \{\tilde{w}_j\}$ IF ($\tilde{W}_{p*} \neq \tilde{W}'_{p*}$) THEN diffuse ($\tilde{w}_j, \tilde{P}, T_{\tilde{W}_r}, (\tilde{w}_j, \tilde{W}_{p*})$); diffuse ($\tilde{w}_j, \tilde{C}_{p*}, T_{\tilde{W}_r}, (\tilde{w}_j, \tilde{W}_{p*})$) ENDIF WHILE ((receiveMsg ($T_{\tilde{P}_r}$)) </pre>	<pre> // W agents search for a partition of fuzzy product agents // save the current partition W_r // if reception of a partition P_r from a fuzzy product agent // update the groups of fuzzy workcenter agents // iteration on r groups of fuzzy product agents // compute k1 (cf. equation 10) // compute k2 (cf. equation 10) // compute k (cf. equation 10), $\alpha \in [0..1]$ // max k founded in the group P^* of fuzzy workcenter agents // assign w_j to the group P^* of fuzzy workcenter agents W_{p*} // inform only if modification // diffuse assigning to fuzzy product agents // diffuse assigning to the fuzzy consensus cluster agents C_{p*} // while information of modifications is sent by agents of P </pre>
<pre> EACH agent $\tilde{C}l_p = (\tilde{P}_p, \tilde{W}_p)$ DO IF (receiveMsg ($T_{\tilde{P}_p}$) OR receiveMsg ($T_{\tilde{W}_p}$)) THEN update (\tilde{P}_p, \tilde{W}_p) ELSE IF (receiveMsg (T_{DESIGNER})) THEN </pre>	<pre> // each consensus cluster agent controls the convergence // if reception of message from product or workcenter agent // update the cluster // if reception of message from a designer </pre>

```

inform (  $\tilde{C}l_p$ , DESIGNER ,  $T_{C1}^-$ , ( $\tilde{P}_p, \tilde{W}_p$ ) ) // transmit to DESIGNER the state of cluster for supervision
ENDIF
WHILE (( receiveMsg (  $T_{P_r}^-$  )) // while information is sent by agents of P or W

```

5 Illustrative case study

5.1 Basic intelligent AVMC formation

A set of fuzzy manufacturing feature agents and their corresponding fuzzy operational feature agents shown in Table 6 are considered. For instance, *Slot milling*, noted \tilde{x}_{21} , is a fuzzy operational feature agent of the manufacturing feature *Slot* (\tilde{x}_2) and results from milling operations.

Table 6. Fuzzy manufacturing features agents and their fuzzy operational features agents

Fuzzy Manufacturing features	Fuzzy Operational features
<i>Through hole</i> (\tilde{x}_1)	Through hole drilling (\tilde{x}_{11}); through hole grinding (\tilde{x}_{12})
<i>Slot</i> (\tilde{x}_2)	Slot milling (\tilde{x}_{21}); slot grinding (\tilde{x}_{22})
<i>Pocket</i> (\tilde{x}_3)	Pocket milling (\tilde{x}_{31})
<i>Opened hole</i> (\tilde{x}_4)	Opened hole milling (\tilde{x}_{41})
<i>Blind slot</i> (\tilde{x}_5)	Blind slot milling (\tilde{x}_{51})
<i>Blind step</i> (\tilde{x}_6)	Blind step milling (\tilde{x}_{61})
<i>Step</i> (\tilde{x}_7)	Step milling (\tilde{x}_{71})
<i>Hole</i> (\tilde{x}_8)	Blind hole drilling (\tilde{x}_{81}); blind hole grinding (\tilde{x}_{82})
<i>Counter-bored-hole</i> (\tilde{x}_9)	Counter-bored-hole drilling (\tilde{x}_{91})

Then, based on manufacturing feature recognition by fuzzy agents' interactions (see §4.2), the relationship between a set of products and their corresponding operational features is shown by the fuzzy product operational feature matrix (Table 7).

Table 7. Fuzzy product–operational feature matrix

	\tilde{x}_{11}	\tilde{x}_{12}	\tilde{x}_{21}	\tilde{x}_{22}	\tilde{x}_{31}	\tilde{x}_{41}	\tilde{x}_{51}	\tilde{x}_{61}	\tilde{x}_{71}	\tilde{x}_{81}	\tilde{x}_{82}	\tilde{x}_{91}
\tilde{p}_1	1	1	0	0	1	0	1	0	0	0	0	0
\tilde{p}_2	0	0	0	0	0	0.8	0	0	0	0	0	0
\tilde{p}_3	0	0	0.8	0.8	0	0	0	0	0	0	0	0
\tilde{p}_4	0	0	0.9	0.9	0	0	0	0	0	0	0	0
\tilde{p}_5	0	0	0.9	0.9	0	0	0	1	0	0	0	0
\tilde{p}_6	0	0	0	0	1	0	1	0	0	1	1	1

Based on workcenter recognition by fuzzy agents' interactions, a fuzzy operational feature–workcenter matrix is built. It indicates the feasibility of the set of operational features over the set of workcenters (Table 8).

Table 8. Fuzzy operational feature–workcenter matrix

	\tilde{x}_{11}	\tilde{x}_{12}	\tilde{x}_{21}	\tilde{x}_{22}	\tilde{x}_{31}	\tilde{x}_{41}	\tilde{x}_{51}	\tilde{x}_{61}	\tilde{x}_{71}	\tilde{x}_{81}	\tilde{x}_{82}	\tilde{x}_{91}
\tilde{w}_1	1	0	0	0	0	0.8	0	0	0	1	0	1
\tilde{w}_2	1	0	0	0	0	1	0	0	0	1	0	1
\tilde{w}_3	0.3	0	0.4	0	0.5	0.2	0.5	0.4	0.8	0.3	0	0.3

\tilde{w}_4	0	1	0	0	0	0	0	0	0	0	0.9	0
\tilde{w}_5	0.2	0	0.8	0	0.3	0.2	0.3	0.8	0.7	0.3	0	0.3
\tilde{w}_6	0	0	0	0.8	0	0	0	0	0	0	0.3	0
\tilde{w}_7	0.2	0	0.8	0	0.3	0.2	0.3	0.8	0.7	0.3	0	0.3

The composition of the two previous fuzzy relationships defines the fuzzy relationship workcenter-product (Table 9.a). Moreover, the application of the consensus algorithm (Table 5) allows obtaining the consensual products visible in the product-cell and workcenter-cell matrices. These partitions show the association stability of products (Table 9.b) or workcenters (Table 9.c) in the same cells in different configurations.

