



HAL
open science

Two Possible Solutions to the Vacuum Catastrophe

Espen Gaarder Norwegian University of Life Sciences Haug

► **To cite this version:**

Espen Gaarder Norwegian University of Life Sciences Haug. Two Possible Solutions to the Vacuum Catastrophe. 2023. hal-04333527v2

HAL Id: hal-04333527

<https://hal.science/hal-04333527v2>

Preprint submitted on 19 Dec 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NoDerivatives 4.0 International License

Two Possible Solutions to the Vacuum Catastrophe

Espen Gaarder Haug*

**Norwegian University of Life Sciences,
Christian Magnus Falsensvei 18, Aas, Norway
Ås, Norway; espenhaug@mac.com
<https://orcid.org/0000-0001-5712-6091>

December 19, 2023

Abstract

We will demonstrate that the vacuum catastrophe can likely be solved by utilizing two different methods, one is by utilizing Bekenstein-Hawking entropy and applying it to black hole type cosmology models, as well as to a large class of $R_H = ct$ models. Additionally, we will examine a recent exact solution to Einstein's field equation (the Haug-Spavieri metric) and explore how it may potentially resolve the vacuum catastrophe rooted in both steady-state universe and growing black hole universe scenarios.

Keywords: Planck energy, Planck mass, Vacuum energy, Cosmology constant problem.

1 Background on the vacuum catastrophe

The vacuum catastrophe, also known as the cosmological constant problem, is related to the fact that the vacuum energy from observations is estimated to be approximately $\rho_{\text{vac}} \approx 5.96 \times 10^{-27} \text{ kg/m}^3$, as reported by the Planck Collaboration [1]. However, according to quantum field theory, a predicted vacuum energy (on mass equivalent form) is given by

$$\rho_{\text{vac}} = \frac{m_p}{\frac{4}{3}\pi l_p^3} \approx 1.23 \times 10^{96} \text{ kg/m}^3, \quad (1)$$

where m_p is the Planck mass and l_p is the Planck length, initially described by Max Planck [2, 3]. This implies that the vacuum energy is overestimated by an order of 120, as highlighted by, for example, [4, 5]. This substantial disparity between predictions and observations is the reason it is termed the vacuum catastrophe, and not merely the vacuum problem. The difference in predictions and observations is remarkably vast. Although this remains an unsolved problem, the following section will explore potential solutions.

2 The black hole entropy solution

Even though the Λ -CMB model stands as the prevailing cosmological model today, the notion that the Hubble sphere can be perceived as a type of black hole presents an alternative theory. This concept was initially proposed in 1972 by Pathria [6] and later revisited by Stuckey in

1994 [7]. The idea of the universe as a black hole remains a topic of active discussion to this day, as evidenced by discussions in literature such as [8–10].

Borges and Carneiro [11] suggest that the vacuum catastrophe can likely be resolved using Friedmann [12] cosmology, but with the additional idea of including decaying vacuum density. They propose a scenario that involves an initial phase dominated by radiation, followed by a dust era long enough to permit structure formation, and an epoch dominated by the cosmological term. This has led to multiple researchers investigating variable vacuum density in relation to also a varying cosmological constant, as the cosmological constant is tied to the vacuum energy in the Λ -CDM model, as seen, for example, in [13]. The decaying vacuum model (DV) has also been interpreted as treating dark energy as a varying vacuum, as discussed in [14].

Various interpretations connect the Hubble sphere to a black hole. One possibility is that it functions as a growing black hole, suggesting that the observable universe originated as a small black hole, evolved into today's universe, and continues to expand. An alternative to the Λ -CDM model discussed actively to this date is the so-called $R_H = ct$ cosmological models (see [15–20]), where the Hubble radius grows at a rate proportional to cosmic time, denoted by t since the universe's inception. A special subclass of $R_H = ct$ models is growing black hole models, as discussed in [21, 22]. Another conception of a black hole universe proposes that the black-hole horizon serves as a form of information horizon everywhere, something we will soon revisit.

