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2 Journal of Theoretical and Computational Acoustics

3 **Scattering from a partially coated shell immersed in water using a subtractive**  
4 **modelling technique**

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17 In this study, we focus on the prediction of the pressure field scattered from an immersed cylindrical  
18 shell partially coated by a soft rubber, impacted by an acoustic plane wave. As the coating covers  
19 only a partial portion along the circumference of the shell, the considered system is not axisym-  
20 metric. As a result, a spectral (Fourier) resolution of the mathematical problem would induce the  
21 coupling of the different circumferential orders, which can lead to prohibitive computing times. To  
22 circumvent this issue, the reverse Condensed Transfer Function (rCTF) method has recently been  
23 developed to decouple vibroacoustic subsystems initially coupled along lines or surfaces. From an  
24 analytical model of the fully coated shell impacted by the acoustic plane wave and a finite element  
25 (FE) model of the missing coating material, the rCTF approach predicts the vibroacoustic behavior  
26 of the coated shell with a voided section instead of the removed part. This voided section can then  
27 be filled by a FE model of the water domain replacing the removed coating material, using the  
28 direct CTF approach. The principle of the rCTF approach, some numerical validations, and results  
29 for the scattering from the partially coated shell are presented in this paper.

30 *Keywords:* Vibroacoustic; Subtractive modelling; Acoustic scattering ; Numerical methods; Cylin-  
31 drical shells.

32 **1. Introduction**

33 Studying the vibroacoustic performances of cylindrical shells finds great interest for indus-  
34 trial applications. They are particularly used in the naval context, in order to model hulls  
35 of underwater vehicles. Indeed, acoustic performances of underwater vehicles must be pre-  
36 dicted in a large frequency range following the increasing capabilities of Sonar antennas. In  
37 order to enhance the vibroacoustic performances of underwater vehicles, acoustic coatings  
38 can be applied at the surface of the hull. **Those coatings are generally made of soft material**  
39 **and can mainly be divided into two categories. Decoupling coatings are used to reduce the**

radiation by decoupling the hull from the external fluid medium. On the other hand, anechoic coatings are used to limit the scattering from the hull by absorbing incoming waves thanks to their impedance match with water.<sup>1</sup>

Over the past decades, a substantive amount of work has been produced to study such technologies and how well they contribute to the mitigation of the noise produced by the structure they are mounted on. Several approaches to model the coating have been investigated, among which we can cite the locally reacting material approach convenient for the simpleness of its implementation. Locally reacting materials have been used to study radiation<sup>2</sup> and scattering<sup>3</sup> from cylindrical shells, but it was observed that this theory is limited in a physical point of view, mainly because it is unable to describe the behavior of the coating in its thickness. A second, more robust, theory is to model the coating using the three-dimensional theory of elasticity.<sup>4,5</sup> This approach gives better results as it accounts for both longitudinal and shear waves travelling in the material, with the main drawback of being much more time consuming. A third approach consists in modelling the coating as an equivalent fluid, meaning that only the longitudinal waves are accounted for in the coating.<sup>6</sup> This approach, particularly indicated when the coating is made of soft material, is a good compromise between the two aforementioned approaches, as it has proven to better capture the physical phenomena in the coating than the locally reacting approach, while being less computationally intensive than the three-dimensional theory of elasticity.

While only the case of homogeneous media has been discussed here, heterogeneous coatings also have been studied in literature. In particular, cylindrically layered media can show great interest as they could allow for example overlaying anechoic and decoupling coatings. A mathematical framework to study cylindrically layered media infinite along the axial direction has been presented by Skelton and James.<sup>1</sup> It is based on a spectral approach where the equations of motion of the shell are solved in the wavenumber domain, by the means of a Fourier transform along the length of the shell and a Fourier series decomposition along its circumference. A global matrix assembly procedure is then used to account for several kinds of layers, which can either be solid or fluid,<sup>7</sup> allowing extending the classical Transfer Matrix Method for so-called *large fd* problems (i.e. at high wavenumber-thickness product) where high numerical instabilities occur. Heterogeneous coatings formed with resonant inclusions have also been studied as way to improve the sound absorption by the coating, either in terms of performances or frequency range. In particular, coatings constituted by a matrix of soft rubber with void inclusions,<sup>6</sup> hard inclusions<sup>8</sup> or both void and hard inclusions<sup>9</sup> have been investigated using analytical homogenisation theories. The previously cited studies were focused on the material technology in itself, but its influence when mounted on a cylindrical shell has also been studied.<sup>10</sup> Other theories can be used such as the layer-multiple-scattering method, which allows to consider the double-periodic problem of anechoic coatings with periodic cavities as a 2-D case because of the invariance in axial direction.<sup>11-13</sup>

If those studies focus on fully coated cylindrical shells, less attention has been paid to the study of partially coated cylindrical shells. Studying such coating configurations can find physical or practical interests, for example to study the impact of a missing coating tile on

82 the vibroacoustic behavior of the system, or to account for areas where the shell is covered  
83 with anechoic coating, and other areas where the shell is covered with decoupling coating.  
84 However, a difficulty arises from the loss of axisymmetry induced by the partial coating.  
85 Indeed, the spectral procedure described in<sup>1</sup> is based on the assumption of axisymmetry of  
86 the system. Hence, the only studies focused on partial coating are restricted to the locally  
87 reacting material approach to describe the coating.<sup>3,14,15</sup>

88 On the other hand, from a numerical point of view, substructuring methods have emerged  
89 during the past decades to overcome the frequency limitations of element-based methods  
90 such as the Finite Element Method (FEM). They allow studying rather complex systems by  
91 separating them into simpler subsystems which can be studied separately, before coupling  
92 the subsystems by the means of frequency transfer functions. In particular, substructuring  
93 methods based on admittance and impedance concepts will be of interest here. In the Patch  
94 Transfer Function (PTF) method,<sup>16</sup> the interface between the subsystem is separated into a  
95 number of elementary surfaces called patches, and computing the transfer functions between  
96 the patches for each subsystem allows coupling them together. This approach has proved  
97 to be popular in the automotive industry,<sup>16–18</sup> but has found applications in other kind of  
98 industries, such as the naval industry<sup>19,20</sup> to study the transmission of sound through the  
99 ballasts of a submarine vessel. Still initially focused for naval applications, the Circumferen-  
100 tial Admittance Approach (CAA)<sup>21</sup> has been developed to couple a model of a cylindrical  
101 shell immersed in water to axisymmetric internal structures such as stiffeners or bulkheads.  
102 In this method, the fluid-loaded cylindrical shell is studied using the spectral procedure  
103 described in.<sup>1</sup> This method has then been applied to study the scattering from a stiffened  
104 cylindrical shell.<sup>22</sup> Later on, the principle of the method was extended to non-axisymmetric  
105 internal frames such as floors or engine foundations by the way of the Condensed Transfer  
106 Function (CTF) method, to investigate their contribution on radiation and scattering from  
107 the shell.<sup>23–25</sup> As such, the CTF method can be seen as a generalisation of both the PTF  
108 and CAA methods.

109 Following these recent advances, the reverse Condensed Transfer Function (rCTF) has  
110 been developed as a reverse formulation of the CTF method to decouple subsystems initially  
111 coupled along lines or surfaces. One of the main advantages of these methods is that it allows  
112 to study non-axisymmetric systems, proving an interesting potential to address the issue  
113 of the partially coated cylindrical shell. The mathematical framework of the method was  
114 proposed in,<sup>26</sup> while the procedure to compute the so-called Condensed Transfer Functions  
115 (necessary to apply the method) from numerical models has been described in.<sup>27</sup> A global  
116 approach of subtractive modelling was then proposed to improve the convergence of the  
117 method by extending the definition of the decoupling interface to interior boundaries of the  
118 initial system.<sup>28</sup> This paper is focused on addressing the problem of the scattering of an  
119 incident plane wave by a partially coated cylindrical shell, by using a coupled CTF-rCTF  
120 approach. Hence, the rCTF formulation is extended to predict the pressure field scattered  
121 by a partially coated cylindrical shell. The theoretical framework is presented in Section  
122 2, whereas the numerical processes used to evaluate the condensed transfer functions are  
123 addressed in Section 3. The numerical results of the proposed approach concerning the

4

124 scattering from the partially coated shell immersed in water are then discussed in Section  
 125 4, before concluding this paper.

