

Abdellah Bouchendouka, Zine El Abiddine Fellah, Cong Truc T Nguyen,

Erick Ogam, Camille Perrot, Arnaud Duval, Claude L. Depollier

▶ To cite this version:

Abdellah Bouchendouka, Zine El Abiddine Fellah, Cong Truc T Nguyen, Erick Ogam, Camille Perrot, et al.. Improving acoustic wave propagation models in highly attenuating porous materials. Journal of the Acoustical Society of America, 2023, p206-217. 10.1121/10.0024008 . hal-04328094

HAL Id: hal-04328094 https://hal.science/hal-04328094

Submitted on 6 Dec 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A. Bouchendouka,¹ Z.E.A. Fellah,¹ C.T. Nguyen,² E. Ogam,¹ C. Perrot,² A. Duval,³ and C. Depollier⁴

¹Aix Marseille Univ, CNRS, Centrale Marseille, LMA UMR 7031, Marseille, France.

²Univ Gustave Eiffel, Univ Paris Est Creteil, CNRS, UMR 8208, MSME,

F-77454 Marne-la-Vallée, France

³Trèves products, services and innovation, 2-4 rue Emile Arqu'es, CS 70017,

51686 Reims Cedex 2, France

⁴Acoustics Laboratory of the University of Le Mans (LAUM), UMR 6613,

Institut d'Acoustique - Graduate School (IA-GS), CNRS, Le Mans University, France.

(Dated: 6 December 2023)

Improving acoustic wave propagation models in highly attenuating porous materials This article presents an improved and extended modeling approach for acoustic wave propagation in rigid porous materials, focusing on examples such as plastic foams used 2 for noise reduction in automotive applications. We demonstrate that the classical 3 model (Johnson-Champoux-Allard) in the asymptotic high frequency limit, widely 4 employed in the literature, fail to accurately reconstruct the transmitted acoustic 5 signal through high absorbant porous materials characterized by significant wave 6 attenuation. The study focuses on the airborne ultrasonic frequency range (30-200 7 kHz). 8

1

To address this limitation, we introduce new non-acoustic parameters Σ and V, 9 and Σ' and V' for thermal effects, with surface and volumetric dimensions, respec-10 tively, allowing for the reconstruction of the transmitted signal and accurate modeling 11 of the pronounced acoustic attenuation within the material. These parameters are 12 incorporated into the expansion on skin depths of the dynamic tortuosity $\alpha(\omega)$ and 13 thermal tortuosity $\alpha'(\omega)$ response functions, which describe the inertial-viscous and 14 thermal interactions between the fluid and the solid, respectively. 15

This novel modeling approach enables a more comprehensive study of high atten-16 uating porous media, which are crucial for effective noise reduction. Additionally, 17 it opens up new possibilities for characterization beyond the capabilities of current 18 models. 19

20 I. INTRODUCTION

The investigation of sound propagation in rigid porous media has garnered significant 21 attention within the acoustic community due to its diverse applications in fields such as 22 sound absorption (Zieliński, 2015), automotive industry (Nguyen, 2021), and industrial 23 noise control (Allard and Atalla, 2009). These fields represent active areas of research. 24 Understanding the inherent microstructural properties that contribute to the models used 25 to describe the acoustic behavior of porous media (e.g., porosity, tortuosity, etc.) is cru-26 cial. Consequently, there has been considerable interest in the pursuit of this knowledge, 27 as evidenced in the existing literature (for a comprehensive review, see Refs (Bonfiglio and 28 Pompoli, 2013; Horoshenkov, 2017)). 29

30

The equivalent fluid model (Allard and Atalla, 2009), which is derived from Biot's theory (Biot, 1956), is employed specifically when the fluid saturating the porous medium is air. Under this circumstance, the porous medium is considered rigid, indicating that the solid structure of the tionary and devoid of vibrations (Allard and Atalla, 2009). Consequently, treating the fluid and solid phases as an effective medium (fluid) in which the medium permittivities are two linear operators, one representing an effective density and the other an effective compressibility (Fellah *et al.*, 2002).

The characterization of porous materials involves determining various parameters related to their inner structure. Many scientists have contributed to the development of models in acoustics of porous media, including Zwiker and Kosten (Zwikker and Kosten, 1949), Biot

(Biot, 1956), Delany and Bazley (Delany and Bazley, 1970), Johnson et al. (Johnson et al., 41 1987), Mikki (Miki, 1990), Champoux and Allard (Champoux and Allard, 1991), Allard and 42 Champoux (Allard and Champoux, 1992), Pride et al. (Pride et al., 1993), Lafarge et al. 43 (Lafarge et al., 1997), and Wilson (Wilson, 1997). In their paper, Johnson et al. (Johnson 44 et al., 1987) analyzed the analytic properties inherent in the response of linear functions, 45 particularly focusing on the high and low frequency limits. This investigation notably in-46 troduced the parameter Λ , a critical element in the high-frequency (frozen limit) domain, 47 referred to as the viscous characteristic length. Expanding upon these foundational insights, 48 Champoux and Allard (Allard and Champoux, 1992; Champoux and Allard, 1991) embarked 49 on an in-depth examination of thermal exchange processes within saturated fluid materi-50 als, thereby introducing the concept of thermal characteristic length Λ' . Further advancing 51 the field, Lafarge et al. (Lafarge et al., 1997) contributed by identifying the thermal static 52 permeability k'_0 , particularly relevant in the context of the relaxed limit (low frequency). 53 The works of Pride et al. (Pride et al., 1993), and Lafarge (Lafarge, 1993) (denoted PL), 54 contributed to a more nuanced comprehension of the low-frequency relaxed limit. Further 55 refinement in the characterization of the low-frequency limit was achieved by Roncen et al. 56 (Roncen et al., 2018), who introduced additional terms to the dynamic tortuosity, employing 57 a Laurent-series expansion on frequency. 58