Table 9. a) Fuzzy workcenter-product matrix, b) and c) consensus-formation

a)	\tilde{p}_1	\tilde{p}_2	\tilde{p}_3	\tilde{p}_4	\tilde{p}_5	\tilde{p}_6
\tilde{w}_1	1	0.8	0	0	0	1
\tilde{w}_2	1	0.8	0	0	0	1
\tilde{w}_3	0.5	0.2	0.4	0.4	0.4	0.5
\tilde{w}_4	1	0	0	0	0	0.9
\tilde{w}_5	0.3	0.2	0.8	0.8	0.8	0.3
\tilde{w}_6	0	0	0.8	0.8	0.8	0.3
\tilde{w}_7	0.3	0.2	0.8	0.8	0.8	0.3

b)	\tilde{p}_1	\tilde{p}_2	\tilde{p}_6	\tilde{p}_3	\tilde{p}_4	\tilde{p}_5
\tilde{c}_1^1	0.8	0.4	0.8	0.1	0.1	0.1
\tilde{c}_2^1	0.3	0.3	0.1	0.8	0.8	0.8

c)	\tilde{w}_1	\tilde{w}_2	\tilde{w}_3	\tilde{w}_4	\tilde{w}_5	\tilde{w}_6	\tilde{w}_7
\tilde{c}_1^1	0.9	0.9	0.6	0.4	0.2	0.2	0.2
\tilde{c}_2^1	0	0	0	0.4	0.8	0.8	0.8

This holonic fuzzy product-workcenter agent relationship $\tilde{w}_7 = (\tilde{P}, \tilde{W})$ is consensual because it does not change the composition of the other holonic fuzzy product-workcenter agent relationship $\tilde{w}_8 = (\tilde{P}, \tilde{W})$. Then, from the attractor agent recognition, the fuzzy product agents and fuzzy workcenter agents interact to form the holonic fuzzy cell agents. The different steps for this formation (five levels in this case study) are shown in Table 10.

Table 10. The different steps of AVMC formation

Parameters	$card(\tilde{P}) = 6$; $\tilde{P} = \{\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4, \tilde{p}_5, \tilde{p}_6\}$ //set of fuzzy product agents
	$card(\tilde{W}) = 7$; $\tilde{W} = \{\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \tilde{w}_4, \tilde{w}_5, \tilde{w}_6, \tilde{w}_7\}$ //set of fuzzy workcenter agents
	$card(\tilde{X})$ for $\tilde{p}_1 = 4$; $card(\tilde{X})$ for $\tilde{p}_2 = 1$ //number of fuzzy feature agents
	$card(\tilde{X})$ for $\tilde{p}_3 = 2$; $card(\tilde{X})$ for $\tilde{p}_4 = 2$ //composing fuzzy product agents
	$card(\tilde{X})$ for $\tilde{p}_5 = 3$; $card(\tilde{X})$ for $\tilde{p}_6 = 5$ // \tilde{p}_1 to \tilde{p}_6
Fuzzy holonic cells of Level 1	$\tilde{C}_1^1 = \{\tilde{W}_1^1 = (\tilde{w}_1, \{\tilde{w}_1, \tilde{w}_2\}), \tilde{P}_1^1 = (\tilde{p}_1, \{\tilde{p}_1, \tilde{p}_6\}), \tilde{R}_1^1\}$ $\tilde{C}_2^1 = \{\tilde{W}_2^1 = (\tilde{w}_7, \{\tilde{w}_7\}), \tilde{P}_2^1 = (\emptyset, \{\emptyset\}), \tilde{R}_2^1\}$ $\tilde{C}_3^1 = \{\tilde{W}_3^1 = (\tilde{w}_6, \{\tilde{w}_6\}), \tilde{P}_3^1 = (\emptyset, \{\emptyset\}), \tilde{R}_3^1\}$ $\tilde{C}_4^1 = \{\tilde{W}_4^1 = (\tilde{w}_5, \{\tilde{w}_5\}), \tilde{P}_4^1 = (\emptyset, \{\emptyset\}), \tilde{R}_4^1\}$ $\tilde{C}_5^1 = \{\tilde{W}_5^1 = (\tilde{w}_4, \{\tilde{w}_4\}), \tilde{P}_5^1 = (\emptyset, \{\emptyset\}), \tilde{R}_5^1\}$ $\tilde{C}_6^1 = \{\tilde{W}_6^1 = (\tilde{w}_3, \{\tilde{w}_3\}), \tilde{P}_6^1 = (\emptyset, \{\emptyset\}), \tilde{R}_6^1\}$ $\tilde{C}_7^1 = \{\tilde{W}_7^1 = (\emptyset, \{\emptyset\}), \tilde{P}_7^1 = (\tilde{p}_5, \{\tilde{p}_5\}), \tilde{R}_7^1\}$ $\tilde{C}_8^1 = \{\tilde{W}_8^1 = (\emptyset, \{\emptyset\}), \tilde{P}_8^1 = (\tilde{p}_4, \{\tilde{p}_4\}), \tilde{R}_8^1\}$ $\tilde{C}_9^1 = \{\tilde{W}_9^1 = (\emptyset, \{\emptyset\}), \tilde{P}_9^1 = (\tilde{p}_3, \{\tilde{p}_3\}), \tilde{R}_9^1\}$ $\tilde{C}_{10}^1 = \{\tilde{W}_{10}^1 = (\emptyset, \{\emptyset\}), \tilde{P}_{10}^1 = (\tilde{p}_2, \{\tilde{p}_2\}), \tilde{R}_{10}^1\}$
Fuzzy holonic cells of Level 2	$\tilde{C}_1^2 = \{\tilde{W}_1^2 = (\tilde{W}_1^1, \{\tilde{W}_1^1, \tilde{W}_5^1\}), \tilde{P}_1^2 = (\tilde{P}_1^1, \{\tilde{P}_1^1, \tilde{P}_5^1\}), \tilde{R}_1^2\}$ $\tilde{C}_2^2 = \{\tilde{W}_2^2 = (\tilde{W}_2^1, \{\tilde{W}_2^1, \tilde{W}_3^1, \tilde{W}_4^1, \tilde{W}_7^1, \tilde{W}_8^1\}), \tilde{P}_2^2 = (\tilde{P}_2^1, \{\tilde{P}_2^1, \tilde{P}_3^1, \tilde{P}_4^1, \tilde{P}_7^1, \tilde{P}_8^1\}), \tilde{R}_2^2\}$ $\tilde{C}_3^2 = \{\tilde{W}_3^2 = (\tilde{W}_6^1, \{\tilde{W}_6^1\}), \tilde{P}_3^2 = (\tilde{P}_6^1, \{\tilde{P}_6^1\}), \tilde{R}_3^2\}$ $\tilde{C}_4^2 = \{\tilde{W}_4^2 = (\tilde{W}_{10}^1, \{\tilde{W}_{10}^1\}), \tilde{P}_4^2 = (\tilde{P}_{10}^1, \{\tilde{P}_{10}^1\}), \tilde{R}_4^2\}$ $\tilde{C}_5^2 = \{\tilde{W}_5^2 = (\tilde{W}_9^1, \{\tilde{W}_9^1\}), \tilde{P}_5^2 = (\tilde{P}_9^1, \{\tilde{P}_9^1\}), \tilde{R}_5^2\}$
Fuzzy holonic cells of Level 3	$\tilde{C}_1^3 = \{\tilde{W}_1^3 = (\tilde{W}_1^2, \{\tilde{W}_1^2\}), \tilde{P}_1^3 = (\tilde{P}_1^2, \{\tilde{P}_1^2\}), \tilde{R}_1^3\}$ $\tilde{C}_2^3 = \{\tilde{W}_2^3 = (\tilde{W}_2^2, \{\tilde{W}_2^2, \tilde{W}_5^2\}), \tilde{P}_2^3 = (\tilde{P}_2^2, \{\tilde{P}_2^2, \tilde{P}_5^2\}), \tilde{R}_2^3\}$ $\tilde{C}_3^3 = \{\tilde{W}_3^3 = (\tilde{W}_3^2, \{\tilde{W}_3^2\}), \tilde{P}_3^3 = (\tilde{P}_3^2, \{\tilde{P}_3^2\}), \tilde{R}_3^3\}$ $\tilde{C}_4^3 = \{\tilde{W}_4^3 = (\tilde{W}_4^2, \{\tilde{W}_4^2\}), \tilde{P}_4^3 = (\tilde{P}_4^2, \{\tilde{P}_4^2\}), \tilde{R}_4^3\}$
Fuzzy holonic cells of Level 4	$\tilde{C}_1^4 = \{\tilde{W}_1^4 = (\tilde{W}_1^3, \{\tilde{W}_1^3, \tilde{W}_3^3\}), \tilde{P}_1^4 = (\tilde{P}_1^3, \{\tilde{P}_1^3, \tilde{P}_3^3\}), \tilde{R}_1^4\}$

