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Improving acoustic wave propagation models in highly attenuating porous materials

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1 This article presents an improved and extended modeling approach for acoustic wave
2 propagation in rigid porous materials, focusing on examples such as plastic foams used
3 for noise reduction in automotive applications. We demonstrate that the classical
4 model (Johnson-Champoux-Allard) in the asymptotic high frequency limit, widely
5 employed in the literature, fail to accurately reconstruct the transmitted acoustic
6 signal through high absorbant porous materials characterized by significant wave
7 attenuation. The study focuses on the airborne ultrasonic frequency range (30-200
8 kHz).

9 To address this limitation, we introduce new non-acoustic parameters Σ and V ,
10 and Σ' and V' for thermal effects, with surface and volumetric dimensions, respec-
11 tively, allowing for the reconstruction of the transmitted signal and accurate modeling
12 of the pronounced acoustic attenuation within the material. These parameters are
13 incorporated into the expansion on skin depths of the dynamic tortuosity $\alpha(\omega)$ and
14 thermal tortuosity $\alpha'(\omega)$ response functions, which describe the inertial-viscous and
15 thermal interactions between the fluid and the solid, respectively.

16 This novel modeling approach enables a more comprehensive study of high atten-
17 uating porous media, which are crucial for effective noise reduction. Additionally,
18 it opens up new possibilities for characterization beyond the capabilities of current
19 models.

I. INTRODUCTION

The investigation of sound propagation in rigid porous media has garnered significant attention within the acoustic community due to its diverse applications in fields such as sound absorption (Zieliński, 2015), automotive industry (Nguyen, 2021), and industrial noise control (Allard and Atalla, 2009). These fields represent active areas of research. Understanding the inherent microstructural properties that contribute to the models used to describe the acoustic behavior of porous media (e.g., porosity, tortuosity, etc.) is crucial. Consequently, there has been considerable interest in the pursuit of this knowledge, as evidenced in the existing literature (for a comprehensive review, see Refs (Bonfiglio and Pompoli, 2013; Horoshenkov, 2017)).

The equivalent fluid model (Allard and Atalla, 2009), which is derived from Biot's theory (Biot, 1956), is employed specifically when the fluid saturating the porous medium is air. Under this circumstance, the porous medium is considered rigid, indicating that the solid structure of the medium is devoid of vibrations (Allard and Atalla, 2009). Consequently, treating the fluid and solid phases as an effective medium (fluid) in which the medium permittivities are two linear operators, one representing an effective density and the other an effective compressibility (Fellah *et al.*, 2002).

The characterization of porous materials involves determining various parameters related to their inner structure. Many scientists have contributed to the development of models in acoustics of porous media, including Zwicker and Kosten (Zwicker and Kosten, 1949), Biot

41 (Biot, 1956), Delany and Bazley (Delany and Bazley, 1970), Johnson *et al.* (Johnson *et al.*,
42 1987), Mikki (Miki, 1990), Champoux and Allard (Champoux and Allard, 1991), Allard and
43 Champoux (Allard and Champoux, 1992), Pride *et al.* (Pride *et al.*, 1993), Lafarge *et al.*
44 (Lafarge *et al.*, 1997), and Wilson (Wilson, 1997). In their paper, Johnson *et al.* (Johnson
45 *et al.*, 1987) analyzed the analytic properties inherent in the response of linear functions,
46 particularly focusing on the high and low frequency limits. This investigation notably in-
47 troduced the parameter Λ , a critical element in the high-frequency (frozen limit) domain,
48 referred to as the viscous characteristic length. Expanding upon these foundational insights,
49 Champoux and Allard (Allard and Champoux, 1992; Champoux and Allard, 1991) embarked
50 on an in-depth examination of thermal exchange processes within saturated fluid materi-
51 als, thereby introducing the concept of thermal characteristic length Λ' . Further advancing
52 the field, Lafarge *et al.* (Lafarge *et al.*, 1997) contributed by identifying the thermal static
53 permeability k'_0 , particularly relevant in the context of the relaxed limit (low frequency).
54 The works of Pride *et al.* (Pride *et al.*, 1993), and Lafarge (Lafarge, 1993) (denoted PL),
55 contributed to a more nuanced comprehension of the low-frequency relaxed limit. Further
56 refinement in the characterization of the low-frequency limit was achieved by Roncen *et al.*
57 (Roncen *et al.*, 2018), who introduced additional terms to the dynamic tortuosity, employing
58 a Laurent-series expansion on frequency.

59 In the present study, we primarily concentrate on the high-frequency limit, wherein parame-
60 ters such as tortuosity and viscous characteristic length come into play. Within this limit, the
61 response functions exhibit expansion into integral power series of viscous and thermal skin
62 depths. Notably, the expansion within the Johnson-Champoux-Allard (JCA) model at high

63 frequencies is limited to second-order terms Λ and Λ' . Advancing beyond this, Kergomard
64 *et al.* (Kergomard *et al.*, 2013) introduced third-order terms in the model, denoted as Σ and
65 Σ' , characterized by surface dimensions. The empirical measurement of these parameters
66 was accomplished by Roncen *et al.* (Roncen *et al.*, 2019) employing a statistical Bayesian
67 approach. Their findings advocate that incorporating these terms into the characterization
68 process enhances the precision in reconstructing experimental signals, especially in materials
69 with high resistive properties.

70 The objectives of our investigation are twofold. Firstly, we propose a novel model for re-
71 sponse functions that incorporates fourth-order terms, labeled as V and V' , representing
72 volume dimensions. These terms, to our knowledge, have not been previously considered.
73 We provide a comprehensive sensitivity analysis for these fourth-order terms, along with Σ
74 and Σ' , given the limited existing information regarding these parameters. Furthermore, we
75 explain the rationale for integrating these terms, and their applicability. The second aim
76 is to empirically validate the fourth-order model utilizing experimental data derived from
77 transmitted waves through foams exhibiting unusual amplitude attenuation. Our findings
78 demonstrate that neither the high-frequency JCA model nor the third-order expansions suf-
79 ficiently predict the experimental signals from these highly attenuated foams, with accuracy
80 achievable only through the inclusion of the fourth-order term.

81 The paper is organized as follows: Section II delineates the experimental procedures and
82 presents the measurements acquired from the transmitted signals of three distinct plastic
83 foams. Subsequently, Section III elaborates on the general theoretical frameworks pertinent
84 to this research, including the introduction of a novel model for the response functions.

85 In Section IV, a comprehensive sensitivity analysis is conducted, elucidating the impact of
86 the higher-order parameters Σ and V on the model's efficacy. Following this, Section V is
87 dedicated to validating the proposed model, particularly the inclusion of the fourth-order
88 term V . This section demonstrates the necessity of this term for accurate characterization
89 of certain samples utilized in the study. The paper concludes with Section VI, where the
90 findings are summarized, and a conclusion is presented.

91 II. EXPERIMENTAL MEASUREMENTS

92 Experiments were performed in air using a pair of broadband Ultrasonics transducers,
93 NCG100-D50 with a central frequency of 100 kHz. Pulses of 400 V are provided by a
94 5058PR Panametrics pulser/receiver. The received signals are filtered above 1 MHz to avoid
95 high-frequency noise. Electronic interference is eliminated by 6000 acquisition averages.
96 (Setup illustrated in Figure 1).

97 In our investigation, we focus on three distinct plastic foams. The first, denoted as foam
98 F, is characterized as a low attenuating material, as evidenced by Figures 2.(a) and 4.(a),
99 and possesses a thickness of $L = 0.025$ m. The latter two foams, named PU1 and PU2, have
100 been integrated with carbon particles and are prevalent in the automotive sector for their
101 noise mitigating properties. For an exhaustive discourse on the attributes and applications
102 of these foams, the reader is directed to references ([Gao et al., 2016](#); [Nguyen, 2021](#); [Nguyen](#)
103 [et al., 2020](#); [Tan Hoang and Perrot, 2013](#); [Trinh et al., 2019](#); [Wang et al., 2018](#); [Xi et al., 2016](#);
104 [Zhai et al., 2015](#); [Zhang et al., 2013](#)). Notably, both PU1 and PU2 manifest pronounced
105 absorbent characteristics, illustrated in Figures 2.(b), 4.(b) for PU1 and Figures 2.(c), 4.(c)

