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Article

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Article

# pELECTRE-Tri: Probabilistic ELECTRE-Tri Method—Application for the Energy Renovation of Buildings

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**Abstract:** This paper proposes the method pELECTRE Tri, a novel probabilistic Multi-Criteria Decision Making (MCDM) method using the traditional ELECTRE Tri, probability distribution, and Monte Carlo simulation to support informed decision-making in complex and uncertain decision environments. The proposed method is illustrated through a case study involving the renovation of three social housing buildings. The paper provides new insights into the application of probabilistic sorting MCDM in the context of energy efficiency in buildings and highlights the benefits of using probabilities rather than crisp values to categorize alternatives enabling stakeholders to make better use of available resources, especially when dealing with a large dataset of energy measures with different features. The methodology implemented in Python (DOI: 10.5281/zenodo.7967655) is available as an open source.

**Keywords:** multi-criteria decision making; ELECTRE Tri; energy retrofitting; sustainability



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## 1. Introduction

Most of the energy in the building sector is consumed by existing buildings, while only 1–3% of existing buildings are replaced by new buildings annually [1,2]. According to the International Energy Agency, the building sector is a highly cost-effective area for reducing energy consumption. As part of the efforts to decrease energy usage and carbon dioxide emissions, 25 policy recommendations have been published focusing on energy efficiency. If these recommendations are adopted globally, it could result in annual savings of 7.6 giga tons of carbon dioxide emissions and 1950 million tons of the oil equivalent in annual energy consumption by the year 2030 [3,4].

Building retrofitting is a process which includes the removal, installation, replacement, and rearrangement of components that the existing building did not possess when constructed initially [5]. Energy use can be reduced significantly in existing buildings through proper retrofitting, ranging from 50% to 85% [6–10]. Building retrofit evaluations are challenging to initiate because the building and its environment are complex systems with multiple criteria to consider. These criteria are related to economic, technical, ecological, social, comfort, and aesthetical aspects in which all subsystems can influence overall energy efficiency and interdependence between subsystems, which plays a significant role [11].

Energy retrofitting typically reduces the energy consumption of a building through energy efficiency measures such as improved thermal insulation, thermal load reduction, solar shading, the replacement of inefficient equipment, and the implementation of renewable energy technologies. Building energy retrofitting benefits the environment, the economy, and the society. Environmental benefits include reduced greenhouse emissions, social benefits include improved occupant health and comfort, and economic benefits can be a cost-effective way of increasing the asset value of an existing building rather than constructing a new building. Existing building retrofitting presents numerous challenges and opportunities. A key challenge is the presence of numerous uncertainties, including

climate change, service change, human behavior, and government policy change, all of which have a direct impact on the choice of retrofit technologies and the project outcome. Other difficulties may include financial constraints and barriers, perceived long payback periods, and operational interruptions. Buildings' subsystems are highly interdependent. On a technical level, different retrofit measures may have varying effects on the associated building subsystems. The quantitative and qualitative nature of various criteria adds to the complexity of the analysis [12–15]. There are trade-offs between retrofitting project benefit categories due to the diverse interests and goals of stakeholders. For example, reducing life-cycle carbon emissions may be the priority of the local government but not the interest of the other stakeholders. When discussing the effectiveness of a retrofit intervention, improving the occupant's comfort, specifically thermal, visual, and acoustic comforts, is frequently an issue. As a result, when discussing building energy retrofits, it is necessary to consider the trade-offs between these objectives in terms of embodied emissions, operational emissions, occupant comfort, and investment costs [16–20]. The energy retrofitting of a residential building in Orlando, USA was carried out with the help of several energy efficiency measures and the results indicated an annual energy saving of up to 29% [21]. In a case study conducted in Australia to convert an existing residential house to a net zero energy house, the results indicated a 50% increase in the thermal efficiency [22].

The multi-criteria decision making (MCDM) approach is gaining popularity in energy retrofit decision processes because of its adaptability and potential to reach a tradeoff among various conflicting criteria and to foster communication among stakeholders [23]. An important challenge is to deal with the probabilistic nature of much data considered in the decision process.

Section 2 provides a review of the literature on the use of MCDM methods in decision-making problems in energy retrofitting of existing buildings and shows the necessity of using probabilistic values in MCDM. Section 3 describes the methodology used to introduce the probabilistic ELECTRE Tri approach. Section 4 illustrates the implementation of the methodology in a real-world case study of a social housing project [24]. Section 5 discusses the results of using the probabilistic ELECTRE Tri method to categorize various energy alternatives in terms of probabilities. Finally, the paper ends with conclusions.

## 2. Literature Review

Multi-criteria decision making is a subfield of operational research that is used to aid decision making in complex scenarios with opposing and conflicting criteria [25–27]. It enables the systematic integration of both quantitative and qualitative analyses to attain informed decisions [28]. When different MCDM methods are applied to the same problem, they produce different results because they deal differently with performance measures and criteria weights. Furthermore, in MCDM problems, the best compromise solution is recommended based on the information available in the given decision situation, rather than the “true” result, as some engineering methods would provide [29,30].

During the last few decades, several new types of MCDM have been developed and the maturity of the existing ones has improved, thus increasing their application to real-world problems. The main differences between these methods are related to the complexity of the problems that can be formulated and solved, the weighting methods of the criteria, the representation of thresholds, the possibility of uncertain data, and finally, the type of data aggregation [31,32]. MCDM shares a number of fundamental principles, which include outlining a set of actions aimed at addressing the problem at hand, establishing at least two different criteria to evaluate each action, the use of thresholds, and having a decision maker responsible for making the final decision [33]. Decisions made through MCDM are justifiable and clear because they are documented and traceable due to them being one of the widely used techniques to support sustainability assessment in the context of energy systems. The ability to consider multiple criteria and objectives at once make MCDM methods well accepted in the domain of energy retrofitting decision processes [34–36]. The ideal solution is a trade-off between a variety of energy and non-energy factors such as

economic, environmental, technical, social, and regulatory factors, among others. The social factors of a sustainable building are the most overlooked, though they are occasionally investigated. However, some studies mentioned building characteristics that promote social sustainability [37,38]. It is not expected of a building to exhibit the best environmental, economic, and social performance at the same time, as these are often contradictory. As a result, balancing the impacts on these dimensions over its entire life cycle is a critical factor in achieving sustainable buildings. Sustainability assessments are designed to collect and disseminate information to help with decision-making [39].

The application of the MCDM approach to energy building renovation has advantages in comparison to the traditional single criterion analysis method because it can give the comprehensive score/ranking of the alternatives under the effect of various multiple criteria, whereas the single criterion analysis is deemed insufficient [40,41]. The stakeholders are likely to turn to intuition in the event of a lack of well-defined practices in the use of decision-making tools [42]. MCDM methods decompose the problem of decision making into steps, compare the relative importance of the criteria, and select the optimal alternative from a set of alternatives in a fuzzy, uncertain, and risky environment [43]. The following steps can be used to describe decision-making [44]: problem identification, goal and objective outlining, criteria establishment, alternative identification, decision-making method selection, criterion assessment of alternatives, review, and outcome validation.

The existing MCDM approaches can be classified into two main school of thoughts [45,46]:

- aggregation methods or the American school of thought;
- outranking methods or the French school of thought.

### 2.1. Aggregation Methods

In the aggregation method, the view of the decision-maker is disaggregated in the sense that the decision-maker is assumed to have a complete preference system. This preference system allows them to express their preferences on all aspects of the decision problem and it can be derived by asking the decision-maker relevant questions.

The aggregation methods use a functional approach that base their methods on the use of value or usability. These approaches typically ignore the possibility of data inaccuracies or uncertainty, as well as decision-maker preferences. This set of methods is closely related to an operational approach that employs a single criterion for synthesis. The primary methods include the Multi-Attribute Value/Utility Theory (MAUT), Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Simple Multi-Attribute Rating Technique (SMART), Utility Additive (UTA), Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH), and Technique for Order Preferences by Similarity to Ideal (TOPSIS) [47].

