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[1]Université Paris-Saclay, CentraleSupélec, Sorbonne Université, CNRS, Group of electrical engineering-Paris, Gif-sur-Yvette, France, alessandro.silvestrini@centralesupelec.fr, maya.hage-hassan@centralesupelec.fr

In this article, the design of a synchronous reluctance motor (SynRM) is subjected to a multi-physical topology optimization (TO) using the Generalized Optimality Criteria (GOC) method. To ensure the structural integrity of the rotor, a two-step magneto-mechanical optimization approach is employed. The objective functions, focusing on magnetic and mechanical aspects, are formulated by considering the magnetic energies obtained from d-axis and q-axis stator currents, as well as the mechanical compliance. The gradients of these objective functions are computed using the adjoint variable method.

***Index Terms*—Density-based method, magneto-mechanical analysis, synchronous reluctance machines, topology optimization.**

I. Introduction

TOPOLOGY optimization (TO) is a design approach that aims to find the optimal material distribution in a defined space. It is different from parametric and shape optimization problems that require an initial design, and this technique also does not rely on the designer's experience. Over the past few decades, TO has been applied in various engineering fields, including electromagnetics [1].

In this study, TO is applied to the design of a synchronous reluctance motor (SynRM). Generally, TO of these machines focuses on improving magnetic performances without considering the mechanical aspects. Recent papers propose to deal with the magneto-mechanical TO [2]–[4]. In these works, the objective function for the magnetostatic problem is usually expressed in terms of average torque. In [5], the authors propose a cost-effective solution in terms of computing time using the ratio of magnetic energies in the d-axis and q-axis to improve the saliency of the machine, based on a discrete algorithm. In this paper, a novel formulation of the objective function based on magnetic energy is proposed, by expressing it as the difference between the non-linear magnetic energies of d- and q-axis. It is important to note that this approach eliminates the need to calculate the average torque, which is typically obtained by computing its instantaneous values for different rotor positions. With this objective function, a magnetic TO is conducted to determine the magnetic optimal geometry. Subsequently, this geometry is employed in a mechanical TO, with the objective of minimizing the compliance of the rotor under a defined rotational speed load, increasing its robustness. These two TO problems are solved using the Solid Isotropic Material with Penalization method (SIMP). The algorithm chosen is the Generalized Optimality Criteria (GOC), which was initially proposed for structural problems in [6] and is applied here, to the best of the authors knowledge, for the first time to an electromagnetic problem. Furthermore, the gradient of the objective functions is analytically determined using the adjoint variable method.

Corresponding author: A. Silvestrini (email: alessandro.silvestrini@centralesupelec.fr).

II. Optimization problems description

The design of SynRM poses a major challenge due to the complexity of their structure. The sizing and optimization process involves a large number of geometric parameters, particularly for the rotor geometry [7]. The rotor is composed of flux barriers as well as radial and tangential bridges to enhance mechanical robustness. To obtain the shapes of the barriers and the arrangement of the bridges, in this study the topology optimization of the rotor is being conducted.

In this optimization process, the geometry of the stator is fixed, and the main parameters of the motor are presented in Table I. The multi-physical problem is applied specifically to the design space Ω corresponding to the rotor (Fig. 1).

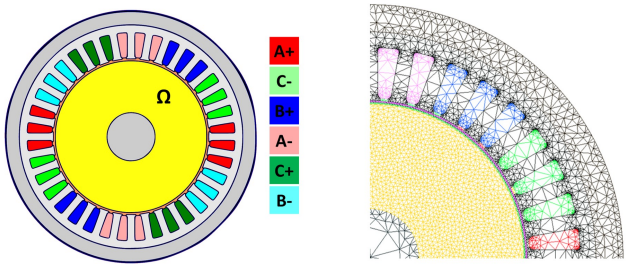

Fig. 1. Geometry and mesh of the motor with the yellow design space Ω.

