

## Comment on “Excitation Spectrum and Superfluid Gap of an Ultracold Fermi Gas”

Reference [1] measures the acoustic excitation branch of a superfluid gas of paired fermionic atoms and determines, depending on the interaction strength, whether this branch is concave or convex at low wavenumber  $q$ . This is a crucial step forward because it provides information about the nature of the gas relaxation mechanisms at low temperatures. It is therefore important to know whether the announced results are definitive or not. This is the purpose of this Comment.

At zero temperature, as in any  $d = 3$  superfluid with short-range interactions, the gas acoustic branch has the Taylor expansion  $\omega_q = cq[1 + \zeta q^2/k_F^2 + O(q^4 \ln q)]$  with  $\omega_q$  the eigenfrequency,  $c$  the speed of sound and  $\zeta$  the curvature parameter (scaled by the Fermi wavenumber  $k_F$ ). In the collisionless regime, the sign of  $\zeta$  determines the damping mechanism of phonons  $\phi$  at low temperature. If  $\zeta > 0$ , it is the well studied three-phonon Beliaev-Landau mechanism. If  $\zeta < 0$ , it is the yet unobserved four-phonon Landau-Khalatnikov mechanism [2, 3]; knowing that  $\zeta < 0$  is crucial for its observation.

We consider here the unitary limit  $1/k_F a = 0$  with  $a$  the  $s$ -wave scattering length. Interactions are strongest and no solid theoretical argument can fix the sign of  $\zeta$ .

Qualitatively, the dispersion relation of fermionic quasiparticles  $\gamma$  (another gas excitation branch) reaches its minimum  $\Delta \simeq 0.44E_F$  at non-zero wavenumber  $k_0 \simeq 0.92k_F$  [4], so the broken-pair continuum has a flat border of value  $2\Delta$  up to  $q = 2k_0$  ( $E_F = \hbar^2 k_F^2 / 2m = k_B T_F$  is the Fermi energy); this border repels the acoustic branch, making it generally concave. However, nothing prevents the branch from remaining convex (as it naturally is when  $1/k_F a \rightarrow +\infty$ ) close to  $q = 0$ , where the continuum repulsion effect is weakest (the effect prevails everywhere when  $1/k_F a \rightarrow -\infty$ ).

Quantitatively,  $\zeta = -\pi^2 (2\xi_B)^{1/2} [c_1 + (3/2)c_2]$  where Bertsch’s parameter gives the chemical potential  $\mu = \xi_B E_F$  and  $c_{1,2}$  quantify gradient corrections to quantum hydrodynamics [5]. Only  $\xi_B$  is well known,  $\xi_B \simeq 3/8$  [6]. The expansion in  $\epsilon = 4 - d$  gives  $c_1 \simeq -0.0624(1 - 2\epsilon/3) + O(\epsilon^2)$  and  $c_2 = O(\epsilon^2)$  [7], so  $\zeta > 0$  to subleading order. Anderson’s RPA, spectrally equivalent to the Gaussian fluctuations approximation of [8], predicts a positive value  $\zeta_{\text{RPA}} \simeq 0.0838$  [9] (for  $c_1 \simeq -0.021$  [7] this gives  $c_2 \simeq 0.0073 \ll |c_1|$ ). The experimental value  $\zeta_{\text{exp}} = -0.085(8)$  [1] is negative.

However, assuming that the RPA is correct, as we will do, actually has no clear incompatibility with the experiment, because the analysis in [1] suffers from two limitations.

(i) The value  $\zeta_{\text{exp}}$ , obtained by cubic fitting of  $\omega_q$  [1], could strongly depend on the fitting interval if too wide. In the RPA, fitting  $\omega_q$  to the interval  $0.22 \leq q/k_F \leq 1.08$  of Figure 1 in [1] gives  $\zeta_{\text{RPA}}^{\text{fit}} \simeq -0.026$ , which even has the wrong sign. Since  $\omega_q^{\text{RPA}}$  has an inflection point at  $q/k_F \simeq 0.5$  [9] the fit blindly mixes convex and concave parts, which also explains the erroneous (negative) value of  $\zeta_{\text{RPA}}$  in [10].

(ii) The rather high temperature,  $T/T_F \simeq 0.13$  [1], could modify the curvature of the acoustic branch by a non-negligible amount  $\delta\zeta^{\phi\phi}$  via interaction with thermal phonons. Treating the cubic phonon-phonon coupling  $H_3^{\phi\phi}$  to second order and the quartic coupling  $H_4^{\phi\phi}$  to first order, then taking the limit  $k_B T/mc^2 \rightarrow 0$ , [11] obtains an expression for the thermal shift of  $\omega_q$ . This gives  $\delta\zeta^{\phi\phi} \sim -[\pi^2/(3\xi_B)^{3/2}](T/T_F)^2$ , or  $\delta\zeta^{\phi\phi} \simeq -0.140$  at the experimental temperature. Since the small parameter used  $k_B T/mc^2 \simeq 0.5$  is not  $\ll 1$ , we abandon the  $T \rightarrow 0$  limit and add corrective curvature factors  $(1 \pm \alpha q^2/k_F^2)$  to the amplitudes  $\rho_q$  and  $\phi_q$  of the superfluid density and phase quantum fluctuations ( $\alpha = \pi^2(\xi_B/2)^{1/2}[c_1 - (3/2)c_2] \simeq -0.136$  [3]). We find  $\delta\zeta^{\phi\phi} \simeq -0.110$ , still negative enough to change the curvature sign in the RPA. Furthermore, thermal pair dissociation creates fermionic quasiparticles that interact with the phonons. Treating to second order  $H_3^{\phi\gamma}$  and to first order  $H_4^{\phi\gamma}$  given in [12] with curvature factors in  $\omega_q$ ,  $\rho_q$  and  $\phi_q$ , we find  $\delta\zeta^{\phi\gamma} \simeq -0.052$ . This is a rough estimate: [12] uses a simple local-density approximation whose small parameter  $(k_B T/m_* c^2)^{1/2}$  is  $\approx 1$  here ( $m_* \simeq 0.56m$  [13] is the  $\gamma$  effective mass). Summing the thermal corrections gives  $\zeta_{\text{RPA}}^{\text{th}} \simeq -0.078$  to compare with  $\zeta_{\text{exp}} = -0.085(8)$ .

To summarize, the experimental work [1], as innovative as it is, may not have accurately measured the coefficient of  $q^3$  of the acoustic branch of the zero-temperature unitary Fermi gas. Further studies seem desirable. We would like to thank Alice Sinatra for her comments on the text.

Y. Castin, LKB-ENS, CNRS, Paris, France.

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