## 126 2. Principle of the coupled CTF-rCTF method

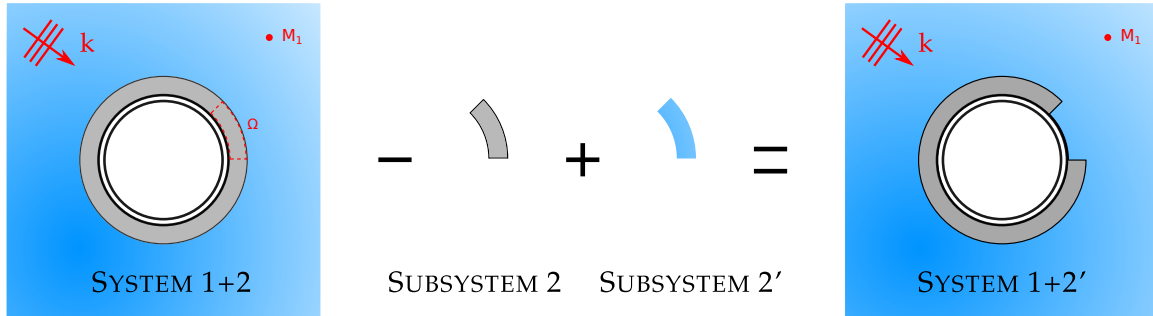


Figure 1. Studying a partially coated cylindrical shell. The '-' sign illustrates the rCTF method; the '+' sign illustrates the CTF method.

127 In order to study the scattering from a partially coated cylindrical shell, **the coupled CTF-**  
 128 **rCTF method, illustrated in Figure 1, is applied. It consists in considering initially** a model  
 129 of the fully coated cylindrical shell impinged by a harmonic acoustic plane wave (system  
 130 1+2 in Figure 1). From this system, a model of the missing part of the coating (subsystem 2  
 131 in Figure 1) is removed. This first operation, called the decoupling procedure, is performed  
 132 using the rCTF method **and is illustrated in Figure 2.** This results in an **intermediary**  
 133 **subsystem: the partially coated cylindrical shell, for which the missing part of the coated has**  
 134 **been replaced by a rigid void (subsystem 1 in Figure 2, where the white area corresponds to**  
 135 **the void).** Then the obtained subsystem is coupled to a model of the water domain replacing  
 136 the removed coating material (subsystem 2' in Figure 1). This second operation is called  
 137 the recoupling procedure, is performed using the CTF method **and is illustrated in Figure**  
 138 **3.** The obtained system is finally the partially coated cylindrical shell immersed in water  
 139 (system 1+2' in Figure 1) impacted by the harmonic acoustic plane wave. In the following  
 140 sections, the mathematical frameworks for the decoupling and recoupling procedures will  
 141 be recalled.

### 142 2.1. Decoupling procedure using the rCTF method

143 In order to consider the decoupling problem (Figure 2), the rCTF method is used. The  
 144 theoretical fundamentals of this procedure have been explained in details in reference,<sup>26</sup>  
 145 but the main steps will be recalled here.

146 For the sake of conciseness, a 2D model of our system is considered. Let us consider  
 147 two systems initially coupled along a line  $\Omega$ . The system 1+2, corresponding to the fully  
 148 coated cylindrical shell, is excited by an external acoustic plane wave, and the responses are

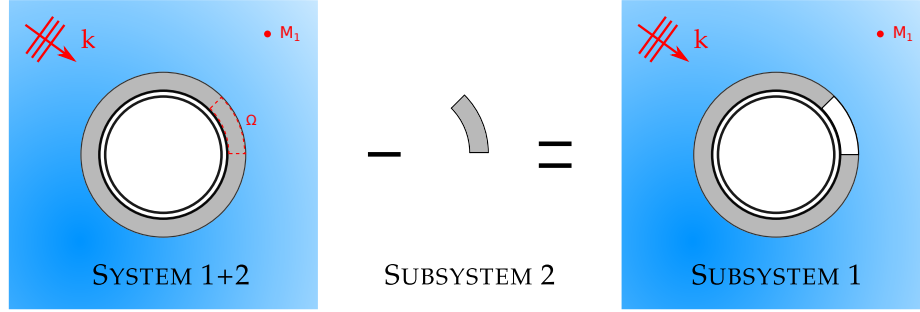


Figure 2. Decoupling procedure using the rCTF method.

149 calculated in harmonic regime. The objective of this decoupling procedure is to obtain the  
 150 response at any point  $M_1$  of subsystem 1. To this end, a set of  $N$  orthonormal functions,  
 151 called condensation functions, is defined on  $\Omega$ :  $\{\varphi^i\}_{1 \leq i \leq N}$ . Then, the pressures  $p_\alpha$  and normal  
 152 velocities  $u_\alpha$  on  $\Omega$  can be approximated, for each (sub)system  $\alpha$ , as a linear combination of  
 153 the condensation functions

$$p_\alpha(x) \simeq \sum_{i=1}^N P_\alpha^i \varphi^i(x) \quad \text{and} \quad u_\alpha(x) \simeq \sum_{i=1}^N U_\alpha^i \varphi^i(x), \quad x \in \Omega \quad (1)$$

154 where  $P_\alpha^i$  and  $U_\alpha^i$  are the unknowns. They can be estimated by defining, for each uncoupled  
 155 subsystem (i.e. subsystems 1 and 2), condensed transfer functions (CTFs) between  $\varphi^i$  and  
 156  $\varphi^j$ . These CTFs are defined by applying a prescribed velocity  $u_\alpha = \varphi^j$  on  $\Omega$ , meaning that  
 157 the CTFs will result in condensed impedances

$$Z_\alpha^{ij} = \frac{\langle \bar{p}_\alpha, \varphi^i \rangle}{\langle u_\alpha, \varphi^j \rangle} = \langle \bar{p}_\alpha, \varphi^i \rangle \quad (2)$$

158 where  $\langle f, g \rangle = \int f(x)g^*(x) dx$  (with  $*$  denoting the complex conjugate), and  $\bar{p}_\alpha$  corresponds  
 159 to the resulting pressure on  $\Omega$  when the subsystem is excited by  $u_\alpha = \varphi^j$ . Concerning  
 160 system 1+2, as the decoupling boundary  $\Omega$  is a fictitious one, a prescribed velocity jump  
 161 corresponding to  $\varphi^j$  must be considered to define its condensed impedances

$$Z_{1+2}^{ij} = \frac{P_{1+2}^i}{\delta U_{1+2}^j} = \frac{\langle \bar{p}_{1+2}, \varphi^i \rangle}{\langle \varphi^j, \varphi^j \rangle} = \langle \bar{p}_{1+2}, \varphi^i \rangle \quad (3)$$

162 The condensed impedances are calculated for each couple of condensation functions  
 163 for each system/subsystem. The condensed impedance matrix for system/subsystem  $\alpha$  can  
 164 subsequently be defined as  $Z_\alpha = [Z_\alpha^{ij}]_{N \times N}$ . Then, following some developments of the  
 165 rCTF approach described in reference,<sup>26</sup> the condensed impedance matrix of subsystem 1  
 166 can be obtained from the condensed impedance matrices of system 1+2 and subsystem 2:

6

$$\mathbf{Z}_1 = \mathbf{Z}_2 (\mathbf{Z}_2 - \mathbf{Z}_{1+2})^{-1} \mathbf{Z}_{1+2} \quad (4)$$

167 The contribution from the plane wave excitation must be introduced through the defi-  
 168 nition of the free condensed pressures of the system 1+2

169

$$P_{1+2}^i = \langle p_{1+2}, \varphi^i \rangle \quad (5)$$

170

171 where  $p_{1+2}$  is the pressure induced by the acoustic plane wave on the the decoupling bound-  
 172 ary  $\Omega$ . Then, to estimate the response at any point  $M_1$  of the uncoupled subsystem 1, the  
 173 quantity  $Z_{1+2}^i(M_1)$ , corresponding to the response at point  $M_1$  in the system 1+2 when  $\Omega$   
 174 is excited by a velocity jump  $\varphi^i$ , must be introduced. This quantity can be inferred from the  
 175 free condensed pressures induced by a monopole of unit volume velocity located at point  
 176  $M_1$ , using a reciprocity principle<sup>29,30</sup>

177

$$Z_{1+2}^i(M_1) = P_{1+2}^{M_1 i *} = \langle p_{1+2}^{M_1 *}, \varphi^i \rangle^* \quad (6)$$

178

179 where  $p_{1+2}^{M_1 *}$  is the complex conjugate of the pressure induced on the decoupling boundary  
 180  $\Omega$  by a monopole of unit volume velocity located at point  $M_1$ . It must be underlined that  
 181 the introduction of the complex conjugates in Eq. 6 comes from the use of the reciprocity  
 182 principle. Indeed, the complex conjugate of the condensation function must appear in the  
 183 definition of a condensed quantity, which is not the case when using the reciprocity princi-  
 184 ple, leading to the necessity of adding the complex conjugates to have a correct correlation  
 185 between the different quantities. The condensed pressure vectors of system 1+2 can subse-  
 186 quently be defined as  $\mathbf{P}_{1+2} = [P_{1+2}^i]_{N \times 1}$  and  $\mathbf{P}_{1+2}^{M_1 *} = [P_{1+2}^{M_1 i *}]_{N \times 1}$ . Finally, the pressure  
 187 at any point  $M_1$  of the uncoupled subsystem 1 can be written as

$$p_1(M_1) = p_{1+2}(M_1) + \left( (\mathbf{I} + \mathbf{Z}_1 \mathbf{Z}_2^{-1}) \mathbf{P}_{1+2}^{M_1 *} \right)^T \mathbf{Z}_2^{-1} \mathbf{P}_{1+2} \quad (7)$$

188 where  $p_{1+2}(M_1)$  is the pressure induced by the acoustic plane wave at point  $M_1$  of system  
 189 1+2.