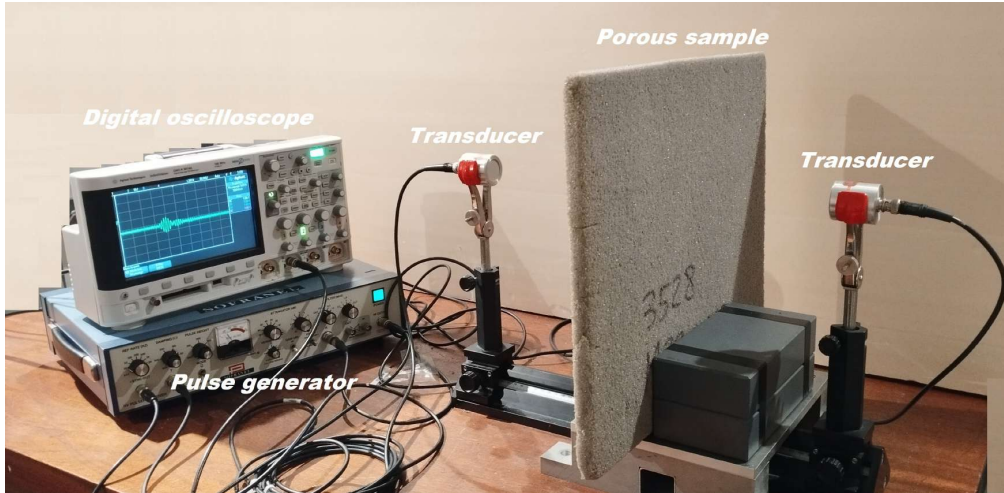
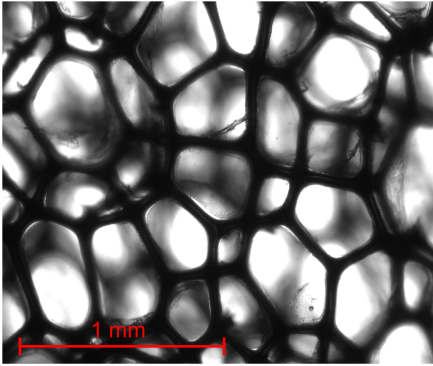


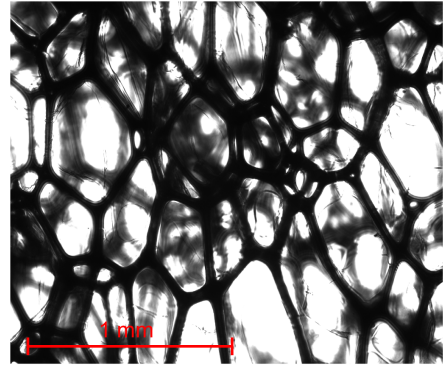
FIG. 1. (color online) Experimental setup

106 for PU2. Their respective thicknesses are quantified as $L = 0.026$ m for PU1 and $L = 0.03$
107 m for PU2.

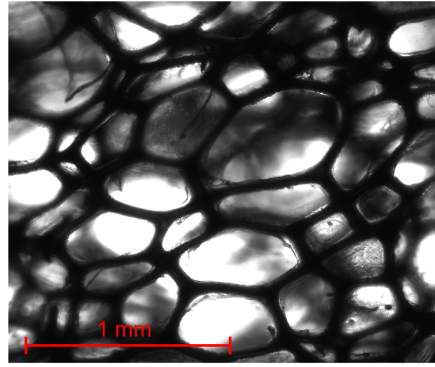
108 In the conducted experiment, the incident signal alongside its spectral representation
109 is represented in Figure 3. The transmitted signals discerned from the various foams are
110 depicted in Figure 4. With regard to foam F, as illustrated in Figure 4.(a), there is a
111 discernible yet moderate attenuation. This is in contrast to the signals from foams PU1
112 and PU2, as shown in Figures 4.(b) and 4.(c), respectively, where a marked attenuation is
113 evident. This reduction in amplitude is further elucidated in Table I. For a more structured
114 comparison, key parameters characterizing the measured signals have been tabulated in
115 Table I.



(a) foam F



(b) foam PU1



(c) Foam PU2

FIG. 2. (color online) Microscopic image of (a) Foam F, (b) foam PU1, and (c) foam PU2, taken by an inverted microscope model CKX53 with x4 objective.

116 In materials exhibiting such pronounced attenuation, it is commonplace to observe ex-
117 ceedingly low values of a specific pore characteristic size, denoted as Λ . This peculiarity
118 often complicates their accurate characterization. As demonstrated in ensuing sections,
119 when Λ is particularly low, the theoretical transmitted signal produced via the Johnson-
120 Champoux-Allard (JCA) model in high frequency regimes may not adequately replicate the
121 experimental signal. This discrepancy suggests that the model does not sufficiently capture
122 certain viscous or thermal effects inherent to these foams. Consequently, we introduce an

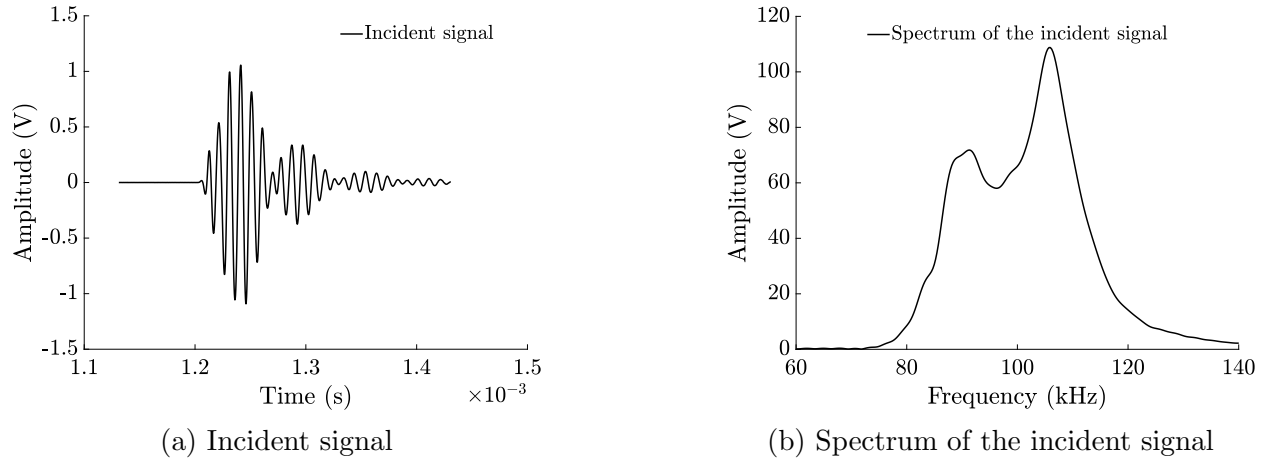


FIG. 3. Measured incident signal (a), and it's spectrum (b)

123 enhanced version of the JCA in the HF model in the following section. This augmented
 124 model incorporates additional terms labeled as Σ and V . A discussion on their consequential
 125 impact on the transmitted signal is subsequently presented.

	Incident signal	Transmitted signal by foam F	Transmitted signal by PU1	Transmitted signal by PU2
Maximum amplitude (V)	1,038	0,66	$2,5 \times 10^{-3}$	$2,9 \times 10^{-4}$
Decrease in amplitude (%)	/	37,46	99,75	99,97
Energy (V^2)	77,48	29,40	$40,6 \times 10^{-5}$	$0,64 \times 10^{-5}$
Delay Δt (μ s)	/	1,24	10,5	15
Front wave velocity c_∞ (m/s)	343	333	301	293
Decrease in front wave velocity (%)	/	3	12	15

TABLE I. Summary of the key parameters of the measured experimental incident and transmitted signals