The MAUT method is based on the fundamental assumption that a single usability function, which takes into account all relevant criteria, can effectively capture the decision-maker's preferences. The MAUT aggregation method was used to evaluate five different energy production alternatives [48]. The method of the AHP is considered the most well-known and widely used functional method within the aggregation methods. The AHP enables decision-makers to prioritize their decision-making problems, whereas the ANP is a more complex variation that allows for the construction of a network model with links between criteria, variants, and feedback. The AHP is a compensatory method in which a high score on one criterion can compensate for a low score on another criterion. In the energy sector, the method of AHP is widely used. The AHP was used to propose an energy efficiency rating system based on criteria for existing buildings in Egypt [49]. The AHP was also used to show how experts prioritize various factors and contexts in their decisions to implement energy efficiency retrofits in the United States [50]. Additionally, the AHP technique was used to select the most suited scenario for electricity generation from four scenarios [51]. The AHP was also used to rank assessment themes and identify the stakeholders' priorities in Malaysia's refurbishment building assessment scheme [52]. In the SMART, the decision-maker uses a value function to mathematically

convert criteria values to a common internal scale. The SMART was used to determine decision-makers preferences for subjective criteria related to the design of sustainable and resilient buildings [53]. The UTA method extracts the decision-makers' preferences from a reference set of variants. The MACBETH compares individual variants in a pairwise comparison matrix and aggregates criterion preferences as a weighted average using qualitative evaluations. The fuzzy MACBETH model was used to explore the inefficiencies and uncertainties related with South European solid waste management systems [54]. The TOPSIS method is constructed on the assumption that the positive ideal alternatives have the best level for all the criteria values and the negative ideal alternatives have the worst level for the criteria values. In a geometrical sense, the optimal alternative should have the shortest distance from the positive ideal solution while having the longest distance from the negative ideal solution. Each criterion is expected to have an increasing or decreasing monotonically utility, making it relatively easy to determine the positive ideal and negative ideal solutions [55]. The TOPSIS was used to propose a novel approach for benchmarking building energy performance using a number of criteria including energy consumption, indoor comfort, and environmental impact [56]. A combination of the TOPSIS and pairwise comparison method was used to rank and select the best renovation option for a given building [57]. The TOPSIS and AHP were used to determine the optimum energy recovery technologies based on financial, environmental, and technical criteria [58].

## 2.2. Outranking Methods

In the outranking method, the solution is closer to the human way of thinking. The basic assumption is that the decision-maker explores the assertion that "alternative  $i$  is at least as good as alternative  $k$ ", and the only pre-existing preferences they have is an idea of the relative importance of the criteria.

The outranking methods employ a relational model that makes use of the outranking relation. This relationship involves comparing pairs of decision options and determining whether one option significantly outperforms the other. The Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) and ELimination et Choix Traduisant la REalité (ELECTRE) methods are two prominent techniques [47].

The PROMETHEE method was first introduced by Brans as an outranking method to rank and choose from a limited set of alternatives. Brans and Vincke further expanded the original method. A finite set of predetermined alternatives is evaluated using multiple criteria, with each criterion assigned a weight and a suitable preference function selected. The preference function expresses the degree of preference for different evaluations [59]. Six methods, each with a specific purpose, have been developed within the PROMETHEE family to solve MCDM problems. PROMETHEE I is intended for partial ranking of alternatives, whereas PROMETHEE II is intended for full ranking. PROMETHEE III improves indifferences by allowing them to rank alternatives based on overlapping intervals computed from interval flows. PROMETHEE IV is used to rank alternatives completely or partially when the set of viable solutions is continuous [60,61]. PROMETHEE V maximizes the total outranking flow of alternatives for a continuous problem by using constraints [62]. PROMETHEE VI incorporates the decision maker preferences, resulting in variations in criteria weights [63]. The PROMETHEE method was used to evaluate the thermal performance of different building renovation alternatives based on multiple criteria, including cost, energy efficiency, environmental impact, and social acceptance using a case study of a masonry building in Algeria. The outcomes were used to rank the alternatives and determine the best renovation option [64]. The PROMETHEE method was used to select energy retrofit measures for buildings and districts in a district of Milan, Italy, involving a comprehensive evaluation of the retrofit measures based on multiple criteria, such as energy performance, cost, environmental impact, and social factors. The proposed method is used to evaluate and rank different retrofit measures for buildings and the district as a whole. The findings of the study indicate that the proposed method can effectively identify



the most appropriate retrofit measures for both individual buildings and the district, taking into account various criteria and stakeholder preferences [65].

ELECTRE is a family of MCDM that was developed in the 1960s by Bernard Roy and his colleagues at the European consultancy company SEMA [66]. The ELECTRE method, which was originally designed to select the best alternative from a given set, has evolved to address three key types of decision problems: choosing, ranking, and sorting [67]. Although there are several versions of the ELECTRE method, they are all based on the same fundamental concepts but differ in their procedures and purpose. For instance, ELECTRE I was developed to solve choice problems [68], whereas ELECTRE IS and ELECTRE IV were developed by introducing indifference and veto thresholds, respectively [66]. ELECTRE II, an updated version of ELECTRE I, addresses ranking problems by defining two outranking relations. The primary distinction between ELECTRE I and ELECTRE II is how the outranking relation is defined. ELECTRE II introduces two outranking relations: weak outranking and strong outranking, whereas ELECTRE I considers only one type of outranking. ELECTRE III, which can be thought of as a fuzzy outranking relation, was developed to address the ranking problem [69]. ELECTRE IV, a variant of ELECTRE III, was developed to deal with situations where criteria weights are difficult to define or are intentionally omitted [70]. The ELECTRE III and ELECTRE IV methods construct outranking relations using different preference domains. The primary difference between the two techniques, however, is found in their distillation procedures. ELECTRE IV uses the number of criteria in each preference domain, whereas ELECTRE III uses a membership function value. A methodology was proposed to assist municipalities in Portugal for improving their energy sustainability through the development of an Energy Action Plan (EAP) using ELECTRE III to evaluate 16 actions in the framework [71].

To address the sorting issue, the most recent members of the ELECTRE family, ELECTRE Tri [72], ELECTRE Tri-C [73], and ELECTRE Tri-nC [74], were developed. ELECTRE Tri was developed to sort and categorize alternatives based on their ability to satisfy specific conditions. It assigns alternatives to different categories based on their similarity between the alternatives and the reference profiles which must be defined properly to ensure a robust categorization process [72,75]. The ELECTRE Tri method was used to classify 16 distinct sustainable energy technologies for electricity generation based on their compliance with the Clean Development Mechanism (CDM) using six different economic, environmental, and social criteria to sort the sixteen alternatives into three priority categories [76]. A comparative evaluation was conducted by using ELECTRE Tri and the Data Envelopment Analysis (DEA) to evaluate 41 biogas industries in Austria. The industries were classified into four categories based on economic, environmental, and social criteria [77]. In Portugal, the ELECTRE Tri method was used to assess the energy efficiency of school buildings [78]. The ELECTRE Tri-nC technique was used to aid decision-making by categorizing alternative energy retrofit measures into three distinct categories for public buildings in the Apulia region of Italy [23].

A multi-criteria analysis is a suitable approach for comparing and defining a retrofit program in order to select alternatives for the renovation of social housing. In this context, the ELECTRE Tri method is regarded as the most appropriate approach due to the following reasons. It is well-suited for complex problems with multiple criteria and alternatives, and makes use of a variety of qualitative and quantitative evaluation scales. It categorizes alternatives based on performance, rather than ranking them from best to worst, which enables the decision-makers to rank them subjectively. The use of boundary reference profiles and categorization provides information on overall performance, allowing for an absolute classification of each alternative. This characteristic is especially important when choosing an energy renovation alternative which should have a high overall performance rather than just relative performance. Finally, the ELECTRE Tri method incorporates various thresholds as well as a user-defined cutting level, which take into account uncertainties in calculations and performance evaluations while avoiding compensation.

Previous research typically used crisp values to represent alternative performance and assumed that evaluation data for decision-making remains constant. However, this assumption does not reflect reality as external factors often cause data to vary. A comprehensive criteria system may include both qualitative and quantitative data in the decision matrix, making it challenging to use crisp numbers to carry out subjective evaluations. Due to the inherent uncertainty of human cognitive capabilities, limited knowledge, data estimations, and the complex nature of environments, decision-makers may find it difficult to express their opinions using exact and definite values. For instance, economic uncertainties may arise due to the fluctuation of raw material prices while environmental uncertainties may be caused by the imprecise measuring of environmental impacts. Often, uncertainty is ignored for avoiding complications in decision-making, resulting in solutions which are far away from reality. In order to address these shortcomings, it is necessary to incorporate uncertainty into decision-making [79]. In practice, decision-making problems are often complex and uncertain, making it impossible to comprehensively understand and consider all aspects of the problem thoroughly. As a result, decision-makers may be presented with incomplete or imprecise information about alternative evaluations or the relative importance of each criterion for an in-depth analysis [80]. Given these complexities, imprecision, and uncertainties, there is a need for the MCDM method that can effectively address these issues in the context of energy systems, which are acceptable to all the stakeholders.