## 190 2.2. Recoupling procedure using the CTF method

191 Once the pressure has been obtained in the decoupled subsystem 1, we are interested in  
 192 recoupling this subsystem to a water tile occupying the missing part of the coating material,  
 193 to obtain the partially coated cylindrical shell (Figure 3). The recoupling procedure is per-  
 194 formed using the CTF method, for which the theoretical fundamentals have been developed  
 195 in<sup>26</sup> for acoustical systems and in<sup>24</sup> for mechanical systems.

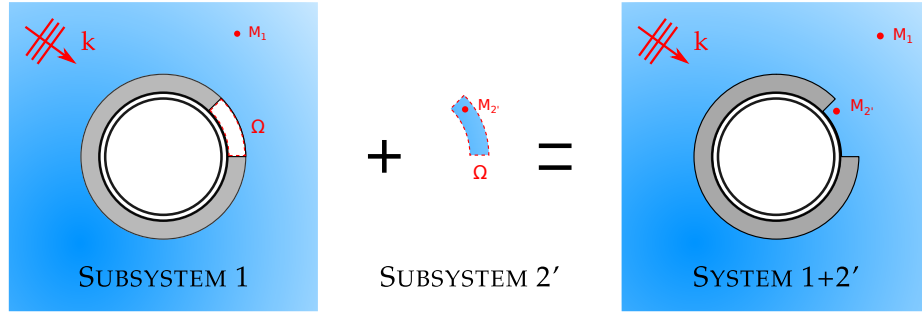


Figure 3. Recoupling procedure using the CTF method.

196 We can find the pressure at any point of the coupled system 1+2', for which the ex-  
 197 pression depends if the considered point initially belonged to subsystem 1 or subsystem  
 198 2'

$$\begin{cases} p_{1+2'}(M_1) = \tilde{p}_1(M_1) + \mathbf{Z}_1^T(M_1)\mathbf{U}_{1+2'} & (8a) \\ p_{1+2'}(M_2') = -\mathbf{Z}_2'^T(M_2')\mathbf{U}_{1+2'} & (8b) \end{cases}$$

199 where the point  $M_1$  initially belongs to subsystem 1, while the point  $M_2'$  initially belongs  
 200 to subsystem 2'.  $\mathbf{Z}_1(M_1)$  is the vector of the point condensed impedances of subsystem 1,  
 201 and its components are defined similarly as  $Z_{1+2}^i(M_1)$ , using a reciprocity principle.<sup>29,30</sup>  
 202 The vector can then be expressed as<sup>26</sup>

$$\mathbf{Z}_1(M_1) = \mathbf{P}_1^{M_1^*} = (\mathbf{I} + \mathbf{Z}_1\mathbf{Z}_2^{-1})\mathbf{P}_{1+2}^{M_1^*} \quad (9)$$

203 Concerning  $\mathbf{U}_{1+2'}$ , it reads

$$\mathbf{U}_{1+2'} = -(\mathbf{Z}_1 + \mathbf{Z}_2')^{-1}\tilde{\mathbf{P}}_1 \quad (10)$$

204 As  $\mathbf{Z}_1$  has already been computed, the necessary quantities here are  $\mathbf{Z}_2'$  (which is as-  
 205 sumed to be known), and  $\tilde{\mathbf{P}}_1$ . The latter corresponds to the condensed pressure at the  
 206 surface  $\Omega$  of the uncoupled subsystem 1 when the excitation is the acoustic plane wave and  
 207 can then be noted  $\tilde{\mathbf{P}}_1$ . It is calculated in a similar way as  $\tilde{\mathbf{P}}_1^{M_1}$  in Equation 9

$$\tilde{\mathbf{P}}_1 = (\mathbf{I} + \mathbf{Z}_1\mathbf{Z}_2^{-1})\mathbf{P}_{1+2} \quad (11)$$

208 From these information, Eq. 8a can be rewritten

$$p_{1+2'}(M_1) = p_{1+2}(M_1) + \left( (\mathbf{I} + \mathbf{Z}_1\mathbf{Z}_2^{-1})\mathbf{P}_{1+2}^{M_1^*} \right)^T \left( \mathbf{Z}_2^{-1}\mathbf{P}_{1+2} - (\mathbf{Z}_1 + \mathbf{Z}_2')^{-1}\tilde{\mathbf{P}}_1 \right) \quad (12)$$



209 where  $\mathbf{Z}_1$  and  $\tilde{\mathbf{P}}_1$  are expressed from Eqs. 4 and 11, respectively. On the other hand, Eq. 8b  
210 gives

$$p_{1+2'}(M_{2'}) = \mathbf{Z}_{2'}^T(M_{2'}) (\mathbf{Z}_1 + \mathbf{Z}_{2'})^{-1} \tilde{\mathbf{P}}_1 \quad (13)$$

211 where  $\mathbf{Z}_{2'}(M_{2'})$  can be computed by evaluating the pressure at point  $M_{2'}$  of subsystem 2'  
212 when the excitation is a condensation function.

213 From Eqs. 12 and 13, the pressure scattered by the partially coated cylindrical shell can  
214 be estimated at any point in the fluid domain, from the condensed impedances of the fully  
215 coated shell (i.e. system 1+2), of the removed coating patch (i.e. subsystem 2) and of the  
216 added water domain (i.e. subsystem 2'). In the next section, we are going to focus on the  
217 numerical computation of these condensed impedances ( $\mathbf{Z}_{1+2}$ ,  $\mathbf{Z}_2$  and  $\mathbf{Z}'_2$ ).

### 218 3. Numerical calculation of the condensed impedances

219 In order to apply the process proposed in Section 2, the condensed impedance matrices  
220  $\mathbf{Z}_{1+2}$ ,  $\mathbf{Z}_2$  and  $\mathbf{Z}'_2$  must be computed. After a preliminary introduction of the considered  
221 system and of the condensed transfer functions in subsection 3.1, the procedure to evaluate  
222 numerically the condensed impedances will be developed in subsections 3.2 and 3.3 for the  
223 system 1+2 and subsystems 2 and 2', respectively.

#### 224 3.1. Description of the system and of the condensation functions

225 In the following, we consider a cylindrical shell of radius  $R$ , thickness  $h$ , Young's modulus  
226  $E_s$ , density  $\rho_s$  and Poisson's ratio  $\nu_s$ . It is excited by a harmonic acoustic plane wave of  
227 oblique incidence  $\theta^i$  compared to the vertical direction (see Figure 4a). The behavior of the  
228 shell are described using Flügge equations for thin shells.<sup>31</sup>

229 The coating is an anechoic coating of thickness  $h_p$  and density  $\rho_p$ , and will be modelled  
230 as an equivalent fluid, meaning that the transverse waves travelling in the coating are not  
231 taken into account. The celerity of longitudinal waves in the coating is noted  $c_p$ , while its  
232 damping loss factor is  $\eta_p$ . The acoustic properties of the coating correspond to the ones used  
233 for the BeTSSi II (Benchmarking of Target Strength Simulations) international workshop.<sup>32</sup>  
234 The coated shell is surrounded by a fluid medium of density  $\rho_f$  and speed of sound  $c_f$ , and  
235 a damping loss factor  $\eta_f$  is introduced to avoid numerical instabilities.<sup>26</sup> Both fluid domains  
236 are modelled using the Helmholtz equation, and the Sommerfeld radiation condition is  
237 verified at infinity

$$\begin{cases} \Delta p(r, \theta, x) + k_f^2 p(r, \theta, x) = 0 \\ \lim_{r \rightarrow \infty} r \left( jk_f p + \frac{\partial p}{\partial r} \right) = 0 \end{cases} \quad (14)$$

238 where  $k_f$  is the acoustic wavenumber.

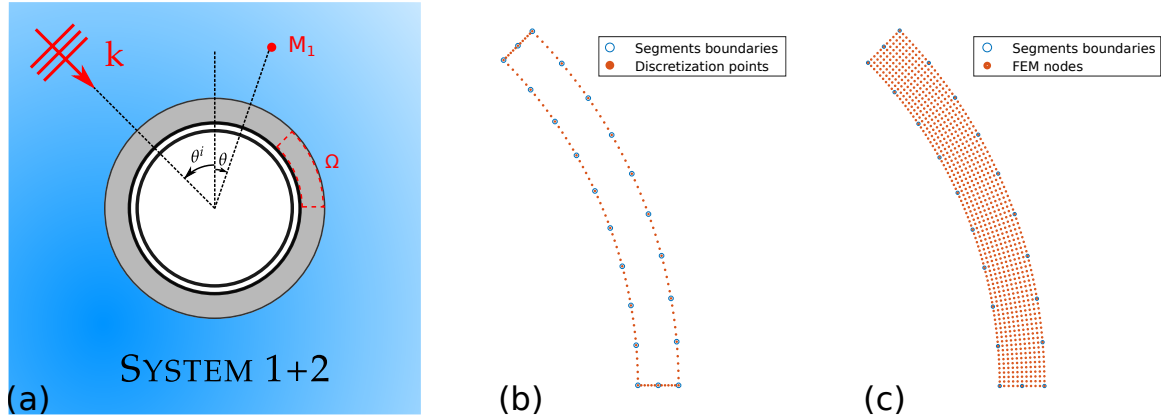


Figure 4. (a) Fully coated cylindrical shell impacted by an acoustic plane wave. (b) Discretization of the decoupling boundary for the system 1+2. (c) FE model for the subsystem 2.