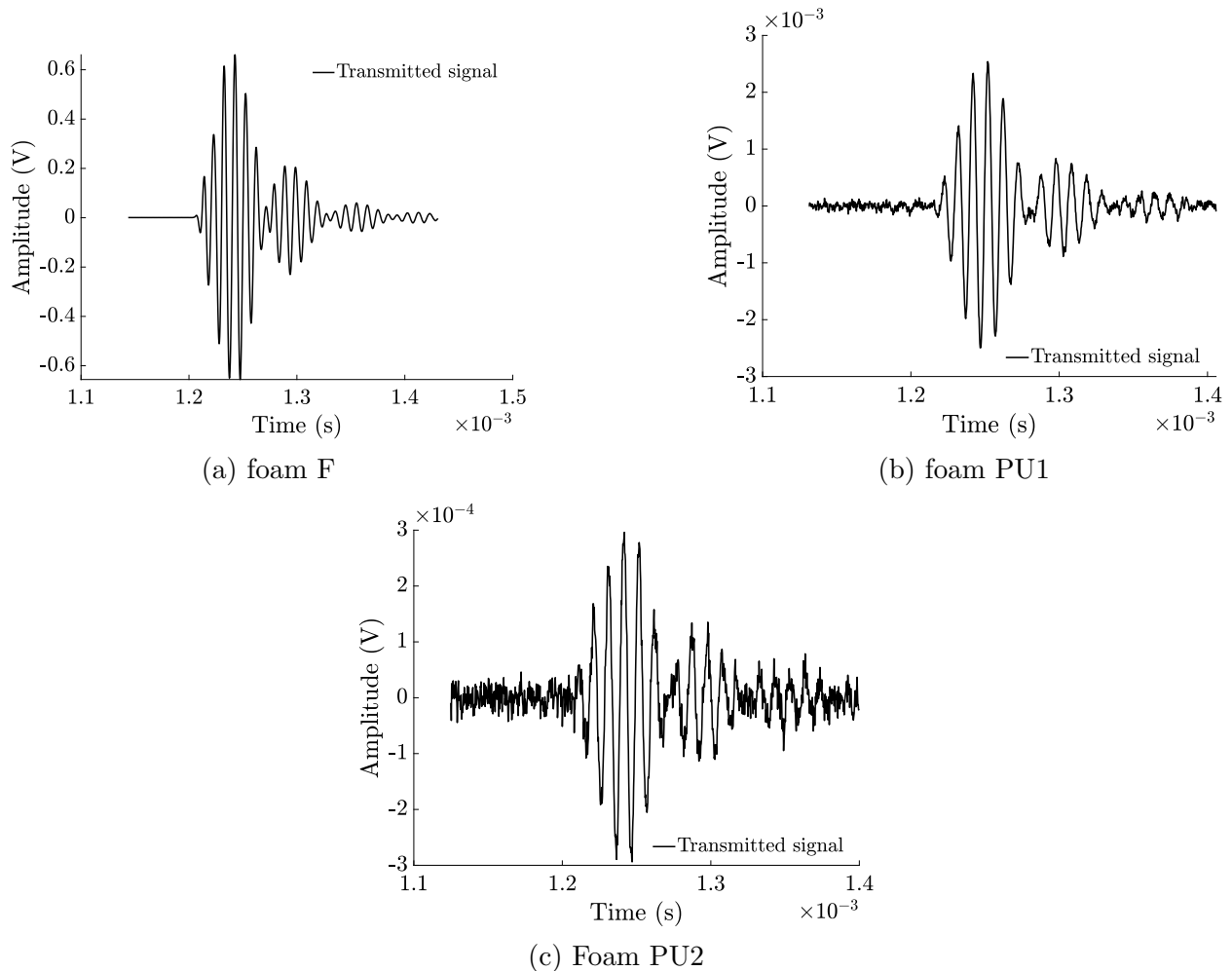


FIG. 4. Measured transmitted signal by foams (a) F, (b) PU1, and (c) PU2

126 III. THEORETICAL FRAMEWORK

127 In instances where the solid framework remains stationary, unaffected by the acoustic
 128 wave, the propagation of the acoustic wave is solely facilitated by the fluid. Under such cir-
 129 cumstances, a specific subset of the Biot theory (Biot, 1956) becomes relevant, termed as the
 130 'equivalent fluid model' (Allard and Atalla, 2009). Within this model, the traditional fluid
 131 parameters (ρ_f and K_f) are substituted by their corresponding equivalent values ($\rho_f\alpha(\omega)$
 132 and $K_f/\beta(\omega)$), and the system, encompassing both solid and fluid phases, is conceptualized

133 as a singular equivalent fluid. The dynamic tortuosity $\alpha(\omega)$ and the dynamic compressibility
 134 $\beta(\omega)$ operate as response functions, epitomizing the visco-inertial and thermal interactions
 135 between the solid and fluid on a microscopic scale, respectively. For the thermal effects, the
 136 dynamic compressibility $\beta(\omega)$ is directly related to a thermal dynamic tortuosity $\alpha'(\omega)$ as
 137 (Lafarge *et al.*, 1997):

$$\beta(\omega) = \gamma - \frac{\gamma - 1}{\alpha'(\omega)}, \quad (1)$$

138 where γ is the adiabatic index.

139 Both functions $\alpha(\omega)$ and $\beta(\omega)$ possess clear definitions at the low and high frequency ex-
 140 tremes, as mentioned previously in the introduction. It is pertinent to note that the primary
 141 emphasis of our investigation is centered on the high-frequency domain.

142 Considering a macroscopically uniform porous medium saturated with a visco-thermal fluid,
 143 and assuming an elementary geometry such that the fluid movement at the pore scale is
 144 divergence-free, we can write the governing equations characterizing the acoustic wave's
 145 behavior within the frequency domain as follows (Allard and Daigle, 1994):

$$\rho_f \alpha(\omega) i \omega \hat{v} = -\nabla \hat{p}, \quad \frac{1}{K_f} \beta(\omega) i \omega \hat{p} = -\nabla \cdot \hat{v}. \quad (2)$$

146 In these equations, \hat{v} is the Fourier coefficient of macroscopic fluid velocity, and \hat{p} is the
 147 Fourier coefficient of acoustic pressure, obtained by averaging the microscopic velocity and
 148 pressure fields over a representative elementary volume.

149 **A. Transmission coefficient**

150 In cases where the incident pressure wave is normal to the material, the transmission
 151 coefficient can be determined by solving a system of four equations, which are derived by
 152 considering the continuity of pressure and velocity on both sides of the material. The solution
 153 to this system (see ref (Roncen *et al.*, 2018)) provides the frequency-domain transmission
 154 coefficient $T(\omega)$ for normal incidence.

$$T(\omega) = \frac{2Z}{2Z \cosh(ik(\omega)L) + (1 + Z)^2 \sinh(ik(\omega)L)}, \quad (3)$$

155 with L being the material thickness, and:

$$Z = \phi \sqrt{\frac{\beta(\omega)}{\alpha(\omega)}}, \quad k(\omega) = \omega \sqrt{\frac{\rho_f \alpha(\omega) \beta(\omega)}{K_f}}, \quad (4)$$

156 where ϕ is the porosity of the material.

157 The mathematical formulation provided in Equation 3 for the transmission coefficient is a
 158 crucial component for characterizing the porous materials considered in this study.

159 **B. Proposed model for the response functions**

160 Assuming that the pore-surface interface appears locally plane at sufficiently high fre-
 161 quencies, that is, meaning that the viscous and thermal boundary layers ($\delta(\omega) = \sqrt{2\eta/\rho_f\omega}$
 162 and $\delta'(\omega) = \sqrt{2\eta/\rho_f\omega P_r}$) become negligible compared with a characteristic radius of curva-
 163 ture of the pore. Then, functions $\alpha(\omega)$ and $\alpha'(\omega)$ expand in integral power series of these
 164 thicknesses (Kergomard *et al.*, 2013), which allows us to write:

$$\alpha(\omega) = \alpha_\infty \left[1 + \frac{2}{\Lambda} \left(\frac{\delta(\omega)}{\sqrt{2i}} \right) + \frac{3}{\Sigma} \left(\frac{\delta(\omega)}{\sqrt{2i}} \right)^2 + \frac{4}{V} \left(\frac{\delta(\omega)}{\sqrt{2i}} \right)^3 \right] + O \left(\frac{1}{i\omega} \right)^2. \quad (5)$$

$$\alpha'(\omega) = \alpha'_\infty \left[1 + \frac{2}{\Lambda'} \left(\frac{\delta'(\omega)}{\sqrt{2i}} \right) + \frac{3}{\Sigma'} \left(\frac{\delta'(\omega)}{\sqrt{2i}} \right)^2 + \frac{4}{V'} \left(\frac{\delta'(\omega)}{\sqrt{2i}} \right)^3 \right] + O \left(\frac{1}{i\omega} \right)^2. \quad (6)$$

Here, α_∞ denotes the ideal fluid tortuosity, whereas $\alpha'_\infty = 1$ signifies the high frequency limit of the the thermal tortuosity. Both Λ and Λ' are representative of the viscous and thermal characteristic lengths, respectively. In the high frequency limit, the Johnson-Champoux-Allard (JCA) model stops at the second-order terms Λ and Λ' . The third order parameters Σ and Σ' , with dimensions representative of a surface, were initially introduced by Kergomard *et al.* (Kergomard *et al.*, 2013) and subsequently explored experimentally by Roncen *et al.* (Roncen *et al.*, 2019). Their findings indicated that for materials with low values of Λ , denoted as resistive materials, these third order terms play a role in the reconstructed signal and can not be neglected. The sensitivity analysis was premised on the assumption $\Sigma \propto \Lambda^2$ and $\Sigma' \propto \Lambda'^2$, drawing its roots from the observation that for cylindrical pores, $\Sigma = \Lambda^2 = R^2$ with R denoting the radius of the cylindrical pore's cross section. Nonetheless, our examination will perceive these parameters as variables that can function independently, thus allowing for a comprehensive insight into their impact. Any further understanding regarding these parameters requires further research. The fourth order parameters (V and V') are innovative, possessing dimensions of a volume. As of our current understanding, they remain unexplored and form the epicenter of this study.