To tackle the multi-physical problem, a two-step approach is adopted. Firstly, a magnetic TO is conducted over the entire domain Ω. Subsequently, the obtained magnetic optimal topology is used as the starting point for further mechanical optimization to ensure a structurally feasible design. For this second step, only the flux barriers composed of air domains constitute the new design domain. This two-step approach has the advantage of simplifying the design process and serves as an initial analysis of the multiphysics problem. The proposed flowchart depicted in Fig. 2 is detailed in the next parts. Both optimization problems are solved using the GOC algorithm, which has several advantages: it is easy to implement, capable of handling multiple constraints, and faster than the commonly used method of moving asymptotes (MMA).

TABLE I
SYNRM PARAMETERS

Parameter	Unit	Value
Phases number		3
Poles number		4
Slots number		36
Stator outer diameter	mm	130
Rotor outer diameter	mm	78.75
Rotor inner diameter	mm	25
Airgap	mm	1
Machine length	mm	120
Current density	A/mm^2	10
Rated speed	rpm	10000
Electrical steel Young's module	GPa	200
Electrical steel yield strength	MPa	400
Electrical steel density	kg/m^3	7800

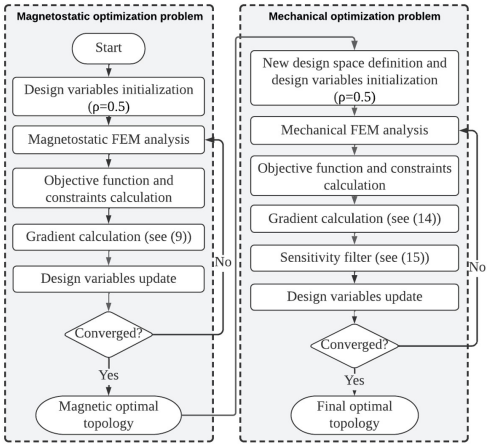

Fig. 2. Flowchart of the proposed two-step optimization procedure.

III. GENERALIZED OPTIMALITY CRITERIA ALGORITHM

Since the 99-line Matlab code for topology optimization has been published [8], the Optimality Criteria (OC) method has been popular. However, the OC method has some limitations, as it only works by minimizing the compliance with a volume fraction constraint. The GOC for TO extends the capability of the OC to multiple inequality constraints and it allows to consider a general objective function. Considering the flowchart in Fig. 2, after the computation of the sensitivity of the objective function and constraints, the design variables ρ are updated at the kth iteration by the following formulation:

$$\rho^{k+1} = \rho^k D^{1/(\beta^k c_0)} \tag{1}$$

where D is the scale factor, β and c_0 two acceleration parameters. In order to consider the general objective function f and multiple constraints g_i, the scale factor is calculated based on the sign of the sensitivities. The numerator consists of all terms with negative sensitivities, while the denominator has all terms with positive sensitivities. Consequently, the scale factor is computed as follows:

$$D = -\frac{\left\langle \frac{\partial f}{\partial \rho} \right\rangle_- + \sum_{i=1}^{NC} \mu_i \left\langle \frac{\partial g_i}{\partial \rho} \right\rangle_-}{\left\langle \frac{\partial f}{\partial \rho} \right\rangle_+ + \sum_{i=1}^{NC} \mu_i \left\langle \frac{\partial g_i}{\partial \rho} \right\rangle_+} \tag{2}$$

where NC is the number of constraints and μ_i the Lagrange multiplier corresponding to constraint g_i. The negative and positive sensitivities are taken as $\langle a \rangle_- = min(0, a)$ and $\langle a \rangle_+ = max(0, a)$. If the numerator and/or the denominator are zero, the scale factor needs to be modified and limited so that it stays close to one. The Lagrange multipliers are iteratively determined during the optimization process to find a value that satisfies the constraints. The method for updating the Lagrange multipliers can vary depending on the specific algorithm and problem formulation. In this case, a bisection method is employed.