⁵⁹ In the present study, we primarily concentrate on the high-frequency limit, wherein parame-⁶⁰ ters such as tortuosity and viscous characteristic length come into play. Within this limit, the ⁶¹ response functions exhibit expansion into integral power series of viscous and thermal skin ⁶² depths. Notably, the expansion within the Johnson-Champoux-Allard (JCA) model at high Improving acoustic wave propagation models in highly attenuating porous materials frequencies is limited to second-order terms Λ and Λ' . Advancing beyond this, Kergomard *et al.* (Kergomard *et al.*, 2013) introduced third-order terms in the model, denoted as Σ and Σ' , characterized by surface dimensions. The empirical measurement of these parameters was accomplished by Roncen *et al.* (Roncen *et al.*, 2019) employing a statistical Bayesian approach. Their findings advocate that incorporating these terms into the characterization process enhances the precision in reconstructing experimental signals, especially in materials with high resistive properties.

The objectives of our investigation are twofold. Firstly, we propose a novel model for re-70 sponse functions that incorporates fourth-order terms, labeled as V and V', representing 71 volume dimensions. These terms, to our knowledge, have not been previously considered. 72 We provide a comprehensive sensitivity analysis for these fourth-order terms, along with Σ 73 and Σ' , given the limited existing information regarding these parameters. Furthermore, we 74 explain the rationale for integrating these terms, and their applicability. The second aim 75 is to empirically validate the fourth-order model utilizing experimental data derived from 76 transmitted waves through foams exhibiting unusual amplitude attenuation. Our findings 77 demonstrate that neither the high-frequency JCA model nor the third-order expansions suf-78 ficiently predict the experimental signals from these highly attenuated foams, with accuracy 79 achievable only through the inclusion of the fourth-order term. 80

The paper is organized as follows: Section II delineates the experimental procedures and presents the measurements acquired from the transmitted signals of three distinct plastic foams. Subsequently, Section III elaborates on the general theoretical frameworks pertinent to this research, including the introduction of a novel model for the response functions. Improving acoustic wave propagation models in highly attenuating porous materials In Section IV, a comprehensive sensitivity analysis is conducted, elucidating the impact of the higher-order parameters Σ and V on the model's efficacy. Following this, Section V is dedicated to validating the proposed model, particularly the inclusion of the fourth-order term V. This section demonstrates the necessity of this term for accurate characterization of certain samples utilized in the study. The paper concludes with Section VI, where the findings are summarized, and a conclusion is presented.

91 II. EXPERIMENTAL MEASUREMENTS

Experiments were performed in air using a pair of broadband Ultran transducers, NCG100-D50 with a central frequency of 100 kHz. Pulses of 400 V are provided by a 5058PR Panametrics pulser/receiver. The received signals are filtered above 1 MHz to avoid high-frequency noise. Electronic interference is eliminated by 6000 acquisition averages. (Setup illustrated in Figure 1).

In our investigation, we focus on three distinct plastic foams. The first, denoted as foam 97 F, is characterized as a low attenuating material, as evidenced by Figures 2.(a) and 4.(a), 98 and possesses a thickness of L = 0.025 m. The latter two foams, named PU1 and PU2, have 99 been integrated with carbon particles and are prevalent in the automotive sector for their 100 noise mitigating properties. For an exhaustive discourse on the attributes and applications 101 of these foams, the reader is directed to references (Gao et al., 2016; Nguyen, 2021; Nguyen 102 et al., 2020; Tan Hoang and Perrot, 2013; Trinh et al., 2019; Wang et al., 2018; Xi et al., 2016; 103 Zhai et al., 2015; Zhang et al., 2013). Notably, both PU1 and PU2 manifest pronounced 104 absorbent characteristics, illustrated in Figures 2.(b), 4.(b) for PU1 and Figures 2.(c), 4.(c) 105



FIG. 1. (color online) Experimental setup

for PU2. Their respective thicknesses are quantified as L = 0.026 m for PU1 and L = 0.03m for PU2.

In the conducted experiment, the incident signal alongside its spectral representation 108 is represented in Figure 3. The transmitted signals discerned from the various foams are 109 depicted in Figure 4. With regard to foam F, as illustrated in Figure 4.(a), there is a 110 discernible yet moderate attenuation. This is in contrast to the signals from foams PU1 111 and PU2, as shown in Figures 4.(b) and 4.(c), respectively, where a marked attenuation is 112 evident. This reduction in amplitude is further elucidated in Table I. For a more structured 113 comparison, key parameters characterizing the measured signals have been tabulated in 114 Table I. 115



(a) foam F









FIG. 2. (color online) Microscopic image of (a) Foam F, (b) foam PU1, and (c) foam PU2, taken by an inverted microscope model CKX53 with x4 objective.