In the forthcoming sections, a detailed exploration of the sensitivity of these novel parameters, focusing predominantly on amplitude, phase velocity and attenuation, will be

183 undertaken. Before embarking on this particular task, it is pertinent to mention the as-
 184 sumption that the viscous and thermal effects are interdependent, which can be expressed
 185 as follows:

$$\frac{\Lambda'}{\Lambda} = 3, \quad \frac{\Sigma'}{\Sigma} = 9, \quad \frac{V'}{V} = 27. \quad (7)$$

186 The ratio for the viscous/thermal characteristic lengths, generally varies from 1.5 to 3
 187 (Dauchez, 1999; Mareze and Lenzi, 2011). Choosing a value of 3 is known to be a credible
 188 approximation for plastic foams such as the ones we're using in this study (Brown *et al.*, 1996;
 189 Fellah *et al.*, 2003). The designated ratios for Σ , Σ' and V , V' align with their corresponding
 190 exponents, specifically 3^2 and 3^3 .

191 IV. SENSITIVITY ANALYSIS:

192 In this section, our objective is to discern the impact of the newly introduced parameters
 193 on the theoretical signal. We initiate our analysis by examining the variations in amplitude
 194 in relation to variation in the values of Σ and V . The goal is to derive a proximate range for
 195 these parameters and ascertain the interval where their influence is at its max. Subsequently,
 196 we analyze the effect on the phase velocity as well as the attenuation of the wave for analogous
 197 reasons. Concluding this section, we compare the JCA_{HF} , 3rd order, and 4th order models,
 198 and comment on the enhancements perceived upon the integration of higher order terms.

199 For the entirety of this section, our focus remains confined to the variations of Σ , and
 200 V against varying values of Λ . Concurrent parameters, namely ϕ and α_∞ , shall remain
 202 invariant, aligning with the values in Table II.

Tortuosity α_∞	Porosity ϕ	Thickness L (cm)
1.3	0.95	2.5

TABLE II. Parameters summary for the sensitivity analysis

203 **A. Sensitivity on the amplitude:**

204 Conducting a sensitivity analysis on amplitude with respect to the parameters Σ and
 205 V can be a challenge due to the complexities involved in assessing their influence over a
 206 substantial range. A simple methodology we can adopt is encapsulating the entire signal
 207 using a singular metric. In this context, the peak-to-peak amplitude, denoted as a_{pp} , serves
 208 as an apt representative. This is mathematically expressed as: $a_{pp} = \text{Max}(S) - \text{Min}(S)$,
 209 where S represents the theoretical transmitted signal. This signal is derived utilizing the
 210 transmission coefficient (see Equation 3), the empirical incident signal as depicted in Figure
 211 3, and computational processing via MATLAB.

212 To ensure clarity in the plotted results, we adopt normalized metrics of a_{pp} , given by:

$$A_{pp} = s \times a_{pp}, \quad (8)$$

213 where s is a scaling factor.

214 In Figure 5 we present a semilogarithmic plot (semi-log x axis) of the variations of the
 215 peak to peak value A_{pp} with respect to (a) the parameter Σ utilizing the 3rd order model
 216 and (b) the parameter V using 4th order model with Σ fixed at $4000 \mu\text{m}^2$, and (c) Σ is
 217 fixed at $40000 \mu\text{m}^2$, for different values of Λ . These plots help in determining the effective

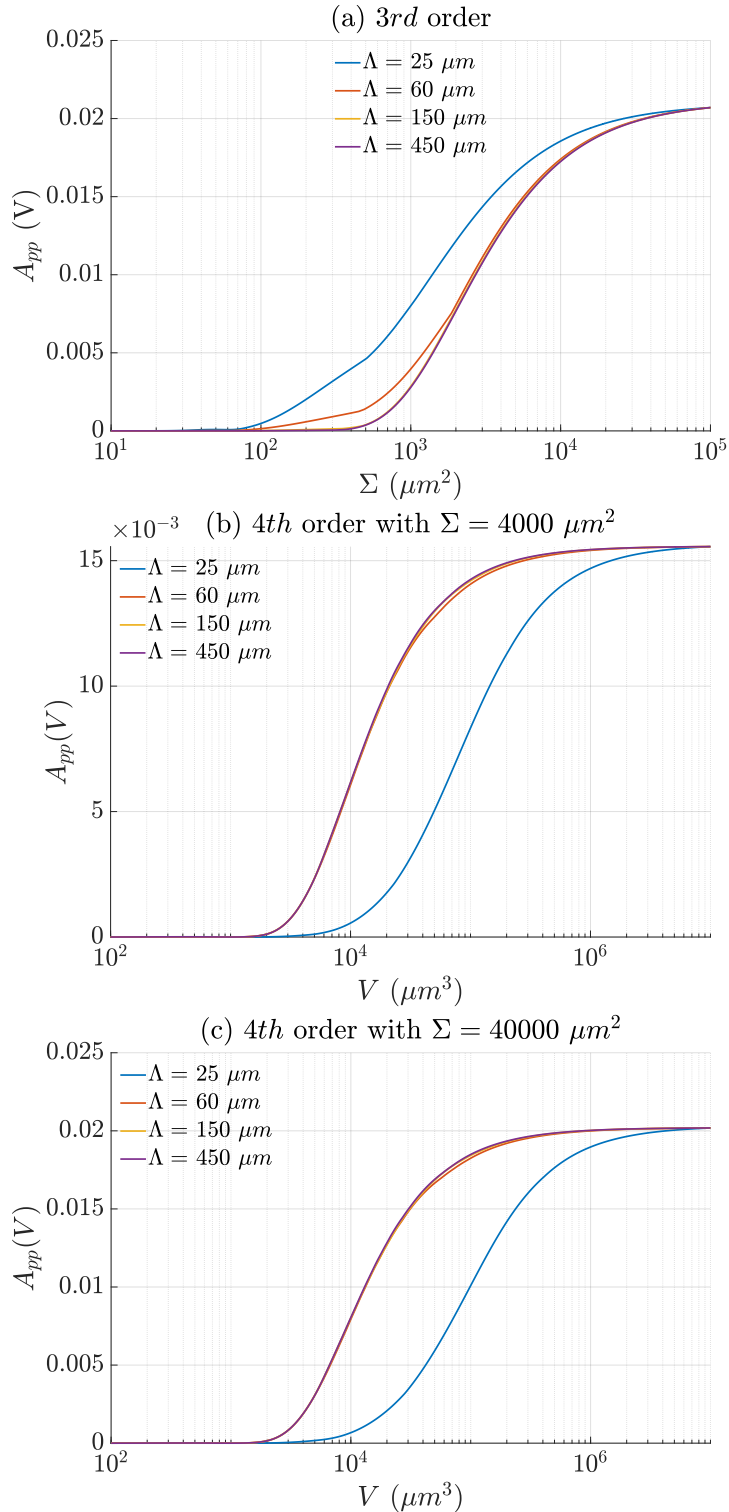


FIG. 5. (color online) Variation of A_{pp} defined by equation 8, (a) with respect to Σ using 3rd order model, (b) with respect to V using 4th order model for a fixed value of $\Sigma = 4000 \mu m^2$, and (c) with respect to V using 4th order model for $\Sigma = 40000 \mu m^2$, for different values of Λ .

218 range of Σ and V where they significantly influence the wave amplitude, and in identifying
 219 a threshold beyond which additional increases in these parameters have a minimal effect.
 220 When altering Λ , the resulting graphs consistently show an initial increase in A_{pp} as Σ and
 221 V increase, followed by stabilization after reaching a certain critical point. The key effective
 222 range for Σ is found to be between $80 \mu\text{m}^2$ and $2 \times 10^5 \mu\text{m}^2$, and for V , it lies approximately
 223 between $700 \mu\text{m}^3$ and $10^7 \mu\text{m}^3$. It seems that the interval of V is not influenced by Σ . These
 224 ranges, however, are subject to change with different Λ values, as is apparent in Figure 5.
 225 A comparison of the rate of change in relation to both parameters (Σ and V) reveals that
 226 V has a considerably greater influence on amplitude than Σ .