3 fuzzy holonic cells are formed	$\tilde{C}_2^4 = \{\tilde{W}_2^4 = (\tilde{W}_2^3, \{\tilde{W}_2^3\}), \tilde{P}_2^4 = (\tilde{P}_2^3, \{\tilde{P}_2^3\}), \tilde{R}_2^4\}$ $\tilde{C}_3^4 = \{\tilde{W}_3^4 = (\tilde{W}_4^3, \{\tilde{W}_4^3\}), \tilde{P}_3^4 = (\tilde{P}_4^3, \{\tilde{P}_4^3\}), \tilde{R}_3^4\}$
Fuzzy holonic cells of Level 5 1 fuzzy holonic cell is formed	$\tilde{C}_1^5 = \{\tilde{W}_1^5 = (\tilde{W}_1^4, \{\tilde{W}_1^4, \tilde{W}_2^4, \tilde{W}_3^4\}), \tilde{P}_1^5 = (\tilde{P}_1^4, \{\tilde{P}_1^4, \tilde{P}_2^4, \tilde{P}_3^4\}), \tilde{R}_1^5\}$

During the recognition of intelligent AVMC, there are five levels of holonic fuzzy cell agents formed. For instance, in the second level, two holonic fuzzy cell agents are recognized: $\tilde{C}_1^2 = (\tilde{P}_1^2, \tilde{W}_1^2, \tilde{R}_1^2)$ and $\tilde{C}_2^2 = (\tilde{P}_2^2, \tilde{W}_2^2, \tilde{R}_2^2)$. The first fuzzy holonic cell agent \tilde{C}_1^2 is composed by holonic product agent \tilde{P}_1^2 and the fuzzy holonic agent workcenter \tilde{W}_1^2 , related by the fuzzy sub-relationship \tilde{R}_1^2 (Table 10).

The holonic fuzzy agent network presented in Figure 2 shows the holonic structure (holarchy) of cellular systems (in this figure, the coloured cells indicate the progressive regrouping of workcenters and products in holonic structures). Therefore, the group of manufacturing feature agents to be manufactured in each cell agent can be inferred. Consequently, each new product agent involving these feature agents can easily find the corresponding cell agent.

								\tilde{P}_1^5					
								\tilde{P}_1^4	\tilde{P}_2^4			\tilde{P}_3^4	
								\tilde{P}_3^3	\tilde{P}_2^3				\tilde{P}_4^3
								\tilde{P}_2^2	\tilde{P}_2^2		\tilde{P}_5^2	\tilde{P}_4^2	
								\tilde{P}_1^1	\tilde{P}_7^1	\tilde{P}_8^1	\tilde{P}_9^1	\tilde{P}_{10}^1	
								\tilde{p}_1	\tilde{p}_6	\tilde{p}_5	\tilde{p}_4	\tilde{p}_3	\tilde{p}_2
\tilde{W}_1^5	\tilde{W}_1^4	\tilde{W}_3^3	\tilde{W}_1^2	\tilde{W}_1^1	\tilde{w}_1	1	1	0	0	0	0,8		
					\tilde{w}_2	1	1	0	0	0	0,8		
				\tilde{W}_5^1	\tilde{w}_4	1	0,9	0	0	0	0		
		\tilde{W}_3^3	\tilde{W}_2^2	\tilde{W}_6^1	\tilde{w}_3	0,5	0,5	0,4	0,4	0,4	0,2		
	\tilde{W}_2^4	\tilde{W}_2^3		\tilde{W}_2^1	\tilde{w}_7	0,3	0,3	0,8	0,8	0,8	0,2		
				\tilde{W}_3^1	\tilde{w}_6	0	0,3	0,8	0,8	0,8	0		
\tilde{W}_4^1			\tilde{w}_5	0,3	0,3	0,8	0,8	0,8	0,2				

Fig. 2. Holonic representation of the intelligent AVMC formed.

5.2 Scenario of adaptive virtual design and manufacturing cell formation

This section illustrates the adaptivity of the previous AVMC, following a scenario in which, starting from the initial situation presented above (Figure 2), four events will occur (Table 11):

- *Event_1*: ‘Unavailability of the workcenter w_2 ’;
- *Event_2*: ‘Adding the new product p_7 ’; product p_7 has four features, and product-workcenter relationship $\tilde{r}_7 = (P, W)$ for product p_7 is expressed by the vector [0.2, 1.0, 0.3, 0.6, 0.5, 0.5];
- *Event_3*: ‘Unavailability of the workcenter w_4 ’;
- *Event_4*: ‘Adding the new workcenter w_8 ’; product-workcenter relationship $\tilde{r}_7 = (P, W)$ for workcenter w_8 is expressed by the vector [0.8, 0.4, 0.3, 0.0, 0.0, 0.8, 0.4].

The simulation of this scenario is based on the following principles:

- The *unavailability of a workcenter* or the *stopping of a product* to be manufactured are detected in the holon containing them through messages exchanged periodically between the holonic fuzzy attractor agent (\tilde{W}_i^1 or \tilde{P}_j^1) and other holonic fuzzy agents of this holon (\tilde{w}_i or \tilde{p}_j); this process is similar to that followed by some routing protocols in communication networks. When a holonic fuzzy agent does not respond, the holonic fuzzy attractor transmits the information to the holonic fuzzy cell agent of that holon. This in turn transmits the information to the holonic fuzzy cell agent of a higher level, and so on. When the holonic fuzzy cell agent of higher level is informed, it causes the reconfiguration of the entire holonic structure. The sequence diagram presented in Figure 3 formalizes all the messages exchanged when a workcenter becomes unavailable.

- The *adding of a product* or the *adding of a workcenter* is accomplished by sending information to the holonic fuzzy cell agent of higher level which, then, causes the reconfiguration of the entire holonic structure. The sequence diagram presented in Figure 4 formalizes all the messages exchanged when a product is added.

