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Searching for the Reference Point

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Abstract. Although reference dependence plays a central role in explaining behavior, little is known about the way that reference points are selected. This paper identifies empirically which reference point people use in decision under risk. We assume a comprehensive reference-dependent model that nests the main reference-dependent theories, including prospect theory, and that allows for isolating the reference point rule from other behavioral parameters. Our experiment involved high stakes with payoffs up to a week’s salary. We used an optimal design to select the choices in the experiment and Bayesian hierarchical modeling for estimation. The most common reference points were the status quo and a security level (the maximum of the minimal outcomes of the prospects in a choice). We found little support for the use of expectations-based reference points.

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Keywords: reference point • reference dependence • Bayesian hierarchical modeling • large-stake experiment

1. Introduction

A key insight of behavioral decision making is that people evaluate outcomes as gains and losses from a reference point. Reference dependence is central in prospect theory, the most influential descriptive theory of decision under risk, and it plays a crucial role in explaining people’s attitudes toward risk (Rabin 2000, Wakker 2010). Evidence abounds, from both the laboratory and the field, that preferences are reference dependent.¹

A fundamental problem of prospect theory and other reference-dependent theories is that they are unclear about the way that reference points are formed. Back in 1952, Markowitz (1952, p. 157) already remarked about customary wealth, which plays the role of the reference point in his analysis: “It would be convenient if I had a formula from which customary wealth could be calculated when this was not equal to present wealth. But I do not have such a rule and formula.” Tversky and Kahneman (1991, pp. 1046–1047) argued that “although the reference point usually corresponds to the decision maker’s current position it can also be influenced by aspirations, expectations, norms, and social comparisons.” This lack of clarity is undesirable, because it creates extra freedom in deriving predictions, making it impossible to rigorously test reference-dependent theories empirically.²

Reviewing the literature more than 60 years after Markowitz (1952), Barberis (2013, p. 192) concludes that addressing the formation of the reference point is still a key challenge to apply prospect theory.

Empirical studies on the formation of reference points are scarce, and their message is mixed. Some evidence³ is consistent with a stochastic reference point that is based on people’s expectations as in the model of Köszegi and Rabin (2006, 2007) and the closely related disappointment model of Delquié and Cillo (2006), but other evidence is not.⁴ Moreover, the interpretation of the available evidence is often unclear, because the data can be consistent with several reference points simultaneously.⁵

This paper explores how people form their reference point in decision under risk. Guided by the available literature, we specified six reference point rules, and we estimated the support for each of these in a high-stakes experiment with payments up to a week’s salary. The selected rules vary depending on whether they are choice specific (the reference point is determined by the choice set) or prospect specific (the reference point is determined by the prospect itself), whether they are stochastic or deterministic, and whether they are defined only by the outcome dimension or by both the outcome and the probability dimension. Out-of-sample

predictions indicated that these rules covered the preferences of our subjects well.

All of the reference points that we consider can be identified through choices, and we work within the revealed preference paradigm. We do not require introspective data, which makes it easy to apply these rules in practical decision analysis. In this, we follow Rabin (2013), who argues that new models are maximally useful if they are “portable” and use the same independent variables as existing models. The core model of decision under risk is expected utility, which only uses probabilities and outcomes as independent variables. All of our reference point rules can also be derived from probabilities and outcomes, and they are, therefore, portable.

We define a comprehensive reference-dependent model that includes the main reference-dependent theories as special cases. In our model, the reference point is a parameter, which we can estimate just as any other model parameter.⁶ This allows for comparing reference point rules *ceteris paribus* (i.e., to isolate the reference point rule from the specification of the other behavioral parameters, like utility curvature, probability weighting, and loss aversion). We use a Bayesian hierarchical model to estimate each subject’s reference point rule. Bayesian modeling estimates the individual-specific parameters by accounting for similarities between individuals in the population. Several recent studies have shown that Bayesian hierarchical modeling leads to more precise estimates of prospect theory’s parameters and prevents inference from being dominated by outliers (Nilsson et al. 2011, Murphy and ten Brincke 2018). We show how Bayesian hierarchical modeling can also be used to estimate the reference point rule that subjects use. Choices were optimally designed to maximize the orthogonality between questions so as to obtain more precise and robust estimates.

Our results indicate that two reference point rules stand out: the Status Quo and MaxMin, a security-based rule according to which subjects adopt the maximum outcome that they can reach for sure as their reference point (Schneider and Day 2018). Together, these two reference points account for the behavior of over 60% of our subjects. We found little support for the use of the prospect itself as a reference point, the only rule in our study with a stochastic reference point, and at most, 20% of our subjects used an expectations-based reference point rule (the prospect itself or the expected value of the prospect).

2. Theoretical Background

A *prospect* is a *probability distribution* over money amounts. *Simple prospects* assign probability 1 to a finite set of outcomes. We denote these simple prospects as $(p_1, x_1; \dots; p_n, x_n)$, which means that they pay $\$x_j$ with probability p_j , $j = 1, \dots, n$. We identify simple prospects with their cumulative distribution functions and denote them with capital Roman letters (F, G). The decision maker has a weak preference relation \succsim over

the set of prospects, and as usual, we denote strict preference by $>$, indifference by \sim , and the reversed preferences by \leq and $<$. The function V defined from the set of simple prospects to the reals represents \succsim if, for all prospects F, G , $F \succsim G$ is equivalent to $V(F) \geq V(G)$.

Outcomes are defined as gains and losses relative to a *reference point* r . An outcome x is a *gain* if $x > r$ and a *loss* if $x < r$.

2.1. Prospect Theory

Under prospect theory (Tversky and Kahneman 1992),⁷ there exist *probability weighting functions* w^+ and w^- for gains and losses and a nondecreasing *gain-loss utility function* $U : \mathbb{R} \rightarrow \mathbb{R}$ with $U(0) = 0$ such that preferences are represented by

$$F \rightarrow PT_r(F) = \int_{x \geq r} U(x - r) dw^+(1 - F) + \int_{x \leq r} U(x - r) dw^-(F). \quad (1)$$

The integrals in Equation (1) are Lebesgue integrals with respect to distorted measures $w^+(1 - F)$ and $w^-(F)$. For losses the weighting applies to the cumulative distribution F , and for gains to the decumulative distribution $1 - F$.

The functions w^+ and w^- are nondecreasing and map probabilities into $[0, 1]$ with $w^i(0) = 0$, $w^i(1) = 1$, $i = +, -$. When the functions w^i are linear, PT reduces to *expected utility* with reference-dependent utility:

$$F \rightarrow EU_r(F) = \int U(x - r) dF. \quad (2)$$

Equation (2) shows that reference dependence by itself does not violate expected utility as long as the reference point is held fixed (see also Schmidt 2003).

Based on empirical observations, Tversky and Kahneman (1992) hypothesized specific shapes for the functions U , w^+ , and w^- . The gain-loss utility U is S shaped: concave for gains and convex for losses. It is steeper for losses than for gains to capture loss aversion, the finding that losses loom larger than gains. The probability weighting functions are inverse S shaped, reflecting overweighting of small probabilities and underweighting of middle and large probabilities.

2.2. Stochastic Reference Points

Tversky and Kahneman (1992) defined prospect theory for a riskless reference point r . Sugden (2003) introduced two modifications to Equation (2). First, he allowed for a stochastic reference point, and second, he suggested a decomposition of utility into a function v , which reflects the decision maker’s absolute evaluation of outcomes (independent of the reference point), and a function U , which reflects his attitude toward gains and losses of utility. Köbberling and Wakker (2005) interpreted v as reflecting the normative component of utility. Following up on the suggestion of Sugden (2003), Köszegi and

Rabin (2006, 2007) proposed the following representation of preferences over prospects F :

$$F \rightarrow KR_R(F) = \int v(x)dF + \iint U(v(x) - v(r))dFdR. \quad (3)$$

The first term $\int v(x)dF$ represents the decision maker's (expected) consumption utility. As in the disappointment models described below, Köszegi and Rabin (2006, 2007) allowed the reference point to be prospect specific (i.e., to differ between the prospects in the choice set). Unlike models with a choice-specific reference point (common to all prospects in the choice set), consumption utility is crucial for models with a prospect-specific reference point to rule out implausible choice behavior. We give an example of such implausible choice behavior in Endnote 8. Köszegi and Rabin (2007) defined consumption utility over final wealth. In our study, the subjects' initial wealth remained constant, and we, therefore, omit it and equate final wealth with outcome.

In empirical applications, consumption utility is usually taken to be linear (e.g., Heidhues and Köszegi 2008, Abeler et al. 2011, Gill and Prowse 2012, Eil and Lien 2014). Even if v is not linear over final wealth, the outcomes used in our study represent marginal increases of wealth, thereby justifying us to approximate $v(x)$ by x . Then, Equation (3) becomes

$$KR_R(F) = \int x dF + \int EU_r(F) dR. \quad (4)$$

Although prospect theory does not specify the reference point, Köszegi and Rabin (2007) presented a theory in which reference points are determined by the decision maker's rational expectations. They distinguish two specifications of the reference point: one prospect specific and one choice specific. In a *choice-acclimating personal equilibrium* (CPE), the reference point is the prospect itself. This prospect-specific reference point gives⁸

$$KR(F) = \int x dF + \int EU_x(F) dF. \quad (5)$$

In a *preferred personal equilibrium* (PPE), the reference point is choice specific and equal to the preferred prospect in the choice set.

There is no probability weighting in Equation (4). It is unclear how the rational expectations reference point should be defined in the presence of probability weighting. Köszegi and Rabin (2006, 2007) do not address this problem and leave out probability weighting, although they acknowledge its relevance (Köszegi and Rabin 2006, footnote 2, p. 1137). For a version of prospect theory with a stochastic reference point and probability weighting, see Schmidt et al. (2008).

2.3. Disappointment Models

The model of Köszegi and Rabin (2006, 2007) is close to the disappointment models of Bell (1985), Loomes and

Sugden (1986), Gul (1991), and Delquié and Cillo (2006). The model of Bell (1985) is equivalent to Equation (3), with $v(r)$ replaced by the expected consumption value of the prospect (although Bell (1985) remarks that this may be too restrictive and also presents a more general model), the model of Loomes and Sugden (1986) is equivalent to Equation (3) with $v(r)$ replaced by the expected consumption utility of the prospect,⁹ and the model of Gul (1991) is equivalent to Equation (3) with $v(r)$ replaced by the certainty equivalent of the prospect. The model of Delquié and Cillo (2006) is identical to the CPE model of Köszegi and Rabin (2007) (Equation (5)). Masatlioglu and Raymond (2016) formally characterize the link between the CPE model of Köszegi and Rabin (2007), the disappointment models, and other generalizations of expected utility. They show that, if the gain-loss utility function U is linear and the decision maker satisfies first-order stochastic dominance, CPE is equal to the intersection between rank-dependent utility (Quiggin 1981, 1982) and quadratic utility (Machina 1982; Chew et al. 1991, 1994).

2.4. General Reference-dependent Specification

To isolate the reference point, we must use the same model specification across all reference point rules. That is, all other behavioral parameters must enter the model in the same way regardless of the reference point rule. To address this *ceteris paribus* principle, we adopt the following general reference-dependent model:

$$F \rightarrow RD(F) = \int x dF + \int PT_r(F) dR. \quad (6)$$

Equation (6) contains prospect theory (Equation (1)), the model of Köszegi and Rabin (2006, 2007) (Equation (4)), and the disappointment models as special cases. In Equation (6), probability weighting plays a role in the psychological part of the model (the second term in the sum), but it does not affect consumption utility (the first term). This seems reasonable, because consumption utility reflects the "rational" part of utility and because probability weighting is usually considered a deviation from rationality. Adjusting the model to also include probability weighting in consumption utility is straightforward.