239 Besides, the continuity of normal velocities and pressures are considered at the shell-  
 240 coating and coating-exterior fluid interfaces. The characteristics of the shell, the coating  
 241 and the surrounding fluid domain are presented in Table 3.1. It must be stressed that, in  
 242 the following, the computations are carried out using 2D models. This choice was made in  
 243 order to be able to compare the obtained results to a FEM reference calculation. Therefore,  
 244 the oblique incidence of the acoustic plane wave in the azimuthal plane was not taken into  
 245 account.

Table 1. Material and fluids characteristics.

Parameter	Notation	Value	Unit
Shell radius	$R$	1	m
Shell thickness	$h_s$	0.01	m
Shell Young's modulus	$E_s$	210	GPa
Shell Poisson's ratio	$\nu_s$	0.3	-
Shell density	$\rho_s$	7800	kg.m <sup>-3</sup>
Shell damping loss factor	$\mu_s$	0.01	-
Coating thickness	$h_p$	0.1	m
Coating density	$\rho_p$	1600	kg.m <sup>-3</sup>
Coating speed of sound	$c_p$	841.5	m.s <sup>-1</sup>
Coating loss factor	$\mu_p$	0.236	-
Exterior fluid density	$\rho_f$	1000	kg.m <sup>-3</sup>
Exterior fluid speed of sound	$c_f$	1500	m.s <sup>-1</sup>
Exterior fluid loss factor	$\mu_f$	0.001	-

246 In this work, as the decoupling boundary  $\Omega$  is a 1D one (because we are working with  
 247 2D models), the condensation functions are chosen to be gate functions. This means that  
 248 the decoupling boundary  $\Omega$ , of length  $\Gamma$ , is divided into  $N$  segments. The condensation

249 functions are then defined depending on their length  $L_s$

$$\varphi^i(s) = \begin{cases} \frac{1}{\sqrt{L_s}} & \text{if } (i-1)L_s \leq s < iL_s, \\ 0 & \text{elsewhere} \end{cases}, \quad i \in \llbracket 1, N \rrbracket \quad 0 \leq s \leq \Gamma \quad (15)$$

250 The size of the segments follows a wavelength-based criterion, and must be smaller than  
 251 half the minimal wavelength (between the acoustic wavelength in the surrounding fluid, the  
 252 acoustic wavelength in the coating and the flexural wavelength of the shell) at the highest  
 253 considered frequency. In the following, the calculations are carried out between 5 Hz and  
 254 1500 Hz. At this frequency, the acoustic wavelength in the coating is the smaller one, and  
 255 is then the one retained to define the size of the segments. Following this criterion, the  
 256 decoupling boundary  $\Omega$  is divided into 22 segments as illustrated in Figure 4b.

### 257 **3.2. Condensed impedances of the fully coated cylindrical shell**

258 According to the definition of the condensed impedances given in Eq. 3, a velocity jump  
 259 must be applied on the decoupling boundary and the resulting pressure must be calcu-  
 260 lated. Besides, to correspond to a condensation function, the velocity jumps will be applied  
 261 successively on each segment of the decoupling boundary according to Eq. 15. To emulate  
 262 these velocity jumps, an integral formulation derived from the indirect BEM formulation  
 263 is retained. Following this formulation, the pressure field from a radiating surface can be  
 264 expressed as

$$p(M) = \int_{\Omega} \nu(P)G(M, P) d\Omega(P), \quad M \in \Omega, \quad P \in \Omega \quad (16)$$

265 where  $\nu(P)$  is the single layer potential due to a layer of monopole sources and represents  
 266 a velocity jump at the crossing of  $\Omega$ . It can be expressed from the velocity jump  $\delta U$  as

$$\nu(P) = j\omega\rho_f\delta U(P) \quad (17)$$

267  $G(M, P)$  is the free-field Green function and corresponds to the pressure at point  $M$   
 268 due to a monopole source located at point  $P$

$$G(M, P) = -\frac{e^{jk_f(1-j\eta_f)|M-P|}}{4\pi|M-P|} \quad (18)$$

269 with  $|M-P|$  being the Euclidian distance between the two points. According to the defini-  
 270 tion of the single layer potential, the velocity jump can then be emulated by placing on  $\Omega$   
 271 an array of monopole sources, which allows to evaluate numerically the integral in Eq. 16.

272 In practice, the formulation to obtain the pressure induced at a single point by a  
 273 monopole excitation must be derived. This calculation is done via a semi-analytical approach  
 274 in the wavenumber domain as described in,<sup>1</sup> by the means of a Fourier series decomposition

275 along the circumferential coordinate. It is worth mentioning that, if we were to work with  
 276 3D models, a Fourier transform along the axial coordinate would also be performed. Finally,  
 277 the total pressure at a single point  $n_i$  due to a monopole excitation located at point  $n_j$  can  
 278 be retrieved by adding the contribution of all the circumferential orders

$$p_{tot}^{n_i n_j} = \sum_{n=-\infty}^{n=+\infty} \tilde{p}_{tot}^{n_i n_j} e^{jn\theta_{n_i}} \quad (19)$$

279 where  $\tilde{p}_{tot}^{n_i n_j}$  is the total spectral pressure obtained using the procedure in.<sup>1</sup> The sum in Eq.  
 280 19 is theoretically infinite, but is in practice truncated to a finite value  $N_{max}$  derived from  
 281 the maximal value of the different wavenumbers of the problem,

$$N_{max} = \text{int} [\kappa_n R \max(k_f, k_p, k_s)] \quad (20)$$

282 with  $k_f$  the acoustical wavenumber in the surrounding fluid medium,  $k_p$  the acoustical  
 283 wavenumber in the anechoic coating, and  $k_s$  the flexural wavenumber of the shell.  $\kappa_n$  is a  
 284 margin coefficient generally fixed to 1.5 for such problems.

285 The condensed impedance between the incident segment  $j$  and the receiving segment  $i$   
 286 is then obtained by summing the resulting pressures at all the points  $N_i$  belonging to the  
 287 segment  $i$ , due to monopole excitations on all the points  $N_j$  belonging to the segment  $j$

$$Z_{1+2}^{ij} = \sum_{n_i=1}^{N_i} \sum_{n_j=1}^{N_j} \frac{p_{tot}^{n_i n_j}}{\sqrt{L_j L_i}} \delta\xi_j \delta\xi_i \quad (21)$$

288 where  $L_i$  and  $L_j$  are the length of the segments  $i$  and  $j$ , respectively, and  $\delta\xi_i$  and  $\delta\xi_j$   
 289 correspond to the discretization step in the segments (which depends on the location of the  
 290 segment on the decoupling boundary).

291 It must be emphasized that the expression of the pressure in Eq. 12 also relies on the  
 292 calculation of the condensed pressures  $\mathbf{P}_{1+2}$  and  $\mathbf{P}_{1+2}^{\mathbf{M}_1}$ . These condensed pressures are  
 293 numerically evaluated in a similar way as  $\mathbf{Z}_{1+2}$ , in the sense that the expressions of the  
 294 pressure induced by the acoustic plane wave (for  $\mathbf{P}_{1+2}$ ) and by the unit monopole (for  
 295  $\mathbf{P}_{1+2}^{\mathbf{M}_1}$ ) are projected on the segments using the discretization points. The details of these  
 296 calculations will not be addressed here, but they can be found in.<sup>33</sup>

### 297 **3.3. Condensed impedances of the missing coating material and the water** 298 **patch**

299 Concerning the condensed impedances of the missing coating material and of the water  
 300 patch added to fill up the voided section, they must be computed by applying a prescribed  
 301 velocity on the decoupling boundary, as defined in Eq. 2. To do so, and given the small  
 302 size of the subsystem, a Finite Element (FE) formulation was used. An example of the FE

303 model of the removed patch can be seen in Figure 4c. As we are working here with acoustical  
 304 systems, the FEM formulation which must be solved yields

$$([\mathbf{K}] - \omega^2 [\mathbf{M}] (1 - 2j\eta)) \{\mathbf{P}\} = \{\mathbf{Q}\} \quad (22)$$

305 where  $[\mathbf{K}]$  and  $[\mathbf{M}]$  are the acoustic stiffness and mass matrices, respectively,  $\eta$  is the  
 306 damping loss factor (either in the coating or in the surrounding fluid),  $\{\mathbf{P}\}$  is the output  
 307 pressure vector, and  $\{\mathbf{Q}\}$  is the input volume velocity vector.