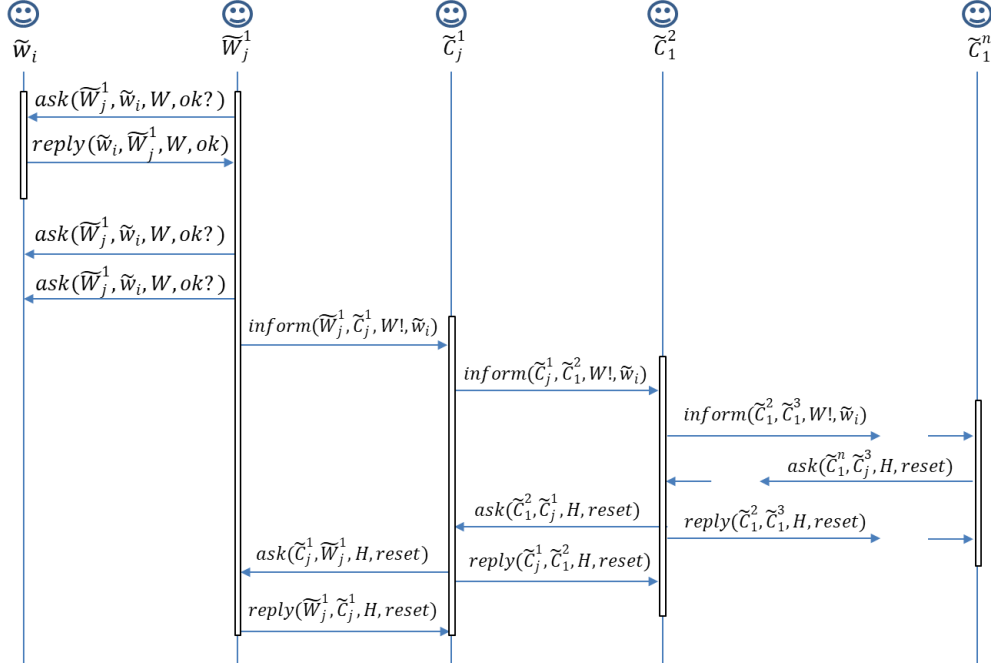


Fig. 3. Sequence diagram in case of “Unavailability of a workcenter”.

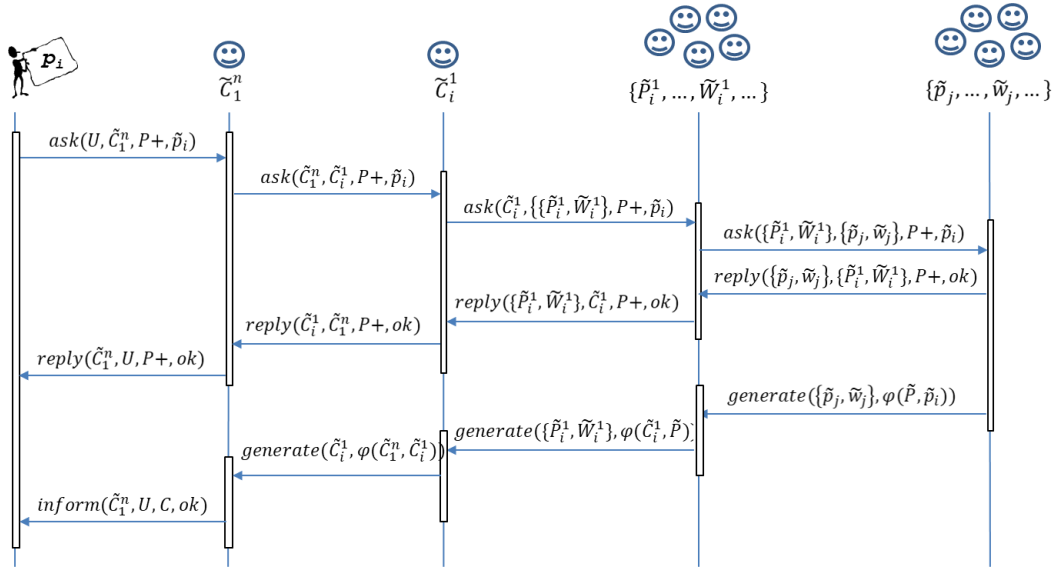


Fig. 4. Sequence diagram in case of “Adding a new product”.

These different events illustrate the adaptation of the initial holonic system (Table 11):

- *Event_1*: The holonic restructuring, following the unavailability of the workcenter w_2 , is realized by the exchanges between the holonic fuzzy agents of all the holonic levels (following the typical sequence diagram in Figure 3). The holonic structure remains at five levels. There is little variation with respect to the preceding structure, except that the holon workcenter \tilde{W}_1^1 of the holon cell \tilde{C}_1^1 has only one fuzzy workcenter agent $\{\tilde{w}_1\}$. This result is

not surprising, since both workcenter w_1 and w_2 have the same manufacturing capabilities. This adaptation scenario gives a new partition of all products and workcenters. Each cell is a part of the partition. Application of the consensus algorithm (see Table 5) allows checking the association stability of workcenters or products in the same cells in different configurations.

- *Event_2*: The holonic structure formed, following the reconfiguration induced by the addition of the new product p_7 , is more important (exchanges between the holonic fuzzy follow the typical sequence diagram in Figure 4): six holonic levels are necessary, as opposed to five levels previously. The fuzzy product agent \tilde{p}_7 is grouped with other fuzzy product agents only from fuzzy holonic cell \tilde{C}_2^4 (i.e., holonic level 4). In this second adaptation scenario, the application of the consensus algorithm also makes it possible to check the association stability of workcenters or products in the same cells in different configurations.
- *Event_3*: The holonic structure formed, following the unavailability of the workcenter w_4 , is again composed of five holonic levels (exchanges between the holonic fuzzy follow the typical sequence diagram in Figure 3). However, the grouping of fuzzy workcenter agents and fuzzy product agents in fuzzy holonic cells is longer, since it takes place mainly at holonic level 4 (\tilde{C}_1^4 and \tilde{C}_2^4). In this new adaptation scenario, the application of the consensus algorithm also makes it possible to check the association stability of workcenters or products in the same cells in different configurations.
- *Event_4*: This time, the holonic structure formed upon the addition of the workcenter w_8 is six holonic levels (exchanges between the holonic fuzzy follow the typical sequence diagram in Figure 4), but the grouping of fuzzy workcenter agents and fuzzy product agents in fuzzy holonic cells are earlier, since made at holonic level three (\tilde{C}_1^3 , \tilde{C}_2^3 and \tilde{C}_3^3). In the latter scenario, the association stability of workcenters or products in the same cells in different configurations is once again verified by the consensus algorithm.