Probability weighting does not affect the (stochastic) reference points either. In this, we follow the literature on stochastic reference points (Sugden 2003; Delquié and Cillo 2006; Köszegi and Rabin 2006, 2007; Schmidt et al. 2008). We will consider alternative specifications in Section 6.4.

3. Reference Point Rules

A reference point rule specifies for each choice situation which reference point is used. Table 1 summarizes the reference point rules that we studied. We distinguish reference point rules along three dimensions. First, we distinguish whether they are *prospect specific*, in which case each prospect has its own reference point, or *choice specific*, in which case there is a common reference point for all

Table 1. The Reference Point Rules Studied in This Paper

	Prospect/choice specific	Stochastic	Uses probability
Status Quo	Choice	No	No
MaxMin	Choice	No	No
MinMax	Choice	No	No
X at Max P	Choice	No	Yes
Expected Value	Prospect	No	Yes
Prospect Itself	Prospect	Yes	Yes

prospects within a choice set. Second, we distinguish whether the reference point is deterministic or stochastic. Third, we distinguish whether the rules use only payoffs to determine the reference point or both payoffs and probabilities.

The first reference point rule is the *Status Quo*, which is often used in experimental studies of reference dependence. Our subjects knew that they would receive a participation fee for sure. Consequently, we took the participation fee as the Status Quo reference point and any extra money that subjects could win if one of their choices was played out for real as a gain. Because all outcomes in our experiment were strictly positive, with this reference point, subjects could suffer no losses. Expected utility maximization is the special case of Equation (6) with the Status Quo reference point where subjects do not weight probabilities. Expected value maximization is the special case of expected utility with the Status Quo as the reference point where subjects have linear utility. The Status Quo is a choice-specific reference point, because it leads to the same reference point for all prospects in a choice set.

MaxMin, the second reference point rule, is based on Hershey and Schoemaker (1985). They found that, when asked for the probability p that made them indifferent between outcome z for sure and a prospect $(p, x_1; 1 - p, x_2)$, with $x_1 > z > x_2$, their subjects took z as their reference point and perceived $x_1 - z$ as a gain and $x_2 - z$ as a loss. Bleichrodt et al. (2001), van Osch et al. (2004, 2006), and Van Osch and Stiggelbout (2008) found similar evidence for such a strategy in medical decisions. For example, van Osch et al. (2006) asked their subjects to think aloud while choosing. The most common reasoning in a choice between life duration z for sure and a prospect $(p, x_1; 1 - p, x_2)$ was “I can gain $x - z$ years if the gamble goes well or lose $z - y$ if it doesn’t” van Osch et al. (2006, table 1).

MaxMin generalizes the above line of reasoning to the choice between any two prospects.¹⁰ It posits that, in a comparison between two prospects, people look at the minimum outcomes of the two prospects and take the maximum of these as their reference point. This reference point is the amount that they can obtain for sure. For example, in a comparison between $(0.50, 100; 0.50, 0)$ and $(0.25, 75; 0.75, 25)$, the minimum outcomes are 0 and 25, and because 25 exceeds 0, MaxMin implies that subjects take 25 as their reference point and view 75 and 100 as gains and 0 as a loss.

MaxMin is a cautious rule and implies that people are looking for security. *MinMax* is the bold counterpart of MaxMin. A MinMax decision maker looks at the maximal opportunities and takes the minimum of the maximum outcomes as his reference point. Hence, MinMax predicts that the decision maker will take 75 as his reference point when choosing between $(0.50, 100; 0.50, 0)$ and $(0.25, 75; 0.75, 25)$ and perceives 100 as a gain and 25 and 0 as losses.

The MaxMin and the MinMax rules both look at extreme outcomes. One reason is that these outcomes are salient. Another salient outcome is the payoff with the highest probability, and our next rule, *X at Max P*, uses this outcome as the reference point. The importance of salience is widely documented in cognitive psychology (Kahneman 2011). Barber and Odean (2008) and Chetty et al. (2009) show the effect of salience on economic decisions. Bordalo et al. (2012) present a theory of salience in decision under risk.

The final two reference points that we considered are the *Expected Value* of the prospect, such as in the disappointment models of Bell (1985) and Loomes and Sugden (1986),¹¹ and the *Prospect Itself*, such as in the disappointment model of Delqu   and Cillo (2006) and the CPE model of K  szegi and Rabin (2007). Unlike the other reference points, these reference points are prospect specific. The prospect itself is the only rule that specifies a stochastic reference point. If the prospect itself is the reference point, then the decision maker will, for example, reframe the prospect $(0.50, 100; 0.50, 0)$ as a 25% chance to gain 100 (if he wins 100 and 0 is the reference point, the probability of this happening is $0.50 \times 0.50 = 0.25$), a 25% chance to lose 100 (if he wins nothing and 100 is the reference point), and a 50% chance that he wins or loses nothing (if either he wins 100 and 100 is the reference point, or he wins nothing and nothing is the reference point). The decision maker’s gain–loss utility is then $w^+ (.25)U(100) + w^- (.25)U(-100)$.

Two points are worth making. First, K  szegi and Rabin (2007) propose the CPE model to describe choices with large time delays between choice and outcome, like for example, in insurance decisions. We use the CPE model outside this specific context as did others before us (e.g., Rosato and Tymula 2016), because it is tractable, both theoretically and empirically. Second, we do not consider the rule that specifies that the

preferred prospect in a choice is used as the reference point, such as in the PPE model of Köszegi and Rabin (2007), because the model in Equation (6) is then defined recursively and cannot be estimated.

4. Experiment

4.1. Subjects and Payoffs

The subjects were 139 students and employees from the Technical University of Moldova (49 females, age range of 17–47 years old, average age 22 years old). They received a 50 Lei participation fee (about \$4, which was \$8 in purchasing power parity at the time of the experiment). To incentivize the experiment, each subject had a one-third chance to be selected to play out one of their choices for real. The choice that was played out for real was randomly determined. Our analysis assumed that subjects consider each choice in isolation from the other choices and from the one-third chance that they would be selected to play out one of their choices for real. This assumption is common in experimental economics, and there exists support for it (Starmer and Sugden 1991, Cubitt et al. 1998, Bardsley et al. 2010).¹² The subjects did not know the outcomes of the prospects to come, preventing them from evaluating the experiment as a single prospect.

The payoffs were substantial. The subjects who played out their choices for real earned 330 Lei on average, which was more than one-half the average week’s salary in Moldova at the time of the experiment. Two subjects won about 600 Lei, the average week’s salary.

4.2. Procedure

The experiment was computer run in group sessions of 10–15 subjects. Subjects took 30 minutes on

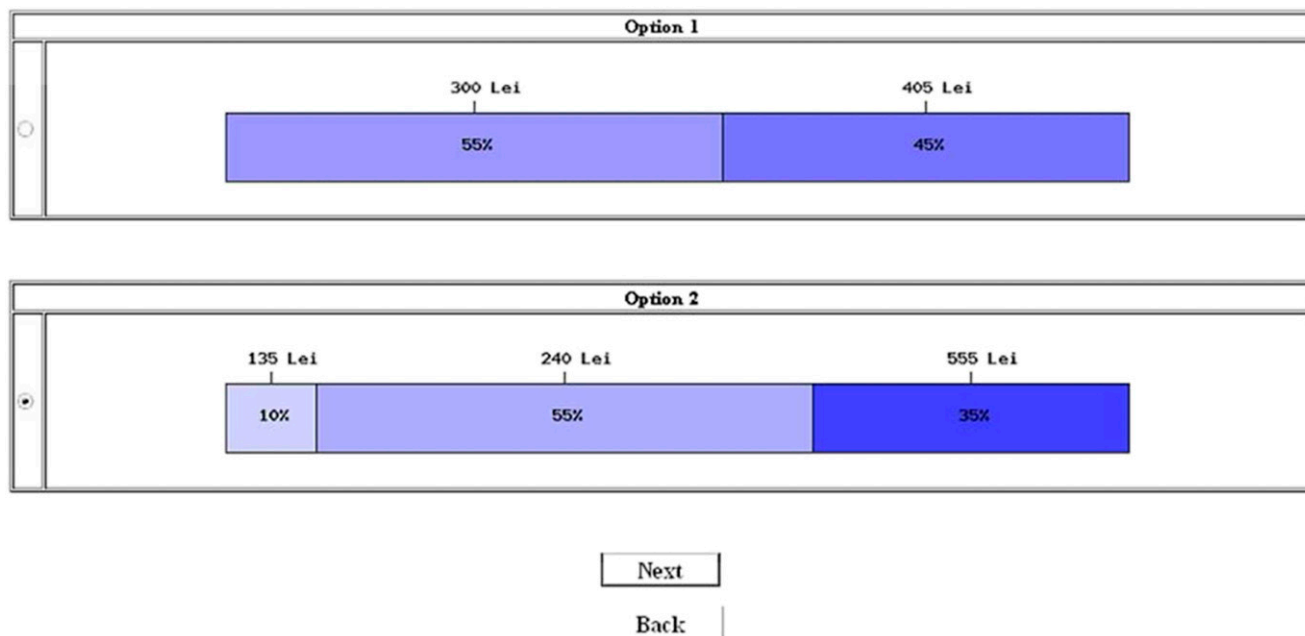
average to complete the experiment, including instructions.

Subjects made 70 choices in total. The 70 choices are listed in Appendix A, including the reference points predicted by each of the rules. The different rules predicted widely different reference points, and the predicted reference points varied substantially across choices (except, of course, for the Status Quo).

Each choice involved two options: Option 1 and Option 2. The options had between one and four possible outcomes, all strictly positive to make sure that subjects would not leave the experiment having lost money. Note that under five of the six reference point rules (the exception is the Status Quo), some strictly positive outcomes will be perceived as losses depending on the reference point. We randomized the order of the choices, and we also randomized whether a prospect was presented as Option 1 or Option 2.

The selection of choices ensured the complete coverage of the outcome and probability space and a balanced pairing of prospects with different numbers of outcomes to avoid favoring specific reference point rules. Eight homogeneous groups of choices were created, with each group containing all possible choices from a 20×20 outcome probability grid. Within each of these groups, a computationally intensive optimal design procedure that minimized the total pairwise correlation between choices was applied to arrive at the final much smaller set of choices. The intuition behind the optimal design procedure is that, just like with orthogonal covariates in linear regression, minimally correlated choices should lead to more efficient and more robust estimates of the behavioral parameters. The procedure

Figure 1. (Color online) Presentation of the Choices in the Experiment



for the construction of the homogeneous groups of questions and the computational details of the optimal design are provided in Appendix B.