308 The procedure to compute the condensed impedances of the subsystems is then similar  
 309 to the one described in subsection 3.2. The nodes of the FE model are firstly associated to  
 310 the different segments to which they belong, in order to define the condensation functions.  
 311 Then, a volume velocity excitation is applied on the nodes belonging to the incident segment  
 312  $j$  according to the definition of the condensation functions in Eq. 15. Following this, the  
 313 input volume velocity vector  $\{\mathbf{Q}^j\}$  associated to the incident segment  $j$  will have  $N_j$  non-null  
 314 components  $Q_n^j$ , with  $N_j$  being the number of nodes belonging to the segment  $j$

$$Q_n^j = \frac{\delta S_n}{\sqrt{L_j}}, \quad n \in \llbracket 1, N_j \rrbracket \quad (23)$$

315 where  $\delta S_n$  is the length around the node  $n$ , and  $L_j$  the length of the segment  $j$ . The resulting  
 316 pressure vector  $\{\mathbf{P}^j\}$  of the FE model is then obtained by inverting directly the dynamic  
 317 matrix of Eq. 22. This operation is not computationally costly as the system has a low  
 318 number of degrees of freedom.

$$\{\mathbf{P}^j\} = ([\mathbf{K}] - \omega^2 [\mathbf{M}] (1 - 2j\eta))^{-1} \{\mathbf{Q}^j\} \quad (24)$$

319 Finally, the condensed impedance between the incident segment  $j$  and the receiving  
 320 segment  $i$  is obtained by summing the resulting pressure at all the  $M_i$  nodes belonging to  
 321 the receiving segment

$$Z_\alpha^{ij} = \frac{1}{\sqrt{L_i}} \sum_{m=1}^{M_i} P_m^j \delta S_m \quad (25)$$

322 where  $P_m^j$  is the pressure at node  $m$  belonging to the segment  $i$  when the system is excited  
 323 on the segment  $j$ ,  $\delta S_m$  is the length around the node  $m$ ,  $L_i$  is the length of the segment  $i$ ,  
 324 and  $\alpha$  corresponds either to subsystem 2 or subsystem 2'.

#### 325 4. Scattering from the partially coated shell immersed in water

326 In Section 3, the formulations to obtain the necessary quantities have been demonstrated.  
 327 We can now proceed with the application of the method to predict the scattering from the  
 328 partially coated cylindrical shell impacted by a plane wave of unit amplitude, for which

329 the missing coating part spreads from 0 to  $\pi/4$  as illustrated in Figure 5. Two loading  
 330 configurations will be considered. In the first one, the acoustic plane wave will be incident  
 331 on the uncoated part of the shell, while for the second one, the acoustic plane wave will  
 332 impinge the other side of the shell.

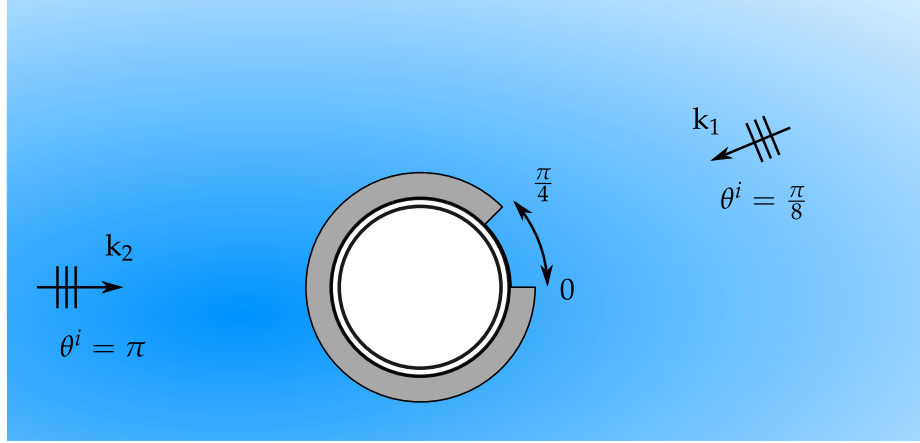


Figure 5. Loading cases and angular spreading of the partial coating.

333 A first paragraph will be dedicated to address some theoretical aspects of the scattering  
 334 problem, in order to understand the quantities that will be studied hereafter. Then, we will  
 335 be interested in investigating the impact of partial coating on the near-field and far-field  
 336 scattered pressure.

#### 337 4.1. Theoretical aspects concerning the scattering problem

338 When dealing with the scattering problem, the total pressure in the domain can be de-  
 339 composed as three different components: the incident pressure  $p_{inc}$  corresponding to the  
 340 pressure induced by the acoustic plane wave, the reflected pressure  $p_{refl}$  corresponding to  
 341 the pressure scattered as though the boundary were hard, and the reradiated pressure  $p_{rad}$   
 342 stemming from the cylinder motion. Generally, when the scattering problem is considered,  
 343 the contribution of the elastic part (i.e. the reradiated pressure) is studied separately from  
 344 the contribution of the rigid part (i.e. the sum of the incident and reflected pressure, also  
 345 called the blocked pressure). This can be summarized in Eq. 26<sup>1</sup>

$$p = \underbrace{p_{inc} + p_{refl}}_{p_{block}} + p_{rad} \quad (26)$$

346 In the following, we will be interested into evaluating the reradiated pressure by the  
 347 cylindrical shell. Indeed, one must keep in mind that the problem investigated here con-  
 348 stitutes a first step towards the study of 3D partially coated, stiffened cylindrical shells.

349 For such studies, we can be interested in the elastic component of the scattered pressure  
 350 (i.e.  $p_{rad}$ ) as the spectra of monostatic backscattered pressures show patterns of periodic  
 351 discontinuities (namely Bragg scattering and scattering from Bloch-Floquet waves) arising  
 352 from the interaction between the incident wave and the elastic stiffeners.<sup>22,34</sup>

353 To obtain the pressure reradiated by the partially coated shell, we have seen in Eq. 12  
 354 that the first step is to compute the pressure reradiated by the fully coated shell,  $p_{1+2}(M_1)$ .  
 355 The procedure to obtain this reradiated pressure is not trivial and will be developed here.

356 Let us consider the problem presented in Figure 4a. The calculation is performed in two  
 357 steps. The first one consists in solving the blocked pressure problem, i.e. the response of  
 358 the system when the shell is considered as rigid. The blocked pressure at the surface of the  
 359 shell is then used as an excitation term to derive the radial displacement of the shell and  
 360 ultimately the reradiated pressure. The blocked pressure satisfies the following conditions

- 361 • Helmholtz equation with a source term in the exterior fluid domain,
- 362 • homogeneous Helmholtz equation in the coating,
- 363 • Sommerfeld radiation condition at infinity.

364 The problem is solved in the wavenumber domain by the means of a Fourier series  
 365 decomposition along the circumference of the shell, allowing to express the pressures as a  
 366 function of the cylindrical harmonics

$$p(r, \theta) = \sum_{n=-\infty}^{\infty} \tilde{p}_n(r) e^{jn\theta} \quad (27)$$

367 The Helmholtz equation can be solved in the wavenumber domain using a linear com-  
 368 bination of Bessel functions. As such, the pressure inside the coating,  $\tilde{p}^1$ , and outside the  
 369 coating,  $\tilde{p}^\infty$ , accounting for the previously stated conditions, can be expressed as<sup>35</sup>

$$\tilde{p}^1(r, n) = A_1 \frac{H_n(k_{pr}r)}{H_n(k_{pr}R)} + B_1 J_n(k_{pr}r) H_n(k_{pr}R_1) \quad \text{if } r \leq R_1 \quad (28a)$$

370

$$\tilde{p}^\infty(r, n) = A_\infty \frac{H_n(k_{fr}r)}{H_n(k_{fr}R_1)} + j^n J_n(k_{fr}r) \quad \text{if } r \geq R_1 \quad (28b)$$

371 where  $A_n$  and  $B_n$  are the unknowns,  $J_n$  and  $H_n$  are the Bessel function of the first kind  
 372 and the Hankel function of the second kind, respectively,  $R$  is the radius of the shell,  $R_1$   
 373 the exterior radius of the coating, and  $k_{ir} = k_i \cos(\theta^i)$  (with  $k_i$  the acoustic wavenumber in  
 374 the domain  $i$ ). The unknowns can be obtained by considering first the Neumann condition  
 375 at the shell surface, the continuity of pressures and velocities at the interface between the  
 376 fluid and the coating, and then solving the subsequent system of equations

$$\begin{bmatrix} -1 & \frac{H_n(k_{pr}R_1)}{H_n(k_{pr}R)} & J_n(k_{pr}R_1)H_n(k_{pr}R_1) \\ \frac{k_{fr}r}{\rho_f} \frac{H'_n(k_{fr}R_1)}{H_n(k_{fr}R_1)} & \frac{k_{pr}}{\rho_p} \frac{H'_n(k_{pr}R_1)}{H_n(k_{pr}R)} & \frac{k_{pr}}{\rho_p} J'_n(k_{pr}R_1)H_n(k_{pr}R_1) \\ 0 & k_{pr} \frac{H'_n(k_{pr}R)}{H_n(k_{pr}R)} & k_{pr} J'_n(k_{pr}R)H_n(k_{pr}R_1) \end{bmatrix} \begin{bmatrix} A_\infty \\ A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} j^n J_n(k_{fr}r) \\ \frac{k_{fr}r}{\rho_f} j^n J'_n(k_{fr}r) \\ 0 \end{bmatrix} \quad (29)$$