ELECTRE Tri deals with fuzziness by assigning alternatives to categories. A fuzzy value represents a value with a degree of membership or truth assigned to it. It reflects the level of certainty or ambiguity associated with the value. Fuzzy values are commonly used in fuzzy logic, which allows for the modeling of vagueness or uncertainty. For instance, in fuzzy sets, a value can be partially true or partially belong to a particular set based on its degree of membership. However, these values are crisp, not imprecise. An imprecise value is characterized by a lack of precision or specificity. It refers to a value that has limited detail or lacks exactness. Imprecise values can arise due to various reasons, such as evaluation errors, approximations, or incomplete information. Unlike fuzzy values, imprecise values do not inherently involve assigning degrees of membership or truth. They are more about expressing a range or uncertainty in the value itself. The precision can be expressed using a probability distribution function (PDF) or a probability density function which provides information about the likelihood of different values occurring within a given range.

Therefore, the main aim of this paper is to introduce imprecision in the expression of values by using probabilistic distributions.

### 3. Method

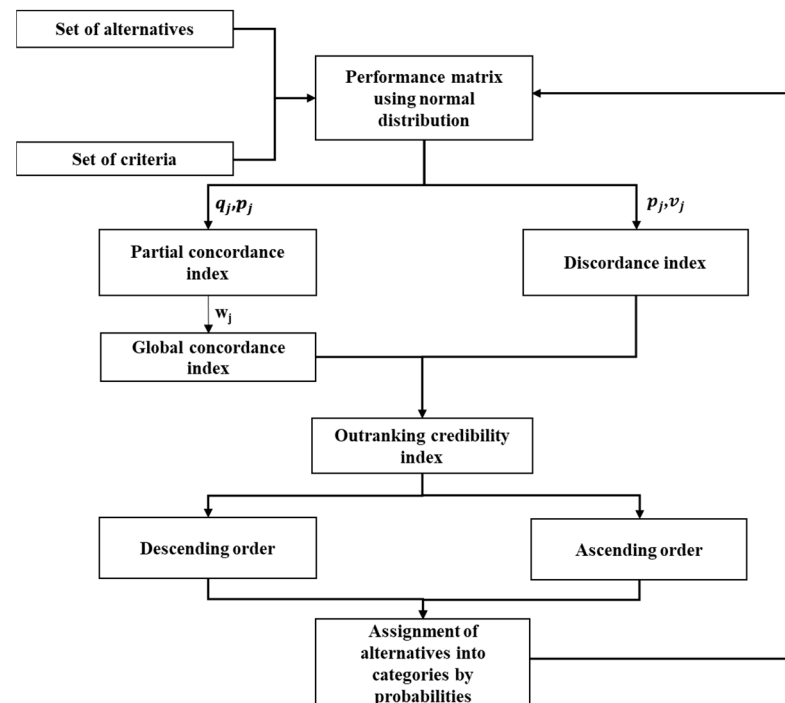
In this section, we provide an overview of the ELECTRE Tri method and Monte Carlo approach; then, the probabilistic extension of the ELECTRE Tri method, proposed in this paper, is explained. The flowchart of the proposed probabilistic ELECTRE Tri procedure is given in Figure 1.

#### 3.1. Description of the ELECTRE Tri Method

ELECTRE Tri is a decision-making method that uses an outranking approach to sort and categorize alternatives based on their ability to satisfy specific conditions. ELECTRE Tri does not require pairwise comparisons between alternatives since they do not compete with each other; instead, it assigns alternatives to different categories based on the similarity between the alternatives and the reference profiles which must be defined properly to ensure a robust categorization process [72,75]. To implement ELECTRE Tri methodology, two primary stages must be accomplished. The initial stage involves establishing an outranking relation between alternatives and reference profiles achieved by calculating concordance and discordance indices using the same computation process as ELECTRE III. The following stage involves assigning alternatives to categories based on the outranking relation established in the initial stage achieved by calculating the pessimistic and optimistic procedures [81]. In order to use ELECTRE Tri, it is necessary to establish distinct and non-



overlapping ordered categories, each of which is represented by reference profiles. These reference profiles serve as both the upper reference profile for a given category and the lower reference profile for the subsequent category [82].



**Figure 1.** Flowchart of the proposed probabilistic ELECTRE Tri method.

Once the fundamental features of the ELECTRE Tri technique have been established, its application requires the following inputs and parameters [83]:

- Alternatives: The set of scenarios or options available from which the decision-maker can choose.
- Criteria: The attributes/factors used to evaluate the alternatives, which can be quantitative or qualitative.
- Weights: The relative importance of each criterion is determined by using weights.
- Performance matrix: A matrix displaying how each alternative performs against each criterion. In the case of a criterion with an increasing performance preference, the higher the evaluation of the alternative on this criterion, the better the alternative performs on this criterion. The criteria with decreasing performance preferences are multiplied by “−1” to align them with the other criteria’s increasing performance preferences, thus unifying the calculation [84].
- Thresholds: Thresholds are used to determine the minimum acceptable performance level for each criterion. If an alternative does not meet the threshold value for a criterion, it is considered unacceptable.
- Preference threshold ( $p$ ): The minimum degree of preference that an alternative must have over another alternative to be considered as outranking it.
- Indifference threshold ( $q$ ): The maximum degree of difference between two alternatives that can be tolerated without affecting the preference relation between them.
- Veto threshold ( $v$ ): The maximum degree of preference that can be given to an alternative despite its poor performance on some criteria.
- Cutting level ( $\lambda$ ): The cutting level is defined as the smallest credibility index value that is consistent with the assertion alternative  $a_i$  which is superior to reference profile  $b_k$ , noted  $a_i S b_k$ . The default value of the cutting level used in the ELECTRE Tri method is 0.75, but it can be adjusted to suit the specific case being studied. To determine the appropriate cutting level, factors such as the desired precision in

ranking the alternatives, the goals to be achieved, and the problem constraints need to be considered.

The steps of ELECTRE Tri method are:

Step 1: Calculate partial concordance indices.

The concordance index  $c_i(a_i, b_k)$  expresses the degree in favor of the hypothesis that alternative  $a_i$  is at least as good as reference profile  $b_k$  [85]. The partial concordance is

$$c_i(a_i, b_k) = \begin{cases} 0 & \text{if } [g_j(b_k) - g_j(a_i)] \geq p_j(b_k) \\ 1 & \text{if } [g_j(b_k) - g_j(a_i)] \leq q_j(b_k) \\ \frac{p_j(b_k) + g_j(a_i) - g_j(b_k)}{p_j(b_k) - q_j(b_k)} & \text{otherwise} \end{cases} \quad (1)$$

where:

$g_j(b_k)$ —the value of the reference profile  $b_k$  for the criterion  $j$ ;

$g_j(a_i)$ —alternative  $a_i$  evaluation on the criterion  $j$ ;

$p_j(b_k)$ —preference threshold indices for the criterion  $j$  and reference profile  $b_k$ ;

$q_j(b_k)$ —indifference threshold indices for the criterion  $j$  and reference profile  $b_k$ .

In order to consider the opposing assertion, and the performance of the reference profile  $b_k$  with respect to alternative  $a_i$ , it is necessary to calculate the concordance indices in the reciprocal way  $c_j(b_k, a_i)$ .

Step 2: Calculate global concordance indices.

The global concordance index  $c(a_i, b_k)$  can be calculated after computing the partial concordance index for all the criteria. The global concordance is

$$c(a_i, b_k) = \frac{\sum_{j=1}^n w_j \cdot c_j(a_i, b_k)}{\sum_{j=1}^n w_j} \quad (2)$$

where  $w_j$  is the weight of the criterion  $j$ . It is necessary to calculate the global concordance indices in the reciprocal way ( $b_k, a_i$ ).

Step 3: Calculate the discordance indices.

The discordance index expresses the degree of evidence against the hypothesis that alternative  $a_i$  is at least as good as reference profile  $b_k$  [85]. The discordance index is

$$d_j(a_i, b_k) = \begin{cases} 0 & \text{if } [g_j(b_k) - g_j(a_i)] \leq p_j(b_k) \\ 1 & \text{if } [g_j(b_k) - g_j(a_i)] > v_j(b_k) \\ \frac{g_j(b_k) - g_j(a_i) - p_j(b_k)}{v_j(b_k) - p_j(b_k)} & \text{otherwise} \end{cases} \quad (3)$$

where  $v_j(b_k)$  are the veto threshold for the criterion  $j$  and reference profile  $b_k$ . In order to consider the opposing assertion, it is necessary to calculate the discordance indices in the reciprocal way  $d_j(b_k, a_i)$ .

Step 4: Calculate the credibility index of the outranking relation.