IV. TWO-STEP TOPOLOGY OPTIMIZATION PROBLEMS

A. Magnetostatic optimization problem

The magnetic optimization problem is formulated as follows:

$$\begin{aligned} \min_{\rho} \quad & f(\rho) = -(W_d(\rho) - W_q(\rho)) \\ \text{s.t.} \quad & \sum_{e=1}^{N_{elem}} \rho_e V_e = V_{max\_mag} \\ & 0.001 \leq \rho_e \leq 1 \end{aligned} \tag{3}$$

where ρ represents the design variables, ρ_e and V_e are the elementary density and volume of the mesh element e, N_{elem} the total number of mesh elements (8712), and V_{max_mag} is the maximum material volume fixed here at 55%. Typically, the rotor core of a SynRM is made up of laminated iron sheets that occupy between 50% and 80% of the rotor volume, depending on the specific motor design and application. In this study,volume constraints are then chosen to achieve a light motor design with a total volume of 60%.

The objective function is expressed as the difference between the magnetic energies W_d and W_q. To determine the two magnetic energies, windings are fed respectively by a d-axis and q-axis current with a density of 10 A/mm² and a respective current angle of 0° and 90°. By formulating the objective function in terms of energy difference, due to the correlation between magnetic energy and inductance, the mean torque T_e is obtained (4). This eliminates the need for numerous calculations required if the objective function was formulated in terms of average torque.

$$\begin{aligned} L_d \simeq \frac{2W_d}{i_d^2} \quad L_q \simeq \frac{2W_q}{i_q^2} \\ T_e = \frac{3}{2} p (L_d - L_q) i_d i_q \end{aligned} \tag{4}$$

where p is the number of pole pairs, L_d, i_d and L_q, i_q are the d- and q-axis inductance and current respectively.

The two-dimensional magnetostatic problem (5) is formulated in terms of magnetic vector potential A and solved on Matlab by a finite element analysis. The non-linear B-H behavior is approximated by Marrocco's expression.

$$K_m(A)A = J \tag{5}$$

where K_m is the magnetic stiffness matrix that contains the magnetic non-linearity of the material and J is the current density vector.

Since the design variable ρ_e varies in a continuous interval, the reluctivity in the design domain material is interpolated such that:

$$\nu_e = \nu_0 + (\nu_f(b_e) - \nu_0)\rho_e \tag{6}$$

where ν_e, ν_0, and ν_f are respectively the element reluctivity, the reluctivity of air and reluctivity of non-linear electrical steel, b_e the elementary magnetic flux density. The non-linear magnetic energy W is expressed by the formulation:

$$W = \sum_e [\nu_0 - \nu_0\rho_e + 2\nu_f(b_e)\rho_e]\frac{b_e^2}{2} - \rho_e W_{co}(b_e) \tag{7}$$

where W_{co} is the non-linear magnetic co-energy. To update the design, the gradient of the objective function is necessary. In this study, using the Adjoint Variable Method (AVM) [9] the derivative of magnetic energy with respect the design variables is analytically evaluated. Once solved (5), the sensitivities are computed using the adjoint state λ, which is determined as the solution of the following linear system, in both d- and q-axis:

$$\left(\frac{\partial(K_m(A)A - J)}{\partial A}\right)^T \lambda = -\frac{\partial W}{\partial A} \tag{8}$$

Then the sensitivities of the non-linear magnetic energy with respect to each design variable ρ can be finally calculated with:

$$\begin{aligned}\frac{\partial f(\rho)}{\partial \rho} =& \frac{\partial(W_d(\rho) - W_q(\rho))}{\partial \rho}\bigg|_A \\ &+ \lambda_d^T \frac{\partial(K_m(A_d)A_d - J_d)}{\partial \rho}\bigg|_\rho \\ &+ \lambda_q^T \frac{\partial(K_m(A_q)A_q - J_q)}{\partial \rho}\bigg|_\rho\end{aligned} \tag{9}$$