377 The blocked pressure at the surface of the shell can then be expressed as

$$\tilde{p}_{bloq}(r = R, n) = A_1 + B_1 J_n(k_{pr}R)H_n(k_{pr}R_1) \quad (30)$$

378 As stated previously, the next step is then to use the expression of Eq. 30 as an excitation  
379 term of the cylindrical shell. To do so, the behavior of the shell will be described using the  
380 Flügge equations<sup>31,36</sup>

$$\tilde{\mathcal{L}}(n) \begin{bmatrix} \tilde{U}(n) \\ \tilde{V}(n) \\ \tilde{W}(n) \end{bmatrix} = \gamma \begin{bmatrix} 0 \\ 0 \\ \tilde{p}_{bloq}(r = R, n) \end{bmatrix} \quad (31)$$

381 where  $\mathcal{L}$  is the Flügge operator,  $\gamma = -\frac{(1-\nu_s^2)R^2}{E_s h_s}$  and  $U$ ,  $V$  and  $W$  are the axial, tangential  
382 and radial displacements, respectively. The resolution of this system of equations allows  
383 expressing the radial displacement as

$$\tilde{W}(n) = -\frac{\gamma \tilde{p}_{bloq}(r = R, n) (\tilde{Z}_{UU}(n)\tilde{Z}_{VV}(n) - \tilde{Z}_{UV}^2(n))}{\tilde{\Delta}(n)} \quad (32)$$

384 with  $\tilde{Z}_{ij}$  being the coefficients of the Flügge operator, and  $\tilde{\Delta}$  is the determinant of the  
385 Flügge matrix taking into account the loading of the fluid and of the coating. It can be  
386 expressed as

$$\begin{aligned} \tilde{\Delta} = & \tilde{Z}_{UW}(\tilde{Z}_{UV}\tilde{Z}_{VW} - \tilde{Z}_{UW}\tilde{Z}_{VV}) + \tilde{Z}_{VW}(\tilde{Z}_{UW}\tilde{Z}_{UV} - \tilde{Z}_{VW}\tilde{Z}_{UU}) + \\ & (\tilde{Z}_{WW} + \gamma\tilde{Z}_f)(\tilde{Z}_{UU}\tilde{Z}_{VV} - \tilde{Z}_{UV}^2) \end{aligned} \quad (33)$$

387 where  $\tilde{Z}_f$  is the effective spectral impedance of the fluid and coating. This quantity can  
388 be obtained similarly as the blocked pressure in the system of Eq. 29, with the following  
389 conditions

- 390 • homogeneous Helmholtz equation in the exterior fluid domain and in the coating,
- 391 • Sommerfeld radiation condition at infinity.
- 392 • Euler equation depending on the radial shell velocity at the shell surface,



393 • continuity of pressures and velocities at the interface between the fluid and the coating.

394 The system of equations which must be solved now reads

$$\begin{bmatrix} -1 & \frac{H_n(k_{pr}R_1)}{H_n(k_{pr}R)} & J_n(k_{pr}R_1)H_n(k_{pr}R_1) \\ \frac{k_{fr}r}{\rho_f} \frac{H'_n(k_{fr}R_1)}{H_n(k_{fr}R_1)} & \frac{k_{pr}}{\rho_p} \frac{H'_n(k_{pr}R_1)}{H_n(k_{pr}R)} & \frac{k_{pr}}{\rho_p} J'_n(k_{pr}R_1)H_n(k_{pr}R_1) \\ 0 & k_{pr} \frac{H'_n(k_{pr}R)}{H_n(k_{pr}R)} & k_{pr} J'_n(k_{pr}R)H_n(k_{pr}R) \end{bmatrix} \begin{bmatrix} A_\infty \\ A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \rho_p \omega^2 \end{bmatrix} \quad (34)$$

395 and the fluid and coating loading impedance is obtained

$$\tilde{Z}_f = A_1 + B_1 J_n(k_{pr}R)H_n(k_{pr}R_1) \quad (35)$$

396 Finally, the shell's radial displacement can be used to obtain the displacement of the  
397 external part of the coating, and, subsequently, the pressure reradiated by the shell (using  
398 the Euler equation)

$$\tilde{W}_{coat}(n) = \frac{k_{pr}}{\rho_p \omega^2} \left( A_1 \frac{H'_n(k_{pr}R_1)}{H_n(k_{pr}R)} + B_1 J'_n(k_{pr}R_1)H_n(k_{pr}R_1) \right) \tilde{W}(n) \quad (36)$$

399 where  $A_1$  and  $B_1$  are the unknowns obtained when computing the effective fluid and coating  
400 impedance in Eq. 34.

401 In the next sections, the influence of the partial coating will be evaluated via two quan-  
402 tities: the evaluation of the near-field reradiated pressure at the ring frequency of the shell,  
403 and the target strength of the shell in the entire frequency spectrum. The target strength is  
404 defined as the ratio between the far-field scattered pressure brought back to 1 m from the  
405 object, and the incident pressure of the acoustic plane wave.

#### 406 **4.2. Evaluation of the near-field pressure at the ring frequency**

407 At first, we are interested in the near-field reradiated pressure at the ring frequency. The  
408 ring frequency corresponds to the frequency at which the longitudinal wavelength in the  
409 shell is equal to the circumference and is defined as

$$f_r = \frac{1}{2\pi R} \sqrt{\frac{E_s}{\rho_s(1 - \nu_s^2)}} \quad (37)$$

410 Around the ring frequency, the radiation efficiency of the cylindrical shell is particularly  
411 high, and this frequency also exhibits a change in behavior of the shell. Indeed, above the  
412 ring frequency, it is generally admitted that the cylindrical shell behaves like a plate,<sup>37</sup> in  
413 the sense that the flexural motions are mainly decoupled from the longitudinal and shear

414 motions. It will then be interesting to study the behavior of the partially coated shell at  
415 this particular frequency. For the case of interest, the ring frequency yields  $f_r = 958$  Hz.

416 The cartography of the near-field reradiated pressure will be presented for two loading  
417 cases, as illustrated in Figure 5. The first one corresponds to an acoustic plane wave having  
418 a normal incidence on the missing coating patch ( $\theta_i = \frac{\pi}{8}$ ), while the second one will be  
419 directed towards the positive x-axis ( $\theta_i = \pi$ ). For each loading case, a comparison is made  
420 between the uncoated shell, the fully coated shell, and the partially coated shell. And for  
421 the latter, the results of the coupled CTF-rCTF approach are compared to a reference  
422 FEM calculation which has been conducted using COMSOL Multiphysics<sup>®</sup>. For the FEM  
423 calculation, it must be mentioned that the elastic radiated pressure cannot be obtained  
424 directly. To obtain this quantity from a FEM calculation, it is necessary to go back to the  
425 definition of the radiated pressure given in Eq. 26. As such, the reradiated pressure can be  
426 inferred by subtracting the blocked pressure from the total pressure. Following this definition,  
427 two FEM calculations must be performed: a full calculation accounting for the elastic motion  
428 of the shell, and a blocked pressure calculation where the shell is not modelled, in order to  
429 be considered as rigid.

430 The first loading case, corresponding to an incident acoustic plane wave impacting the  
431 shell at the location of the missing coating material, is shown in Figure 6. The direction  
432 of the incident plane wave is specified in Figure 6a. The first observation that can be  
433 made regarding Figures 6a (uncoated shell) and 6b (fully coated shell) is that the anechoic  
434 coating has a beneficial impact as the values of the reradiated pressure are globally much  
435 lower when the coating is applied on the shell. Then, regarding Figures 6c (FEM reference  
436 calculation) and 6d (CTF-rCTF calculation) which show the pressure field for the partially  
437 coated shell, **we can see that there is a very good agreement between the two calculations,**  
438 **which validates the formulation proposed in Section 2. The different patterns of maxima**  
439 **and minima of pressure are correctly localized, and the amplitudes are well correlated. It**  
440 **can also be said that the absence of coating on the impact area of the plane wave increases**  
441 **the reradiated pressure, especially at the vicinity of the shell, as it could have been expected.**  
442 **Finally, it is important to stress that the symmetry of the system with respect to the angle**  
443 **of incidence of the plane wave has been well complied with.**

444 Concerning the second loading case, corresponding to an acoustic plane wave arriving  
445 towards the positive x-axis, the reradiated pressure is shown in Figure 7 for the four config-  
446 urations. It can be observed that the fully coated shell and the partially coated shell exhibit  
447 a very similar behavior. This shows that the partial coating has a very low influence on  
448 the behavior of the shell when the missing coating material is located in the shadow area  
449 of the shell. This figure also highlights that the CTF-rCTF and FEM calculations exhibit  
450 almost the same results, **as it was observed in Figure 6 for the previous loading case.** This  
451 is associated to the previous observation concerning the low impact of the partial coating  
452 for this configuration, hence limiting the influence of the sensitivity of the rCTF method.