Table 11. AVMC formation following each event of the scenario

Event_1: "Unavailability of the workcenter w_2 "																																																																														
5 levels of holonic fuzzy cell agents are formed: <ol style="list-style-type: none"> 1) $\{\tilde{C}_1^1, \tilde{C}_2^1, \dots, \tilde{C}_{10}^1\}$, 2) $\{\tilde{C}_1^2, \tilde{C}_2^2, \dots, \tilde{C}_5^2\}$, 3) $\{\tilde{C}_1^3, \tilde{C}_2^3, \dots, \tilde{C}_4^3\}$, 4) $\{\tilde{C}_1^4, \tilde{C}_2^4, \tilde{C}_3^4\}$, 5) $\{\tilde{C}_1^5\}$ 						<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><th colspan="6" style="text-align: center;">\tilde{P}_1^5</th></tr> <tr><td style="text-align: center;">\tilde{P}_1^4</td><td colspan="4" style="text-align: center;">\tilde{P}_2^4</td><td style="text-align: center;">\tilde{P}_3^4</td></tr> <tr><td style="text-align: center;">\tilde{P}_2^2</td><td colspan="4" style="text-align: center;">\tilde{P}_2^2</td><td style="text-align: center;">\tilde{P}_5^2</td></tr> <tr><td style="text-align: center;">\tilde{P}_2^2</td><td colspan="2" style="text-align: center;">\tilde{P}_2^2</td><td style="text-align: center;">\tilde{P}_3^2</td><td colspan="2" style="text-align: center;">\tilde{P}_4^2</td></tr> <tr><td style="text-align: center;">\tilde{P}_1^1</td><td style="text-align: center;">\tilde{P}_7^1</td><td style="text-align: center;">\tilde{P}_8^1</td><td style="text-align: center;">\tilde{P}_9^1</td><td colspan="2" style="text-align: center;">\tilde{P}_{10}^1</td></tr> <tr><td style="text-align: center;">\tilde{p}_1</td><td style="text-align: center;">\tilde{p}_6</td><td style="text-align: center;">\tilde{p}_5</td><td style="text-align: center;">\tilde{p}_4</td><td style="text-align: center;">\tilde{p}_3</td><td style="text-align: center;">\tilde{p}_2</td></tr> </table>						\tilde{P}_1^5						\tilde{P}_1^4	\tilde{P}_2^4				\tilde{P}_3^4	\tilde{P}_2^2	\tilde{P}_2^2				\tilde{P}_5^2	\tilde{P}_2^2	\tilde{P}_2^2		\tilde{P}_3^2	\tilde{P}_4^2		\tilde{P}_1^1	\tilde{P}_7^1	\tilde{P}_8^1	\tilde{P}_9^1	\tilde{P}_{10}^1		\tilde{p}_1	\tilde{p}_6	\tilde{p}_5	\tilde{p}_4	\tilde{p}_3	\tilde{p}_2																															
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5) $\{\tilde{C}_1^5, \tilde{C}_2^5\}$	\tilde{W}_1^6	\tilde{W}_1^5	\tilde{W}_1^4	\tilde{W}_1^3	\tilde{W}_1^2	\tilde{W}_1^1	\tilde{w}_1	1	1	0,2	0	0	0	0,8
6) $\{\tilde{C}_1^6\}$			\tilde{W}_6^1	\tilde{W}_4^1	\tilde{W}_2^1	\tilde{W}_3^1	\tilde{w}_4	1	0,9	0,3	0	0	0	0
			\tilde{W}_3^3	\tilde{W}_2^2	\tilde{W}_2^1	\tilde{W}_3^1	\tilde{w}_3	0,5	0,5	1	0,4	0,4	0,4	0,2
			\tilde{W}_2^4	\tilde{W}_2^3	\tilde{W}_3^1	\tilde{W}_7^1	\tilde{w}_7	0,3	0,3	0,6	0,8	0,8	0,8	0,2
			\tilde{W}_2^4	\tilde{W}_2^3	\tilde{W}_4^1	\tilde{W}_6^1	\tilde{w}_6	0	0,3	0,5	0,8	0,8	0,8	0
			\tilde{W}_5^1	\tilde{W}_5^1	\tilde{w}_5	0,3	0,3	0,5	0,8	0,8	0,8	0,2		

Event_3: "Unavailability of the workcenter w_4 "

5 levels of holonic fuzzy cell agents are formed:

- 1) $\{\tilde{C}_1^1, \tilde{C}_2^1, \dots, \tilde{C}_9^1\}$,
- 2) $\{\tilde{C}_1^2, \tilde{C}_2^2, \dots, \tilde{C}_5^2\}$,
- 3) $\{\tilde{C}_1^3, \tilde{C}_2^3, \tilde{C}_3^3\}$,
- 4) $\{\tilde{C}_1^4, \tilde{C}_2^4\}$,
- 5) $\{\tilde{C}_1^5\}$

		\tilde{P}_1^5											
		\tilde{P}_1^4					\tilde{P}_2^4						
		\tilde{P}_1^3			\tilde{P}_2^3			\tilde{P}_3^3			\tilde{P}_2^2		
		\tilde{P}_1^2		\tilde{P}_4^2		\tilde{P}_3^2		\tilde{P}_5^2		\tilde{P}_2^1			
		\tilde{P}_1^1		\tilde{P}_9^1		\tilde{P}_6^1		\tilde{P}_7^1		\tilde{P}_8^1		\tilde{P}_2^1	
		\tilde{p}_1		\tilde{p}_6		\tilde{p}_2		\tilde{p}_5		\tilde{p}_4		\tilde{p}_3	\tilde{p}_7
\tilde{W}_1^5	\tilde{W}_1^4	\tilde{W}_1^3	\tilde{W}_1^2	\tilde{W}_1^1	\tilde{w}_1	1	1	0,8	0	0	0	0,2	
	\tilde{W}_3^3	\tilde{W}_3^3	\tilde{W}_5^1	\tilde{w}_3	0,5	0,5	0,2	0,4	0,4	0,4	1		
	\tilde{W}_2^4	\tilde{W}_2^3	\tilde{W}_2^2	\tilde{W}_2^1	\tilde{w}_2	0,3	0,3	0,2	0,8	0,8	0,8	0,5	
	\tilde{W}_2^4	\tilde{W}_2^3	\tilde{W}_3^1	\tilde{W}_6^1	\tilde{w}_6	0	0,3	0	0,8	0,8	0,8	0,5	
	\tilde{W}_2^4	\tilde{W}_2^3	\tilde{W}_4^1	\tilde{W}_5^1	\tilde{w}_5	0,3	0,3	0,2	0,8	0,8	0,8	0,6	

Event_4: "Adding the new workcenter w_8 "

6 levels of holonic fuzzy cell agents are formed:

- 1) $\{\tilde{C}_1^1, \tilde{C}_2^1, \dots, \tilde{C}_{10}^1\}$,
- 2) $\{\tilde{C}_1^2, \tilde{C}_2^2, \dots, \tilde{C}_6^2\}$,
- 3) $\{\tilde{C}_1^3, \tilde{C}_2^3, \dots, \tilde{C}_4^3\}$,
- 4) $\{\tilde{C}_1^4, \tilde{C}_2^4, \tilde{C}_3^4\}$,
- 5) $\{\tilde{C}_1^5, \tilde{C}_2^5\}$
- 6) $\{\tilde{C}_1^6\}$

		\tilde{P}_1^6											
		\tilde{P}_1^5					\tilde{P}_2^5						
		\tilde{P}_1^4			\tilde{P}_4^4			\tilde{P}_3^4			\tilde{P}_3^3		
		\tilde{P}_1^3		\tilde{P}_5^3		\tilde{P}_3^3		\tilde{P}_6^3		\tilde{P}_2^2			
		\tilde{P}_1^2		\tilde{P}_5^2		\tilde{P}_3^2		\tilde{P}_6^2		\tilde{P}_2^1			
		\tilde{P}_1^1		\tilde{P}_{10}^1		\tilde{P}_7^1		\tilde{P}_8^1		\tilde{P}_9^1		\tilde{P}_2^1	
		\tilde{p}_1		\tilde{p}_6		\tilde{p}_2		\tilde{p}_5		\tilde{p}_4		\tilde{p}_3	\tilde{p}_7
\tilde{W}_1^6	\tilde{W}_1^5	\tilde{W}_1^4	\tilde{W}_1^3	\tilde{W}_1^2	\tilde{W}_1^1	\tilde{w}_1	1	1	0,8	0	0	0	
	\tilde{W}_4^4	\tilde{W}_3^3	\tilde{W}_3^3	\tilde{W}_4^1	\tilde{W}_3^1	\tilde{w}_8	0,8	0,8	0,4	0	0	0,3	0,4
	\tilde{W}_3^4	\tilde{W}_3^3	\tilde{W}_2^2	\tilde{W}_2^1	\tilde{w}_3	0,5	0,5	0,2	0,4	0,4	0,4	0,4	
	\tilde{W}_2^4	\tilde{W}_2^3	\tilde{W}_2^2	\tilde{W}_4^1	\tilde{w}_7	0,3	0,3	0,2	0,8	0,8	0,8	0,8	
	\tilde{W}_2^4	\tilde{W}_2^3	\tilde{W}_3^1	\tilde{W}_5^1	\tilde{w}_6	0	0,3	0	0,8	0,8	0,8	0,8	
			\tilde{W}_6^1	\tilde{W}_5^1	\tilde{w}_5	0,3	0,3	0,2	0,8	0,8	0,8	0,8	

5.3 Consensus-searching

The two following tables will allow us to analyze and understand the scenario of adaptive virtual design and manufacturing cell formation proposed as case study. It is thus possible to verify that adaptivity is indeed a characteristic of the AVMC modelled with our fuzzy holon agent approach.