If the Hubble sphere indeed represents any type of black hole, whether a growing black hole or a steady-state information horizon black hole, then the Bekenstein-Hawking entropy [23] or similar entropies may aid in addressing the vacuum catastrophe. This is a relatively old idea, see for example Shalyt-Margolin [24] (2010). Tatum et. al [25, 26] also discusses how Bekenstein-Hawking entropy in models incorporating scaling vacuum energy density can likely resolve the vacuum catastrophe. They describes a method that seems to lead to the right vacuum energy based on the principle of vacuum energy scaling down over the cosmic epoch in his growing black hole model (FSC growing black hole model). In this section, we take a slightly different angle and are presenting multiple new equations on how to predict the vacuum energy, predicting a vacuum energy close to what has been observed.

The Bekenstein-Hawking black hole entropy is expressed as:

$$S_{BH} = \frac{k_b 4\pi r^2}{4l_p^2} = \frac{k_b \pi r^2}{l_p^2}, \quad (2)$$

and when applied to the Hubble sphere, the Bekenstein-Hawking entropy becomes:

$$S_{BH,H} = \frac{k_b \pi R_H^2}{l_p^2}, \quad (3)$$

where R_H represents the Hubble radius defined as $R_H = \frac{c}{H_0}$.

Somewhat speculatively Haug [27] (in a brief section 6, November 8), suggested that the predicted Planck energy quantum field vacuum energy likely must be adjusted by the entropy within the black hole Hubble sphere. After all, entropy is inherently connected to how energy disperses over time. Haug proposed the following adjustment:

$$\rho_{vac} = \frac{\frac{m_p}{\frac{4}{3}\pi l_p^3}}{S_{BH,H}} k_b = \frac{\frac{m_p}{\frac{4}{3}\pi l_p^3}}{\frac{k_b \pi R_H^2}{l_p^2}} k_b = \frac{m_p}{\frac{4}{3}\pi^2 l_p R_H^2} \approx 5.31 \times 10^{-27} \text{ kg/m}^3 \quad (4)$$

Where k_b is the Boltzmann constant. This formula provides predictions quite close to the measured vacuum density of $\rho_{vac} \approx 5.96 \times 10^{-27}$. Equation (4) can be explained from a physical standpoint under different models. The exact value could then change, as the black

hole entropy of Hawking used here is rooted in the Schwarzschild metric. The entropy could simply represent how energy spreads out over time in a growing black hole $R_H = ct$ model. The formula that also covers earlier times of the cosmic epoch would then be:

$$\rho_{vac} = \frac{\frac{m_p}{\frac{4}{3}\pi l_p^3}}{S_{BH,H}} k_b = \frac{\frac{m_p}{\frac{4}{3}\pi l_p^3}}{\frac{\pi(ct)^2}{l_p^2}} = \frac{m_p}{\frac{4}{3}\pi^2 l_p (ct)^2} = \frac{m_p}{\frac{4}{3}\pi^2 l_p (nct_p)^2} \quad (5)$$

where n is the number of Planck times since the start of the black hole universe and t_p is the Planck time. Today, $R_H = ct = \frac{c}{H_0}$, but if we look back in time, it will be $R_H = ct$, where t is the time since the beginning of the black hole observable universe, one such model likely fitting this view is the Tatum et al growing black hole model rooted in Schwarzschild type black holes. We will soon also look at another new metric from Einstein's field equation.

Alternatively if we simply use $\frac{m_p}{l_p^3}$ as the Planck mass density as often is done, then we get

$$\rho_{vac} = \frac{\frac{m_p}{l_p^3}}{S_{BH,H}} k_b = \frac{\frac{m_p}{l_p^3}}{\frac{k_b \pi R_H^2}{l_p^2}} k_b = \frac{m_p}{\frac{4}{3}\pi R_H^2 l_p} \approx 16.67 \times 10^{-27} \text{ kg/m}^3 \quad (6)$$

This is way too high for the observed vacuum energy, but if we divide this by 3 we get, $5.58 \times 10^{-27} \text{ kg/m}^3$, which is very close to the observed. But where did we get to that we just should divide by three, is this then not just numerology? In the next section we will see from a different approach how one get to this formula from a logical point of view.