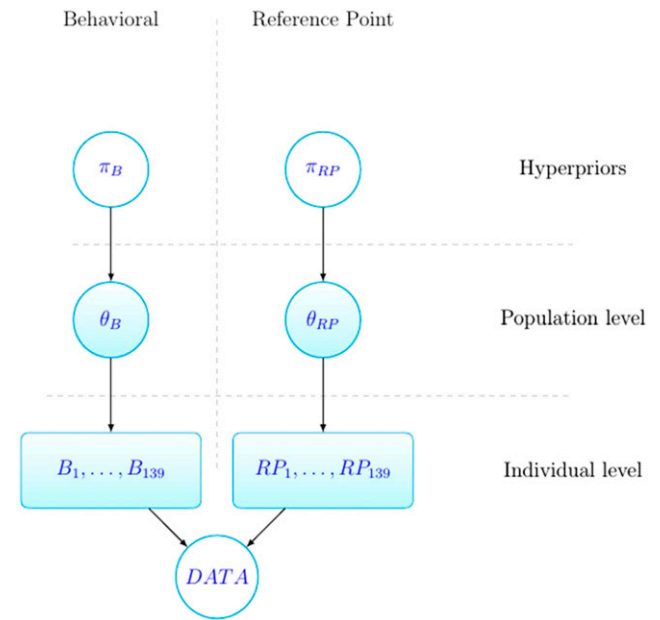
Figure 1 shows how the choices were displayed. Prospects were presented as horizontal bars with as many parts as there were different payoffs. The size of each part corresponded with the probability of the payoff. The intensity of the color (blue) of each part increased with the size of the payoff. The payoffs were presented in increasing order. Subjects were asked to click on a bullet to indicate their preferred option (Figure 1 illustrates a choice for Option 2).

5. Bayesian Hierarchical Modeling

We analyzed the data using Bayesian hierarchical modeling. Economic analyses of choice behavior usually estimate models either by treating all data as generated by the representative agent or by independent estimation of each subject's parameters from the data collected from that specific subject. Both approaches have their limitations. Representative agent aggregation ignores individual heterogeneity and may result in estimates that are not representative of any individual in the sample. Individual-level estimation relies on relatively few data points, which may lead to unreliable results. Hierarchical modeling is an appealing compromise between these two extremes (Rouder and Lu 2005). It estimates the model parameters for each subject separately, but it assumes that subjects share similarities and that their individual parameter values come from a common (population-level) distribution. Hence, the parameter estimates for one individual benefit from the information obtained from all others. This improves the precision of the estimates (in Bayesian statistics, this is known as *collective inference*), and it reduces the impact of outliers. Individual parameters are shrunk toward the group mean, an effect that is stronger for individuals with noisier behavior or individuals with fewer data points, thus making the overall estimation more robust. This is particularly true for parameters that are estimated with lower precision. An example is the loss aversion coefficient in prospect theory, for which the standard error of the parameter estimates is usually high. Nilsson et al. (2011) and Murphy and ten Brincke (2018) illustrate that Bayesian hierarchical modeling leads to more accurate, efficient, and reliable estimates of loss aversion than the commonly used maximum likelihood estimation.

Figure 2 shows a schematic representation of our statistical model. Details of the estimation procedure are in Appendix C. The model consists of two parts: first, the specification of the behavioral parameters $B_i, i \in \{1, \dots, 139\}$ in Equation (6), which includes utility, probability weighting, and the loss aversion parameter; second, the specification of the reference point rule $RP_i, i \in \{1, \dots, 139\}$. The reference point rule is one of the candidates listed in Table 1. Our analysis will

Figure 2. (Color online) Graphical Representation of Our Model



Note. Nonshaded nodes are known or predefined quantities, and shaded nodes are the unknown latent parameters.

estimate the posterior probabilities of a subject using each of the different reference point rules. In that sense, our analysis does allow for the possibility that subjects use a mixture of reference point rules.

The distributions of the behavioral parameters and the reference point rules in the population are parameterized by unknown vectors θ_B and θ_{RP} , respectively. Both θ_B and θ_{RP} are estimated from the data. The parameters θ_B and θ_{RP} also follow a distribution but with a known shape. This final layer in the hierarchical specification is commonly referred to as a hyperprior. The hyperpriors are denoted by π_B and π_{RP} , respectively.

It is worth pointing out that the above joint prior for all unknown parameters presumes that the latent variable RP_i is independent from the behavioral parameter B_i . Although this is true for the prior distribution, RP_i and B_i are not independent in the posterior.¹³ Just like we specify a flat (noninformative) prior for parameters for which we are agnostic about their values in the real world, we specify a joint independence for multivariate dependencies for which we do not know the true relationship. Because estimation in the Bayesian framework provides us with the full joint distribution of unknown parameters, investigating correlations in the joint posterior can provide useful insights into the relationship of these parameters in the real-world systems.

We assume that the utility function U in Equation (6) is a power function:

$$U(x) = \begin{cases} (x - r)^\alpha & \text{if } x \geq r \\ -\lambda(r - x)^\alpha & \text{if } x < r. \end{cases} \quad (7)$$

In Equation (7), α reflects the curvature of utility, and λ indicates loss aversion. We assumed the same curvature for gains and losses, because the estimations of loss aversion can be substantially biased when utility curvature for gains and losses can both vary freely (Nilsson et al. 2011).

For probability weighting, we assumed the one-parameter specification of Prelec (1998):

$$w(p) = e^{(-\ln p)^\gamma}. \quad (8)$$

We used the same probability weighting for gains and losses. Empirical studies usually find that the differences in probability weighting between gains and losses are relatively small (Tversky and Kahneman 1992, Abdellaoui 2000, Kothiyal et al. 2014).

To account for the probabilistic nature of people's choices, we used the logistic choice rule of Luce (1959). Let $RD(F)$ and $RD(G)$ denote the values of prospects F and G , respectively, according to our general reference-dependent model (Equation (6)). Luce's choice rule (1959) says that the probability $P(F, G)$ of choosing prospect F over prospect G equals

$$P(F, G) = \frac{1}{1 + e^{\xi[RD(G) - RD(F)]}}. \quad (9)$$

In Equation (9), $\xi > 0$ is a precision parameter that measures the extent to which the decision maker's choices are determined by the differences in value between the prospects. In other words, the ξ parameter signals the quality of the decision. Larger values of ξ imply that choice is driven more by the value difference between prospects F and G . If $\xi = 0$, choice is random, and if ξ goes to infinity, choice essentially becomes deterministic. In his comprehensive exploration of prospect theory specifications, Stott (2006) concluded that power utility, the one-parameter probability weighting function of Prelec (1998), and the choice rule of Luce (1959) gave the best fit to his data for gains. We, therefore, selected these specifications.

To test for robustness, we also ran our analysis with exponential utility, the two-parameter specification of the weighting function of Prelec (1998), and an alternative *incomplete regularized β function (IBeta)* probability weighting function (Wilcox 2012). IBeta is a flexible, two-parameter family that can accommodate many shapes (convex, concave, S shaped, and inverse S shaped); see Appendix D and the online appendix for details. The robustness analyses confirmed our main conclusions. The results of these analyses are in the online appendix.

6. Results

6.1. Consistency

To test for consistency, five choices were asked twice. In 68.7% of these repeated choices, subjects made the same choice. Reversal rates up to one-third are

common in experiments (Stott 2006). Moreover, our choices were complex, involving more than two outcomes and with expected values that were close. The median number of reversals was one; 20% of the subjects made at least three reversals, and the number of reversals that they made accounted for 41% of the total number of reversals. We also recorded the time that subjects spent on making their choices. The average time that they spent was not correlated with the reversal rate. An advantage of Bayesian analysis is that subjects who were particularly prone to make errors received little weight. For these subjects, the estimated parameters will be closer to the population averages.

Two questions had one option stochastically dominating the other; 108 subjects always chose the dominant options, 27 chose it once, and 4 never chose it. The time spent on these questions (and also the time spent on all questions) by subjects violating stochastic dominance at least once did not significantly differ from the time spent by the subjects who never violated stochastic dominance.

6.2. Reference Points

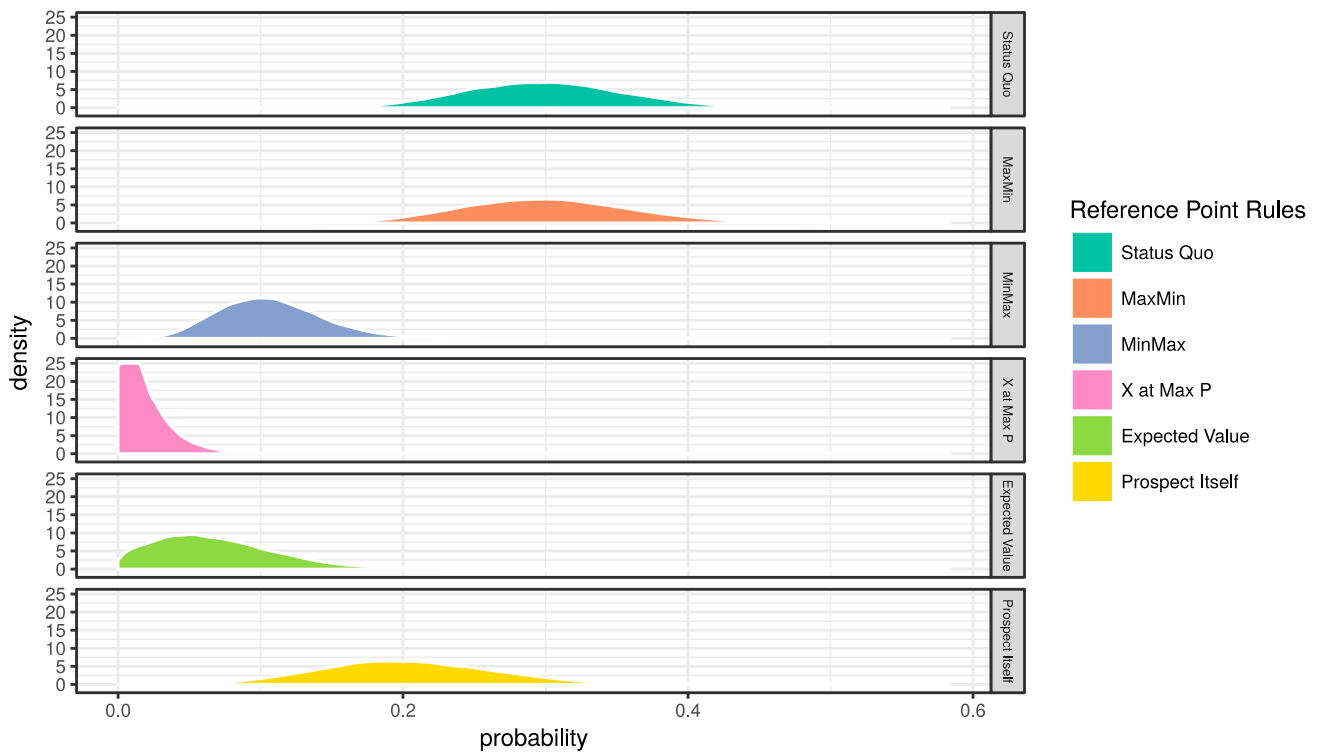
We first report the estimates of θ_{RP} , which indicate for each reference point rule the probability that a randomly chosen subject behaved in agreement with it. Figure 3 shows for each RP rule the marginal posterior distribution of θ_{RP} in the population. Table 2 reports the medians and standard deviations of these distributions.¹⁴

The reference points that were most likely to be used were the Status Quo and MaxMin. According to our median estimates, each of these two rules was used by 30% of the subjects. The prospect itself (the rule suggested by Delqu   and Cillo (2006) and K  szegi and Rabin (2006, 2007)) was used by 20% of the subjects. The other three rules were used rarely.