In the ELECTRE Tri method, a degree of credibility is calculated to establish the relationship between the alternatives and the reference profiles. This value measures the proximity of each alternative to each reference profile and is compared to the cutting level  $\lambda$  to ascertain if a preference for an alternative over a reference profile can be established or not [86]. The credibility index is

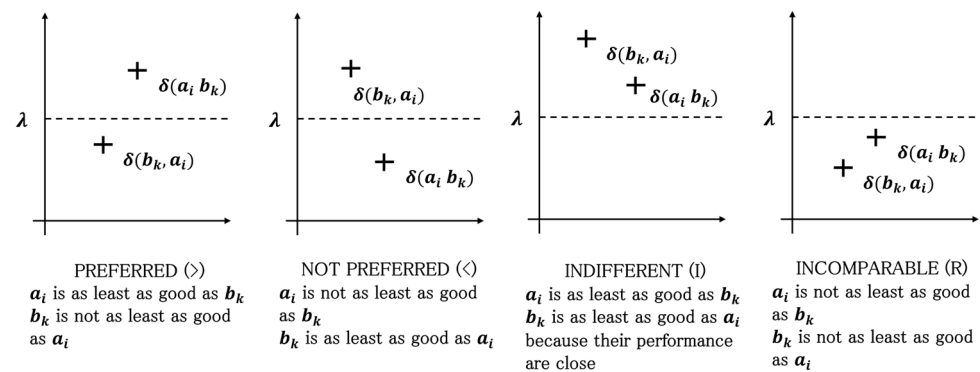
$$\sigma(a_i, b_k) = C(a_i, b_k) \prod_{j \in J} \frac{1 - D_j(a_i, b_k)}{1 - C(a_i, b_k)} \quad (4)$$

where  $j$  stands for all the criteria for which  $D_j(a_i, b_k) \geq C(a_i, b_k)$ .

When the decision-maker considers that it is not appropriate to define the veto threshold, the credibility index coincides with the global concordance index. Once all credibility indexes  $\sigma(a_i, b_k)$  have been calculated, they are compared to the cutting level  $\lambda$  which has a value in the range of  $[0.5 - 1.0]$  and must be greater than  $[1 - \text{highest weight} / \text{total weight}]$  [87,88]. This relationship allows the definition of the outranking relation that each alternative has with the reference profile.

The degree of credibility for  $\sigma(b_k, a_i)$  is calculated from the global concordance  $C(b_k, a_i)$  and the discordance  $D(b_k, a_i)$ .

There are four types of relationships that can be established between each performance alternative  $a_i$  and each reference profile  $b_k$  by the value of  $\sigma(a_i, b_k)$ ,  $\sigma(b_k, a_i)$ , and  $\lambda$ , represented in Figure 2.



**Figure 2.** Schematic relationship between the alternative and the profile.

- If  $\sigma(a_i, b_k) > \lambda$  and  $\sigma(b_k, a_i) > \lambda$ , then  $a_i I b_k$ :  $a_i$  is indifferent to  $b_k$ .
- If  $\sigma(a_i, b_k) > \lambda$  and  $\sigma(b_k, a_i) < \lambda$ , then  $a_i > b_k$ :  $a_i$  is preferred to  $b_k$ .
- If  $\sigma(a_i, b_k) < \lambda$  and  $\sigma(b_k, a_i) > \lambda$ , then  $a_i < b_k$ :  $a_i$  is not preferred to  $b_k$ .
- If  $\sigma(a_i, b_k) < \lambda$  and  $\sigma(b_k, a_i) < \lambda$ , then  $a_i R b_k$ :  $a_i$  is incomparable to  $b_k$ .

The second stage of the ELECTRE Tri method involves assigning alternatives to classes based on the outranking relation obtained in the first stage. The optimistic and pessimistic rules are the two assignment procedures proposed by ELECTRE Tri. The outcomes of the two procedures do not necessarily coincide. The optimistic rule assigns the alternative  $a_i$  to the lowest class  $C_h$  for which the upper profile  $b_k$  is preferred to  $a_i$ , i.e.,  $b_k > a_i$ . The pessimistic rule assigns the alternatives to the highest class such that  $a_i$  outranks  $b_k$ , i.e.,  $a_i S b_k$ .

### 3.2. Description of the Monte Carlo Method

The Monte Carlo method is a statistical technique for estimating operations in complex systems that rely on random sampling and statistical modeling. It can be used to apply a method to a set of probability functions by generating a set of data repeatedly. The method is applied to the generated data set; one data point is sampled from each distribution. By repeating this process a large number of times, the method generates results that are representative of the underlying distributions of the input data, allowing for the incorporation of the uncertainty present in the data into the decision-making process.

The Probability Density Function (PDF) is used to integrate data fluctuation into the performance matrix [89]. A probability density function is a mathematical expression that describes the probability distribution of a continuous random variable [90]. PDFs can be used to represent the distribution of various types of phenomena, including the normal,

uniform, Poisson, and exponential distributions. These curves allow the expression and measurement of knowledge of the input data based on their parameters. Although any PDF can be used, in what follows, the normal distribution is used without reducing the generality of the method. The decision to use the normal distribution is based on the following two main reasons:

- The normal distribution is commonly used to represent real-world values, especially in fields such as economics, social sciences, and natural sciences, where the true distribution of the data is unknown exactly.
- When a large number of samples are taken, the normal distribution is a frequent approximation for the distribution of data, even if it is not the true underlying distribution [91]. This has been demonstrated by statisticians such as Pierre-Simon Laplace through experimentation in various fields, where the normality of the data is observed to increase with sample size.

The normal distribution, noted  $N(\mu, \sigma^2)$ , is described by two parameters: the mean value  $\mu$  and the standard deviation  $\sigma$ . It is worth noting that the variance is  $v = \sigma^2$ . As a result, the calculations will be done with the mean value  $\mu$  and the variance  $v$  as parameters.

The following steps need to be followed in order to implement the proposed method.

Step 1: Select values from Probability Distribution Functions (PDFs).

The underlying assumption is that the values are provided in the form of Probability Distribution Functions rather than crisp values. Therefore, the performance of each alternative with respect to each criterion is considered as a probability distribution, taking into consideration the data uncertainty. The performance of an alternative  $a$  regarding a criterion  $j$  is:

$$S_a(C_j) = F_j(x) \quad (5)$$

where  $F_j$  is the probability density function of the criterion  $j$ .

The first step is to select a value from each distribution. As a result, a new performance matrix is formed using the values selected from the distributions.

Step 2: ELECTRE Tri implementation.

Once the data set is obtained, ELECTRE Tri is used to calculate the optimistic and pessimistic ranking of each alternative. In the traditional ELECTRE Tri, the alternatives are assigned to a single category in the optimistic and pessimistic rankings (Table 1).

**Table 1.** Example of optimistic and pessimistic rankings obtained in single categories.

Alternative	C1	C2	C3	C4	C5	Alternative	C1	C2	C3	C4	C5
a1.1		X				a1.1			X		
a1.2				X		a1.2				X	
a1.3	X					a1.3	X				

Step 3: Obtaining a set of outcomes.

The final step is to repeat the first two steps several times to generate a large number of optimistic and pessimistic rankings. The number of times each alternative is classified in each category is calculated by memorizing the ranking of each alternative. As a result, the outcomes are a distribution of alternatives in the categories. These two sets of rankings for each alternative are then converted into percentages to obtain tables that display the probability of each alternative ranking in each category (Table 2).

**Table 2.** Example of optimistic and pessimistic rankings obtained in multiple categories.

Alternative	C1	C2	C3	C4	C5	Alternative	C1	C2	C3	C4	C5
a1.1	10.2	25.9	63.9	0	0	a1.1	0	15.7	63.6	21	0
a1.2	0	0	100	0	0	a1.2	0	0	6.4	93.6	0
a1.3	0	2.5	45.7	39.3	12.5	a1.3	0	0	13.2	62.3	24.5

One of the primary benefits of the outlined procedure is the ability to obtain more detailed results. These tables not only categorize the alternatives, but also show their sensitivity to changes in the input data.

A computational algorithm has been developed to automate the probabilistic ELECTRE Tri method enabling rapid and efficient processing [92]. The method assigns alternatives into different ranking categories named as  $C_k$ , which are defined by upper and lower boundary reference profiles named as  $B_k$ . The decision makers have the flexibility to define the boundary reference profiles; their performance is used to establish the ranking categories. The Python programming language is selected for the implementation of the method due to its readability, user-friendliness, popularity, and versatility in terms of computer support.

#### 4. Case Study

##### 4.1. Description

The proposed methodology is illustrated by using a social housing project consisting of sixty-seven apartments made up of three adjacent buildings (called B1, B2 and B3) located in the region of Lyon, France and built in 1973. The apartments range in size from one to five rooms and cover a total living area of 4815 m<sup>2</sup> (Figures 3 and 4). An independent consultancy office carried out a thermal diagnosis (Figure 5), which involved calculating the primary energy consumption of the residences using a regulatory calculation approach. The buildings were classified with respect to regulatory energy and greenhouse gas emission labels. Prospective areas of improvement to enhance the energy label were identified by the consultants.