In materials exhibiting such pronounced attenuation, it is commonplace to observe exceedingly low values of a specific pore characteristic size, denoted as Λ . This peculiarity often complicates their accurate characterization. As demonstrated in ensuing sections, when Λ is particularly low, the theoretical transmitted signal produced via the Johnson-Champoux-Allard (JCA) model in high frequency regimes may not adequately replicate the experimental signal. This discrepancy suggests that the model does not sufficiently capture certain viscous or thermal effects inherent to these foams. Consequently, we introduce an



FIG. 3. Measured incident signal (a), and it's spectrum (b)

enhanced version of the JCA in the HF model in the following section. This augmented model incorporates additional terms labled as Σ and V. A discussion on their consequential impact on the transmitted signal is subsequently presented.

	Incident signal	Transmitted signal by foam F	Transmitted signal by PU1	Transmitted signal by PU2
Maximum amplitude (V)	1,038	0,66	$2,5 imes 10^{-3}$	$2,9\times 10^{-4}$
Decrease in amplitude (%)	/	37,46	99,75	99,97
Energy (V^2)	77,48	29,40	$40, 6 \times 10^{-5}$	$0,64\times 10^{-5}$
Delay $\Delta t \ (\mu \ s)$	/	1, 24	10, 5	15
Front wave velocity c_{∞} (m/s)	343	333	301	293
Decrease in front wave velocity (%)	/	3	12	15

TABLE I. Summary of the key parameters of the measured experimental incident and transmitted

signals



FIG. 4. Measured transmitted signal by foams (a) F, (b) PU1, and (c) PU2

126 III. THEORETICAL FRAMEWORK

In instances where the solid framework remains stationary, unaffected by the acoustic wave, the propagation of the acoustic wave is solely facilitated by the fluid. Under such circumstances, a specific subset of the Biot theory (Biot, 1956) becomes relevant, termed as the 'equivalent fluid model' (Allard and Atalla, 2009). Within this model, the traditional fluid parameters (ρ_f and K_f) are substituted by their corresponding equivalent values ($\rho_f \alpha(\omega)$ and $K_f/\beta(\omega)$), and the system, encompassing both solid and fluid phases, is conceptualized Improving acoustic wave propagation models in highly attenuating porous materials as a singular equivalent fluid. The dynamic tortuosity $\alpha(\omega)$ and the dynamic compressibility $\beta(\omega)$ operate as response functions, epitomizing the visco-inertial and thermal interactions between the solid and fluid on a microscopic scale, respectively. For the thermal effects, the dynamic compressibility $\beta(\omega)$ is directly related to a thermal dynamic tortuosity $\alpha'(\omega)$ as (Lafarge *et al.*, 1997):

$$\beta(\omega) = \gamma - \frac{\gamma - 1}{\alpha'(\omega)},\tag{1}$$

¹³⁸ where γ is the adiabatic index.

Both functions $\alpha(\omega)$ and $\beta(\omega)$ possess clear definitions at the low and high frequency extremes, as mentioned previously in the introduction. It is pertinent to note that the primary emphasis of our investigation is centered on the high-frequency domain.

Considering a macroscopically uniform porous medium saturated with a visco-thermal fluid, and assuming an elementary geometry such that the fluid movement at the pore scale is divergence-free, we can write the governing equations characterizing the acoustic wave's behavior within the frequency domain as follows (Allard and Daigle, 1994):

$$\rho_f \alpha(\omega) i \omega \hat{v} = -\nabla \hat{p}, \qquad \frac{1}{K_f} \beta(\omega) i \omega \hat{p} = -\nabla . \hat{v}.$$
⁽²⁾

In these equations, \hat{v} is the Fourier coefficient of macroscopic fluid velocity, and \hat{p} is the Fourier coefficient of acoustic pressure, obtained by averaging the microscopic velocity and pressure fields over a representative elementary volume.

149 A. Transmission coefficient

In cases where the incident pressure wave is normal to the material, the transmission coefficient can be determined by solving a system of four equations, which are derived by considering the continuity of pressure and velocity on both sides of the material. The solution to this system (see ref (Roncen *et al.*, 2018)) provides the frequency-domain transmission coefficient $T(\omega)$ for normal incidence.

$$T(\omega) = \frac{2Z}{2Z\cosh(ik(\omega)L) + (1+Z)^2 \sinh(ik(\omega)L)},$$
(3)

with L being the material thickness, and:

$$Z = \phi \sqrt{\frac{\beta(\omega)}{\alpha(\omega)}}, \qquad k(\omega) = \omega \sqrt{\frac{\rho_f \alpha(\omega)\beta(\omega)}{K_f}}, \tag{4}$$

where ϕ is the porosity of the material.

The mathematical formulation provided in Equation 3 for the transmission coefficient is a crucial component for characterizing the porous materials considered in this study.