We also estimated for each subject the likelihood that they used a specific reference point by looking at their posterior distribution. Figure 4 shows, for example, the posterior distributions of subjects 17, 50, and 100. Subject 17 has about 60% probability to use the prospect itself as reference point and a 25% probability to use MinMax. Subject 50 almost surely uses MaxMin, and subject 100 almost surely uses the Status Quo as reference point.

Subjects were classified *sharply* if they had a posterior probability of at least 50% to use one of the six reference point rules. For example, subjects 17, 50, and 100 were all classified sharply. Subjects who could not be classified sharply might use different rules across choices, or they might not behave according to Equation (6): for example, because they used some choice heuristic. Of the 139 subjects, 107 could be classified sharply.¹⁵

Figure 5 shows the distribution of the sharply classified subjects over the six reference point rules. The dominance of the Status Quo and MaxMin increased further, and around 70% of the sharply classified subjects used one of these two rules.

Figure 3. (Color online) Marginal Posterior Distributions of Each Reference Point Rule

6.3. Behavioral Parameters

Figure 6 shows the gain-loss utility function in the psychological (PT) part of Equation (6) based on the estimated behavioral population-level parameters (θ_B). The utility function was S shaped: concave for gains and convex for losses. We found more utility curvature than most previous estimations of gain-loss utility (for an overview, see Fox and Poldrack 2014), but our estimated utility function is no outlier. It is, for example, close to the functions estimated by Wu and Gonzalez (1996), Gonzalez and Wu (1999), and Toubia et al. (2013). The loss aversion coefficient was equal to 2.34, which is consistent with other findings in the literature.

Figure 7 shows the estimated probability weighting function in the subject population. The function has the commonly observed inverse S shape, which reflects overweighting of small probabilities and underweighting of intermediate and large probabilities.¹⁶ Our estimated

probability weighting function is close to the estimated functions in Gonzalez and Wu (1999), Bleichrodt and Pinto (2000), and Toubia et al. (2013).

Bayesian hierarchical modeling expresses the uncertainty in the individual parameter estimates by means of the posterior densities. To illustrate, Figure 8 shows the posterior densities of subject 17. As the graph shows, subject 17's parameter estimates varied considerably, although it is safe to say that he had concave utility and inverse S-shaped probability weighting.

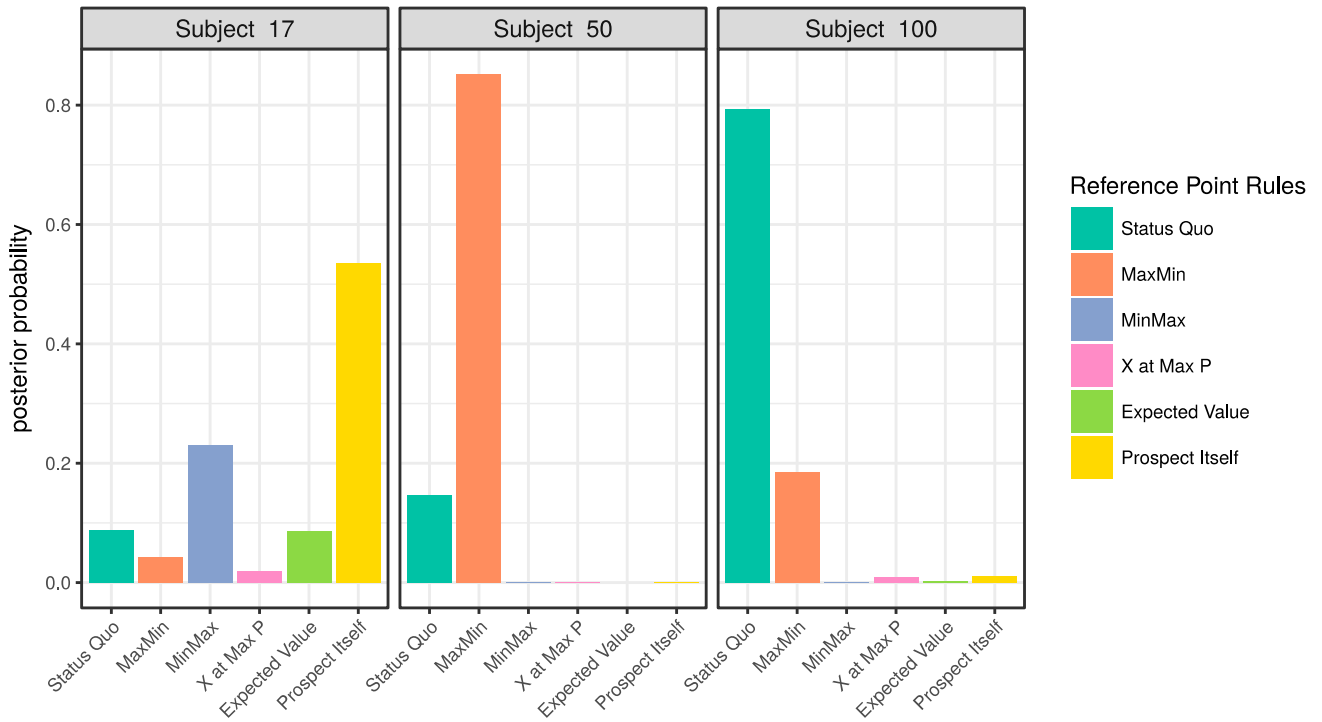
Table 3 shows the quantiles of the posterior point estimates of all 139 subjects. The table shows that utility curvature and to a lesser extent, probability weighting were rather stable across subjects. Loss aversion varied much more, although the estimates of more than 75% of the subjects were consistent with loss aversion.

Table 4 shows the median behavioral parameters of the sharply classified subjects subdivided by reference point rule. A priori, it seemed plausible that subjects who used different rules might also have different behavioral parameters, in particular loss aversion. The table confirms this conjecture. Although utility curvature and probability weighting were rather stable across the groups, the loss aversion coefficients varied from 0.50 in the MinMax group to 2.44 in the Expected Value group. The loss aversion coefficient of 0.50 in the MinMax group has the interesting interpretation that these optimistic subjects weight gains twice as much as losses, and they exhibit what might be seen as the reflection of

Table 2. Medians and Standard Deviations of the Marginal Posterior Distributions of the Reference Point Rules

	Median	Standard deviation
Status Quo	0.30	0.06
MaxMin	0.30	0.06
MinMax	0.10	0.04
X at Max P	0.01	0.02
Expected Value	0.06	0.04
Prospect Itself	0.20	0.06

Figure 4. (Color online) Posterior Distributions of Subjects 17, 50, and 100



the preferences of the cautious MaxMin subjects who weight losses more than twice as much as gains.

Table 4 also shows that subjects who used the Status Quo as their reference point were typically not expected

utility maximizers, because there was substantial probability weighting in this group. Table 5 gives a more detailed overview. It shows the subdivision of the subjects who used the Status Quo as their reference point

Figure 5. (Color online) Proportion of Sharply Classified Respondents Satisfying a Particular Reference Point Rule

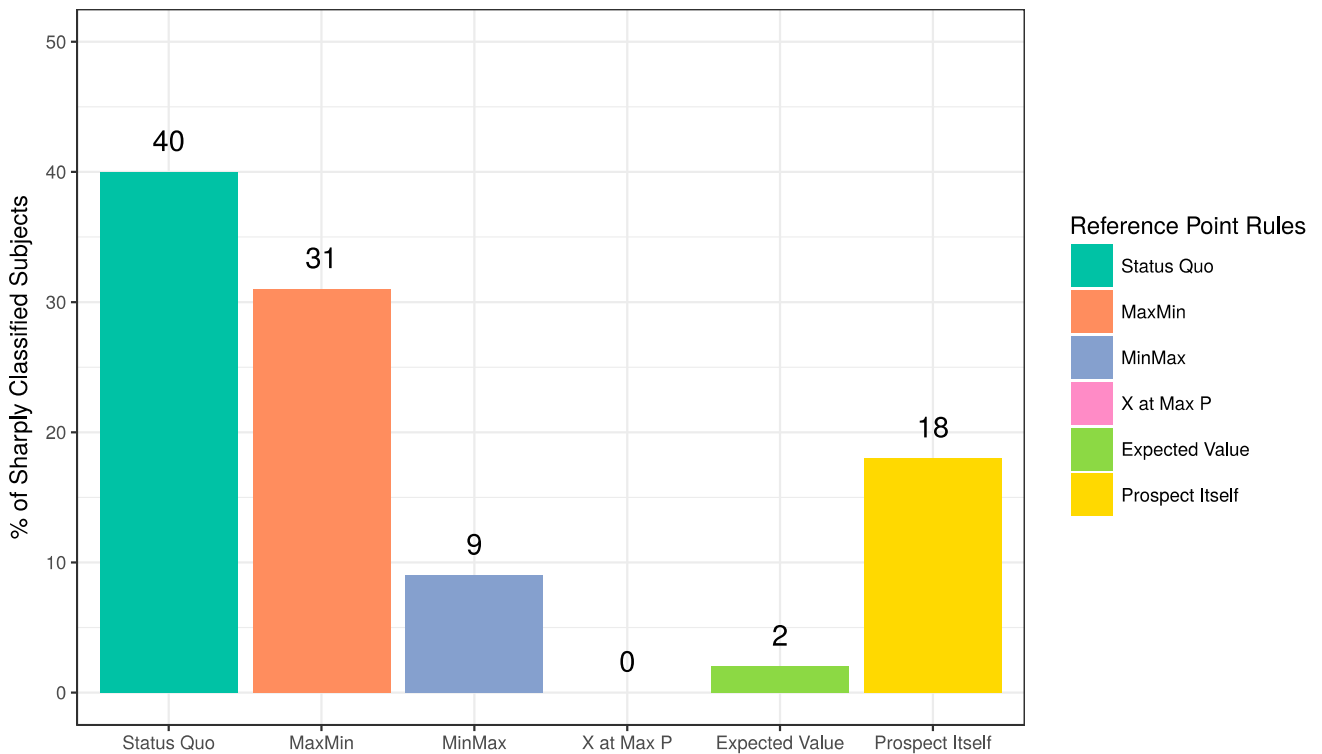
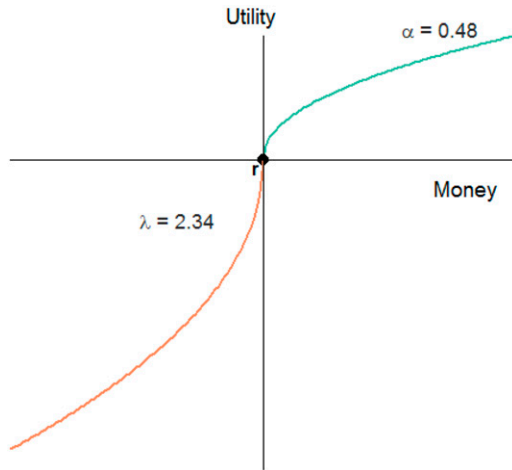


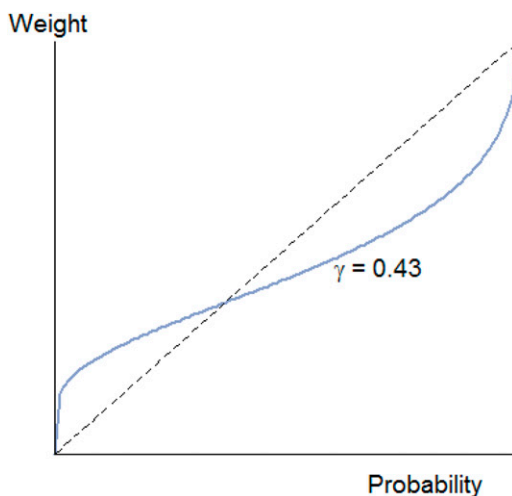
Figure 6. (Color online) The Gain-Loss Utility Function Based on the Estimated Group Parameters



based on the 95% Bayesian credible intervals of their estimated utility curvature and probability weighting parameters. Twelve subjects (those with $\gamma = 1$) behaved according to expected utility, three of whom (those with $\alpha = 1$ and $\gamma = 1$) were expected value maximizers. Thus, less than 10% of our subjects were expected utility maximizers.