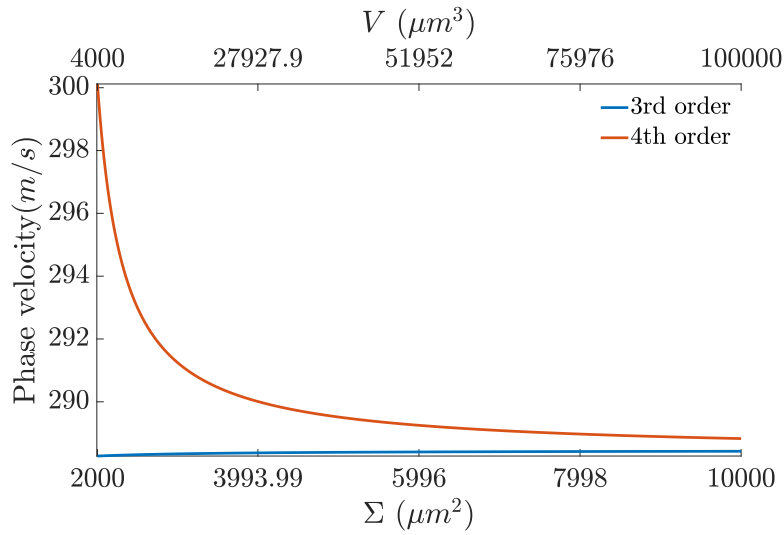
227 **B. Sensitivity on phase velocity and attenuation of the wave:**

228 In a porous medium, the wave number can be a complex quantity because of the presence
 229 of attenuation and dispersion effects. The real part is directly related to the phase velocity
 230 of the wave, and the imaginary part represents the attenuation due to the medium (Allard
 231 and Daigle, 1994). The complex wave number, can be represented as follows(Allard and
 232 Daigle, 1994):

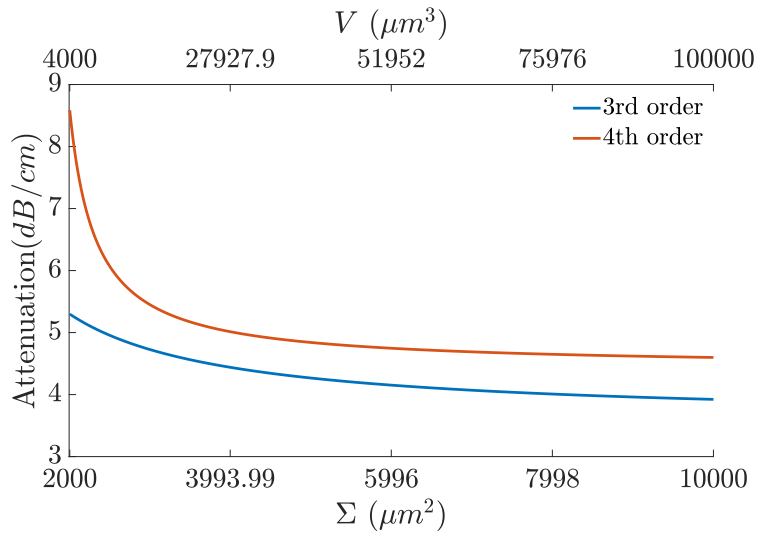
$$k(\omega) = \sqrt{\frac{K_f}{\rho_f}(\alpha(\omega)\beta(\omega))^{1/2}} = \frac{\omega}{V_p(\omega)} - iA(\omega), \quad (9)$$

233 where $V_p(\omega)$ represents the phase velocity of the wave, and $A(\omega)$ is the attenuation of the
 234 wave.

235 In Figure 6 we present a graphical representation of the (a) phase velocity, and (b)
 236 attenuation of the wave using 3rd and 4th order models at a fixed frequency of $f = 100$



(a) Phase velocity $V_p(f = 100 \text{ kHz})$



(b) Attenuation $A(f = 100 \text{ kHz})$

FIG. 6. (color online) Representation of the (a) phase velocity and (b) Attenuation with respect to Σ using 3rd order model (blue line), and V using 4th order model (red line)

237 kHz. The phase velocity (V_p) and attenuation (A) are depicted as functions of the parameter
 238 Σ (bottom axes) for the 3rd order model (illustrated by the blue curve). For 4th order model
 239 (red curve), variations are observed with respect to V (top axes), while maintaining Λ at a
 240 constant value of $100 \mu\text{m}$. For the phase velocity, as shown in Figure 6.(a), the parameter Σ
 241 appears to exert no influence, which aligns with theoretical expectations, given its exclusive
 242 association with the imaginary component of the response functions (refer to Equations
 243 5 and 6), which limits its influence. The parameter V exhibit a significant effect on the
 244 velocity, especially for lower values of V , where a decrease in V corresponds to an increase
 245 in wave velocity.

246 When examining attenuation in Figure 6.(b), it is evident that the inclusion of parameters
 247 Σ and V contributes to the wave's attenuation. However, the impact of Σ is marginal when
 248 contrasted with the effects V , thus ascribing limited significance to the third-order term Σ
 249 in both the phase velocity and attenuation of the wave.

250 C. Comparison between JCA in high frequency limit model with higher order 251 models:

252 In the investigation of the theoretical signal transmission models, the rationale for the in-
 253 tegration of the novel parameters Σ and V becomes evident upon comparative analysis. This
 254 comparison is predicated upon the distortion of the simulated transmitted signal, which, in
 255 this context, refers to the deviation of the signal's waveform from its original (incident) con-
 256 figuration upon the variations of the parameter Λ . To quantitatively assess this distortion,

257 the autocorrelation function of the predicted transmitted signal will be employed as a metric
 258 across a spectrum of Λ values:

$$R(\Lambda, k) = \sum_n S(\Lambda, n).S(\Lambda, n + k), \quad (10)$$

259 where $R(\Lambda, k)$ is the autocorrelation function, S the predicted transmitted signal, n the
 260 discrete time index, and k represents the time lag between the two signals being correlated.
 261 The choice of the autocorrelation function is simply because of its interesting symmetry
 262 property, facilitating a better visualization of the signal shape alterations. For the sake of
 263 clarity, we use scaled values of $R(\Lambda, k)$:

$$R_s(\Lambda, k) = sR(\Lambda, k), \quad (11)$$

264 where s is a scaling factor. In this context, since scaled values are employed, the absolute
 265 magnitudes of R_s are of secondary interest compared to the contour of the autocorrelation
 266 function, which serves as the primary indicator of signal integrity.

268 In Figure 7 we represent the results of the scaled autocorrelation function R_s for (a)
 269 JCA_{HF} model, (b) 3rd order model, and (c) 4th order model. The gradation from blue to
 270 yellow represents a range from low or negative correlation to high or positive correlation,
 271 respectively. An undistorted signal is anticipated to exhibit periodic peaks at varying k
 272 values, which do not necessarily have to be of equal magnitude. A signal not exhibiting such
 273 features is considered distorted.

274 Within the confines of the JCA_{HF} model (Figure 7.(a)), three distinct zones are discernible.
 275 Initially, an undistorted (baseline) zone is observed, characterized by clear periodic peaks