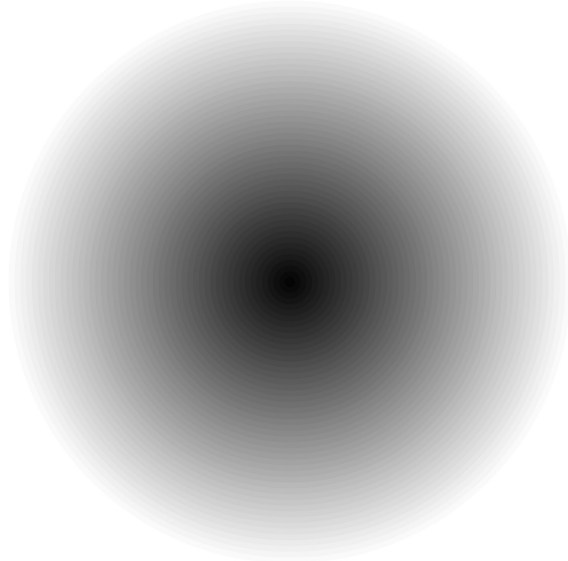


Figure 1: This figure illustrates how the energy density decreases over time in a black hole growth model. The idea is that the universe started with Planck mass density, and entropy over cosmic time is at least part of the explanation for why the density is much lower today. Entropy in a growing black hole universe is also simply a consequence of the universe growing over time. If the Hubble sphere is the result of a growing black hole then the mass is growing linearly in $R_H = ct$ type growing black holes, while the volume is increasing as a function of $R_H^3 = (ct)^3$

3 Steady state black hole universe with center everywhere and information horizon equal to the Hubble radius everywhere used to solve the vacuum catastrophe

Another solution to the vacuum catastrophe can be derived from the steady-state black hole view that we will introduce here. For thousands of years, it was assumed that the universe extended infinitely in both time and space. This perspective was held by Einstein, Lorentz, Poincaré, and other great physicists until around 1930 when the cosmological red-shift observations by Lemaître [28] and Hubble [29] were interpreted as indicating the universe's expansion. However, there could be alternative explanations for the cosmological red-shift, which we will touch upon shortly, even though it is not the focus here.

Let's consider a universe extending infinitely in space and time, assuming there was no Big Bang and no expansion of space. However, just as in the standard model, we assume there is a vacuum energy density. As long as there is a nonzero energy density, every point in such a universe will have an event horizon. The specific nature of the event horizon depends on the metric solution used. Let's begin with the Schwarzschild metric. Here, we can rewrite the Schwarzschild radius as a function simply of the equivalent mass density. We use the term "equivalent" because energy can also be treated as rest mass, given that we have $M = \frac{E}{c^2}$. Thus, we must have:

$$\begin{aligned}
 R_s &= \frac{2GM}{c^2} \\
 R_s &= \frac{8\pi G \frac{M}{\frac{4}{3}\pi R_s^3} R_s^3}{3c^2} \\
 R_s &= \frac{8\pi G \rho R_s^3}{3c^2} \\
 R_s &= \frac{8\pi \rho R_s^3}{3c^2} \\
 3c^2 &= 8\pi G \rho R_s^2 \\
 \frac{3c^2}{8\pi G \rho} &= R_s^2 \\
 R_s &= \sqrt{\frac{3c^2}{8\pi G \rho}}
 \end{aligned} \tag{7}$$

For example the density in the critical Friedmann universe is given by:

$$\begin{aligned}
 \rho_{cr} &= \frac{M_c}{\frac{4}{3}\pi R_s^3} \\
 \rho_{cr} &= \frac{\frac{R_H c^2}{2G}}{\frac{4}{3}\pi R_s^3} \\
 \rho_{cr} &= \frac{3H_0^2}{8\pi G} \approx 8.38 \times 10^{-27} \text{ kg/m}^3
 \end{aligned} \tag{8}$$

Inserted in equation 7 we get:

$$\begin{aligned}
R_s &= \sqrt{\frac{3c^2}{8\pi G\rho_c}} \\
R_s &= \sqrt{\frac{3c^2}{8\pi G\frac{3H_0^2}{8\pi G}}} \\
R_s &= R_H \approx 1.38 \times 10^{26} \text{ m}
\end{aligned} \tag{9}$$

In the recent new exact solution to Einstein's [30] field equation given by Haug and Spavieri [31], the energy density of the observable universe is exactly twice that of the critical Friedman universe:

$$\rho_{HS} = \frac{3H_0^2}{4\pi G} \approx 1.68 \times 10^{-27} \text{ kg/m}^3 \tag{10}$$

However, the event horizon for twice the mass density is equal to the event horizon of the Schwarzschild metric because the event horizon, as a function of energy density in this model, is given by:

$$R_H = \sqrt{\frac{3c^2}{8\pi G\rho_{HS}}} \approx 1.38 \times 10^{26} \text{ m} \tag{11}$$

It is important to be aware that decaying vacuum density is an additional constraint that is ad hoc added to the Friedmann universe for models working out from this model. However in the Haug-Spavieri metric, when applied to a steady-state universe, it automatically predicts that the black hole mass increases exactly by the Planck mass for every Planck length moved from the center (the observer).