¹⁵⁹ B. Proposed model for the response functions

Assuming that the pore-surface interface appears locally plane at sufficiently high frequencies, that is, meaning that the viscous and thermal boundary layers ($\delta(\omega) = \sqrt{2\eta/\rho_f \omega}$ and $\delta'(\omega) = \sqrt{2\eta/\rho_f \omega P_r}$) become negligible compared with a characteristic radius of curvature of the pore. Then, functions $\alpha(\omega)$ and $\alpha'(\omega)$ expand in integral power series of these thicknesses (Kergomard *et al.*, 2013), which allows us to write:

$$\alpha(\omega) = \alpha_{\infty} \left[1 + \frac{2}{\Lambda} \left(\frac{\delta(\omega)}{\sqrt{2i}} \right) + \frac{3}{\Sigma} \left(\frac{\delta(\omega)}{\sqrt{2i}} \right)^2 + \frac{4}{V} \left(\frac{\delta(\omega)}{\sqrt{2i}} \right)^3 \right] + O\left(\frac{1}{i\omega} \right)^2.$$
(5)

$$\alpha'(\omega) = \alpha'_{\infty} \left[1 + \frac{2}{\Lambda'} \left(\frac{\delta'(\omega)}{\sqrt{2i}} \right) + \frac{3}{\Sigma'} \left(\frac{\delta'(\omega)}{\sqrt{2i}} \right)^2 + \frac{4}{V'} \left(\frac{\delta'(\omega)}{\sqrt{2i}} \right)^3 \right] + O\left(\frac{1}{i\omega} \right)^2.$$
(6)

Here, α_{∞} denotes the ideal fluid tortuosity, whereas $\alpha'_{\infty} = 1$ segnifies the high frequency 165 limit of the thermal tortuosity. Both Λ and Λ' are representative of the viscous and ther-166 mal characteristic lengths, respectively. In the high frequency limit, the Johnson-Champoux-167 Allard (JCA) model stops at the second-order terms Λ and Λ' . The third order parameters 168 Σ and Σ' , with dimensions representative of a surface, were initially introduced by Kergor-169 mard et al. (Kergomard et al., 2013) and subsequently explored experimentally by Roncen 170 et al. (Roncen et al., 2019). Their findings indicated that for materials with low values of Λ , 171 denoted as resistive materials, these third order terms play a role in the reconstructed signal 172 and can not be neglected. The sensitivity analysis was premised on the assumption $\Sigma\propto\Lambda^2$ 173 and $\Sigma' \propto \Lambda'^2$, drawing its roots from the observation that for cylindrical pores, $\Sigma = \Lambda^2 = R^2$ 174 with R denoting the radius of the cylindrical pore's cross section. Nonetheless, our exam-175 ination will perceive these parameters as variables that can function independently, thus 176 allowing for a comprehensive insight into their impact. Any further understanding regard-177 ing these parameters requires further research. The fourth order parameters (V and V) 178 are innovative, possessing dimensions of a volume. As of our current understanding, they 179 remain unexplored and form the epicenter of this study. 180

In the forthcoming sections, a detailed exploration of the sensitivity of these novel parameters, focusing predominantly on amplitude, phase velocity and attenuation, will be Improving acoustic wave propagation models in highly attenuating porous materials undertaken. Before embarking on this particular task, it is pertinent to mention the assumption that the viscous and thermal effects are interdependent, which can be expressed as follows:

$$\frac{\Lambda'}{\Lambda} = 3, \quad \frac{\Sigma'}{\Sigma} = 9, \quad \frac{V'}{V} = 27.$$
(7)

The ratio for the viscous/thermal characteristic lengths, generally varies from 1.5 to 3 (Dauchez, 1999; Mareze and Lenzi, 2011). Choosing a value of 3 is known to be a credible approximation for plastic foams such as the ones we're using in this study (Brown *et al.*, 1996; Fellah *et al.*, 2003). The designated ratios for Σ , Σ' and V, V' align with their corresponding exponents, specifically 3^2 and 3^3 .

¹⁹¹ IV. SENSITIVITY ANALYSIS:

In this section, our objective is to discern the impact of the newly introduced parameters on the theoretical signal. We initiate our analysis by examining the variations in amplitude in relation to variation in the values of Σ and V. The goal is to derive a proximate range for these parameters and ascertain the interval where their influence is at its max. Subsequently, we analyze the effect on the phase velocity as well as the attenuation of the wave for analogous reasons. Concluding this section, we compare the JCA_{HF} , 3rd order, and 4th order models, and comment on the enhancements perceived upon the integration of higher order terms.

For the entirety of this section, our focus remains confined to the variations of Σ , and V against varying values of Λ . Concurrent parameters, namely ϕ and α_{∞} , shall remain invariant, aligning with the values in Table II.

Tortuosity α_{∞}	Porosity ϕ	Thickness L (cm)
1.3	0.95	2.5

TABLE II. Parameters summary for the sensitivity analysis

A. Sensitivity on the amplitude:

Conducting a sensitivity analysis on amplitude with respect to the parameters Σ and 204 V can be a challenge due to the complexities involved in assessing their influence over a 205 substantial range. A simple methodology we can adopt is encapsulating the entire signal 206 using a singular metric. In this context, the peak-to-peak amplitude, denoted as a_{pp} , serves 207 as an apt representative. This is mathematically expressed as: $a_{pp} = Max(S) - Min(S)$, 208 where S represents the theoretical transmitted signal. This signal is derived utilizing the 209 transmission coefficient (see Equation 3), the empirical incident signal as depicted in Figure 210 3, and computational processing via MATLAB. 211

To ensure clarity in the plotted results, we adopt normalized metrics of a_{pp} , given by:

$$A_{pp} = s \times a_{pp},\tag{8}$$

where s is a scaling factor.

In Figure 5 we present a semilogarithmic plot (semi-log x axis) of the variations of the peak to peak value A_{pp} with respect to (a) the parameter Σ utilizing the 3rd order model and (b) the parameter V using 4th order model with Σ fixed at 4000 μ m², and (c) Σ is fixed at 40000 μ m², for different values of Λ . These plots help in determining the effective



FIG. 5. (color online) Variation of A_{pp} defined by equation 8, (a) with respect to Σ using 3rd order model, (b) with respect to V using 4th order model for a fixed value of $\Sigma = 4000 \ \mu m^2$, and (c) with respect to V using 4th order model for $\Sigma = 40000 \ \mu m^2$, for different values of Λ .