Table 12 shows the Presence Matrix. This matrix makes it possible to follow the evolution of the presence of a workcenter in the different AVMCs which are formed following the four events of the scenario described in the previous section: if Presence Matrix cell $(\tilde{w}_k, \tilde{W}_j^i)$ equal to 1 then workcenters \tilde{w}_k is part of AVMC \tilde{W}_j^i , otherwise it is not part of it. We can notice in this matrix that the workcenter \tilde{w}_8 , which is not part of the initial configuration, appears with *Event_4*.

Table 13 presents the Consensus Matrix. This matrix shows the formation of two consensual Holon Cluster (\tilde{C}_1 and \tilde{C}_2). We can notice a very high stability/robustness of the five AVMCs constituting the consensual Holon Cluster \tilde{C}_2 . These AVMCs include the workcenters \tilde{w}_5 , \tilde{w}_6 and \tilde{w}_7 . The workcenters grouped in the consensual Holon Cluster \tilde{C}_1 are more flexible/adaptive (i.e., flexibility and adaptivity when restructuring the AVMCs that contains them):

- The workcenter \tilde{w}_1 is very stable in this consensual Holon Cluster.
- The workcenter \tilde{w}_2 disappears from consensual Holon Cluster \tilde{C}_1 following *Event_1*.
- The workcenter \tilde{w}_3 (workcenter very interesting for its flexibility) can change consensual Holon Cluster. In our scenario, it moves three times: firstly, following the *Event_2* which proposes to manufacture a new product; secondly, following the

Event_3 when disappears the workcenter \tilde{w}_4 ; and third, following the *Event_4* when the workcenter \tilde{w}_8 appears.

- The workcenter \tilde{w}_4 disappears from the cluster following *Event_3*.
- The workcenter \tilde{w}_8 appears in the cluster following *Event_4*.

Table 12. Presence Matrix

	AVMC (Fig 2)		AVMC Event 1		AVMC Event 2		AVMC Event 3		AVMC Event 4	
	\tilde{W}_1^4	\tilde{W}_2^4	E1. \tilde{W}_1^4	E1. \tilde{W}_2^4	E2. \tilde{W}_1^4	E2. \tilde{W}_2^4	E3. \tilde{W}_1^4	E3. \tilde{W}_2^4	E4. \tilde{W}_1^5	E4. \tilde{W}_2^5
\tilde{w}_1	1	0	1	0	1	0	1	0	1	0
\tilde{w}_2	1	0	0	0	0	0	0	0	0	0
\tilde{w}_3	1	0	1	0	0	1	1	0	0	1
\tilde{w}_4	1	0	1	0	1	0	0	0	0	0
\tilde{w}_5	0	1	0	1	0	1	0	1	0	1
\tilde{w}_6	0	1	0	1	0	1	0	1	0	1
\tilde{w}_7	0	1	0	1	0	1	0	1	0	1
\tilde{w}_8	0	0	0	0	0	0	0	0	1	0

Table 13. Consensus Matrix

	First Consensual Holon Cluster: \tilde{C}_1					Second Consensual Holon Cluster: \tilde{C}_2				
	\tilde{W}_1^4	E1. \tilde{W}_1^4	E2. \tilde{W}_1^4	E3. \tilde{W}_1^4	E4. \tilde{W}_1^5	\tilde{W}_2^4	E1. \tilde{W}_2^4	E2. \tilde{W}_2^4	E3. \tilde{W}_2^4	E4. \tilde{W}_2^5
\tilde{w}_1	1	1	1	1	1	0	0	0	0	0
\tilde{w}_2	1	0	0	0	0	0	0	0	0	0
\tilde{w}_3	1	1	0	1	0	0	0	1	0	1
\tilde{w}_4	1	1	1	0	0	0	0	0	0	0
\tilde{w}_8	0	0	0	0	1	0	0	0	0	0
\tilde{w}_5	0	0	0	0	0	1	1	1	1	1
\tilde{w}_6	0	0	0	0	0	1	1	1	1	1
\tilde{w}_7	0	0	0	0	0	1	1	1	1	1

6 Discussion

The powerful role of the CAD features is exploited to organize and integrate product design and product manufacturing engineering knowledge in such a way that a set of CAD products, modelled by set of features, can be manufactured in adaptive intelligent virtual manufacturing cells. The concepts of holon, attractor, and uncertainty are used to model the adaptive intelligent virtual manufacturing cells in CBDM. The concepts of the holon and the attractor allow multi-scale cell formation. This multi-scale formation overcomes the lack of adaptivity of the traditional cell formation problem based on manufacturing process data [Burbidge, 1975; Wei and Gaither, 1990; Sundaram and Lian, 1990; Tam, 1990; Gupta and Seifoddini, 1990; Chandrasekharan and Rajagopalan, 1986; Al-Quattan, 1990; Kusiak and Chow, 1987; Aalaei and Davoudpour, 2016; Ameli and Arkat, 2008; Saidi-Mehrabad and Safei, 2007; Mak et al., 2000; Yasuda et al., 2005; Lei and Wu, 2006] or product data [Xu and Wang, 1989; Ben-Arieh and Triantaphyllou, 1992; Ben-Arieh et al., 1996; Mutuel and Ostrosi, 2002]. Indeed, the fuzzy cell holons also overcome the distinction continuous-discontinuous of traditional cell formation problem. Indeed holonic fuzzy agents form a communication networks. The communication follows the fundamental property of the holonic structure. A nested hierarchical order, called a *holarchy of manufacturing cells*, is formed. A hierarchy of fuzzy cells holons is thus a holarchy. The holarchic model modernise the cell formation problem. Since a fuzzy cell holon is embedded in larger whole fuzzy cell holons, it is influenced by and also influencing these larger wholes. Thus, fuzzy cell holon is a whole fuzzy cell that is a part of a greater whole fuzzy cell, which

at the same time contains fuzzy cells, of which it is composed. Horizontal self-similarity relates to self-similarity across different fuzzy cells on a given level of aggregation. Vertical self-similarity refers to self-similarity across different levels of aggregation fuzzy cells. Thus, from horizontal and vertical communication, holonic fuzzy cell agents can reconfigure partially or totally the manufacturing systems.

The proposed cell holon formation model is driven by the integrative tendencies of cell formation: the product design knowledge and product manufacturing knowledge. The proposed model shows that adaptive intelligent virtual manufacturing cells, with holonic structure, emerge from the interaction of fuzzy workcenter agents and the fuzzy product agents with fuzzy agent attractors.

Virtual cell formation is not an orderly and well-behaved design problem. Sudden transformations and unpredictable changes characterize the virtual cell formation problem. The consensus-searching, as the problem of searching for the best overlap in-between fuzzy cell holons, allows holonic fuzzy agents to search for the most consensual space.