In a steady-state black hole universe, there is a center everywhere with an information horizon equal to R_H everywhere. This is due to a density limitation arising from the metric when one seeks to avoid imaginary event horizons, as discussed in [32]. But in the steady state black hole the scaling of the vacuum energy do not need to be do to a growing black hole, but is a constrain on the energy density coming out from the metric solution itself, see [32] section. This corresponds to the black hole increasing by a Planck mass for every Planck time t_p moved with the speed of light away from the central singularity. This leads to a current prediction of vacuum density:

$$\rho_{vac} = \frac{m_p}{\frac{4}{3}\pi R_H^3 - \frac{4}{3}\pi(R_H - l_p)^3} \approx \frac{m_p}{4\pi R_H^2 l_p} \approx 5.57 \times 10^{-27} \text{ kg/m}^3 \tag{12}$$

With a one-standard deviation of $5.34 \times 10^{-27} \text{ kg/m}^3$ to $6.32 \times 10^{-27} \text{ kg/m}^3$ when using the Hubble parameter value found by the recent study by Kelly et al. [33] of $66.6_{-3.3}^{+4.1} (km/s)/Mpc$. Equation 12 can either be seen as the current (now) vacuum density in a growing black hole model or as the vacuum density close to the observer in a steady-state black hole universe. Equation (12) can also be approximated as:

$$\rho_{vac} = \frac{m_p}{\frac{4}{3}\pi R_H^3 - \frac{4}{3}\pi(R_H - l_p)^3} \approx \frac{m_p}{4\pi R_H^2 l_p} \approx 5.56 \times 10^{-27} \text{ kg/m}^3 \tag{13}$$

We can then see the only difference between this and our other equation (4) to predict the vacuum energy based on Bekenstein-Hawking entropy is that in the denominator, we have 4π instead of $\frac{4}{3}\pi^2$. When we talk about "close to the observer", we are naturally in this

context talking about cosmic distances relative to the Hubble radius. This model can also be extended to observations further away in time as we then must have:

$$\rho = \frac{m_p}{\frac{4}{3}\pi(ct)^3 - \frac{4}{3}\pi(ct - l_p)^3} \quad (14)$$

where ct is the distance the photons used for observations have traveled to reach us, and t is the time from when the photon was sent to reach us. Additional gravitational time dilation effects may occur over long distances.

Instead of a steady state black hole universe, one can also try to formulate the Haug-Spavieri metric consistent with a growing black hole universe. The mass would then start with the Planck mass and grow at the Planck mass for every Planck time, similar to the Tatum et al. [21] model. However, the latter model is rooted in the Schwarzschild metric and does not automatically give constraints on the mass density from the metric solution, as all the mass in a Schwarzschild metric can end up in the central singularity. However Tatum et. al has added an extra assumption to that the Black-Hole grow with more than a Half a Planck mass per Planck time. That all the mass can end in the center singularity seems to be an impossibility in the Haug-Spavieri metric. Actually the Haug-Spavieri metric indicates that at the Planck length distance from the center of the “black-hole” there can only be a Planck mass inside, due to the density constrain given by the metric to get real values of the horizon radius. And the singularity itself should simply be interpreted as no mass can be inside a zero volume, so then there are no gravitational effect as mass causes space-time to curve in general relativity theory, so without mass there is no space-time curvature and no gravity.

A Haug-Spavieri growing black hole would mean today’s mass (energy-equivalent mass) is exactly identical to twice the mass in the critical Friedman universe. However, at the current state, I personally lean more towards a steady-state universe, despite the consensus theory still being the Λ -CDM model. Actually it seems possible impossible to distinguish from observations a growing black-hole or steady state black hole when one interpret through the Haug-Spavieri metric.