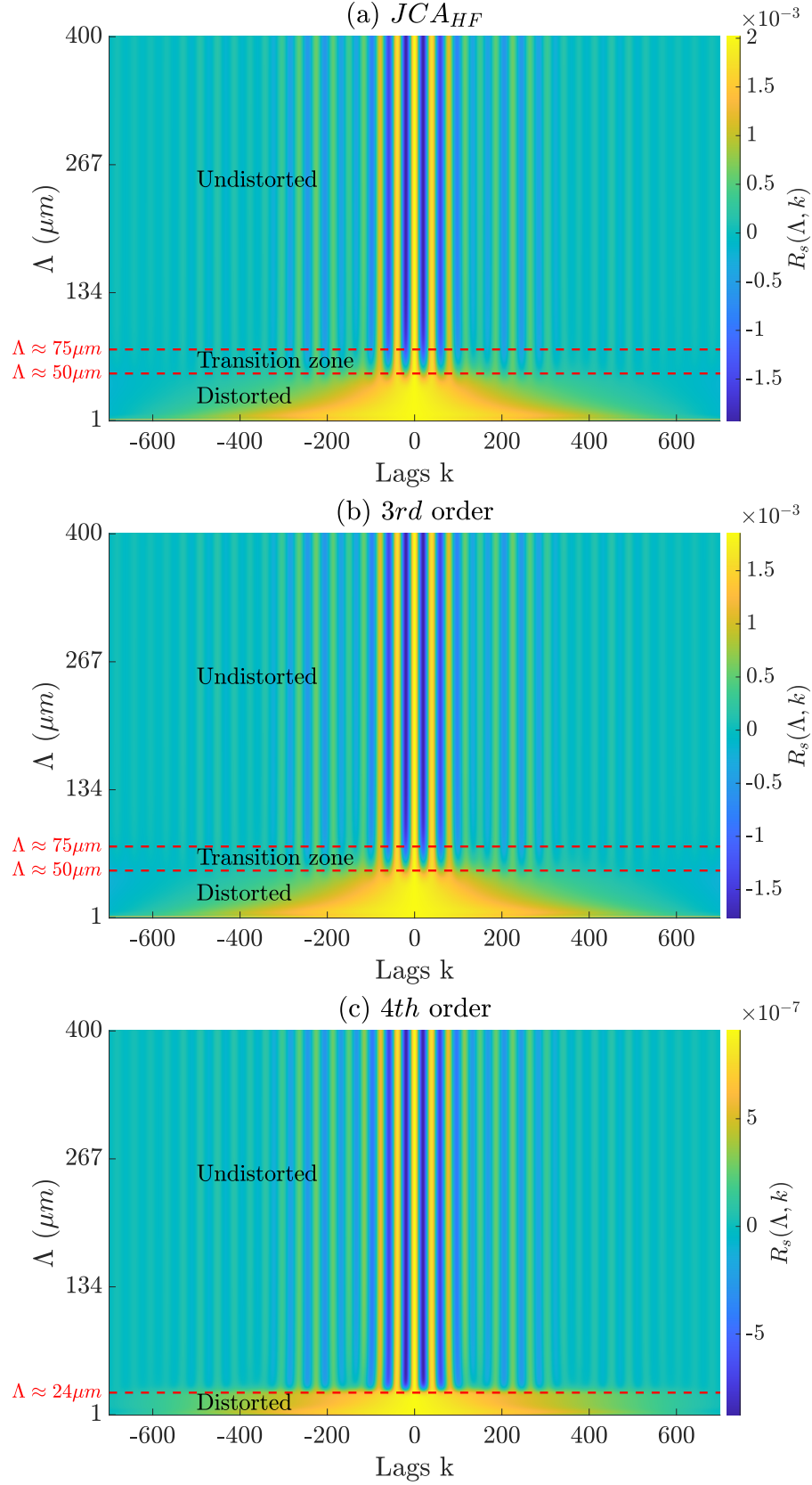


FIG. 7. (color online) Color-scaled image of R_s (Equation 11) using (a) JCA_{HF} , (b) 3rd order, and (c) 4th order models.

276 indicative of signal constancy. This is followed by a transition zone, spanning from $\Lambda \approx 50$
 277 μm to $\Lambda \approx 75 \mu\text{m}$, where these periodic peaks are somewhat less visible. Finally, a distorted
 278 zone is identified, in which periodicity is absent, and signal deformation is evident. Regard-
 279 ing the *3rd* order model (Figure 7.(b)), computed at a constant $\Sigma = 4000 \mu\text{m}^2$, there appears
 280 to be no substantial enhancement in signal integrity, which is expected, as we have seen in
 281 Section IV that the influence of Σ is marginal. Conversely, the *4th* order model (Figure
 282 7.(c)), calculated for $\Sigma = 4000 \mu\text{m}^2$ and $V = 2000 \mu\text{m}^3$, manifests a notable augmentation
 283 in the signal's integrity, with the distorted zone compressed to $\Lambda \approx 24 \mu\text{m}$.
 284 This demonstration is critical as it underpins the limitation of the JCA_{HF} and *3rd* order
 285 models for applications wherein the viscous characteristic length of a porous medium en-
 286 croaches upon the distorted zone, thereby mandating the inclusion of the term V .
 287 It is imperative for the reader to note that the Λ values representing the different zones for
 288 the JCA_{HF} and higher-order terms are not general but are contingent upon the specific val-
 289 ues of α_∞ , ϕ , the material thickness L , along with the characteristics of the incident signal,
 290 such as its frequency, used in this research. For illustrative purposes, Figure 8 delineates
 291 the same R_s function albeit utilizing a non-resistive material parameter set with $\phi = 0.99$,
 292 $\alpha_\infty = 1.03$, and $L = 1 \text{ cm}$.

293 The observations from Figure 8 suggest a contracted distorted zone, specifically around
 294 $\Lambda \approx 18 \mu\text{m}^2$ for both JCA_{HF} and *3rd* order models. However, an incremental enhancement
 295 is noted upon the introduction of the term V ($\Lambda \approx 11 \mu\text{m}$). This implies that in the
 296 context of materials with pronounced resistive characteristics, the term V is essential for
 297 accurate characterization. The forthcoming section will substantiate the efficacy of the *4th*

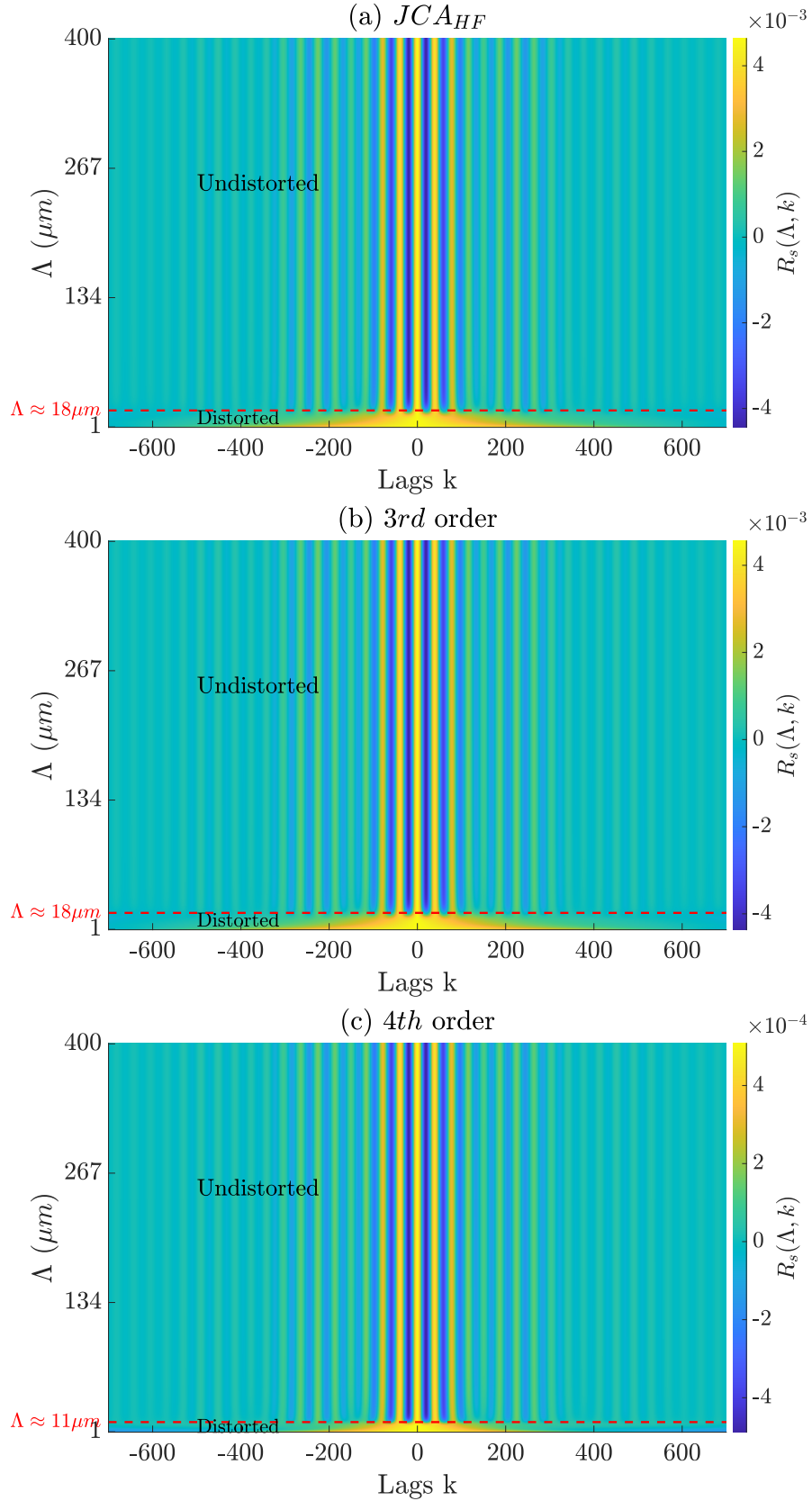


FIG. 8. (color online) Color-scaled image of R_s (Equation 11) using (a) JCA_{HF} , (b) 3rd order, and (c) 4th order models, for $\phi = 0.99$, $\alpha_\infty = 1.03$, and $L = 1$ cm.