Improving acoustic wave propagation models in highly attenuating porous materials range of Σ and V where they significantly influence the wave amplitude, and in identifying 218 a threshold beyond which additional increases in these parameters have a minimal effect. 219 When altering Λ , the resulting graphs consistently show an initial increase in A_{pp} as Σ and 220 V increase, followed by stabilization after reaching a certain critical point. The key effective 221 range for Σ is found to be between 80 μ m² and 2 × 10⁵ μ m², and for V, it lies approximately 222 between 700 μm^3 and 10⁷ μm^3 . It seems that the interval of V is not influenced by Σ . These 223 ranges, however, are subject to change with different Λ values, as is apparent in Figure 5. 224 A comparison of the rate of change in relation to both parameters (Σ and V) reveals that 225 V has a considerably greater influence on amplitude than Σ . 226

B. Sensitivity on phase velocity and attenuation of the wave:

In a porous medium, the wave number can be a complex quantity because of the presence of attenuation and dispersion effects. The real part is directly related to the phase velocity of the wave, and the imaginary part represents the attenuation due to the medium (Allard and Daigle, 1994). The complex wave number, can be represented as follows(Allard and Daigle, 1994):

$$k(\omega) = \sqrt{\frac{K_f}{\rho_f}} (\alpha(\omega)\beta(\omega))^{1/2} = \frac{\omega}{V_p(\omega)} - iA(\omega), \qquad (9)$$

where $V_p(\omega)$ represents the phase velocity of the wave, and $A(\omega)$ is the attenuation of the wave.

In Figure 6 we present a graphical representation of the (a) phase velocity, and (b) attenuation of the wave using 3rd and 4th order models at a fixed frequency of f = 100



(b) Attenuation A(f = 100 kHz)

FIG. 6. (color online) Representation of the (a) phase velocity and (b) Attenaution with respect to Σ using 3rd order model (blue line), and V using 4th order model (red line)

kHz. The phase velocity (V_n) and attenuation (A) are depicted as functions of the parameter 237 Σ (bottom axes) for the 3rd order model (illustrated by the blue curve). For 4th order model 238 (red curve), variations are observed with respect tp V (top axes), while maintaining Λ at a 239 constant value of 100 μ m. For the phase velocity, as shown in Figure 6.(a), the parameter Σ 240 appears to exert no influence, which aligns with theoretical expectations, given its exclusive 241 association with the imaginary component of the response functions (refer to Equations 242 5 and 6), which limits its influence. The parameter V exhibit a significant effect on the 243 velocity, especially for lower values of V, where a decrease in V corresponds to an increase 244 in wave velocity. 245

²⁴⁶ When examining attenuation in Figure 6.(b), it is evident that the inclusion of parameters ²⁴⁷ Σ and V contributes to the wave's attenuation. However, the impact of Σ is marginal when ²⁴⁸ contrasted with the effects V, thus ascribing limited significance to the third-order term Σ ²⁴⁹ in both the phase velocity and attenuation of the wave.

²⁵⁰ C. Comparison between JCA in high frequency limit model with higher order ²⁵¹ models:

In the investigation of the theoretical signal transmission models, the rationale for the integration of the novel parameters Σ and V becomes evident upon comparative analysis. This comparison is predicated upon the distortion of the simulated transmitted signal, which, in this context, refers to the deviation of the signal's waveform from its original (incident) configuration upon the variations of the parameter Λ . To quantitatively assess this distortion, Improving acoustic wave propagation models in highly attenuating porous materials ²⁵⁷ the autocorrelation function of the predicted transmitted signal will be employed as a metric ²⁵⁸ across a spectrum of Λ values:

$$R(\Lambda, k) = \sum_{n} S(\Lambda, n) \cdot S(\Lambda, n+k), \qquad (10)$$

where $R(\Lambda, k)$ is the autocorrelation function, S the predicted transmitted signal, n the discrete time index, and k represents the time lag between the two signals being correlated. The choice of the autocorrelation function is simply because of its interesting symmetry property, facilitating a better visualization of the signal shape alterations. For the sake of clarity, we use scaled values of $R(\Lambda, k)$:

$$R_s(\Lambda, k) = sR(\Lambda, k), \tag{11}$$

where s is a scaling factor. In this context, since scaled values are employed, the absolute magnitudes of R_s are of secondary interest compared to the contour of the autocorrelation function, which serves as the primary indicator of signal integrity.

In Figure 7 we represent the results of the scaled autocorrelation function R_s for (a) JCA_{HF} model, (b) 3rd order model, and (c) 4th order model. The gradation from blue to yellow represents a range from low or negative correlation to high or positive correlation, respectively. An undistorted signal is anticipated to exhibit periodic peaks at varying kvalues, which do not necessarily have to be of equal magnitude. A signal not exhibiting such features is considered distorted.