**Figure 3.** Overview of the social housing complex used to illustrate the methodology [24].

The assessment of the thermal state of the residential buildings also revealed shortcomings in the envelope of all the three buildings (Figure 5). An estimation of the overall heat transfer coefficient,  $H$ , was conducted and compared to a reference coefficient  $H_{max}$  which serves as a benchmark for the overall heat transfer coefficient relative to the envelope. The comparison clearly showed that the buildings are insufficiently insulated. The initial assessment revealed that the thermal losses of the three residential buildings were evenly distributed. The two most significant sources of loss were discovered to be air renewal and windows, which contributed 28% and 25%, respectively. This was due to the use of obsolete single-glazed wooden windows in the building's envelope and a high rate of air change

in the single-flow ventilation system with self-regulating extract units. Thermal bridges and external walls made the second most significant contribution, accounting for 21% and 19% of the losses, respectively. The thermal bridges are critical because the presence of an electric underfloor heating system causes major losses at the points where the walls and floors converge. According to the evaluation, the initial steps in any renovation project should include implementing passive measures to reduce heat loss. These measures are especially desirable as they are usually inexpensive and easy to implement. These measures may include thermal insulation enhancement, efficient ventilation system installation, or thermal bridge removal with external insulation.



Figure 4. Top view of the social housing complex [24].

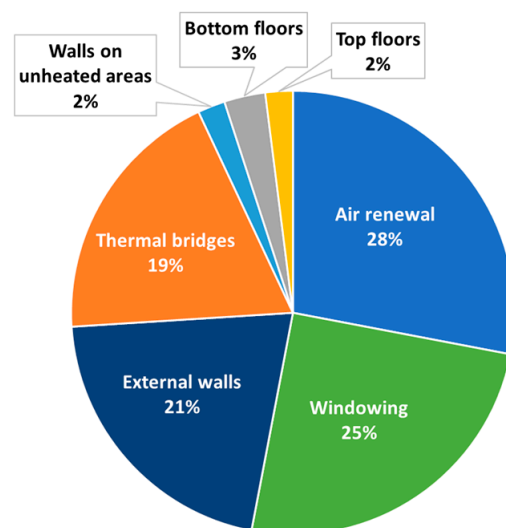


Figure 5. Heat loss distribution of the buildings [24].

#### 4.2. Identification of Potential Renovation Scenarios

Several renovation scenarios, consisting of elementary actions aimed at achieving specific goals, were considered. These elementary actions were chosen in collaboration with designers and manufacturers to ensure alignment with the project objectives. The



elementary actions were combined to create global renovation scenarios, with a particular emphasis on cohesion and compatibility. The goal is to create renovation scenarios that improve thermal comfort and usability for tenants while reducing energy consumption and greenhouse gas emissions. To accomplish this, the renovation options are divided into seven distinct classes, each with its own specific set of individual energy performance improvement actions, as outlined in the European Energy Performance of Buildings Directive 2010/31/EU and Energy Efficiency Directive 2012/27/EU.

Seven global renovation scenarios were created by combining various elementary renovation actions (Table 3) in a variety of coherent ways:

- S1: Maintain existing individual electric heating and domestic hot water systems.
- S2: Improve individual electric heating and domestic hot water systems in version 1.
- S3: Improve individual electric heating and domestic hot water systems in version 2.
- S4: Individual electric heating and individual thermodynamic domestic hot water.
- S5: Individual gas heating and domestic hot water.
- S6: Individual electric heating and collective thermodynamic domestic hot water.
- S7: Individual electric heating and collective solar domestic hot water.

**Table 3.** The basic elementary actions [24].

Class	Options
Existing electric floor heating	Connected Disconnected Existing electric radiant heater
Auxiliary heating	Smart and connected electric radiant heater Smart and connected electric storage heater Low temperature hot water radiators Automated towel warmer Existing individual electric hot water tank Smart and connected individual electric hot water tank
Domestic hot water production	Smart and connected individual thermodynamic hot water tank Individual condensing gas boiler Collective thermodynamic Individual solar hot water tank with centralized solar hot water tank
Ventilation system	Existing controlled mechanical ventilation Controlled mechanical ventilation with single-flow and a humidity-sensitive extract unit None
Local energy production	Roof photovoltaic solar panels Roof thermal solar panels Roof hybrid solar panels None
External joinery replacement	All external joinery double glazing All external joinery triple glazing Only on the balcony windows double glazing
External wall insulation	Existing wall insulation External thermal insulation

These seven classes of global renovation scenarios were further subdivided into four variants designated as  $S_{i,1}$  to  $S_{i,4}$  where  $i = 1, 2, \dots, 7$ . In order to analyze their effect on performance, in each variant, the type of renewable energy production and joineries were modified while the heating and domestic hot water systems remained the same. For example, in some renovation scenarios, the installation of solar photovoltaic energy production or only the replacement of balcony windows were considered, whereas in others, all exterior windows were replaced and solar thermal energy production was installed. A total of 28 possible scenarios were considered (Figure 6).

N°	Basic renovation actions	Global renovation scenarios																											
		S <sub>1,1</sub>	S <sub>1,2</sub>	S <sub>1,3</sub>	S <sub>1,4</sub>	S <sub>2,1</sub>	S <sub>2,2</sub>	S <sub>2,3</sub>	S <sub>2,4</sub>	S <sub>3,1</sub>	S <sub>3,2</sub>	S <sub>3,3</sub>	S <sub>3,4</sub>	S <sub>4,1</sub>	S <sub>4,2</sub>	S <sub>4,3</sub>	S <sub>4,4</sub>	S <sub>5,1</sub>	S <sub>5,2</sub>	S <sub>5,3</sub>	S <sub>5,4</sub>	S <sub>6,1</sub>	S <sub>6,2</sub>	S <sub>6,3</sub>	S <sub>6,4</sub>	S <sub>7,1</sub>	S <sub>7,2</sub>	S <sub>7,3</sub>	S <sub>7,4</sub>
a <sub>1</sub>	<b>Existing electric floor heating</b>																												
a <sub>1,1</sub>	Connected	•	•	•	•	•	•	•	•																				
a <sub>1,2</sub>	Disconnected																												
a <sub>2</sub>	<b>Auxiliary heating</b>																												
a <sub>2,1</sub>	Existing electric radiant heater	•	•	•	•																								
a <sub>2,2</sub>	Smart and connected electric radiant heater					•	•	•	•																				
a <sub>2,3</sub>	Smart and connected electric storage heater									•	•	•	•	•	•	•	•												
a <sub>2,4</sub>	Low temperature hot water radiators																	•	•	•	•								
a <sub>2,5</sub>	Automated towel warmer					•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a <sub>3</sub>	<b>Domestic hot water production</b>																												
a <sub>3,1</sub>	Existing individual electric hot water tank	•	•	•	•																								
a <sub>3,2</sub>	Smart and connected individual electric hot water tank					•	•	•	•	•	•	•	•	•	•	•	•												
a <sub>3,3</sub>	Smart and connected individual thermodynamic hot water tank																												
a <sub>3,4</sub>	Individual condensing gas boiler																	•	•	•	•								
a <sub>3,5</sub>	Collective thermodynamic																					•	•	•	•				
a <sub>3,6</sub>	Individual solar centralized hot water tank																									•	•	•	
a <sub>4</sub>	<b>Controlled mechanical ventilation system</b>																												
a <sub>4,1</sub>	Existing controlled mechanical ventilation	•																											
a <sub>4,2</sub>	CMV with single flow and humidity-sensitive extract unit	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a <sub>5</sub>	<b>Local energy production</b>																												
a <sub>5,1</sub>	None	•	•	•	•	•				•								•				•							
a <sub>5,2</sub>	Roof Photovoltaic solar panels					•	•	•		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a <sub>5,3</sub>	Roof Thermal solar panels																									•			
a <sub>5,4</sub>	Roof Hybrid solar panels																									•	•	•	
a <sub>6</sub>	<b>Exterior joinery replacement</b>																												
a <sub>6,1</sub>	None	•																											
a <sub>6,2</sub>	All exterior joinery double glazing		•			•	•	•		•	•		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a <sub>6,3</sub>	All exterior joinery triple glazing			•								•												•				•	
a <sub>6,4</sub>	Only on the balcony windows double glazing				•							•												•				•	
a <sub>7</sub>	<b>External wall insulation</b>																												
a <sub>7,1</sub>	Maintaining the existing wall insulation	•																											
a <sub>7,2</sub>	External thermal insulation	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	

Figure 6. Identification of global renovation scenarios from the basic elementary actions [24].