453 To conclude on this part, the results presented in this paragraph show that the partial  
454 coating can have an impact on the reradiated pressure, compared to the fully coated shell.  
455 But this change in behavior is restricted to the case where the incident plane wave impinges

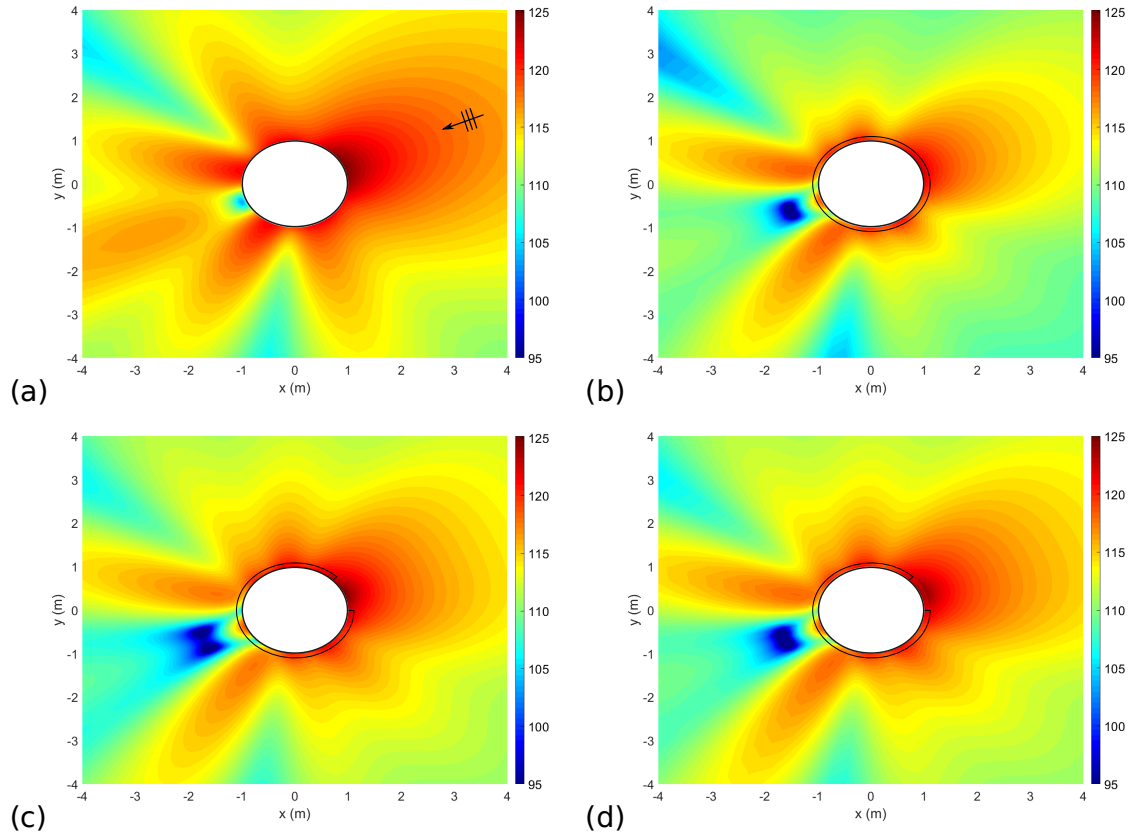


Figure 6. Reradiated pressure (in dB ref.  $1e-6$  Pa) for  $\theta_i = \frac{\pi}{8}$ . (a) Uncoated shell. (b) Fully coated shell. (c) Partially coated shell - reference calculation. (d) Partially coated shell - CTF-rCTF calculation.

456 directly the shell around the uncoated area. When the plane wave is incident on a coated  
 457 part of the shell, the impact of the partial coating on the reradiated pressure is very limited.

### 458 **4.3. Impact of the partial coating on the target strength of the shell**

459 We are now interested into evaluating the elastic component of the target strength of the  
 460 shell, and the impact of the partial coating on this quantity, for the two lading configurations.  
 461 As stated earlier, the target strength is defined as the ratio between the far-field scattered  
 462 pressure brought back to 1 m from the object, and the incident pressure of the acoustic  
 463 plane wave. It can be evaluated at the same angle as the incident angle of the plane wave  
 464 (monostatic target strength) or at a different angle (bistatic target strength). In this study,  
 465 we are only interested in the monostatic target strength for the two loading cases.

466 The elastic component of the target strength corresponds to the evaluation of the target  
 467 strength when only the reradiated pressure  $p_{rad}$  is accounted for. In practice, it can be  
 468 obtained for a 2-D case by using the asymptotic expression of the Hankel function for large

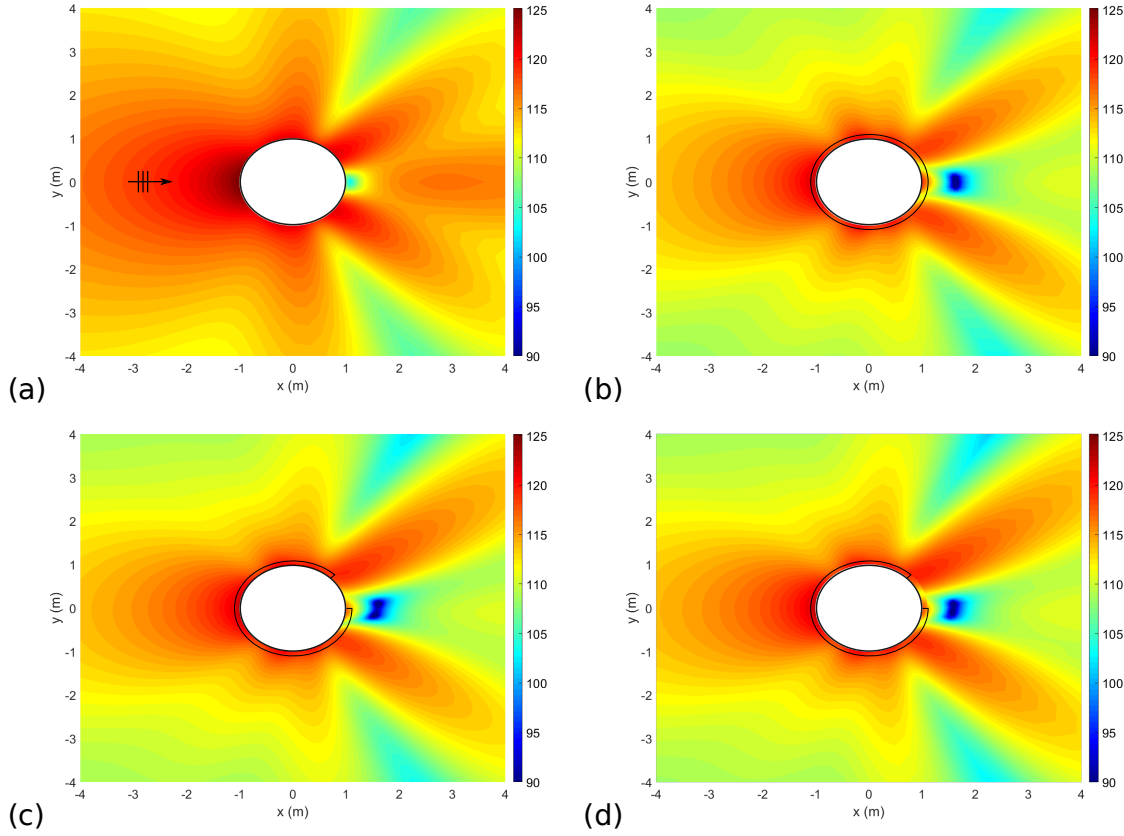


Figure 7. Reradiated pressure (in dB ref.  $1e-6$  Pa) for  $\theta_i = \pi$ . (a) Uncoated shell. (b) Fully coated shell. (c) Partially coated shell - reference calculation. (d) Partially coated shell - CTF-rCTF calculation.