Within the confines of the JCA_{HF} model (Figure 7.(a)), three distinct zones are discernible. Initially, an undistorted (baseline) zone is observed, characterized by clear periodic peaks



FIG. 7. (color online) Color-scaled image of Rs (Equation 11) using (a) JCA_{HF} , (b) 3rd order, 21 and (c) 4th order models.

indicative of signal constancy. This is followed by a transition zone, spanning from $\Lambda \approx 50$ 276 μm to $\Lambda \approx 75 \ \mu m$, where these periodic peaks are somewhat less visible. Finally, a distorted 277 zone is identified, in which periodicity is absent, and signal deformation is evident. Regard-278 ing the 3rd order model (Figure 7.(b)), computed at a constant $\Sigma = 4000 \ \mu m^2$, there appears 279 to be no substantial enhancement in signal integrity, which is expected, as we have seen in 280 Section IV that the influence of Σ is marginal. Conversely, the 4th order model (Figure 281 7.(c)), calculated for $\Sigma = 4000 \ \mu \text{m}^2$ and $V = 2000 \ \mu \text{m}^3$, manifests a notable augmentation 282 in the signal's integrity, with the distorted zone compressed to $\Lambda \approx 24 \ \mu m$. 283

This demonstration is critical as it underpins the limitation of the JCA_{HF} and 3rd order models for applications wherein the viscous characteristic length of a porous medium encroaches upon the distorted zone, thereby mandating the inclusion of the term V.

It is imperative for the reader to note that the Λ values representing the different zones for the JCA_{HF} and higher-order terms are not general but are contingent upon the specific values of α_{∞} , ϕ , the material thickness L, along with the characteristics of the incident signal, such as its frequency, used in this research. For illustrative purposes, Figure 8 delineates the same R_s function albeit utilizing a non-resistive material parameter set with $\phi = 0.99$, $\alpha_{\infty} = 1.03$, and L = 1 cm.

The observations from Figure 8 suggest a contracted distorted zone, specifically around $\Lambda \approx 18 \ \mu \text{m}^2$ for both JCA_{HF} and 3rd order models. However, an incremental enhancement is noted upon the introduction of the term V ($\Lambda \approx 11 \ \mu \text{m}$). This implies that in the context of materials with pronounced resistive characteristics, the term V is essential for accurate characterization. The forthcoming section will substantiate the efficacy of the 4th



FIG. 8. (color online) Color-scaled image of Rs (Equation 11) using (a) JCA_{HF} , (b) 3rd order, 23 and (c) 4th order models, for $\phi = 0.99$, $\alpha_{\infty} = 1.03$, and L = 1 cm.

Improving acoustic wave propagation models in highly attenuating porous materials order model by contrasting it with empirical data obtained from signals transmitted through highly attenuative materials.

300 V. EXPERIMENTAL VALIDATION:

In line with the previous discussions, the application of the 4th order model becomes per-301 tinent in cases involving porous materials whose viscous characteristic length, Λ , is situated 302 within the distorted zone delineated in Figure 7(a)-(b). This is particularly true for foams 303 PU1 and PU2, which we will demonstrate possess Λ values that reside within this zone, 304 thereby rendering the JCA_{HF} and 3rd order models inadequate for their characterization. 305 Initially, our analysis begins with foam F, demonstrating that the JCA_{HF} model suffices 306 for precise characterization, obviating the need for higher-order terms. Subsequently, we 307 encounter the issue when characterizing foams PU1 and PU2 with the JCA_{HF} and 3rd or-308 der models, underscoring the necessity to incorporate the term V into the characterization 300 process. This requirement arises from the fact that the viscous characteristic length Λ for 310 these foams is approximately 30 μ m, placing them squarely within the distorted zone. 311

312 Foam F

In Figure 9, the results of the JCA_{HF} model's application for the acoustic characterization of Foam F are shown. The signal reconstruction was predicated for the values $\phi = 0.99$, $\alpha_{\infty} = 1.028$, and $\Lambda = 214.71 \ \mu m$. The adequacy of the JCA_{HF} model is substantiated by its precision in describing the acoustic properties of the foam. This precision is attributable to Improving acoustic wave propagation models in highly attenuating porous materials $_{317}$ the value of Λ being within the designated unaltered region, thereby negating the necessity $_{318}$ for integrating higher-order terms into the model for this specific material characterization.



FIG. 9. (color online) Reconstructed signal using JCA_{HF} model, compared with the experimental signal accuired from foam F

319 Foams PU1 and PU2

In Figure 10 we compare the experimental signal accuired from foam PU1 with simulated signals generated using the JCA_{HF} (Figure 10.(a)) and 3rd order (Figure 10.(b)-(c)) models for viscous characteristic lengths $\Lambda = 25\mu$ m and $\Lambda = 40\mu$ m. The selection of these specific Λ values is necessary to account for the notably low amplitude observed in the experimental transmitted signal, which implicates the need for lesser Λ values. The graphical representation illustrates that both models, JCA_{HF} and 3rd order, are ineffectual in replicating the experimental signal's characteristics.