### 4.3. Identification and Weighting of the Evaluation Criteria

This step involves developing a comprehensive set of criteria to evaluate the performance of the compared scenarios in terms of achieving the project objectives. Table 4 contains a set of criteria divided into four categories: technical, economic, environmental, and social. Each criterion will be evaluated using qualitative, quantitative, or binary methods, depending on the evaluation unit. The decision-makers must decide on the criteria that will be used to evaluate the effectiveness of the compared scenarios. The criteria serve as performance indicators and they must be quantifiable in the sense that they can be measured, at least qualitatively, in order to assess the expected performance of each scenario in relation to the criterion under consideration. It is beneficial to classify the criteria into families, especially given their large number. This classification allows for a more in-depth investigation of the issue by emphasizing how the scenarios balance one another. It also simplifies the process of determining weights because weights can be allocated first between criteria within the same family and then between families themselves. Furthermore, grouping by family allows for the retention of a greater number of criteria for

a decision analysis. One of the difficulties in selecting criteria is that their number must be kept to a minimum in order to avoid introducing redundancy or repeatability phenomena. A sound decision should be based on few, distinct, complementary criteria [65,93].

**Table 4.** Identification of assessment criteria with their respective weights.

Category	Weight	Criteria	Description	Units	Absolute Weight
Economic	45%	g1.1	Investment cost	€ excl. taxes	12.27
		g1.2	Reinvestment cost over a period of 30 years	€ excl. taxes	10.64
		g1.3	Possibility of financial subsidy	€ excl. taxes	4.09
		g1.4	Operating cost	€ excl. taxes	12.27
		g1.5	Energy cost effectiveness of the solution	€/kWh/(m <sup>2</sup> ·year)	5.73
Technical	25%	g2.1	Ease to integrate into existing building	5-point scale	7.14
		g2.2	Implementation in occupied space	Yes/No	10.71
		g2.3	Maintenance/Serviceability	5-point scale	3.57
		g2.4	Ease of monitoring/Energy management	5-point scale	3.57
Social	25%	g3.1	Impact of the cost on the tenant	€ excl. taxes	6.14
		g3.2	Level of thermal comfort	5-point scale	4.77
		g3.3	Level of acoustic comfort	5-point scale	2.05
		g3.4	Aesthetics and space requirements	5-point scale	2.05
Environmental	15%	g4.1	Energy consumption reduction	kWh/m <sup>2</sup> ·year	7.50
		g4.2	Carbon dioxide emissions avoided	Ton of CO <sub>2</sub> /year	5.00
		g4.3	Production place	Km	2.50

Once the assessment criteria have been defined, it is imperative to establish their relative importance in the context of decision-making. This involves determining whether criterion “A” is more important to the decision-maker’s final choice than criterion “B” and quantifying the ratio of prevalence between them. To accomplish this, weights must be assigned to each of the selected criteria in the multi-criteria analysis. There are several methods for determining the appropriate weight to be assigned to each criterion. The Simos procedure or card method is a popular method, particularly in the context of outranking methods [94]. This method allows decision-makers, even those without prior knowledge of the multi-criteria analysis, to express their preferences for weighing criteria in each context. It involves presenting a set of cards corresponding to each criterion and asking decision-makers or expert groups to rank the criteria on a grid in increasing order of importance. It is possible to group together several criteria of equal importance and leave gaps between two consecutive cards to indicate an importance difference. Following this ranking, normalized weightings are assigned to the criteria [95]. The Simos procedure was revised and renamed the SRF procedure in 2002. The updated method introduced the Z ratio, which represents the weight of the most important criterion in relation to the least important. The ratio Z allows the normalization of the obtained criterion weights. As a result, changes in Z cause changes in the criterion weights [96].

Three independent steps are involved in determining the weights of the criteria. The first step is to assign an absolute weight to each of the categories of criteria using the SRF weighting procedure. The second step involves independently ranking the sub-criteria within each family and assigning them a relative weight. Finally, the absolute weights of each criterion are determined by multiplying their relative weights with those of the corresponding family weights. This method ensured that the weights were normalized and the significance of each criterion in the decision-making process was reflected.

#### 4.4. Performance Matrix

Twenty-eight energy retrofit scenarios were developed by combining elementary renovation measures. To determine the economic viability of each scenario, information was gathered from suppliers and maintenance companies on the purchase, installation, and

maintenance costs of each component. Furthermore, the reinvestment costs over a period of thirty years were calculated based on the average lifespan of each component in the scenarios. This thirty-year period corresponds to standard building maintenance intervals. Engineering and sociology consultants (specialists in building and energy transition) were consulted to assess the technical and social performance of the energy retrofit scenarios. Qualitative criteria were evaluated for each elementary action within the same system category (such as domestic hot water production) using a scoring system ranging from 0 to 5. This approach was used for all the classes of elementary actions and criteria to generate an overall score on a linear distribution scale for each scenario. This results in a score on a scale of 5 representing the performance of each scenario for the criteria g2.1, g2.3, g2.4, g3.2, g3.3, and g3.4.

Collaboration with engineering consultants was used to compare the energy performance of the scenarios. The modeling tool employed simulates the behavior of the building, its various components, and systems. Multiple simulations were run to determine the primary and final energy consumption of each scenario by varying the input parameters such as the heating system, domestic hot water production, window type, and local energy production.

The evaluation process, with the collaboration and assistance of technical equipment suppliers and installers, technical engineering offices, a specialized sociology firm, and rental management experts, enabled the construction of the performance matrix. It includes the evaluations of each global renovation scenario shown based on the various criteria.

The data uncertainty is accounted for by representing it as probability density functions, specifically the normal distribution. The mean value  $\mu$  and the variance  $v = \sigma^2$  are required to define the normal distribution for each alternative related to each criterion. The mean value is derived from the performance matrix, whereas the variance fluctuates based on the criterion under consideration.

Variance is denoted by Var in the performance matrix in Tables 5 and 6. To ensure uniform treatment of uncertainty in the decision-making process, a variance value of 0.1 is applied to all criteria, which helps to maintain model consistency and minimizes complexity. The variance value can be reassessed using statistical models that consider the data's evolution over time.

**Table 5.** Performance matrix of the global renovation scenarios—(Part 1).

Scenarios	Economical					Technical			
	g1.1	g1.2	g1.3	g1.4	g1.5	g2.1	g2.2	g2.3	g2.4
	€ excl.	€ excl.	€ excl.	€ excl.	€/kwh/m <sup>2</sup> ·year	0 ... 5	Yes/No	0 ... 5	0 ... 5
S1.1	0.00	1,757,134	0.00	82,701	0.00	5.00	Yes	4.38	3.57
S1.2	1,008,654	551,661	952,088	41,346	5933	3.33	Yes	5.00	3.57
S1.3	1,260,874	711,361	991,697	36,119	6591	3.00	Yes	5.00	3.57
S1.4	905,165	551,661	917,604	42,268	5457	3.67	Yes	5.00	3.57
S2.1	1,239,757	551,661	955,885	40,971	6771	2.33	Yes	4.38	4.29
S2.2	1,397,247	625,566	996,168	28,230	6736	1.67	Yes	3.75	3.57
S2.3	1,636,981	785,266	1,035,489	23,290	7185	1.33	Yes	3.75	3.57
S2.4	1,282,394	390,966	995,521	28,762	6255	2.00	Yes	3.75	3.57
S3.1	1,344,232	610,011	957,187	41,692	7140	2.00	Yes	4.38	5.00
S3.2	1,522,330	683,916	1,031,776	28,197	7087	1.33	Yes	3.75	4.29
S3.3	1,751,844	843,616	1,070,386	23,760	7526	1.00	Yes	3.75	4.29
S3.4	1,385,515	449,316	1,030,933	28,758	6545	1.67	Yes	3.75	4.29
S4.1	1,531,907	775,596	1,070,213	32,638	6586	1.67	Yes	3.75	5.00
S4.2	1,635,756	849,501	1,110,816	19,664	6342	1.00	Yes	3.13	4.29
S4.3	1,899,185	1,009,201	1,149,305	15,369	6895	0.67	Yes	3.13	4.29
S4.4	1,517,360	614,901	1,109,597	20,507	5980	1.33	Yes	3.13	4.29
S5.1	1,427,140	787,239	1,124,178	26,341	5284	1.33	No	2.50	2.86
S5.2	1,555,978	861,144	1,131,706	12,639	5229	0.67	No	1.88	2.14

Table 5. Cont.