469 arguments. This allows predicting the far-field reradiated pressure from the radial spectral  
 470 displacement of the shell for uncoated shells, or of the radial spectral displacement of the  
 471 external boundary of the coating for coated shells. Its expression for a coated shell yields

$$p_{rad}^{far}(r, \theta) = \frac{\rho_f \omega^2}{k_0} \sqrt{\frac{2}{\pi r k_0}} \sum_{n=-\infty}^{n=+\infty} \frac{\tilde{W}_{coat}(n)}{H_n^{(2)'}(R_1 k_0)} e^{-j r k_0 + j \frac{(n+1)\pi}{2} + j n \theta}, \quad r \gg \frac{2\pi}{k_0} \quad (38)$$

472 where  $\tilde{W}_n^{coat}$  is the spectral radial displacement of the external boundary of the coating  
 473 obtained in Eq. 36, and  $H_n^{(2)'}$  is the derivative of the Hankel function of the second kind  
 474 and of order  $n$ . It must be noted that Eq. 38 exhibits an infinite sum. In practice, this sum  
 475 is truncated to a maximal circumferential order, and the choice of this value is explained in  
 476 details in.<sup>22</sup>

477 For the partially coated shell, however, we have seen in Section 2 that the final quantity  
 478 that is obtained using the CTF-rCTF method is the radiated pressure, and there is no

479 information concerning the radial displacement of the shell or of the coating. Hence, the far-  
 480 field pressure will be evaluated in the same way as the near-field pressure, using Eq. 12. To  
 481 obtain the reradiated far-field pressure, the process is the same as for the FEM calculation.  
 482 Two calculations are performed to obtain the total pressure and the blocked pressure, and  
 483 the reradiated pressure is obtained by subtracting the latter from the former. When the  
 484 far-field reradiated pressure has been computed for all of the coating configurations, the  
 485 monostatic target strength can be obtained from the following expression

$$TS_{mono}(\theta) = 10 \log \left( \frac{\left( \sqrt{r} p_{rad}^{far}(r, \theta) \right)^2}{p_0^2} \right) \quad (39)$$

486 with  $p_0$  being the amplitude of the incident plane wave.

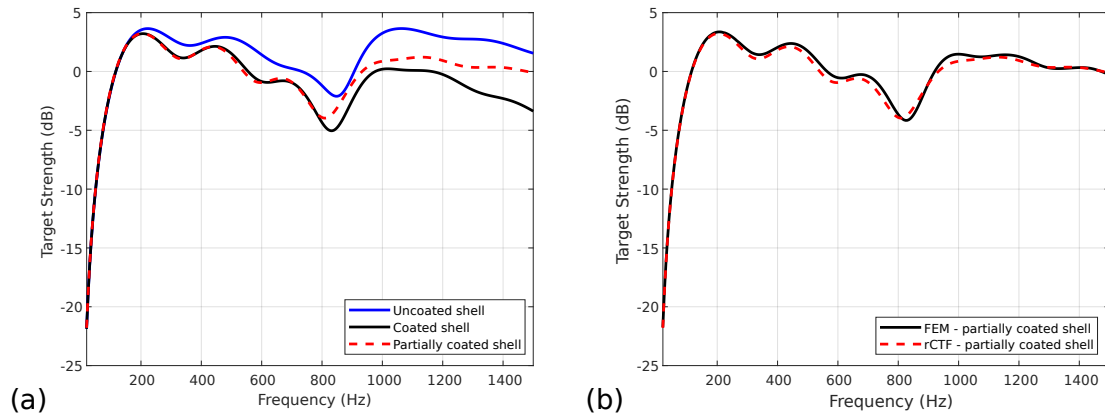


Figure 8. Monostatic target strength when the plane wave is incident on the uncoated part of the shell ( $\theta_i = \pi/8$ ). (a) Comparison between the coating situations. (b) Comparison between the rCTF and FEM calculations.

487 The monostatic target strength for the three different coating situations (uncoated,  
 488 fully coated and partially coated) is shown in Figure 8a, for the first loading case (i.e. the  
 489 acoustic plane wave impinges the missing coating area of the shell). We can observe that  
 490 the target strength of the uncoated shell is higher than for the fully and partially coated  
 491 shells, which shows that the anechoic coating also proves its interest when it comes to  
 492 limiting the scattering in the far-field. It is also interesting to notice that there is not much  
 493 difference between the fully coated and the partially coated shell up to 800 Hz. Then the  
 494 target strength tends to be slightly higher for the partially coated shell up to 1200 Hz,  
 495 and the gap increases for frequencies above 1200 Hz. To analyze this result, one could look  
 496 for a relation between the size of the opening and the acoustic wavelength in the coating.  
 497 Indeed, we could assume that, if the acoustic wavelength is greater than the opening, then

498 the plane wave will be attenuated by the coating nevertheless. With the opening spreading  
 499 from 0 to  $\pi/4$ , its length is then  $\pi/4$  m (as the shell's radius is 1 m). From this value, we  
 500 can define a limit frequency  $f_l$  above which the acoustic wavelength in the coating will be  
 501 smaller than the opening. In this case, we obtain  $f_l=1070$  Hz. This is hence consistent with  
 502 the hypothesis stating that the partial coating will mainly have an impact at frequencies  
 503 where the acoustic wavelength in the coating is smaller than the opening. Besides, it can  
 504 be observed in Figure 8b that there is an excellent agreement between the rCTF and the  
 505 FEM calculations, **which is consistant with the previous observations made in Section 4.2**  
 506 **and validates the coupled CTF-rCTF approach.**

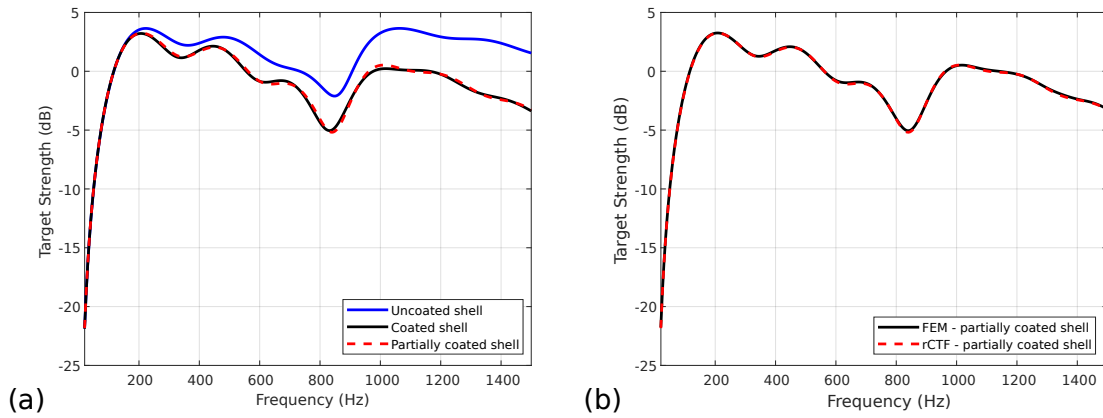


Figure 9. Monostatic target strength when the plane wave is incident on the coated part of the shell ( $\theta_i = \pi$ ). (a) Comparison between the coating situations. (b) Comparison between the rCTF and FEM calculations.

507 Concerning the second loading case, the comparison between the different coating situa-  
 508 tions is shown in Figure 9a. The results displayed in this figure are very consistent with the  
 509 analysis made from Figure 7, which state that the absence of coating on a given part of the  
 510 shell does not have a significant impact on the scattering of the shell when this specific part  
 511 is not impinged by the plane wave. Hence, as the behavior of the fully coated and partially  
 512 coated shells are similar for this loading case, the CTF-rCTF exhibits no calculation error  
 513 compared to the FEM calculation. The conclusions drawn in this study will have to be  
 514 investigated in more details in further studies, and particularly for 3D stiffened cylindrical  
 515 shells for which very specific scattering phenomena can be observed.

## 516 5. Conclusion

517 In this paper, the scattering problem of an acoustic plane wave by a partially coated cylin-  
 518 drical shell has been addressed using the CTF and rCTF methods. The theoretical funda-  
 519 mentals of the method which were previously developed for studying radiation problems  
 520 have been extended in the present paper to the case of scattering problems. A theoretical

521 framework for the calculation of elastic radiated pressure and elastic target strength has  
522 been proposed for the cases of fully coated and partially coated cylindrical shells.

523 A comparison with FEM simulations on a 2D model validates the proposed coupled  
524 CTF-rCTF approach, while it is expected that the latter will require much less computing  
525 time to deal with 3D models than with a FEM simulation. **The comparison with the FEM  
526 calculation exhibits accurate results for both the near-field pressure and the far-field pressure  
527 (represented by the elastic target strength).** These first results show that the influence of  
528 the partial coating is particularly highlighted when the incident acoustic plane wave is  
529 heading towards the area where the coating material is missing, and for frequencies where  
530 the acoustic wavelength in the coating is smaller than the opening.

531 It is important to mention that these results, although giving a first insight into the  
532 possibilities brought by the rCTF method, are not completely of interest for naval applica-  
533 tions, as the calculations were performed on a 2D model of an unstiffened shell. However,  
534 this study must be seen as a first step towards the possibility of studying the scattering  
535 from a 3D partially coated, stiffened cylindrical shell.

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