³²⁷ Conversely, Figure 11 extends this comparison to foam PU2, arriving at a similar conclusion ³²⁸ regarding the performance of the aforementioned models. However, an interesting outcome ³²⁹ is presented in Figure 12, where the adoption of the 4th order model yields a simulation ³³⁰ that closely aligns with the experimental signals for both foam specimens, PU1 and PU2. ³³¹ This improved correlation substantiates the 4th order model's efficacy in the accurate sim-³³² ulation of transmitted signals for these particular foam materials. The values obtained for ³³³ the parameters for all three foams are tabulated in Table III

Foam specimen	${ m F}$	PU1	PU2
Model used	JCA_{HF}	4th order	4th order
ϕ	0,99	0,98	0,98
$lpha_\infty$	1,028	1,13	1.18
$\Lambda~(\mu{ m m})$	214,71	38,7	36,7
$\Sigma ~(\mu { m m}^2)$	/	6470	4076
$V~(\mu { m m}^3)$	/	7790	5717

TABLE III. Summary of the results obtained for the different foam specimens

334 VI. CONCLUSION

A new model has been proposed that extends the high-frequency expansion on the skin depths of response functions by incorporating higher-order terms. This model introduces two



FIG. 10. (color online) Reconstructed signal for values of $\Lambda = 25 \ \mu \text{m}$ and 40 μm compared with experimental signal for foam PU1 using models: (a) JCA_{HF} , (b) 3rd order for $\Sigma = 4000 \ \mu \text{ m}^2$, and (c) 3rd order for $\Sigma = 40000 \ \mu \text{m}^2$

³³⁷ non-acoustical parameters, Σ and V. It has been observed that V exerts a more substantial ³³⁸ effect on wave amplitude, phase velocity, attenuation, and overall signal integrity compared ³³⁹ to Σ . Particularly under low values of Λ , the simulated transmitted signal using both ³⁴⁰ the Johnson-Champoux-Allard (JCA) model at high frequencies and the third-order model ³⁴¹ (which only includes up to Σ) shows distortion. Conversely, incorporating the V term



FIG. 11. (color online) Reconstructed signal for values of $\Lambda = 25 \ \mu m$ and 40 μm compared with experimental signal for foam PU2 using models: (a) JCA_{HF} , (b) 3rd order for $\Sigma = 4000 \ \mu m^2$, and (c) 3rd order for $\Sigma = 40000 \ \mu m^2$

significantly improves signal integrity, leading to more precise material characterization. This model's effectiveness was confirmed through experiments on two plastic foams, PU1 and PU2, which possess high attenuation properties. The process of characterizing these two foams highlighted the importance of including the fourth-order term V and demonstrated the limitations of both the JCA model at high frequencies and the third-order model.



FIG. 12. (color online) Reconstructed signal using 4th order model compared with experimental signal for foams (a) PU1 and (b) PU2.

In summary, incorporating these terms is crucial for accurately characterizing rigid porous
 materials with high attenuation properties and should not be overlooked.

The implications of these findings are far-reaching, impacting various industrial sectors such as engineering, manufacturing, performance optimization, product development, and innovation. Moreover, they extend to acoustic applications, broadening the study's practical significance and potential influence.

353 VII. AUTHOR DECLARATIONS

The authors declare that they have no conflicts of interest to disclose in the context of this research. They also affirm that their work adheres to ethical approval, indicating that their research does not involve experiments on animal subjects and/or human participants.

357 VIII. DATA AVAILABILITY

Data is available upon request from the authors. The information supporting the findings of this study is accessible from the corresponding author upon reasonable request. 360

- Allard, J., and Atalla, N. (2009). Propagation of sound in porous media: modelling sound *absorbing materials* (John Wiley & Sons).
- Allard, J.-F., and Champoux, Y. (1992). "New empirical equations for sound propagation
- in rigid frame fibrous materials," The Journal of the Acoustical Society of America 91(6),
 3346–3353.
- Allard, J.-F., and Daigle, G. (1994). "Propagation of sound in porous media: Modeling
 sound absorbing materials" .
- Biot, M. A. (1956). "Theory of propagation of elastic waves in a fluid-saturated porous solid. ii. higher frequency range," The Journal of the acoustical Society of america 28(2), 179–191.
- Bonfiglio, P., and Pompoli, F. (**2013**). "Inversion problems for determining physical parameters of porous materials: Overview and comparison between different methods," Acta Acustica united with Acustica **99**(3), 341–351.
- 374 Brown, N., Melon, M., Montembault, V., Castagnède, B., Lauriks, W., and Leclaire, P.
- ³⁷⁵ (1996). "Evaluation of the viscous characteristic length of air-saturated porous materials
- ³⁷⁶ from the ultrasonic dispersion curve," Comptes rendus dAcademie des sciences. Serie IIb,
- $_{377}$ Mecanique **322**(2), 122–127.
- ³⁷⁸ Champoux, Y., and Allard, J.-F. (**1991**). "Dynamic tortuosity and bulk modulus in air-³⁷⁹ saturated porous media," Journal of applied physics **70**(4), 1975–1979.
- 380 Dauchez, N. A. R. (1999). Etude vibroacoustique des matériaux poreux par éléments finis.

- Delany, M., and Bazley, E. (1970). "Acoustical properties of fibrous absorbent materials," 381
- Applied acoustics $\mathbf{3}(2)$, 105–116. 382
- Fellah, Z., Depollier, C., and Fellah, M. (2002). "Application of fractional calculus to the 383 sound waves propagation in rigid porous materials: validation via ultrasonic measure-384 ments," Acta Acustica united with Acustica 88(1), 34–39. 385
- Fellah, Z. E. A., Fellah, M., Lauriks, W., and Depollier, C. (2003). "Direct and inverse 386 scattering of transient acoustic waves by a slab of rigid porous material," The Journal of 387 the Acoustical Society of America 113(1), 61-72. 388
- Gao, K., Van Dommelen, J., and Geers, M. (2016). "Microstructure characterization and 389 homogenization of acoustic polyurethane foams: Measurements and simulations," Interna-390 tional Journal of Solids and Structures 100, 536–546. 391
- Horoshenkov, K. V. (2017). "A review of acoustical methods for porous material character-392
- isation," Int. J. Acoust. Vib **22**(1), 92–103. 393