Figure 9 displays the correlation matrix of the estimated behavioral parameters and the reference point rules. The correlations between the behavioral parameters (α, γ, λ , and ξ) are small. There is a slight tendency for more loss-averse subjects to choose more randomly. The correlations between the reference point rules and the behavioral parameters are largely consistent with the findings reported in Table 4. Status Quo and MinMax are associated with lower loss aversion, whereas the opposite is true for MaxMin and Prospect Itself. The reported correlations for X at Max P and

Figure 7. (Color online) The Probability Weighting Function Based on the Estimated Group Parameters



Expected Value should be interpreted with caution, because the probability of using these rules was very low.

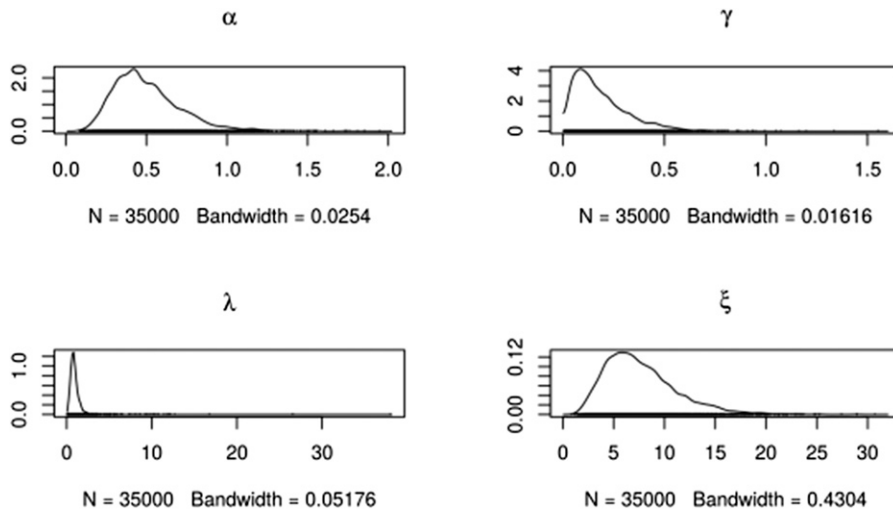
6.4. Robustness

In the main analysis, we assumed Equation (6) for all reference point rules, allowing us to keep all behavioral parameters constant when comparing reference point rules. We also tried several other specifications, which are summarized in Table 6. Model 1 corresponds to the results reported in Sections 6.2 and 6.3. The two variables that we varied in the robustness checks were the inclusion of consumption utility and probability weighting. Although models with prospect-specific reference points need consumption utility to rule out implausible choice behavior, models with a choice-specific reference point do not. Prospect theory, for example, does not include consumption utility. Consequently, we estimated the models with a choice-specific reference point both with and without consumption utility.

In Equation (6), we assumed that subjects weight probabilities when they evaluate prospects relative to a reference point, but following the literature on stochastic reference points, we abstracted from probability weighting in the determination of the stochastic reference point. This may be arbitrary, and we, therefore, also estimated the models without probability weighting. We performed two sets of estimations: one in which the models with a choice-specific reference point included probability weighting, but the models with a prospect-specific reference point did not (Models 3 and 4) and one in which no model had probability weighting (Models 5 and 6).

The results of the robustness checks were as follows. First, our main conclusion that the Status Quo and MaxMin were the dominant reference points remained valid. The behavior of 60%–75% of the subjects was best described by a model with one of these two reference points. Second, excluding consumption utility from models with a choice-specific reference point (Models 2 and 4) led to a substantial increase in the precision parameter ξ . This suggests that there is no need to include consumption utility in models like prospect theory. Third, probability weighting played a crucial role. Excluding probability weighting from the models with a prospect-specific reference point (Model 3) decreased the share of the Prospect Itself as a reference point to 10% (8% if we only include the sharply classified subjects) and increased the share of the MaxMin reference point to 44% (52% if we only include the sharply classified subjects). The shares of the other rules changed only little. Hence, prospect-specific models, like the disappointment aversion models and the model of Köszegi and Rabin's (2006, 2007), benefit from including probability weighting. Ignoring probability

Figure 8. Posterior Densities of the Behavioral Parameters for Subject 17



Notes. N denotes the number of simulations on which the densities are based. Bandwidth denotes the smoothing parameter for the kernel density estimation.

weighting altogether, as in Models 5 and 6, led to unstable estimation results.

The behavioral parameters were comparable across all models that we estimated. The power utility coefficient was approximately 0.50 in all models, the probability weighting parameter varied between 0.40 and 0.60 (except, of course, when no probability weighting was assumed), and the loss aversion coefficient varied between 2 and 2.50. Detailed results of the robustness analyses are in the online appendix.

6.5. Crossvalidation

Throughout the paper, we considered six reference point rules. Although these rules cover many of the rules that have been proposed in the literature and used in empirical research, it might be that subjects adopted another rule. In that case, the model would be misspecified, and it would poorly predict subjects' choices. Part of this is captured by restricting the analysis to the sharply classified subjects, which as we explained above, gave similar results.

To explore the predictive ability of our reference points, we performed the following crossvalidation exercise. We estimated the model on 69 questions and predicted the choice made by each of the 139 subjects for the remaining question. This out-of-sample prediction procedure was repeated 70 times, and each

Table 3. Quantiles of the Point Estimates of the Behavioral Parameters of the 139 Subjects

	2.5%	25%	50%	75%	97.5%
α	0.31	0.40	0.44	0.50	0.60
λ	0.36	1.19	1.59	2.25	4.63
γ	0.09	0.14	0.24	0.44	1.66
ξ	6.11	8.26	10.89	14.41	25.76

question was used once as the choice to be predicted. The reference point rules predicted around 70% of the choices correctly. Given that the consistency rate was also around 70%, we conclude that the rules that we included captured our subjects' preferences well and that there is no indication that the model was misspecified. The part that could not be explained probably reflected noise.

7. Discussion

Empirical studies often assume that subjects take the Status Quo as their reference point. Our results help to assess the validity of that assumption; 30%–40% of our subjects adopted the Status Quo as their reference point. A majority used a different reference point rule, in particular MaxMin. Our data suggest ways to increase the likelihood that subjects use the Status Quo as their reference point. For example, experiments involving mixed prospects could include a prospect with zero as its minimum outcome in each choice. Then, MaxMin subjects will also use zero as

Table 4. Median Individual-Level Behavioral Parameters for the Sharply Classified Subjects in Each Group

	α	γ	λ	ξ
Status Quo	0.42	0.28	1.51	11.75
MaxMin	0.46	0.24	2.24	10.30
MinMax	0.40	0.15	0.50	14.34
Expected Value	0.36	0.25	2.44	6.14
Prospect Itself	0.45	0.16	2.23	10.89

Notes. The reason that λ is not equal to one for subjects who were sharply classified as using the Status Quo rule was that, even for those subjects, there was a nonnegligible probability that they used any of the other reference point rules and were loss averse. X at Max P is not in this table because there were no sharply classified subjects who behaved according to this rule.

Table 5. Behavioral Parameters of the Subjects Using the Status Quo as Their Reference Points (Classification into Groups is Based on the 95% Bayesian Credible Intervals)

Utility	Probability weighting			Total
	$\gamma < 1$	$\gamma = 1$	$\gamma > 1$	
$\alpha < 1$	28	9	0	37
$\alpha = 1$	3	3	0	6
$\alpha > 1$	0	0	0	0
Total	31	12	0	43

their reference point, and our results suggest that, consequently, a large majority of the subjects will use zero as their reference point.

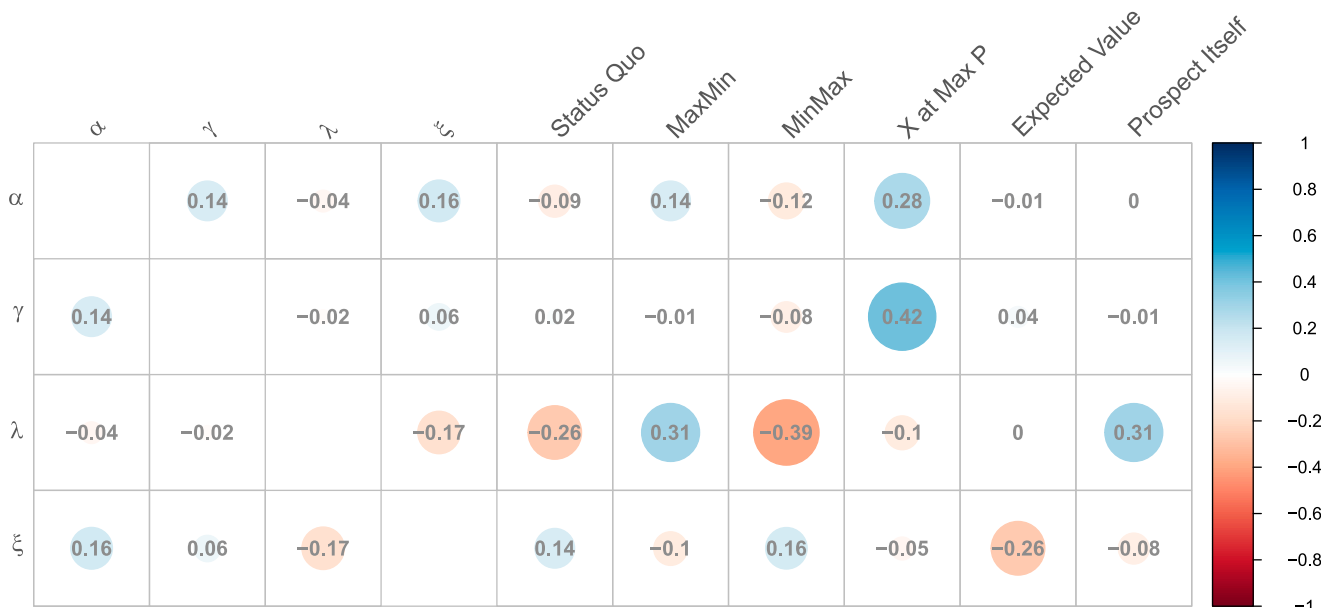
We used a Bayesian hierarchical approach to analyze the data. Bayesian analysis strikes a nice balance between representative agent and independent per subject estimation, and it leads to more precise parameter estimates. A potential limitation of Bayesian analysis is that the selected priors may in principle affect the estimations, but the choice of priors, as is common, had a negligible impact on the estimates in our analyses.