An important question in a steady state cosmological model is how we then can explain the cosmological red-shift. We notice that:

$$z \approx \frac{dH_0}{c} = \frac{1}{\frac{GM_u}{c^2 d}} \quad (15)$$

where $M_u = \frac{c^3}{GH_0}$, that is the mass (mass equivalent) in the Hubble sphere one get from the Haug-Spavieri cosmology [32]. This is exactly twice of the critical mass in the Friedmann model $M_c = \frac{c^3}{2GH_0}$. Exactly half of this mass in the Haug-Spavieri cosmological model is due to gravitational energy. The way to write the cosmological red-shift in equation (15) means one possible explanation is that cosmological red-shift has nothing to do with expanding space, but possibly is just a special kind of gravitational red-shift due to how close the photons are sent out relative to the information horizon, which is the Hubble radius.

4 Comparison with the average mass density in whole Hubble sphere

In the Haug-Spavieri solution, we have:

$$\frac{M_u}{\frac{4}{3}\pi R_H^3} = \frac{M_u}{\frac{4}{3}\pi R_h^3} = \frac{3H_0^2}{4\pi G} \quad (16)$$

where the mass in the Hubble sphere is given by $M_u = \frac{c^3}{GH_0}$ and the event horizon $R_h = \frac{GM}{c^2} = \frac{1}{2}R_s = R_H$. This we can compare with the shell density in the Hubble sphere black hole, as this new metric puts a constraint on the maximum Planck mass per shell inside the black hole and therefore also in the Hubble sphere if we treat it as such an object. This we calculated already in Section 3, but here we can compare it in more detail with the average mass density, we find:

$$\begin{aligned}
& \text{Shell density/ cosmic epoch shell density :} & \text{Current average density :} \\
& \frac{\frac{m_p}{\frac{4}{3}\pi R_s^3 - \frac{4}{3}\pi(R_s - l_p)^3}}{\frac{m_p}{4n^2 l_p^3}} \approx \frac{\frac{m_p}{4R_s^2 l_p}}{4n^2 l_p^2 l_p} & , \quad \frac{\frac{M_{BH}}{\frac{4}{3}\pi R_H^3}}{\frac{nm_p}{\frac{4}{3}\pi n^3 l_p^3}} \\
& \frac{m_p}{4n^2 l_p^3} \approx 5.56 \times 10^{-27} \text{ kg/m}^3 & , \quad 3 \frac{m_p}{4\pi n^2 l_p^3} \approx 16.67 \times 10^{-27} \text{ kg/m}^3 \quad (17)
\end{aligned}$$

This means the mass density in the outer shell is almost exactly identical to $\frac{1}{3}$ of the average mass density in the entire Hubble sphere. We can perform additional analyses in the Schwarzschild solution and the critical Friedmann universe. The average mass density in the Hubble sphere is well-known:

$$\frac{M_c}{\frac{4}{3}\pi R_H^3} = \frac{M_c}{\frac{4}{3}\pi R_s^3} = \frac{3H_0^2}{8\pi G} \quad (18)$$

where the critical Friedmann mass is given by $M_c = \frac{c^3}{2GH_0}$. This can be compared with the shell density if we assume there is a half Planck mass per Planck length extending from the center of the black hole (a growing black hole $R_H = ct$ model). Here, we must assume that there is half a Planck mass in each shell if we want to analyze such a scenario; there is nothing in the Schwarzschild solution itself giving such a constraint on mass density in each shell. We obtain:

$$\begin{aligned}
& \text{Shell density/ cosmic epoch shell density :} & \text{Current average density :} \\
& \frac{\frac{\frac{1}{2}m_p}{\frac{4}{3}\pi R_s^3 - \frac{4}{3}\pi(R_s - l_p)^3}}{\frac{\frac{1}{2}m_p}{4n^2 l_p^3}} \approx \frac{\frac{\frac{1}{2}m_p}{4R_s^2 l_p}}{\frac{\frac{1}{2}nm_p}{\frac{4}{3}\pi n^3 l_p^3}} & , \quad \frac{\frac{M_{BH}}{\frac{4}{3}\pi R_s^3}}{\frac{nm_p}{\frac{4}{3}\pi n^3 l_p^3}} \\
& \frac{1}{2} \frac{m_p}{4n^2 l_p^3} \approx 2.78 \times 10^{-27} \text{ kg/m}^3 & , \quad \frac{3}{2} \frac{m_p}{4\pi n^2 l_p^3} \approx 8.33 \times 10^{-27} \text{ kg/m}^3 \quad (19)
\end{aligned}$$