298 order model by contrasting it with empirical data obtained from signals transmitted through
 299 highly attenuative materials.

300 V. EXPERIMENTAL VALIDATION:

301 In line with the previous discussions, the application of the *4th* order model becomes per-
 302 tinent in cases involving porous materials whose viscous characteristic length, Λ , is situated
 303 within the distorted zone delineated in Figure 7(a)-(b). This is particularly true for foams
 304 PU1 and PU2, which we will demonstrate possess Λ values that reside within this zone,
 305 thereby rendering the JCA_{HF} and *3rd* order models inadequate for their characterization.
 306 Initially, our analysis begins with foam F, demonstrating that the JCA_{HF} model suffices
 307 for precise characterization, obviating the need for higher-order terms. Subsequently, we
 308 encounter the issue when characterizing foams PU1 and PU2 with the JCA_{HF} and *3rd* or-
 309 der models, underscoring the necessity to incorporate the term V into the characterization
 310 process. This requirement arises from the fact that the viscous characteristic length Λ for
 311 these foams is approximately $30 \mu\text{m}$, placing them squarely within the distorted zone.

312 Foam F

313 In Figure 9, the results of the JCA_{HF} model's application for the acoustic characterization
 314 of Foam F are shown. The signal reconstruction was predicated for the values $\phi = 0.99$,
 315 $\alpha_\infty = 1.028$, and $\Lambda = 214.71 \mu\text{m}$. The adequacy of the JCA_{HF} model is substantiated by its
 316 precision in describing the acoustic properties of the foam. This precision is attributable to

317 the value of Λ being within the designated unaltered region, thereby negating the necessity
 318 for integrating higher-order terms into the model for this specific material characterization.

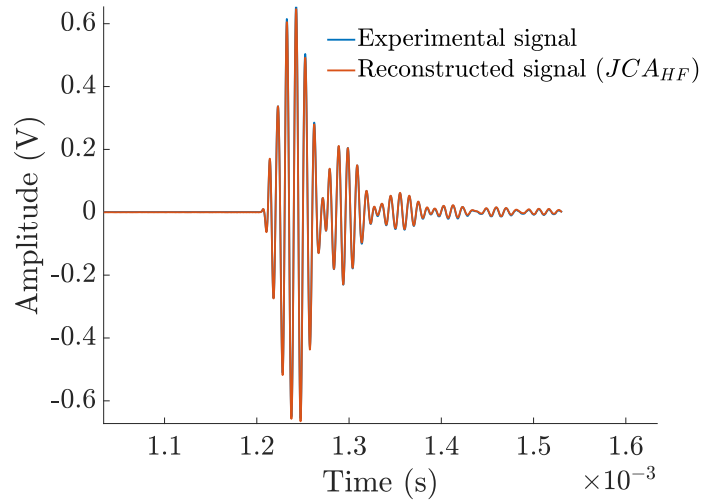


FIG. 9. (color online) Reconstructed signal using JCA_{HF} model, compared with the experimental signal acquired from foam F

319 Foams PU1 and PU2

320 In Figure 10 we compare the experimental signal acquired from foam PU1 with simulated
 321 signals generated using the JCA_{HF} (Figure 10.(a)) and 3rd order (Figure 10.(b)-(c)) models
 322 for viscous characteristic lengths $\Lambda = 25\mu\text{m}$ and $\Lambda = 40\mu\text{m}$. The selection of these specific
 323 Λ values is necessary to account for the notably low amplitude observed in the experimental
 324 transmitted signal, which implicates the need for lesser Λ values. The graphical representa-
 325 tion illustrates that both models, JCA_{HF} and 3rd order, are ineffectual in replicating the
 326 experimental signal's characteristics.

327 Conversely, Figure 11 extends this comparison to foam PU2, arriving at a similar conclusion
 328 regarding the performance of the aforementioned models. However, an interesting outcome
 329 is presented in Figure 12, where the adoption of the 4th order model yields a simulation
 330 that closely aligns with the experimental signals for both foam specimens, PU1 and PU2.
 331 This improved correlation substantiates the 4th order model’s efficacy in the accurate sim-
 332 ulation of transmitted signals for these particular foam materials. The values obtained for
 333 the parameters for all three foams are tabulated in Table III

Foam specimen	F	PU1	PU2
Model used	JCA_{HF}	4th order	4th order
ϕ	0,99	0,98	0,98
α_∞	1,028	1,13	1.18
Λ (μm)	214,71	38,7	36,7
Σ (μm^2)	/	6470	4076
V (μm^3)	/	7790	5717

TABLE III. Summary of the results obtained for the different foam specimens

334 **VI. CONCLUSION**

335 A new model has been proposed that extends the high-frequency expansion on the skin
 336 depths of response functions by incorporating higher-order terms. This model introduces two

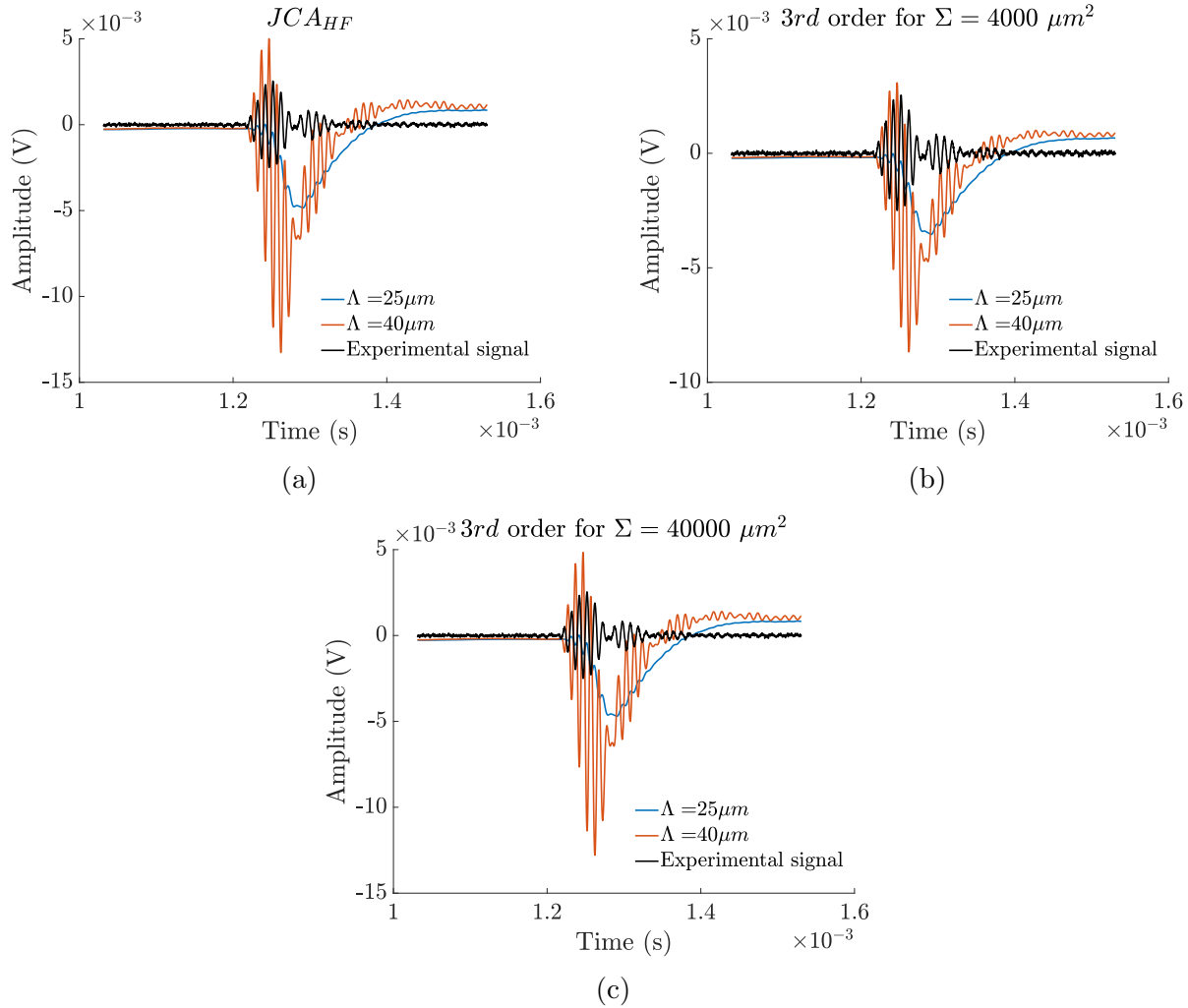


FIG. 10. (color online) Reconstructed signal for values of $\Lambda = 25 \mu\text{m}$ and $40 \mu\text{m}$ compared with experimental signal for foam PU1 using models: (a) JCA_{HF} , (b) 3rd order for $\Sigma = 4000 \mu\text{m}^2$, and (c) 3rd order for $\Sigma = 40000 \mu\text{m}^2$