These fuzzy cell holons also overcome the continuous-discontinuous distinction of the traditional cell formation problem. Indeed, holonic fuzzy agents form communication networks. The communication follows the fundamental property of the holonic structure. A nested hierarchical order, called a *holarchy of manufacturing cells*, is formed. A hierarchy of fuzzy cell holons is, thus, a holarchy. The holarchic model modernises the cell formation problem. Since a fuzzy cell holon is embedded in larger whole fuzzy cell holons, it is influenced by, and also influences, these larger wholes. Thus, a fuzzy cell holon is a whole fuzzy cell that is a part of a greater whole fuzzy cell, which, at the same time, contains the fuzzy cells of which it is composed. Horizontal self-similarity relates to self-similarity across different fuzzy cells on a given level of aggregation. Vertical self-similarity refers to self-similarity across different levels of aggregate fuzzy cells. Thus, from horizontal and vertical communication, holonic fuzzy cell agents can partially or totally reconfigure the manufacturing systems.

7 Conclusion

Research presented in this paper assumed that cloud-based design and manufacturing can be referred to as a dynamic service-oriented network where a set of *CAD* products, modelled by a set of features, can be manufactured in virtual manufacturing cells under certain constraints. Using the concepts of the intelligent manufacturing features and integrating the uncertainty in modelling of holonic fuzzy agents, this paper presented an approach to addressing adaptive virtual design and manufacturing cell formation in cloud-based design and manufacturing. In addition, the paper proposed that products, manufacturing systems as well as their models, should be designed holistically as adaptive objects.

Manufacturing features, modelled as holonic fuzzy agents, provide a means to integrate the product data and manufacturing data. The presentation of a scenario causing different changes in workcenters (unavailability of machines or addition of new products) has illustrated the behaviour of holonic fuzzy agents during the reconfiguration of the virtual manufacturing cells' holonic structure, and thus has demonstrated the adaptive capacity of this kind of holonic structure. Thus, the product-feature-workcenter network modelled by holonic fuzzy agents offers a promising adaptive solution. Fuzzy cell holons also overcome the continuous-discontinuous distinction of the traditional cell formation problem. Horizontal and vertical communication of fuzzy cell agents allows the partial or total reconfiguration of virtual cellular manufacturing systems.

Virtual cell formation is not an orderly and well-behaved design problem because sudden transformations and unpredictable changes characterize virtual cell formation problem. The consensus searching as the problem of searching for the best overlap in-between fuzzy cell holons allows for searching and the evaluation of the adaptability of intelligent *AVMC*.

Further developments include the study of the dynamic behaviour of intelligent *AVMC* in digitalized design and manufacturing network through analysis of the creation or destruction of attractors that are identified and defined in the holonic fuzzy cell agents.

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Author Biographies

Alain-Jérôme Fougères graduated in computer engineering. He received his PhD and HDR in artificial intelligence from the University of Technology of Compiègne – (UTC, France). He is Professor of Computer Science at ECAM Rennes where he leads a research team on IOT. His first research areas of interests were: 1) natural language processing for writing software specifications, and 2) design of agent-based systems, considering interactions, communication, and cooperation problems. Over the last ten years, his research has focused on agent-based modelling and simulation of complex systems, mainly using the concepts of intelligent entities, fuzzy agents and quantum agents.

Egon Ostrosi holds a Ph.D. from the University of Strasbourg, and is currently Associate Professor at the University of Technology of Belfort-Montbéliard (UTBM, France). His research concerns intelligent mechanical product design and modelling, knowledge engineering and its application in mechanical design and manufacturing systems. He has been actively involved with the development of methods for intelligent product design, CAD product modelling, product configuration, and collaborative and distributed design process.

Appendix – Complete traces of adaptive virtual cells formation

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#-----
#Holonic architecture of the initial situation
#-----
#Cells of level 1
C1.1 = {W1.1 = {w1,w2}, P1.1 = {p1,p6}}
C1.2 = {W1.2 = {w7}, P1.2 = {null}}
C1.3 = {W1.3 = {w6}, P1.3 = {null}}
C1.4 = {W1.4 = {w5}, P1.4 = {null}}
C1.5 = {W1.5 = {w4}, P1.5 = {null}}
C1.6 = {W1.6 = {w3}, P1.6 = {null}}
C1.7 = {W1.7 = {null}, P1.7 = {p5}}
C1.8 = {W.8 = {null}, P1.8 = {p4}}
C1.9 = {W.9 = {null}, P1.9 = {p3}}
C1.10 = {W1.10 = {null}, P1.10 = {p2}}
#Cells of level 2
C2.1 = {W2.1 = {W1.1,W1.5}, P2.1 = {P1.1,P1.5}}
C2.2 = {W2.2 = {W1.2,W1.3,W1.4,W1.7,W1.8}, P2.2 = {P1.2,P1.3,P1.4,P1.7,P1.8}}
C2.3 = {W2.3 = {W1.6}, P2.3 = {P1.6}}
C2.4 = {W2.4 = {W1.10}, P2.4 = {P1.10}}
C2.5 = {W2.5 = {W1.9}, P2.5 = {P1.9}}
#Cells of level 3
C3.1 = {W3.1 = {W2.1}, P3.1 = {P2.1}}
C3.2 = {W3.2 = {W2.2,W2.5}, P3.2 = {P2.2,P2.5}}
C3.3 = {W3.3 = {W2.3}, P3.3 = {P2.3}}
C3.4 = {W3.4 = {W2.4}, P3.4 = {P2.4}}
#Cells of level 4
C4.1 = {W4.1 = {W3.1,W3.3}, P4.1 = {P3.1,P3.3}}
C4.2 = {W4.2 = {W3.2}, P4.2 = {P3.2}}
C4.3 = {W4.3 = {W3.4}, P4.3 = {P3.4}}
#Cells of level 5
C5.1 = {W5.1 = {W4.1,W4.2,W4.3}, P5.1 = {P4.1,P4.2,P4.3}}
#-----
#Event_1: "Unavailability of the workcenter w2"
#-----
#Cells of level 1
C1.1 = {W1.1 = {w1}, P1.1 = {p1, p6}}
C1.2 = {W1.2 = {w7}, P1.2 = {null}}
C1.3 = {W1.3 = {w6}, P1.3 = {null}}
C1.4 = {W1.4 = {w5}, P1.4 = {null}}
C1.5 = {W1.5 = {w4}, P1.5 = {null}}
C1.6 = {W1.6 = {w3}, P1.6 = {null}}
C1.7 = {W1.7 = {null}, P1.7 = {p5}}
C1.8 = {W1.8 = {null}, P1.8 = {p4}}
C1.9 = {W1.9 = {null}, P1.9 = {p3}}
C1.10 = {W1.10 = {null}, P1.10 = {p2}}
#Cells of level 2
C2.1 = {W2.1 = {W1.1,W1.5}, P2.1 = {P1.1,P1.5}}
C2.2 = {W2.2 = {W1.2,W1.3,W1.4,W1.7,W1.8}, P2.2 = {P1.2,P1.3,P1.4,P1.7,P1.8}}
C2.3 = {W2.3 = {W1.6}, P2.3 = {P1.6}}
C2.4 = {W2.4 = {W1.10}, P2.4 = {P1.10}}
C2.5 = {W2.5 = {W1.9}, P2.5 = {P1.9}}
#Cells of level 3
C3.1 = {W3.1 = {W2.1}, P3.1 = {P2.1}}
C3.2 = {W3.2 = {W2.2,W2.5}, P3.2 = {P2.2,P2.5}}
C3.3 = {W3.3 = {W2.3}, P3.3 = {P2.3}}
C3.4 = {W3.4 = {W2.4}, P3.4 = {P2.4}}
#Cells of level 4
C4.1 = {W4.1 = {W3.1,W3.3}, P4.1 = {P3.1,P3.3}}
C4.2 = {W4.2 = {W3.2}, P4.2 = {P3.2}}
C4.3 = {W4.3 = {W3.4}, P4.3 = {P3.4}}
#Cells of level 5
C5.1 = {W5.1 = {W4.1,W4.2,W4.3}, P5.1 = {P4.1,P4.2,P4.3}}
#-----
#Event_2: "Adding a new product p7"
#-----
#Cells of level 1
C1.1 = {W1.1 = {w1}, P1.1 = {p1, p6}}
C1.2 = {W1.2 = {w3}, P1.2 = {p7}}
C1.3 = {W1.3 = {w7}, P1.3 = {null}}
C1.4 = {W1.4 = {w6}, P1.4 = {null}}
C1.5 = {W1.5 = {w5}, P1.5 = {null}}
C1.6 = {W1.6 = {w4}, P1.6 = {null}}
C1.7 = {W1.7 = {null}, P1.7 = {p5}}
C1.8 = {W1.8 = {null}, P1.8 = {p4}}