Also, in this case, the mass density in the outer shell model is $\frac{1}{3}$ of the average density in the Hubble sphere. The predicted density in a growing black hole is only $2.78 \times 10^{-27} \text{ kg/m}^3$ in the outer shell if one assumes the black hole grows with half a Planck mass per Planck time. Tatum et al [34] assume an additional dark energy of 50%, so they obtain the same predicted density as the Haug-Spavieri metric of $5.56 \times 10^{-27} \text{ kg/m}^3$. However, in the Haug-Spavieri metric, there is exactly 50% gravitational energy that is not accounted for in the Schwarzschild metric.

5 Conclusion

We have investigated how the utilization of Hawking-Bekenstein black hole entropy could potentially explain the vacuum catastrophe. Entropy describes the dispersion of energy over time and distance. Inside a black hole, it appears that when considering Hawking-Bekenstein black hole entropy, the quantum field-predicted Planck energy results in an energy level close to the observed vacuum energy. Additionally, we have demonstrated that both steady-state and growing black-hole universes, when analyzed through the Haug-Spavieri metric, also appear to resolve the vacuum catastrophe.

References

- [1] Planck Collaboration and et al. Planck intermediate results – xxiv. constraints on variations in fundamental constants. *Astronomy & Astrophysics*, 580, 2015.
- [2] M. Planck. *Natuerliche Masseinheiten*. Der Königlich Preussischen Akademie Der Wissenschaften: Berlin, Germany, 1899. URL <https://www.biodiversitylibrary.org/item/93034#page/7/mode/1up>.
- [3] M. Planck. *Vorlesungen über die Theorie der Wärmestrahlung*. Leipzig: J.A. Barth, p. 163, see also the English translation “The Theory of Radiation” (1959) Dover, 1906.
- [4] C. G. Adler, B. Casey, and O. C. Jacob. Vacuum catastrophe: An elementary exposition of the cosmological constant problem. *American Journal of Physics*, 63:620, 1993. URL <https://doi.org/10.1119/1.17850>.
- [5] S.E. Rugh and H. Zinkernagel. The quantum vacuum and the cosmological constant problem. *hilosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 33(4):663, 2002. URL [https://doi.org/10.1016/S1355-2198\(02\)00033-3](https://doi.org/10.1016/S1355-2198(02)00033-3).
- [6] R. K. Pathria. The universe as a black hole. *Nature*, 240:298, 1972. URL <https://doi.org/10.1038/240298a0>.
- [7] W. M. Stuckey. The observable universe inside a black hole. *American Journal of Physics*, 62:788, 1994. URL <https://doi.org/10.1119/1.17460>.
- [8] N. Popławski. The universe in a black hole in Einstein–Cartan gravity. *The Astrophysical Journal*, 832:96, 2016. URL <https://doi.org/10.3847/0004-637X/832/2/96>.
- [9] O. Akhavan. The universe creation by electron quantum black holes. *Acta Scientifical Applied Physics*, 2:34, 2022. URL <https://actascientific.com/ASAP/pdf/ASAP-02-0046.pdf>.
- [10] C. H. Lineweaver and V. M. Patel. All objects and some questions. *American Journal of Physics*, 91(819), 2023.
- [11] H. A. Borges and S. Carneiro. Clues to the fundamental nature of gravity, dark energy and dark matter. *General Relativity and Gravitation*, 37:1385, 2005. URL <https://doi.org/10.1007/s10714-005-0122-z>.
- [12] A. Friedmann. Über die krüing des raumes. *Zeitschrift für Physik*, 10:377, 1922. URL <https://doi.org/10.1007/BF01332580>.