To make inferences about the different reference point rules, we used a comprehensive model, which allowed for isolating the impact of the reference point rule from the other behavioral parameters. This approach is cleaner and easier to interpret than the common practices of using mixture models, where each model is specified separately and parameterizations can differ across models, or horse races between models based on criteria, such as the Akaike Information Criterion. Hierarchical models have the additional advantage that inference can be done both at the aggregate level and for each subject individually.

Our robustness tests have two interesting implications for reference-dependent models. First, they indicate that models with a choice-specific reference point do not benefit from including consumption utility. This suggests that the role of the absolute amounts of money was limited and that our subjects were mainly concerned about changes from the reference point. Kahneman and Tversky (1979, p. 277) conjectured that, although an individual’s attitudes to money depend on both his asset position and changes from his reference point, a utility function that is only defined over changes from the reference point generally provides a satisfactory approximation.¹⁷ Our results support their conjecture. Second, we conclude that probability weighting could not be ignored. The fit of expectation-based models, like the disappointment aversion models and the model of Köszegi and Rabin (2007), which in their original form, are linear in probabilities, clearly improved when probability weighting was included.

In models with prospect-specific reference points, the gain-loss utility component (the second term of Equation (6)) can violate stochastic dominance. The inclusion of consumption utility (the first term of Equation (6)) mitigates this problem but does not solve it. Masatlioglu and Raymond (2016) showed that models that have the prospect itself as the reference point, such as the models of Köszegi and Rabin (2007) and of Delquie and Cillo (2006), can still suffer from violations of first-order stochastic dominance (unless, for instance, utility is piecewise linear and $0 \leq \lambda \leq 2$). The same problem actually occurs when subjects use the expected value as their reference point. Unlike models with a prospect-specific reference point, models with a choice-specific

Figure 9. (Color online) Correlations Between Behavioral Parameters and Estimated Reference Point Rules



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Table 6. Estimated Models

Model	Choice-specific reference point		Prospect-specific reference point	
	Consumption utility	Probability weighting	Consumption utility	Probability weighting
1	Yes	Yes	Yes	Yes
2	No	Yes	Yes	Yes
3	Yes	Yes	Yes	No
4	No	Yes	Yes	No
5	Yes	No	Yes	No
6	No	No	Yes	No

reference point always satisfy first-order stochastic dominance. Two choices in our experiment tested first-order stochastic dominance. Subjects who violated first-order stochastic dominance at least once were more likely to use a prospect-specific reference point (they had a 12% chance to use Expected Value and a 30% chance to use Prospect Itself) than subjects who never violated it (5% chance Expected Value, 18% chance Prospect Itself). The model of Köszegi and Rabin (2007) has become the main model in applications of reference dependence, particularly in economics. Our findings challenge this: first, because we find that it is only used by a small fraction of subjects, and second, because those subjects who use it are particularly likely to violate first-order stochastic dominance.

There are several ways in which our study can be extended. First, it may be interesting to explore whether our results can be replicated for choices involving more than just two prospects. Second, another extension would be to look at prospects with continuous distributions, which are often relevant in applied decision analysis. Third, the minimum probability that we included was 5%, whereas real-world decisions frequently involve smaller probabilities (e.g., the annual risk of contracting a fatal disease). It is, for instance, unclear whether MaxMin would perform as well if the lowest outcome occurred with only a very small probability.

We did not test all reference points that have been proposed in the literature. As we explained in Section 1, we studied reference point rules that used the same independent variables as the core theory of decision under risk: expected utility. This implied, for example, that we did not test explicitly for subjects' goals (Heath et al. 1999) or their aspirations (Diecidue and Van de Ven 2008), because these require other inputs based on introspection. We did not test reference point rules that are based on previous choices either. Such rules would introduce extra degrees of freedom, like which

information from these past choices to use, how far to look back, and how to update the reference point based on new information. The rules that we included fitted our subjects' preferences well, and a large majority of our subjects (around 75%) could be sharply classified, suggesting that they used one of the selected rules.

In our paper, we have concentrated on decision under risk, mainly because prospect theory was formulated for this context. There is a rich literature that studies reference dependence in other domains, such as time preference (Loewenstein and Prelec 1992), consumer choice (Tversky and Kahneman 1991), and marketing (Winer 1986, Hardie et al. 1993, Kopalle and Winer 1996). It is not immediately obvious that our results carry over to these domains. As mentioned above, none of our reference point rules looked at previous choices. Past experiences may be important, in intertemporal choice, to understand habit formation and in consumer choice, to determine reference prices. In marketing, past prices and past purchases will probably shape reference prices and reference alternatives. Studies of reference points in choices between alternatives that have more than one attribute (e.g., price and quality) face an additional challenge: is there a reference point for each attribute, or is there a reference alternative (such as in Tversky and Kahneman 1991 and Hardie et al. 1993) with which multiattribute alternatives are compared?¹⁸ If there is a reference point for each attribute, then it is straightforward to translate, for example, MaxMin to multiattribute choice. However, if there is a reference alternative, this seems more complex.

8. Conclusion

Reference dependence is a key concept in explaining people's choices, but little insight exists into the question of which reference point people use. Reference-dependent theories give little guidance about this question. This paper has estimated the prevalence of six reference point rules using a unique data set in which we used stakes up to a week's salary. We modeled the reference point rule as a latent categorical variable, which we estimated using Bayesian hierarchical modeling. Our results indicate that the Status Quo and MaxMin were the most commonly used reference points. We found little support for the use of a stochastic reference point, and at most, 20% of our subjects used an expectations-based reference point. Adding consumption utility does not improve models with a choice-specific reference point (like prospect theory), but adding probability weighting improves models with a prospect-specific reference point.

Appendix A. The Experimental Questions and the Predicted Reference Points

Table A.1 describes the 70 choices between prospects $x = (p_1, x_1; p_2, x_2; p_3, x_3; 1 - p_1 - p_2 - p_3, x_4)$ and $y = (q_1, y_1; q_2, y_2; q_3, y_3; 1 - q_1 - q_2 - q_3, y_4)$ used in the experiment. The last five columns give the choice-specific reference points of the MaxMin, MinMax, and X at Max P rules and the prospect-specific reference points of the Expected Value rule. The reference point of the Status Quo

used in the experiment. The last five columns give the choice-specific reference points of the MaxMin, MinMax, and X at Max P rules and the prospect-specific reference points of the Expected Value rule. The reference point of the Status Quo

Table A.1. Choices and Reference Points

No.	x_1	x_2	x_3	x_4	p_1	p_2	p_3	y_1	y_2	y_3	y_4	q_1	q_2	q_3	MaxMin	MinMax	X at Max P	Expected value	
																		x	y
1	267	313	453	546	0.1	0.8	0.05	127	220	406		0.15	0.05	0.8	267	406	313	327.05	354.85
2	159	221	408		0.7	0.1	0.2	34	97	346		0.1	0.3	0.6	159	346	159	215	240.1
3	183	233	384	485	0.7	0.05	0.1	32	132	334		0.15	0.05	0.8	183	334	334	250.9	278.6
4	223	263	383		0.4	0.5	0.1	143	183	343		0.1	0.4	0.5	223	343	263	259	259
5	127	255	287		0.7	0.05	0.25	95	191	223		0.15	0.05	0.8	127	223	223	173.4	202.2
6	103	213	377		0.6	0.15	0.25	48	158	267	322	0.3	0.1	0.05	103	322	103	188	220.65
7	92	245			0.85	0.15		16	130	206		0.1	0.7	0.2	92	206	92	114.95	133.8
8	135	290	329		0.55	0.35	0.1	96	213	251		0.25	0.05	0.7	135	251	251	208.65	210.35
9	209	309	459		0.35	0.55	0.1	159	259	359	409	0.05	0.55	0.1	209	409	309	289	309
10	221	504			0.85	0.15		80	292	434		0.05	0.7	0.25	221	434	221	263.45	316.9
11	64	188	313		0.4	0.1	0.5	2	126	251	375	0.25	0.4	0.1	64	313	313	200.9	169.75
12	122	270	418		0.15	0.8	0.05	48	196	344	492	0.1	0.35	0.45	122	418	270	255.2	277.4
13	224	416			0.55	0.45		95	352	480		0.25	0.7	0.05	224	416	352	310.4	294.15
14	100	211			0.2	0.8		64	137	285		0.2	0.5	0.3	100	211	211	188.8	166.8
15	257	427			0.8	0.2		143	370	484		0.35	0.45	0.2	257	427	257	291	313.35
16	223	416			0.45	0.55		159	287	544		0.05	0.7	0.25	223	416	287	329.15	344.85
17	219	448			0.2	0.8		143	296	372	601	0.1	0.1	0.7	219	448	448	402.2	364.4
18	99	225			0.8	0.2		16	141	183	266	0.1	0.4	0.45	99	225	99	124.2	153.65
19	94	187			0.3	0.7		64	125	156	248	0.25	0.3	0.05	94	187	187	159.1	160.5
20	203	317			0.75	0.25		127	241	279	354	0.35	0.05	0.45	203	317	203	231.5	235.15
21	138	245			0.55	0.45		30	84	191	352	0.05	0.05	0.85	138	245	191	186.15	185.65
22	118	200			0.8	0.2		64	91	173	228	0.2	0.1	0.6	118	200	118	134.4	148.5
23	232	374			0.4	0.6		91	161	303	515	0.05	0.1	0.6	232	374	374	317.2	331.2
24	233	344			0.7	0.3		159	196	307	381	0.3	0.2	0.1	233	344	233	266.3	270
25	251	358			0.7	0.3		143	304	412	465	0.05	0.85	0.05	251	358	304	283.1	309.4
26	105	278			0.25	0.75		48	163	336	394	0.25	0.4	0.1	105	278	278	234.75	209.3
27	183	302			0.6	0.4		64	242	361	421	0.15	0.7	0.1	183	302	242	230.6	236.15
28	61	179			0.45	0.55		22	101	218	257	0.4	0.05	0.5	61	179	179	125.9	135.7
29	147	367			0.6	0.4		0	74	367		0.25	0.05	0.7	147	367	367	235	260.6
30	99	251			0.6	0.4		48	251			0.4	0.6		99	251	99	159.8	169.8
31	259	558			0.75	0.25		159	359	558		0.15	0.7	0.15	259	558	259	333.75	358.85
32	168	397			0.6	0.4		16	92	397		0.05	0.4	0.55	168	397	168	259.6	255.95
33	209	407			0.75	0.25		143	407			0.5	0.5		209	407	209	258.5	275
34	120	243			0.75	0.25		80	161	243		0.15	0.7	0.15	120	243	120	150.75	161.15
35	142	209	277		0.7	0.05	0.25	74	108	277		0.4	0.1	0.5	142	277	142	179.1	178.9
36	151	230	348		0.5	0.15	0.35	111	269	348		0.25	0.6	0.15	151	348	269	231.8	241.35
37	140	200	261		0.85	0.05	0.1	80	110	261		0.05	0.55	0.4	140	261	140	155.1	168.9
38	79	170	308		0.25	0.7	0.05	33	216	308		0.15	0.8	0.05	79	308	216	154.15	193.15
39	192	341			0.15	0.85		192	390	439		0.55	0.4	0.05	192	341	341	318.65	283.55
40	15	290			0.3	0.7		15	382			0.5	0.5		15	290	290	207.5	198.5
41	95	443			0.3	0.7		95	327	559		0.1	0.8	0.1	95	443	327	338.6	327
42	102	311			0.15	0.85		102	381	450		0.55	0.25	0.2	102	311	311	279.65	241.35
43	127	284			0.2	0.8		127	336			0.45	0.55		127	284	284	252.6	241.95
44	54	259			0.3	0.7		54	191	328		0.05	0.85	0.1	54	259	191	197.5	197.85
45	127	259	390		0.05	0.4	0.55	127	456	521		0.45	0.15	0.4	127	390	390	324.45	333.95
46	57	221	331		0.3	0.1	0.6	57	167	386		0.1	0.6	0.3	57	331	331	237.8	221.7
47	111	194	277		0.1	0.05	0.85	111	318	359		0.5	0.3	0.2	111	277	277	256.25	222.7
48	6	229	377		0.05	0.8	0.15	6	155	451		0.1	0.7	0.2	6	377	229	240.05	199.3
49	100				1			13	186			0.45	0.55		100	100	100	100	108.15
50	224				1			12	294			0.25	0.75		224	224	224	224	223.5
51	276				1			80	374	472		0.35	0.45	0.2	276	276	276	276	290.7
52	203				1			106	154	299		0.45	0.05	0.5	203	203	203	203	204.9
53	196				1			95	146	246	297	0.3	0.05	0.5	196	196	196	196	203.35
54	383				1			171	453			0.25	0.75		383	383	383	383	382.5