B. Mechanical optimization problem

Next, the optimal geometry resulting from the magnetic optimization (Fig. 4a) is considered as the initial design for the mechanical TO. Note that no symmetry was imposed, in order to give maximum freedom to the topological optimization. The new optimization space is formed by the air regions of flux barriers, highlighted in yellow in Fig. 3. In this step, to achieve a robust rotor geometry, a mechanical analysis is conducted. Subsequently, the compliance C is evaluated as follows:

$$C(\rho) = F(\rho)^T u = u^T K_s(\rho) u \tag{10}$$

where K_s is the global structural stiffness matrix, u the global vector of nodal displacements, and $F(\rho)$ the mechanical loading corresponding to the global inertial forces. These forces depend on the density ρ and act along the radial direction. Considering one single element, the force is evaluated as:

$$F_e = \rho_e m_e r_e \omega^2 \tag{11}$$

where m_e is the mass of the element, r_e its distance from the center of rotation, and ω the angular speed of the rotor, equal to 10000 rpm. Similar to the previous problem, ρ_e is taken from a continuous interval. The constitutive equation of the material is expressed by interpolating Young's modulus:

$$E_e = E_{min} + (E_{fe} - E_{min})\rho_e^q \tag{12}$$

where E_{fe} is the Young's modulus of electrical steel, E_{min} an imposed Young's modulus minimum value, and q the SIMP penalization factor, here equal to 3. The optimization problem is then formulated as:

$$\begin{aligned}\min_\rho \quad & C(\rho) = u^T K_s u \\ \text{s.t.} \quad & \sum_{e=1}^{N_{air}} \rho_e V_e = V_{max\_mec} \\ & 0.001 \le \rho_e \le 1\end{aligned} \tag{13}$$

where N_{air} is the total number of air mesh elements (4099), and V_{max_mec} is the maximum additional material volume, fixed here at 10% to obtain a final rotor structure of 60% of total iron volume. The gradient of the objective function is also obtained using the AVM, it is calculated considering the structural compliance that has design-dependents loads and it is formulated as follows:

$$\frac{\partial C(\rho)}{\partial \rho} = -u^T \frac{\partial K_s}{\partial \rho} u + 2u^T \frac{\partial F}{\partial \rho} \tag{14}$$

Once the gradient is computed, a sensitivity filter is applied to ensure a feasible solution and to avoid the formation of checkerboard patterns that are present for mechanical TO. The formulation of the filtered gradient is given by:

$$\frac{\partial \tilde{C}(\rho)}{\partial \rho_e} = \frac{\sum_{i \in N_e} w(x_i)\rho_i \frac{\partial C}{\partial \rho_i}/v_i}{\rho_e / v_e \sum_{i \in N_e} w(x_i)} \tag{15}$$

where N_e represents the neighborhood of the element e, ρ_i the density of the neighbor element i, v_i the volume of the element i, and $w(x_i)$ a weighting function that depends on the distance between the neighboring element i and the target element e. The function $w(x_i)$ is defined as follows:

$$w(x_i) = 1 - \|x_i - x_e\|/r_{filter} \tag{16}$$

where r_{filter} is a parameter, chosen here to be equal to 3 mm, which defines the neighborhood size, and $\|x_i - x_e\|$ represents the distance between the center of the ith element and the center of the eth element. Therefore, the size of the set N_e depends on the parameter r_{filter}. This radius should be chosen based on the mesh size.

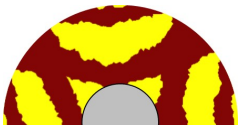

Fig. 3. Rotor domain (in yellow) for the mechanical optimization.

V. Results and Discussion

Final optimized designs are given in Fig. 4, with the magnetic optimization design (a) and the mechanical optimization design (b) showing iron bridges. To analyze the magnetic behavior of the machines, the average torque is determined in post-processing by the average of its instantaneous values, calculated using Arkkio's method [10]. An optimal current angle has been determined after the optimization process. The mechanical displacement of the geometry obtained in Fig. 4b is given in Fig. 5a, obtained with a zero-displacement boundary condition applied on the rotor shaft.