337 non-acoustical parameters, Σ and V . It has been observed that V exerts a more substantial
 338 effect on wave amplitude, phase velocity, attenuation, and overall signal integrity compared
 339 to Σ . Particularly under low values of Λ , the simulated transmitted signal using both
 340 the Johnson-Champoux-Allard (JCA) model at high frequencies and the third-order model
 341 (which only includes up to Σ) shows distortion. Conversely, incorporating the V term

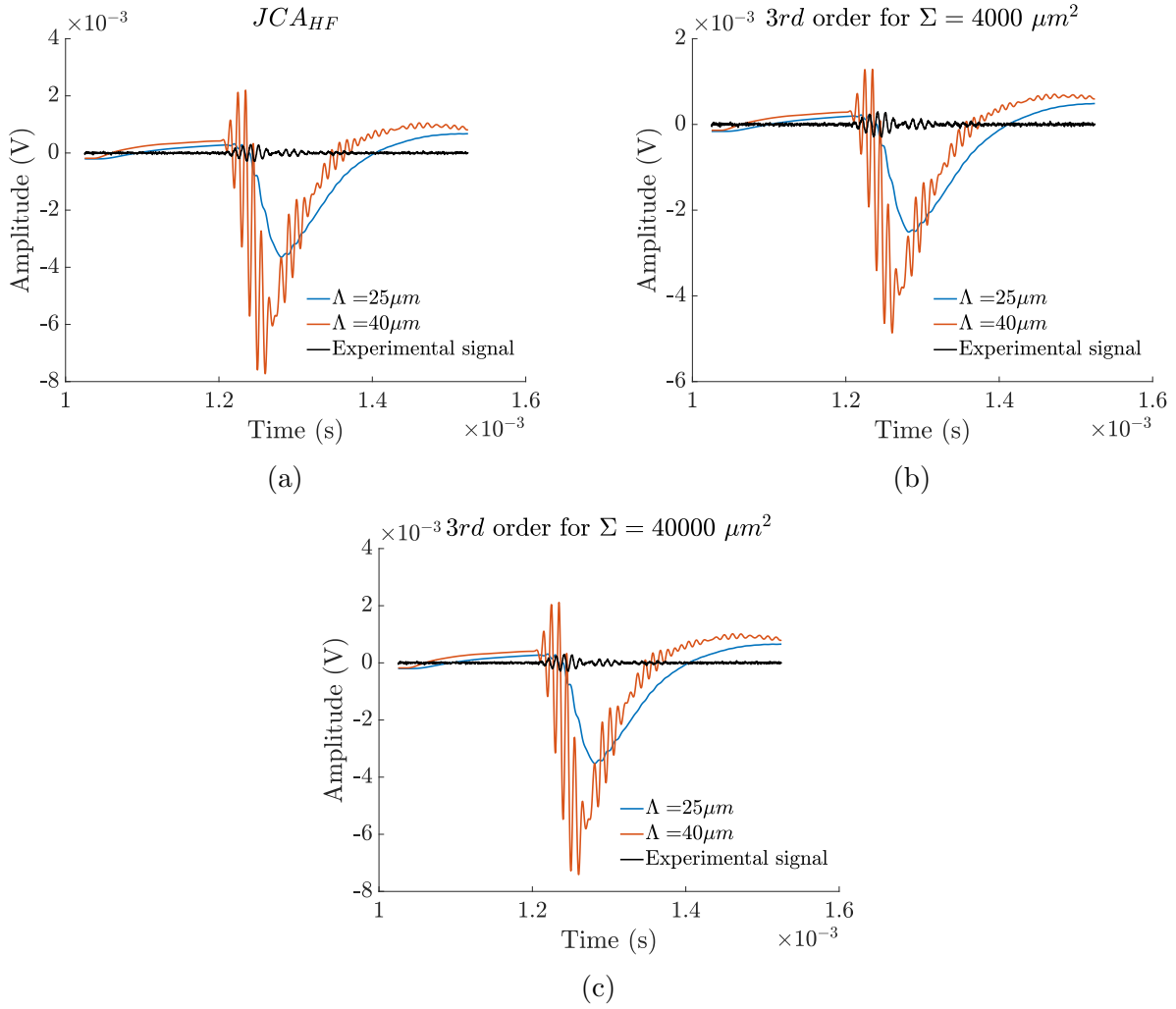


FIG. 11. (color online) Reconstructed signal for values of $\Lambda = 25 \mu\text{m}$ and $40 \mu\text{m}$ compared with experimental signal for foam PU2 using models: (a) JCA_{HF} , (b) 3rd order for $\Sigma = 4000 \mu\text{m}^2$, and (c) 3rd order for $\Sigma = 40000 \mu\text{m}^2$

342 significantly improves signal integrity, leading to more precise material characterization.
 343 This model's effectiveness was confirmed through experiments on two plastic foams, PU1 and
 344 PU2, which possess high attenuation properties. The process of characterizing these two
 345 foams highlighted the importance of including the fourth-order term V and demonstrated
 346 the limitations of both the JCA model at high frequencies and the third-order model.

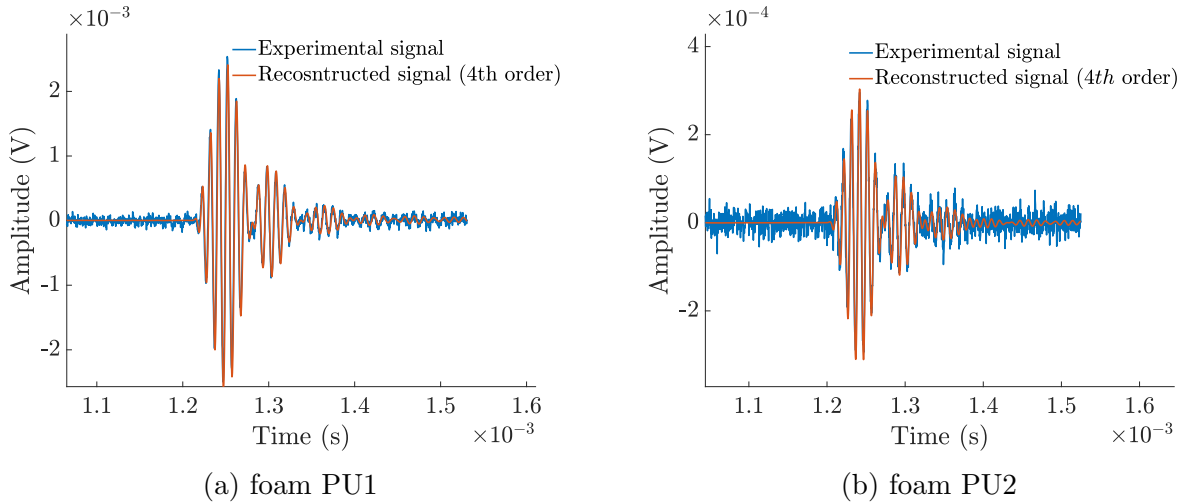


FIG. 12. (color online) Reconstructed signal using 4th order model compared with experimental signal for foams (a) PU1 and (b) PU2.

347 In summary, incorporating these terms is crucial for accurately characterizing rigid porous
 348 materials with high attenuation properties and should not be overlooked.

349 The implications of these findings are far-reaching, impacting various industrial sectors
 350 such as engineering, manufacturing, performance optimization, product development, and
 351 innovation. Moreover, they extend to acoustic applications, broadening the study's practical
 352 significance and potential influence.

353 VII. AUTHOR DECLARATIONS

354 The authors declare that they have no conflicts of interest to disclose in the context of
 355 this research. They also affirm that their work adheres to ethical approval, indicating that
 356 their research does not involve experiments on animal subjects and/or human participants.

357 **VIII. DATA AVAILABILITY**

358 Data is available upon request from the authors. The information supporting the findings
359 of this study is accessible from the corresponding author upon reasonable request.

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