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Table A.1. (Continued)

No.	x_1	x_2	x_3	x_4	p_1	p_2	p_3	y_1	y_2	y_3	y_4	q_1	q_2	q_3	MaxMin	MinMax	X at Max P	Expected value	
																		x	y
55	297	404			0.55	0.45		189	243	350	511	0.05	0.05	0.85	297	404	350	345.15	344.65
56	220	338			0.45	0.55		181	260	377	416	0.4	0.05	0.5	220	338	338	284.9	294.7
57	238	329	467		0.25	0.7	0.05	192	375	467		0.15	0.8	0.05	238	467	375	313.15	352.15
58	301	368	436		0.7	0.05	0.25	233	267	436		0.4	0.1	0.5	301	436	301	338.1	337.9
59	259				1			172	345			0.45	0.55		259	259	259	259	267.15
60	362				1			265	313	458		0.45	0.05	0.5	362	362	362	362	363.9
61	213	418			0.3	0.7		213	350	487		0.05	0.85	0.1	213	418	350	356.5	356.85
62	223	347	472		0.4	0.1	0.5	161	285	410	534	0.25	0.4	0.1	223	472	472	359.9	328.75
63	306	526			0.6	0.4		159	233	526		0.25	0.05	0.7	306	526	526	394	419.6
64	251	358			0.7	0.3		143	304	412	465	0.05	0.85	0.05	251	358	304	283.1	309.4
65	95	443			0.3	0.7		95	327	559		0.1	0.8	0.1	95	443	327	338.6	327
66	223	416			0.45	0.55		159	287	544		0.05	0.7	0.25	223	416	287	329.15	344.85
67	209	407			0.75	0.25		143	407			0.5	0.5		209	407	209	258.5	275
68	138	245			0.55	0.45		30	84	191	352	0.05	0.05	0.85	138	245	191	186.15	185.65
69	111	207	223		0.5	0.4	0.1	80	95	207		0.1	0.4	0.5	111	207	111	160.6	149.5
70	111	175	207		0.1	0.4	0.5	80	159	191		0.25	0.25	0.5	111	191	207	184.6	155.25

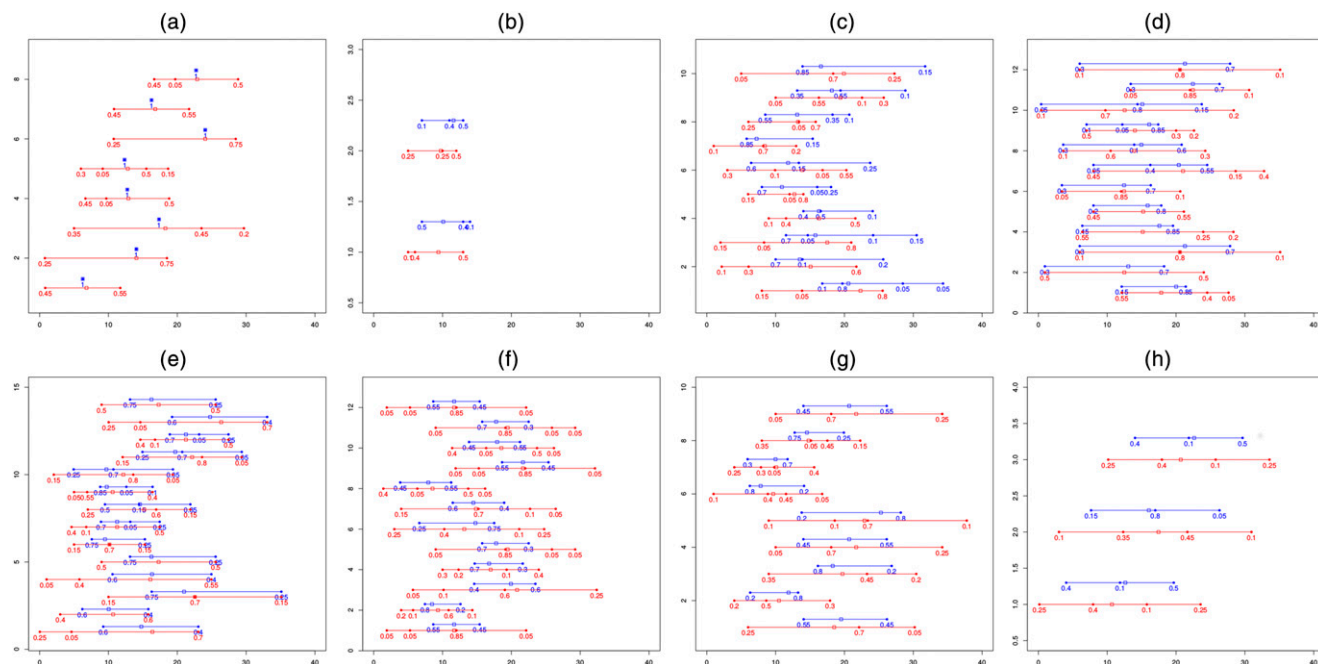
rule is always zero, and the prospect-specific reference points of the Prospect Itself rule were x and y themselves.

Appendix B. The Procedure to Construct the Experimental Choices

The selection of experimental questions was guided by the following contrasting principles.

- Questions must be diverse in terms of number of outcomes and magnitudes of probabilities involved.
- Questions within each choice must have nonmatching maximal or minimal outcomes.
- Questions must be diverse in terms of relative positioning in the outcome space (also known as shifting; see the description below).

Figure B.1. (Color online) Choices Used in the Experiment



Notes. Each subfigure represents a group of homogeneous choices. Each question consists of two prospects, blue (dark in black-and-white print) and red (light). The x axis represents the amounts in euros, and the y axis has no quantitative meaning. Numbers below the prospect lines are the outcome probabilities. Small squares are the expectations of the prospects. (a) Group with certainty equivalents. (b) Stochastic dominance group. (c) Shifted group (extremes of blue (dark) prospect are shifted with respect to the red (light) prospect). (d) Minima of blue (dark) and red (light) prospects coincide. (e) Maxima of blue (dark) and red (light) prospects coincide. (f)–(h) Three groups for which the range of the blue (dark) object is inside the range of the red (light) prospect.

- Questions must have similar expected value to avoid trivial or statistically noninformative choice situations.
- Question pairs must be “orthogonal” in some sense to maximize statistical efficiency.

Our question set (Table A.1) consists of six homogeneous groups that are illustrated graphically in Figure B.1. The first group is a set of eight choices, where one of the prospects is certain and the other option is a two- to four-outcome prospect (Figure B.1(a)). The second set consists of two choices, where one prospect stochastically dominates the other (Figure B.1(b)). The third set comprises 10 choices, where one prospect is relatively shifted—both minimum and maximum are relatively higher than for the other prospect (Figure B.1(c)). The fourth group consists of 12 choices for which the prospects in a choice have the same minimum outcomes (Figure B.1(d)). The fifth group consists of 14 choices for which the prospects in a choice have the same maximum outcome (Figure B.1(e)). The last three groups (Figure B.1, (f)–(h)) consist of 24 choices, where the range of one prospect is within the range of the other prospect. This group is further split into three homogeneous subgroups determined by the number of outcomes in the smaller prospect (two versus three) and the shift of the smaller prospect with respect to the bigger one (one or two outcomes). Choices in all groups are roughly balanced with respect to the relative shift (there are both one outcome- and two outcome-shifted questions on either side of the prospects).

To maximize statistical efficiency and minimize redundancy, within each group of questions we perform the exhaustive search that minimizes the sum of the pairwise crosschoice covariance within that group. We defined the crosschoice covariance for a choice pair $(A1, B1), (A2, B2)$ as

$$\left(\frac{(\text{cor}(A1, A2) + \text{cor}(B1, B2))}{2} \right)^2.$$

This is an intuitive counterpart of the statistical covariance. For each subgroup of choices, we optimized the sum of all pairwise crosschoice covariances within that group.

Appendix C. Details of the Bayesian Hierarchical Estimation Procedure

The vector of the observed choices (data) of individual i is denoted by $D_i = (D_{i1}, \dots, D_{i70})$. Each of the 139 subjects in the experiment had his own parameter vector $B_i = (\alpha_i, \lambda_i, \gamma_i, \xi_i)$. We assumed that each parameter in B_i came from a log-normal distribution: $\alpha_i \sim \log N(\mu_\alpha, \sigma_\alpha^2)$, $\lambda_i \sim \log N(\mu_\lambda, \sigma_\lambda^2)$, $\gamma_i \sim \log N(\mu_\gamma, \sigma_\gamma^2)$, and $\xi_i \sim \log N(\mu_\xi, \sigma_\xi^2)$. Thus, the complete vector of unknown parameters at the population level is $\theta_G = (\mu_\alpha, \mu_\lambda, \mu_\gamma, \mu_\xi, \sigma_\alpha^2, \sigma_\lambda^2, \sigma_\gamma^2, \sigma_\xi^2)$. For the hyperpriors, $\pi^* = (\mu_*, \sigma_*^2)$, $*$ $\in \{\alpha, \lambda, \gamma, \xi\}$ of the parent distributions, we made the usual assumption of conjugate NormalGamma prior: the μ_* follow a normal distribution (conditional on σ_*^2), and the σ_*^2 follow an inverse Gamma distribution. We centered the hyperpriors at linearity (expected value) and chose the variances such that the hyperpriors were diffuse and would have a negligible impact on the posterior estimation.