- [13] R.K. Tiwari and R. Singh. Flat universe with decaying cosmological constant. *The European Physics Journal Plus*, 129:253, 2014. URL <https://doi.org/10.1140/epjp/i2014-14253-5>.
- [14] M. Tong and H. Noh. Observational constraints on decaying vacuum dark energy model. *The European Physical Journal C*, 71:1586, 2011. URL <https://doi.org/10.1140/epjc/s10052-011-1586-5>.
- [15] M. V. John. $R_H = ct$ and the eternal coasting cosmological model. *Monthly Notices of the Royal Astronomical Society*, 484. URL <https://doi.org/10.1093/mnras1/sly243>.
- [16] M. V. John and K. B. Joseph. Generalized Chen-Wu type cosmological model. *Physical Review D*, 61:087304. URL <https://doi.org/10.1103/PhysRevD.61.087304>.
- [17] M. V. John and J. V. Narlikar. Comparison of cosmological models using bayesian theory. *Physical Review D*, 65:043506. URL <https://doi.org/10.1103/PhysRevD.65.043506>.
- [18] F. Melia. The $R_H = ct$ universe without inflation. *Astronomy & Astrophysics*, 553, 2013. URL <https://doi.org/10.1051/0004-6361/201220447>.
- [19] F. Melia. The linear growth of structure in the $R_H = ct$ universe. *Monthly Notices of the Royal Astronomical Society*, 464:1966, 2017. URL <https://doi.org/10.1093/mnras/stw2493>.
- [20] F. Melia and Shevchuk A. S. H. The $R_H = ct$ universe. *Monthly Notices of the Royal Astronomical Society*, 419:2579, 2012. URL <https://doi.org/10.1111/j.1365-2966.2011.19906.x>.
- [21] E. T. Tatum, U. V. S. Seshavatharam, and S. Lakshminarayana. The basics of flat space cosmology. *International Journal of Astronomy and Astrophysics*, 5:16, 2015. URL <http://dx.doi.org/10.4236/ijaa.2015.52015>.
- [22] E. T. Tatum and Seshavatharam U. V. S. How a realistic linear $R_H = ct$ model of cosmology could present the illusion of late cosmic acceleration. *Journal of Modern Physics*, 9:1397, 2018.
- [23] S. Hawking. Particle creation by black holes. *Communications in Mathematical Physics*, 43:199, 1975. URL <https://doi.org/10.1007/BF02345020>.
- [24] A. E. Shalyt-Margolin. Entropy in the present and early universe and vacuum energy. *AIP Conf. Proc.*, 1205:160, 2010. URL <https://doi.org/10.1140/epjp/i2014-14253-5>.
- [25] E. T. Tatum. Why flat space cosmology is superior to standard inflationary cosmology. *Journal of Modern Physics*, 9:1867, 2018.
- [26] E. T. Tatum and Seshavatharam U. V. S. Clues to the fundamental nature of gravity, dark energy and dark matter. *Journal of Modern Physics*, 8:1469, 2018.
- [27] E. G. Haug. Cmb, hawking, Planck and Hubble scale relations consistent with recent quantization of general relativity theory. *Hal archive, November 9, version 1*, 2023. URL <https://hal.science/hal-04271060>.

- [28] G. Lemaître. Un univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques. *Annales de la Société Scientifique de Bruxelles*, page 49, 1927. URL <http://dx.doi.org/10.4006/0836-1398-34.4.502>.
- [29] E. Hubble. Extragalactic nebulae. *Astrophysical Journal*, 64:321, 1926. URL <https://doi.org/0.1086/143018>.
- [30] A. Einstein. Näherungsweise integration der feldgleichungen der gravitation. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin*, 1916.
- [31] E. G. Haug and G. Spavieri. Mass-charge metric in curved spacetime. *International Journal of Theoretical Physics*, 62:248, 2023. URL <https://doi.org/10.1007/s10773-023-05503-9>.
- [32] E. G. Haug and G. Spavieri. New exact solution to Einsteins field equation gives a new cosmological model. *Researchgate.org Pre-print*, 2023. URL <http://dx.doi.org/10.13140/RG.2.2.36524.44161>.
- [33] P. L. Kelly and et. al. Constraints on the Hubble constant from supernova Refsdal's reappearance. *Science*, 380:6649, 2023. URL <https://doi.org/10.1126/science.abh1322>.
- [34] E. T. Tatum. A universe comprised of 50% matter mass-energy and 50% dark energy. *Journal of Modern Physics*, 10:1144, 2019. URL <https://doi.org/10.4236/jmp.2019.109074>.