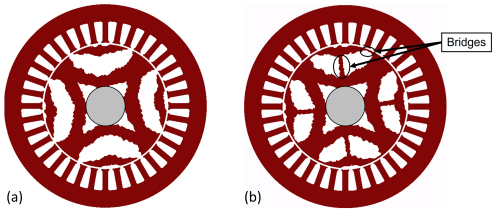

Fig. 4. Magnetic (a) and mechanical (b) optimization result.

TABLE II
MAGNETIC AND MECHANICAL RESULTS

Optimization problem	W_d - W_q $[J]$	Average torque $[N.m]$	Compliance $[J]$	Max. displacement $[m]$
Magnetostatic	0.0366	25.6	-	-
Mechanical (10%)	0.0301	18.2	0.42	1.62e-6
Mechanical (7%)	0.0315	20.3	0.52	2.55e-6

To analyze the impact of bridges, another mechanical TO is performed with the constraint V_{max_mec} fixed at 7%. As expected, the average torque has increased, while the mechanical displacement has almost doubled, as in Fig. 5b. The results are summarized in Table II, allowing a comparison of average torque and compliance. The bridges reduce the compliance of the rotor and limit the maximum deformation in the air gap due to centrifugal forces, as shown in Fig. 5. Furthermore, considering the material yield strength and a safety factor of 2, the resulting admissible Von Mises stress is 200 MPa. In Fig. 6, it can be observed that the target stress value is never reached, with a maximum stress of 26 MPa (Fig. 6a) and 28.6 MPa (Fig. 6b).

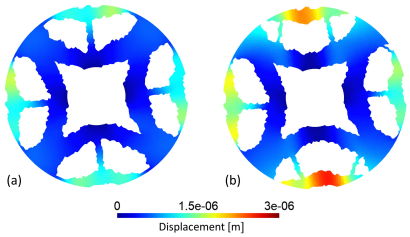

Fig. 5. Rotor displacement [m] with V_{max_mec} 10% (a) and 7% (b).

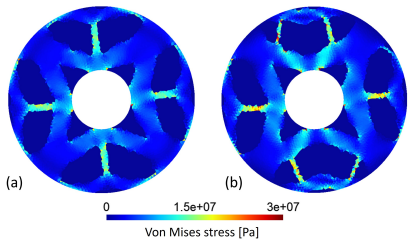

Fig. 6. Von Mises stress distribution with V_{max_mec} 10% (a) and 7% (b).

A relationship has also been observed between the volume constraint, the compliance, and the average torque. The results of this analysis are shown in Fig. 7. We varied the volume constraint from 3% to 15%, and processed each resulting geometry with the proposed TO, calculating the average torque

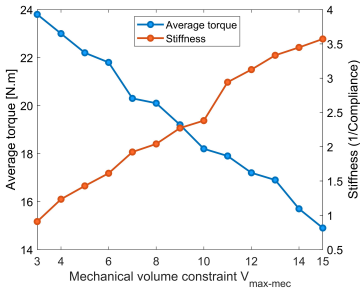

Fig. 7. Relationship between mechanical volume constraint, average torque and compliance.

with the optimal current angle. These results highlight the inverse correlation between compliance and average torque with a quasi-linear evolution vs V_{max_mec}. Indeed, compliance is the inverse of stiffness, as the volume constraint increases, the structure becomes more rigid but at the disadvantage of torque.

VI. CONCLUSION

In this article, the GOC algorithm is used to solve a magneto-mechanical TO problem applied to a SynRM. A novel formulation of the magnetic objective function is proposed and the utilised two-step approach simplifies the design process of the rotor and determines the optimal position of both flux barriers and iron bridges. After further investigations, a trade-off must be considered between the magnetic performance and the rotor's mechanical integrity. The proposed algorithm will be extended to a simultaneous magneto-mechanical optimization, and compared to the approach presented in this paper.

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