The joint probability distribution of the behavioral parameters $\mathbf{B} = (B_1, \dots, B_{139})$ and θ_B is

$$P(\mathbf{B}, \theta_B | \pi_B) = \left(\prod_{i=1}^{139} P(B_i | \theta_B) \right) P(\theta_B | \pi_B). \quad (C.1)$$

Given reference point rule RP_i , the likelihood of subject i 's responses is

$$P(D_i | B_i, RP_i) = \prod_{q=1}^{70} P(D_{i,q} | B_i, RP_i). \quad (C.2)$$

The probability of each choice $D_{i,q}$ is computed using Luce's (1959) choice rule (Equation (9)). From Equations (C.1) and (C.2), it follows that the joint probability distribution of all of the unknown behavioral parameters \mathbf{B} and θ_B and all of the observed choices $\mathbf{D} = (D_1, \dots, D_{139})$ is

$$P(\mathbf{D}, \mathbf{B}, \theta_B | \mathbf{RP}, \pi_B) = \left(\prod_{i=1}^{139} \prod_{q=1}^{70} P(D_{i,q} | B_i, RP_i) \right) \cdot \left(\prod_{i=1}^{139} P(B_i | \theta_B) \right) P(\theta_B | \pi_B). \quad (C.3)$$

In Equation (C.3), $\mathbf{RP} = (RP_1, \dots, RP_{139})$ is the vector of individual specific reference point rules.

For each of the six reference point rules of Table 1, we estimated the posterior probability that a subject used it given the data: $P(RP_i | \mathbf{D})$. RP_i is a (six-dimensional) categorical variable. For categorical variables, it is common to use the Dirichlet distribution: $\theta_{RP} \sim \text{Dirichlet}(\pi_{RP})$, where θ_{RP} is a probability vector in a six-dimensional simplex and π_{RP} is a diffuse hyperprior parameter for the Dirichlet distribution. Then, the joint probability density of \mathbf{RP} and θ_{RP} becomes

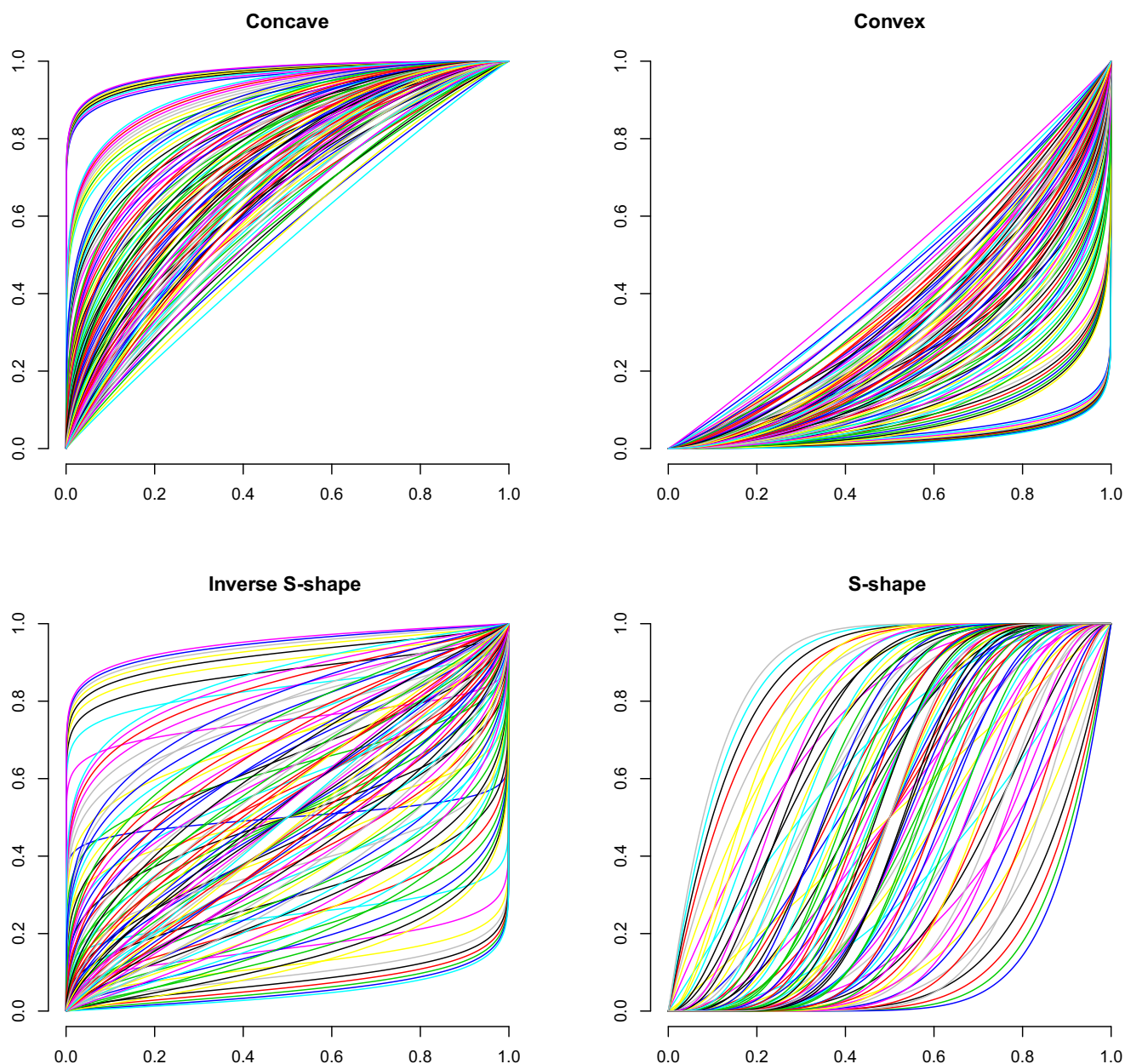
$$P(\mathbf{RP}, \theta_{RP} | \pi_{RP}) = \left(\prod_{i=1}^{139} P(RP_i | \theta_{RP}) \right) P(\theta_{RP} | \pi_{RP}). \quad (C.4)$$

Combining Equations (C.3) and (C.4) gives the complete specification of our statistical model:

$$P(\mathbf{D}, \mathbf{B}, \theta_B, \mathbf{RP}, \theta_{RP} | \pi_B, \pi_{RP}) = \left(\prod_{i=1}^{139} \prod_{q=1}^{70} P(D_{i,q} | B_i, RP_i) \right) \left(\prod_{i=1}^{139} P(B_i | \theta_B) \right) \cdot \left(\prod_{i=1}^{139} P(RP_i | \theta_{RP}) \right) P(\theta_B | \pi_B) P(\theta_{RP} | \pi_{RP}). \quad (C.5)$$

To compute the marginal posterior distributions $P(B_i | \mathbf{D}, \pi_B, \pi_{RP})$, $P(RP_i | \mathbf{D}, \pi_B, \pi_{RP})$, $P(\theta_B | \mathbf{D}, \pi_B, \pi_{RP})$, and $P(\theta_{RP} | \mathbf{D}, \pi_B, \pi_{RP})$, we used Markov Chain Monte Carlo (MCMC) sampling (Gelfand and Smith 1990) with blocked Gibbs sampling.¹⁹ We first used 10,000 burn-in iterations with adaptive MCMC and then, 20,000 standard MCMC burn-in iterations. The results are based on the subsequent 50,000

Figure C.1. (Color online) Various Shapes of the *IBeta* Function



iterations, of which the first 15,000 iterations were used for warmup until convergence was achieved and the last 35,000 iterations were used for the reported estimations.

Appendix D. *IBeta*

The *incomplete regularized β function* (*IBeta*) is a very flexible monotonically increasing $[0, 1] \rightarrow [0, 1]$ function. It can capture a wide range of convex, concave, S-shape and inverse S-shaped functions without favoring specific shapes or inflection points. The family is symmetric in the sense that

$IBeta(x; a, b) = 1 - IBeta(1 - x; a, b)$. Various shapes of *IBeta* function are illustrated in Figure C.1.

Endnotes

¹Examples of real-world evidence for reference dependence are the equity premium puzzle, the finding that stock returns are too high relative to bond returns (Benartzi and Thaler 1995), the disposition effect, the finding that investors hold losing stocks and property too long and sell winners too early (Odean 1998, Genesove and Mayer 2001), default bias in pension and insurance choice (Samuelson and Zeckhauser 1988, Thaler and Benartzi 2004) as well as organ donation

(Johnson and Goldstein 2003), the excessive buying of insurance (Sydnor 2010), the annuitization puzzle, the fact that at retirement people allocate too little of their wealth to annuities (Benartzi et al. 2011), the behavior of professional golf players (Pope and Schweitzer 2011) and poker players (Eil and Lien 2014), and the bunching of marathon finishing times just ahead of round numbers (Allen et al. 2017).

²For example, different assumptions about the reference point are required to explain two well-known anomalies from finance: the equity premium puzzle demands that the reference point adjusts over time, whereas adjustments in the reference point weaken the disposition effect (Meng and Weng 2018).

³For example, Abeler et al. (2011), Card and Dahl (2011), Crawford and Meng (2011), Gill and Prose (2012), and Bartling et al. (2015).

⁴For example, Baucells et al. (2011), Heffetz and List (2014), Lien and Zheng (2015), Wenner (2015), and Allen et al. (2017).

⁵To illustrate, in the online appendix, we show that the data of Abeler et al. (2011), which they interpret as supporting the model of Köszegi and Rabin (2007), are also consistent with the MaxMin rule that we study in this paper.

⁶This contrasts with the standard practice in mixture modeling, where different models are aggregated through a mixing distribution. In mixture models, the formal specification and parameterization of the submodels can differ substantially.

⁷When we refer to prospect theory, we mean cumulative prospect theory, the version that Tversky and Kahneman (1992) proposed in their 1992 article in the *Journal of Risk and Uncertainty*.

⁸If consumption utility was excluded, the CPE model of Köszegi and Rabin (2007) implies that any prospect that gives x with probability 1 has a value of zero, regardless of the size of x . Therefore, the decision maker should be indifferent between \$1 for sure and \$1,000 for sure. Consumption utility prevents this.

⁹With nonlinear v .

¹⁰This rule was suggested in an unpublished working paper by Bleichrodt and Schmidt (2005). See also Birnbaum and Schmidt (2010) and Schneider and Day (2018).

¹¹The equivalence with Loomes and Sugden (1986) follows, because we assume that $v(x) = x$.

¹²Cox et al. (2015) found evidence against isolation.

¹³In the probabilistic graphical models literature, such dependency relationships are known as “V-structures” $B \rightarrow D \leftarrow R$. B and R are unconditionally independent ($B \perp R$) but dependent conditionally on D ($B \not\perp R | D$).

¹⁴Note that the medians need not add to 100%.

¹⁵Terzi et al. (2016) compared four expectations-based reference points and found that around 40% of their subjects used no reference point or used multiple reference points. This is higher than the proportion of our subjects who could not be sharply classified in our study. The difference can be explained by the larger and broader set of reference point rules in our study.

¹⁶The Prelec one-parameter probability weighting function only allows for inverse or S-shaped weighting. However, the two-parameter Prelec function and the IBeta function allow for all shapes, and their estimated shapes were also inverse S.

¹⁷In their 1975 working paper version of prospect theory, Kahneman and Tversky (1975, p. 15) write that $[U]$ “is a function in two arguments: current wealth and magnitudes of change. It seems, however, that the preference relation between gambles is relatively insensitive to wealth and highly sensitive to changes.”

¹⁸See Bleichrodt and Miyamoto (2003) and Bleichrodt et al. (2009) for theoretical analyses of reference dependence in multiattribute utility.

¹⁹For the behavioral parameters B_1, \dots, B_{139} , we used Metropolis–Hasting MCMC with symmetric normal proposal on the log scale. For

the block RP_1, \dots, RP_{139} , we used Metropolis–Hasting MCMC with uniform proposal, and the group-level blocks θ_G and θ_{RP} were sampled directly from the conjugate NormalGamma and Dirichlet–Categorical distributions, respectively.

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