

# Comment on “Excitation Spectrum and Superfluid Gap of an Ultracold Fermi Gas”

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At zero temperature, as any three-dimensional superfluid with short-range interactions, a gas of paired fermionic atoms exhibits an acoustic excitation branch of low-wavenumber expansion  $\omega_q = cq[1 + \zeta q^2/k_F^2 + O(q^4 \ln q)]$  with  $c$  the speed of sound and  $\zeta$  the curvature parameter (scaled by the Fermi wavenumber  $k_F$ ). Reference [1] claims to have experimentally determined whether the branch has a convex  $\zeta > 0$  or concave  $\zeta < 0$  start, depending on the interaction strength. This is crucial information that dictates the nature of the gas relaxation mechanisms at low temperatures (the well studied three-phonon Beliaev-Landau or the yet unobserved four-phonon Landau-Khalatnikov mechanism [2, 3]). However, in this Comment, we argue that the high wavenumbers  $q$  and temperature  $T$  used in the experiment introduce a large bias capable of turning a convex branch into a concave one, so that further measurements at lower  $q$  and  $T$  are required to give a definitive answer.

To fix ideas, we consider the unitary limit  $1/k_F a = 0$  with  $a$  the  $s$ -wave scattering length, where the interactions are strongest and no solid argument can predict the sign of  $\zeta$ . Theoretically, one has  $\zeta = -\pi^2 (2\xi_B)^{1/2} [c_1 + (3/2)c_2]$  where Bertsch’s parameter gives the chemical potential in units of the Fermi energy  $\mu = \xi_B E_F$  and  $c_{1,2}$  quantify gradient corrections to quantum hydrodynamics [4]. Only  $\xi_B$  is well known,  $\xi_B \simeq 3/8$  [5]. The dimensional expansion in powers of  $\epsilon = 4 - d = 1$  gives  $c_1 \simeq -0.0624(1 - 2\epsilon/3) + O(\epsilon^2)$  and  $c_2 = O(\epsilon^2)$  [6], so  $\zeta > 0$  to subleading order. Anderson’s RPA, spectrally equivalent to the Gaussian fluctuations approximation of [7], also predicts a positive value  $\zeta_{\text{RPA}} \simeq 0.0838$  [8] (for  $c_1 \simeq -0.021$  [6] this gives  $c_2 \simeq 0.0073 \ll |c_1|$ ). The experimental value  $\zeta_{\text{exp}} = -0.085(8)$  [1] is negative. However, assuming that the RPA is correct and that the branch start is convex, as we will do, actually has no clear incompatibility with the experiment, because the analysis in [1] suffers from two serious limitations.

First, the value  $\zeta_{\text{exp}}$ , obtained by cubic fitting of  $\omega_q$  [1], could strongly depend on the fitting interval if too wide. In the RPA, fitting  $\omega_q$  e.g. to the interval  $0.22 \leq q/k_F \leq 1.08$  of Figure 1 in [1] gives  $\zeta_{\text{RPA}}^{\text{fit}} \simeq -0.026$ , which even has the wrong sign. Since  $\omega_q^{\text{RPA}}$  has an inflection point at  $q/k_F \simeq 0.5$  [8] the fit blindly mixes convex and concave parts, which also explains the erroneous (negative) value of  $\zeta_{\text{RPA}}$  in [9].

Second, The high temperature  $T \simeq 0.13T_F \simeq 0.8T_c$  ( $T_c$  is the superfluid transition temperature) in [1] could modify the curvature of the acoustic branch by a non-negligible amount  $\delta\zeta^{\phi\phi}$  via interaction with thermal

phonons  $\phi$ . Treating the cubic phonon-phonon coupling  $H_3^{\phi\phi}$  to second order and the quartic coupling  $H_4^{\phi\phi}$  to first order, then taking the limit  $k_B T/mc^2 \rightarrow 0$  ( $m$  is the fermion mass), [10] obtains an expression for the thermal shift of  $\omega_q$ . This gives  $\delta\zeta^{\phi\phi} \sim -[\pi^2/(3\xi_B)^{3/2}](T/T_F)^2$ , or  $\delta\zeta^{\phi\phi} \simeq -0.140$  at the experimental temperature. Since the small parameter used  $k_B T/mc^2 \simeq 0.5$  is not  $\ll 1$ , we abandon the  $T \rightarrow 0$  limit and add corrective curvature factors  $(1 \pm \alpha q^2/k_F^2)$  to the amplitudes  $\rho_q$  and  $\phi_q$  of the superfluid density and phase quantum fluctuations ( $\alpha = \pi^2(\xi_B/2)^{1/2}[c_1 - (3/2)c_2] \simeq -0.136$  [3]). We find  $\delta\zeta^{\phi\phi} \simeq -0.110$ , still negative enough to change the sign of curvature in the RPA. Furthermore, thermal pair dissociation creates fermionic quasiparticles  $\gamma$  (another gas excitation branch) that interact with the phonons. Treating to second order the coupling  $H_3^{\phi\gamma}$  and to first order the coupling  $H_4^{\phi\gamma}$  given in [11] with curvature factors in  $\omega_q$ ,  $\rho_q$  and  $\phi_q$ , we find  $\delta\zeta^{\phi\gamma} \simeq -0.052$ . This is a rough estimate: [11] uses a simple local-density approximation whose small parameter  $(k_B T/m_* c^2)^{1/2}$  is  $\approx 1$  here (we use the  $\gamma$  dispersion relation of [12] with the effective mass  $m_* \simeq 0.56m$  and an energy minimum  $\Delta = 0.44E_F$  located at wavenumber  $k_0 = 0.92k_F$  [13]). Summing the thermal corrections gives  $\zeta_{\text{RPA}}^{\text{th}} \simeq -0.078$  to compare with  $\zeta_{\text{exp}} = -0.085(8)$ .

The experiment, at first sight at odds with the RPA, could therefore very well be in agreement with it due to large finite-momentum and thermal bias, and the sign of curvature announced in [1] may differ from the zero-temperature low-momentum one relevant for phonon damping.

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