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C1.9 = {W1.9 = {null}, P1.9 = {p3}}
C1.10 = {W1.10 = {null}, P1.10 = {p2}}
#Cells of level 2
C2.1 = {W2.1 = {W1.1,W1.6}, P2.1 = {P1.1,P1.6}}
C2.2 = {W2.2 = {W1.2}, P2.2 = {P1.2}}
C2.3 = {W2.3 = {W1.3,W1.4,W1.5,W1.7,W1.8}, P2.3 = {P1.3,P1.4,P1.5,P1.7,P1.8}}
C2.4 = {W2.4 = {W1.10}, P2.4 = {P1.10}}
C2.5 = {W2.5 = {W1.9}, P2.5 = {P1.9}}
#Cells of level 3
C3.1 = {W3.1 = {W2.1}, P3.1 = {P2.1}}
C3.2 = {W3.2 = {W2.3,W2.5}, P3.2 = {P2.3,P2.5}}
C3.3 = {W3.3 = {W2.2}, P3.3 = {P2.2}}
C3.4 = {W3.4 = {W2.4}, P3.4 = {P2.4}}
#Cells of level 4
C4.1 = {W4.1 = {W3.1}, P4.1 = {P3.1}}
C4.2 = {W4.2 = {W3.2,W3.3}, P4.2 = {P3.2,P3.3}}
C4.3 = {W4.3 = {W3.4}, P4.3 = {P3.4}}
#Cells of level 5
C5.1 = {W5.1 = {W4.1,W4.2}, P5.1 = {P4.1,P4.2}}
C5.2 = {W5.2 = {W4.3}, P5.2 = {P4.3}}
#Cells of level 6
C6.1 = {W6.1 = {W5.1,W5.2}, P6.1 = {P5.1,P5.2}}
#-----
#Event_3 "Unavailability of the workcenter w4"
#-----
#Cells of level 1
C1.1 = {W1.1 = {w1}, P1.1 = {p1, p6}}
C1.2 = {W1.2 = {w3}, P1.2 = {p7}}
C1.3 = {W1.3 = {w7}, P1.3 = {null}}
C1.4 = {W1.4 = {w6}, P1.4 = {null}}
C1.5 = {W1.5 = {w5}, P1.5 = {null}}
C1.6 = {W1.6 = {null}, P1.6 = {p5}}
C1.7 = {W1.7 = {null}, P1.7 = {p4}}
C1.8 = {W1.8 = {null}, P1.8 = {p3}}
C1.9 = {W1.9 = {null}, P1.9 = {p2}}
#Cells of level 2
C2.1 = {W2.1 = {W1.1}, P2.1 = {P1.1}}
C2.2 = {W2.2 = {W1.2}, P2.2 = {P1.2}}
C2.3 = {W2.3 = {W1.3,W1.4,W1.5,W1.6,W1.7}, P2.3 = {P1.3,P1.4,P1.5,P1.6,P1.7}}
C2.4 = {W2.4 = {W1.9}, P2.4 = {P1.9}}
C2.5 = {W2.5 = {W1.8}, P2.5 = {P1.8}}
#Cells of level 3
C3.1 = {W3.1 = {W2.1,W2.4}, P3.1 = {P2.1,P2.4}}
C3.2 = {W3.2 = {W2.3,W2.5}, P3.2 = {P2.3,P2.5}}
C3.3 = {W3.3 = {W2.2}, P3.3 = {P2.2}}
#Cells of level 4
C4.1 = {W4.1 = {W3.1}, P4.1 = {P3.1}}
C4.2 = {W4.2 = {W3.2,W3.3}, P4.2 = {P3.2,P3.3}}
#Cells of level 5
C5.1 = {W5.1 = {W4.1,W4.2}, P5.1 = {P4.1,P4.2}}
#-----
#Event_4: "Adding a new workcenter w8"
#-----
#Cells of level 1
C1.1 = {W1.1 = {w1}, P1.1 = {p1, p6}}
C1.2 = {W1.2 = {w3}, P1.2 = {p7}}
C1.3 = {W1.3 = {w8}, P1.3 = {null}}
C1.4 = {W1.4 = {w7}, P1.4 = {null}}
C1.5 = {W1.5 = {w6}, P1.5 = {null}}
C1.6 = {W1.6 = {w5}, P1.6 = {null}}
C1.7 = {W1.7 = {null}, P1.7 = {p5}}
C1.8 = {W1.8 = {null}, P1.8 = {p4}}
C1.9 = {W1.9 = {null}, P1.9 = {p3}}
C1.10 = {W1.10 = {null}, P1.10 = {p2}}
#Cells of level 2
C2.1 = {W2.1 = {W1.1}, P2.1 = {P1.1}}
C2.2 = {W2.2 = {W1.2}, P2.2 = {P1.2}}
C2.3 = {W2.3 = {W1.4,W1.5,W1.6,W1.7,W1.8}, P2.3 = {P1.4,P1.5,P1.6,P1.7,P1.8}}
C2.4 = {W2.4 = {W1.3}, P2.4 = {P1.3}}
C2.5 = {W2.5 = {W1.10}, P2.5 = {P1.10}}
C2.6 = {W2.6 = {W1.9}, P2.6 = {P1.9}}
#Cells of level 3
C3.1 = {W3.1 = {W2.1,W2.4}, P3.1 = {P2.1,P2.4}}
C3.2 = {W3.2 = {W2.3,W2.6}, P3.2 = {P2.3,P2.6}}
C3.3 = {W3.3 = {W2.2}, P3.3 = {P2.2}}
C3.4 = {W3.4 = {W2.5}, P3.4 = {P2.5}}

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#Cells of level 4
C4.1 = {W4.1 = {W3.1}, P4.1 = {P3.1}}
C4.2 = {W4.2 = {W3.2,W3.4}, P4.2 = {P3.2,P3.4}}
C4.3 = {W4.3 = {W3.3}, P4.3 = {P3.3}}
#Cells of level 5
C5.1 = {W5.1 = {W4.1}, P5.1 = {P4.1}}
C5.2 = {W5.2 = {W4.2,W4.3}, P5.2 = {P4.2,P4.3}}
#Cells of level 6
C6.1 = {W6.1 = {W5.1,W5.2}, P6.1 = {P5.1,P5.2}}
```