395

- Johnson, D. L., Koplik, J., and Dashen, R. (1987). "Theory of dynamic permeability and 394 tortuosity in fluid-saturated porous media," Journal of fluid mechanics 176, 379–402.
- Kergomard, J., Lafarge, D., and Gilbert, J. (2013). "Transients in porous media: Exact 396 and modelled time-domain green's functions," Acta Acustica united with Acustica 99(4), 397 557-571. 398
- Lafarge, D. (1993). "Propagation du son dans les matériaux poreux à structure rigide 399 saturés par un fluide viscothermique: Définition de paramètres géométriques, analogie 400 electromagnétique, temps de relaxation," Ph.D. thesis, Le Mans. 401

- ⁴⁰² Lafarge, D., Lemarinier, P., Allard, J. F., and Tarnow, V. (1997). "Dynamic compressibility
- ⁴⁰³ of air in porous structures at audible frequencies," The Journal of the Acoustical Society
- 404 of America 102(4), 1995–2006.
- ⁴⁰⁵ Mareze, P. H., and Lenzi, A. (**2011**). "Characterization and optimization of rigid-frame ⁴⁰⁶ porous material," ICSV18, Rio de Janeiro, Brazil 10–14.
- Miki, Y. (1990). "Acoustical properties of porous materials-modifications of delany-bazley
 models," Journal of the Acoustical Society of Japan (E) 11(1), 19–24.
- ⁴⁰⁹ Nguyen, C. T. (2021). "Acoustic foams with pore size distributions and controlled intercon-
- nections: Relationships between structures, properties, fabrication," Ph.D. thesis, Université Gustave Eiffel.
- ⁴¹² Nguyen, C. T., Guilleminot, J., Detrez, F., Langlois, V., Bornert, M., Duval, A., and
 ⁴¹³ Perrot, C. (2020). "Micro-macro acoustic modeling of heterogeneous foams with nucleation
 ⁴¹⁴ perturbation," SAE Technical Paper 01–1526.
- ⁴¹⁵ Pride, S. R., Morgan, F. D., and Gangi, A. F. (**1993**). "Drag forces of porous-medium ⁴¹⁶ acoustics," Physical review B **47**(9), 4964.
- ⁴¹⁷ Roncen, R., Fellah, Z. E., Lafarge, D., Piot, E., Simon, F., Ogam, E., Fellah, M., and
 ⁴¹⁸ Depollier, C. (2018). "Acoustical modeling and bayesian inference for rigid porous media
 ⁴¹⁹ in the low-mid frequency regime," The Journal of the Acoustical Society of America 144(6),
- 419 In the low line nequency regime, The Southar of the Reoustical Society of America 144(e 420 3084–3101.
- 421 Roncen, R., Fellah, Z. E. A., Piot, E., Simon, F., Ogam, E., Fellah, M., and Depollier, C.
- (2019). "Inverse identification of a higher order viscous parameter of rigid porous media
 in the high frequency domain," The Journal of the Acoustical Society of America 145(3),

1629 - 1639.424

- Tan Hoang, M., and Perrot, C. (2013). "Identifying local characteristic lengths governing 425 sound wave properties in solid foams," Journal of Applied Physics **113**(8), 084905. 426
- Trinh, V. H., Langlois, V., Guilleminot, J., Perrot, C., Khidas, Y., and Pitois, O. (2019). 427
- "Tuning membrane content of sound absorbing cellular foams: Fabrication, experimental 428 evidence and multiscale numerical simulations," Materials & Design 162, 345-361. 429
- Wang, S., Qian, L., and Xin, F. (2018). "The synergistic flame-retardant behaviors of 430
- pentaerythritol phosphate and expandable graphite in rigid polyurethane foams," Polymer 431
- Composites 39(2), 329-336. 432

444

- Wilson, D. K. (1997). "Simple, relaxational models for the acoustical properties of porous 433 media," Applied Acoustics 50(3), 171–188. 434
- Xi, W., Qian, L., Huang, Z., Cao, Y., and Li, L. (2016). "Continuous flame-retardant 435
- actions of two phosphate esters with expandable graphite in rigid polyurethane foams," 436
- Polymer Degradation and Stability 130, 97–102. 437
- Zhai, T., Li, D., Fei, G., and Xia, H. (2015). "Piezoresistive and compression resistance re-438 laxation behavior of water blown carbon nanotube/polyurethane composite foam," Com-439 posites Part A: Applied Science and Manufacturing 72, 108–114. 440
- Zhang, L., Zhang, M., Zhou, Y., and Hu, L. (2013). "The study of mechanical behavior 441 and flame retardancy of castor oil phosphate-based rigid polyurethane foam composites 442 containing expanded graphite and triethyl phosphate," Polymer Degradation and Stability 443 **98**(12), 2784–2794.

- ⁴⁴⁵ Zieliński, T. G. (2015). "Normalized inverse characterization of sound absorbing rigid porous
- ⁴⁴⁶ media," The Journal of the Acoustical Society of America **137**(6), 3232–3243.
- ⁴⁴⁷ Zwikker, C., and Kosten, C. W. (**1949**). *Sound absorbing materials* (Elsevier publishing ⁴⁴⁸ company).