Scenarios	Economical					Technical			
	g1.1	g1.2	g1.3	g1.4	g1.5	g2.1	g2.2	g2.3	g2.4
	€ excl.	€ excl.	€ excl.	€ excl.	€/kwh/m <sup>2</sup> ·year	0 ... 5	Yes/No	0 ... 5	0 ... 5
S5.3	1,796,800	1,020,844	1,204,438	10,007	5867	0.33	No	1.88	2.14
S5.4	1,492,598	626,544	1,131,267	12,444	5027	1.00	No	1.88	2.14
S6.1	1,372,026	811,447	1,030,582	35,809	6507	1.67	Yes	3.13	2.86
S6.2	1,526,016	885,352	1,071,164	22,827	6466	1.00	Yes	2.50	2.14
S6.3	1,717,070	1,045,052	1,109,732	18,439	6767	0.67	Yes	2.50	2.14
S6.4	1,377,529	650,752	1,070,150	23,678	5934	1.33	Yes	2.50	2.14
S7.1	1,404,394	847,315	995,961	37,880	6790	1.00	No	0.63	0.71
S7.2	1,533,952	921,220	1,070,209	24,669	6603	0.33	No	0.00	0.00
S7.3	1,771,679	1,080,920	1,109,007	20,125	7057	0.00	No	0.00	0.00
S7.4	1,432,939	686,620	1,035,825	25,267	6251	0.67	No	0.00	0.00
Var	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 6. Performance matrix of the global renovation scenarios—(Part 2).

Scenarios	Social				Environmental		
	g3.1	g3.2	g3.3	g3.4	g4.1	g4.2	g4.3
	€ excl.	0 ... 5	0 ... 5	0 ... 5	kWh/m <sup>2</sup> ·Year	Ton of CO <sub>2</sub> /Year	km
S1.1	368	0.00	5.00	4.00	0.00	0.00	0.00
S1.2	182	2.78	5.00	4.00	170	65	418
S1.3	159	3.33	5.00	4.00	191	74	690
S1.4	186	2.22	5.00	4.00	166	64	418
S2.1	168	4.44	4.00	4.00	183	71	2368
S2.2	141	4.44	4.00	4.00	207	80	2448
S2.3	118	5.00	4.00	4.00	228	88	2720
S2.4	144	3.89	4.00	4.00	205	79	2448
S3.1	162	2.22	3.00	2.00	188	73	2368
S3.2	133	2.22	3.00	2.00	215	83	2448
S3.3	113	2.78	3.00	2.00	233	90	2720
S3.4	136	1.67	3.00	2.00	212	82	2448
S4.1	114	2.22	1.00	0.00	233	90	2314
S4.2	86	2.22	1.00	0.00	258	99	2394
S4.3	66	2.78	1.00	0.00	275	106	2666
S4.4	91	1.67	1.00	0.00	254	98	2394
S5.1	164	0.56	0.00	1.00	270	5	2425
S5.2	0	0.56	0.00	1.00	298	33	2505
S5.3	113	1.11	0.00	1.00	306	50	2777
S5.4	137	0.00	0.00	1.00	297	32	2505
S6.1	138	3.89	5.00	5.00	211	81	2618
S6.2	110	3.89	5.00	5.00	236	91	2698
S6.3	90	4.44	5.00	5.00	254	98	2970
S6.4	114	3.33	5.00	5.00	232	89	2698
S7.1	142	4.44	3.00	3.00	207	80	2548
S7.2	114	4.44	3.00	3.00	232	89	2628
S7.3	93	5.00	3.00	3.00	251	97	2900
S7.4	117	3.89	3.00	3.00	229	88	2628
Var	0.1	0.1	0.1	0.1	0.1	0.1	0.1

#### 4.5. Identification of Thresholds

To compare each alternative based on each criterion, it is essential to establish thresholds. These thresholds determine whether an alternative is preferred, equivalent, inferior, or incomparable to a boundary reference profile. These thresholds take into account the imperfect nature of data when evaluating alternative performances, as well as the subjectivity

that may affect criterion definition. As a result, it is necessary to define these thresholds for each criterion.

The ELECTRE Tri method allows decision-makers to define these thresholds based on their preferences and the importance of performance gaps, such as investment costs. Decision-makers, on the other hand, may lack expertise in certain criteria and struggle to define these thresholds. These tolerance thresholds are expressed as a percentage of the average performance for each criterion. The decision was motivated by practical considerations such as the large number of criteria and the experimental nature of the methodology. The thresholds can easily be modified through the parameter adjustment  $\gamma$ , thereby facilitating the analysis of the impact on scenario rankings. The indifference threshold is

$$q_j = \gamma_q \cdot \mu_j \quad (6)$$

where:

- $\gamma_q = 5\%$  is the indifference threshold;
- $\mu_j$  represents the average performance of all the scenarios on a given criterion defined by

$$\mu_j = \mu(g_j) = \frac{\sum_{i=1}^m g_j(S_i)}{m} \quad (7)$$

The preference threshold is

$$p_j = \gamma_p \cdot \mu_j \quad (8)$$

where  $\gamma_p = 15\%$  is the preference threshold.

The veto threshold is

$$v_j = \gamma_v \cdot \mu_j \quad (9)$$

where  $\gamma_v = 30\%$  is the veto threshold.

#### 4.6. Identification of Boundary Reference Profiles

The ranking categories are defined by boundary reference profiles and are denoted by  $b^0, b^1, \dots, b^k, \dots, b^q$ . The boundary reference profiles represent the boundary between two consecutive ranking categories, thus the lower boundary reference profile of a better category is also the upper boundary reference profile of a worse consecutive category. It is important to note that  $b^0$  is the lowest boundary reference profile and  $b^q$  is the highest boundary reference profile.

Six boundary reference profiles have been identified:  $B = \{b^0, b^1, b^2, b^3, b^4, b^5\}$  in order to establish and define five distinct categories:  $C = \{C^1, C^2, C^3, C^4, C^5\}$ , where  $b^1$  is the boundary between  $C^1$  and  $C^2$  and  $b^2$  is the boundary between  $C^2$  and  $C^3$ , and so on. The set of boundary reference profiles is typically constructed through an interactive process between the analyst and the decision-maker, making the use of the ELECTRE Tri method more time-consuming and complex (Table 7).

#### 4.7. Degree of Credibility Index

The degree of credibility is calculated to measure the proximity of each alternative to each boundary reference profile and is compared to a cutting level  $\lambda$ . Through trial and error, it was determined that a cutting level of  $\lambda = 0.75$  was appropriate. It is worth noting that a higher cutting level may result in alternatives that cannot be compared.

#### 4.8. Sorting Procedure

In the final step, two sorting procedures, specific to the ELECTRE Tri method, are carried out to assign the alternatives to ranking categories: the "pessimistic ranking" and the "optimistic ranking". The treatment of the incomparability (R) relationship varies between the two procedures. In the optimistic ranking, an incomparability relationship moves an alternative to the next higher performance category, while in the pessimistic ranking, it moves an alternative to the next lower performance category (Table 8).



**Table 7.** Identification of the boundary reference profiles and thresholds.

Criteria	$b^0$	$b^1$	$b^2$	$b^3$	$b^4$	$b^5$	$q_j$	$P_j$	$v_j$
g1.1	-4,224,020	-1,667,495	-1,528,955	-1,418,833	-1,330,025	212,825	70,942	212,825	468,215
g1.2	-3,749,830	-928,689	-837,382	-680,856	-602,593	-273,185	39,260	117,781	259,118
g1.3	-304,916	963,728	1,031,134	1,071,156	1,111,178	1,356,896	50,819	152,458	335,408
g1.4	-174,020	-38,545	-28,729	-24,522	-18,913	-5698	1436	4309	9480
g1.5	-16,904	-6853	-6717	-6376	-5491	926	309	926	2037
g2.1	-0.46	0.71	1.20	1.54	2.09	5.00	0.08	0.23	0.51
g2.2	0.00	0.00	0.00	1.00	1.00	1.00	0.00	1.00	2.00
g2.3	-0.90	1.93	2.60	3.26	4.00	5.00	0.15	0.45	0.98
g2.4	-0.90	1.98	2.35	3.30	4.28	5.00	0.15	0.45	0.99
g3.1	-778	-160	-137	-115	-99	-46	7.00	21	45
g3.2	-0.86	1.78	2.23	3.74	4.19	5.00	0.14	0.43	0.94
g3.3	-0.90	1.00	2.10	3.36	4.08	5.00	0.15	0.45	0.99
g3.4	-0.82	0.59	1.87	3.14	4.41	5.00	0.14	0.41	0.90
g4.1	-66	194	223	234	257	340	11	33	74
g4.2	-22	60	74	82	90	117	4.00	11	24
g4.3	-6616	-2737	-2657	-2496	-2376	338	113	338	744

**Table 8.** Optimistic and pessimistic rankings obtained with the probabilistic ELECTRE Tri procedure.

Alternative	C1	C2	C3	C4	C5	Total	Alternative	C1	C2	C3	C4	C5	Total
S1.1		100				100	S1.1	100					100
S1.2		28.4	71	0.6		100	S1.2	28.4	71	0.6			100
S1.3		0.7	65.8	33.3	0.2	100	S1.3	0.7	65.8	33.3	0.2		100
S1.4		41.9	58	0.1		100	S1.4	41.9	58	0.1			100
S2.1		14.5	82.6	2.9		100	S2.1	14.5	82.6	2.9			100
S2.2		0.2	3.1	53.9	42.8	100	S2.2	0.2	3.1	53.9	42.8		100
S2.3		2.2	7.3	65.7	24.8	100	S2.3	2.2	7.3	65.7	24.8		100
S2.4		0.1	3	57.8	39.1	100	S2.4	0.1	3	57.8	39.1		100
S3.1		14.4	82.3	3.3		100	S3.1	14.4	82.3	3.3			100
S3.2		1.3	4.3	94.4		100	S3.2	1.3	4.3	94.4			100
S3.3		7.4	34.9	57.7		100	S3.3	7.4	34.9	57.7			100
S3.4			6.4	93.6		100	S3.4			6.4	93.6		100
S4.1			100			100	S4.1		100				100
S4.2		0.4	99.6			100	S4.2	0.4	99.6				100
S4.3		33	67			100	S4.3	33	67				100
S4.4			100			100	S4.4	100					100
S5.1		100				100	S5.1	100					100
S5.2		100				100	S5.2	100					100
S5.3		92.9	7.1			100	S5.3	92.9	7.1				100
S5.4		100				100	S5.4	100					100
S6.1		0.1	40.7	58.1	1.1	100	S6.1	0.1	40.7	58.1	1.1		100
S6.2		0.3	11.6	87.9	0.2	100	S6.2	0.3	11.6	87.9	0.2		100
S6.3		21.7	75.9	2.4		100	S6.3	21.7	75.9	2.4			100
S6.4		0.3	0.7	94	5	100	S6.4	0.3	0.7	94	5		100
S7.1		100				100	S7.1	100					100
S7.2		100				100	S7.2	100					100
S7.3		100				100	S7.3	100					100
S7.4		100				100	S7.4	100					100

### 5. Results and Discussion

The purpose of this section is to compare the results of the newly implemented procedure to those obtained by using the traditional ELECTRE Tri method. The results are presented in the form of percentage rankings of the alternatives in each category. Firstly, the results obtained with the new procedure will be compared to those of the traditional ELECTRE Tri method to determine the additional information provided and the appropriate

methods of analysis. Secondly, the impact of incorporating variance into the data will be investigated to determine if all alternatives are affected in the same way.

The resulting table shows the percentage of each alternative classified in each category. This new way of representing the results adds significant information. The results obtained with and without the new procedure are compared in Table 9. This was applied to the case study, with the emphasis on the optimistic sorting of the four scenarios: ‘S2.1’, ‘S2.2’, ‘S2.3’, and ‘S2.4’. This analysis holds true for the other alternatives as well as for the pessimistic ranking.

**Table 9.** Optimistic ranking obtained with and without the probabilistic ELECTRE Tri procedure for four scenarios.

Alternative	C1	C2	C3	C4	C5	Alternative	C1	C2	C3	C4	C5	Total
S2.1			X			S2.1	0	14.5	82.6	2.9	0	100
S2.2					X	S2.2	0	0.2	3.1	53.9	42.8	100
S2.3				X		S2.3	0	2.2	7.3	65.7	24.8	100
S2.4				X		S2.4	0	0.1	3	57.8	39.1	100

The probabilistic results shown on the righthand side of Table 9 give useful information about the alternatives.

Focusing on the alternative S2.1, it is clear that it is classified in category 3 in the traditional ELECTRE Tri method (lefthand side of Table 9). The new probabilistic procedure demonstrates that, by incorporating data fluctuation, this alternative is classified as 82.6% in category 3, 14.5% in category 2, and 2.9% in category 4.

This probabilistic classification gives information on the possibility of ranking alternatives in the face of uncertainty and variance. When comparing two alternatives that appear to be ranked equally, the percentage of ranking in other categories allows for differentiation. Let us consider two alternatives classified in different categories, ‘S2.2’ and ‘S2.4’, by using the traditional ELECTRE Tri method. These two alternatives are also classified in categories 2, 3, and 5 due to the incorporation of fluctuation. However, the alternative ‘S2.2’ is classified in category 5 more frequently at 42.8% than the alternative ‘2.4’, which is only 39.1% in C5. These findings may lead to the conclusion that the ‘S2.2’ alternative outperforms the ‘S2.4’ alternative, allowing decision-makers to distinguish between two alternatives that initially appeared to be identical. This allocation enables a more detailed examination of elementary actions. The alternatives are carefully constructed to examine the impact of elementary actions on rankings. This additional information allows us to determine whether an elementary action improves the overall performance of an alternative.

Table 10 summarizes the outcomes of four alternatives: S2.1, S2.2, S3.1, and S3.2. All Group 2 options include an electric radiant panel and an existing electric floor, while all Group 3 options include electric storage heating but not electric floor heating. Otherwise, these two families of alternatives are identical, with solar panels on the roof distinguishing the index 2 alternatives from the index 1 alternatives. Alternatives with index 1 do not have an autonomous energy system. By analyzing these results, it is clear that the alternatives, which include solar panels, are better classified.

**Table 10.** Optimistic ranking obtained for four scenarios.

Alternative	C1	C2	C3	C4	C5	Total
S2.1	0	14.5	82.6	2.9	0	100
S2.2	0	0.2	3.1	53.9	42.8	100
S3.1	0	14.4	82.3	3.3	0	100
S3.2	0	1.3	4.3	94.4	0	100

Alternatives do not respond uniformly to the distributions applied by criteria. Some alternatives have been classified into four distinct categories, whereas others have only

been classified into one. Table 11 corresponds to the optimistic sorting obtained with the new procedure for the alternatives of family 2: 'S2.1', 'S2.2', 'S2.3', and 'S2.4', and family 7: 'S7.1', 'S7.2', 'S7.3', and 'S7.4'. Both alternative families were subjected to fluctuations with similar standard deviations.

**Table 11.** Optimistic ranking of the alternatives of family 2 and 7's global renovation scenarios.

Alternative	C1	C2	C3	C4	C5	Total	Alternative	C1	C2	C3	C4	C5	Total
S2.1	0	14.5	82.6	2.9	0	100	S7.1	0	100	0	0	0	100
S2.2	0	0.2	3.1	53.9	42.8	100	S7.2	0	100	0	0	0	100
S2.3	0	2.2	7.3	65.7	24.8	100	S7.3	0	100	0	0	0	100
S2.4	0	0.1	3	57.8	39.1	100	S7.4	0	100	0	0	0	100

Based on their fluctuation values, the alternatives of family 2 can be classified into four distinct categories, whereas the alternatives of family 7 are consistently classified in the same category. This classification discrepancy is influenced by a variety of factors, including the performance of the alternatives' proximity to the established thresholds. Even minor data deviations can cause the threshold to be exceeded, resulting in a different classification. Furthermore, the weight assigned to each criterion being evaluated is important, because a change in performance for a criterion with a high weight is more likely to affect the overall results.

## 6. Validation

It is necessary to ensure that the results obtained are consistent in order to validate the new procedure. To accomplish this, the results with a very low variance should be identical to those obtained by implementing only the crisp values. The algorithm was first implemented by using the traditional ELECTRE Tri method without the Monte Carlo procedure. The algorithm was then implemented a second time, this time incorporating the new procedure and using a very low variance ( $v = 0.000001$ ) with a repetition number of 10,000. Because the variance was so low, each alternative was classified in the same category as the one obtained with the traditional ELECTRE Tri method.

## 7. Conclusions

Multi-criteria decision analysis (MCDA) methods provide a valuable tool for understanding complex environmental problems by taking into account a variety of evaluation criteria and the classification of different alternatives. Despite the availability of data, it is critical to recognize the inherent uncertainty and variability of this information. To account for these uncertainties, it is proposed that data in the analysis be represented by probability distributions. It is possible to effectively address these probability distributions and fluctuations in data by integrating multi-criteria decision analysis methods, such as ELECTRE Tri, with a Monte Carlo implementation, resulting in a more robust ranking of alternatives.

The proposed new procedure adds significant Information by providing the percentage of rankings for each alternative within each category, allowing for a more detailed differentiation of alternatives. This method also makes it easier to compare alternatives that were previously ranked at the same level, allowing for a more detailed examination of the performance of individual actions within scenarios. To fully reap the benefits of this method, decision-makers must first understand the process, including how different data and parameters affect the results.

Furthermore, because they use the same performance matrix to group the input data for the analysis process, this procedure can be combined effectively with other multi-criteria decision analysis methods to ensure that no important information is overlooked. This method allows for a more comprehensive understanding of the sensitivity of the results to the input data and can help identify a wider range of potential outcomes by generalizing the use of probability distributions rather than crisp values.

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