Elastic energy in locomotion: Spring-mass vs. poly-articulated models

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Original Article


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The human is often modeled as a Poly-Articulated Model (PAM) with rigid segments while some authors use a Spring Mass Model (SMM) for modeling locomotion. These two models are considered independent, and the objective of this study was to link them in order to enlighten the origin of the elasticity in locomotion.

Using the characteristics of the two models, a theoretical relationship demonstrates that the variation of elastic energy of the SMM equals the variation of the internal kinetic energy minus internal forces work of the PAM. This theoretical relationship was experimentally investigated among 19 healthy participants walking and running on a treadmill.
The results showed that the equality is verified except during the double support phase at 0.56 m.s\(^{-1}\), at high walking speeds (1.67 and 2.22 m.s\(^{-1}\)) or during the aerial phase of running.

The formal relationship showed that the global stiffness of the SMM is directly related to the work of the internal forces of the PAM, and thus, to the characteristics of the musculoskeletal system. It also showed the relevance of taking into account the participation of each joint in the global stiffness. Finally, the coordination of internal forces work to produce a global stiffness may be considered as a new criterion of movement optimization for clinical purposes or motion planning for humanoid robots.

**Keywords**: Elastic energy, work of internal forces, mechanical energy theorem, energy transfers, human gait.
1. Introduction

The human locomotion can be modeled as a Spring Mass Model (SMM, Fig. 1) for both walking [1,2] and running [3]. The SMM is represented as a body mass at the center of mass (CoM) oscillating at the end of a massless spring (Fig. 1). This model has been originally promoted for running gait since it takes into account elastic energy, which seems to play an important role in the mechanical energy conservation [4]. Therefore, the use of this model to characterize walking suggests an equivalent role of elastic energy in this locomotion mode. Indeed, the SMM is a conservative system inducing no change in mechanical energy ($E_{M}^{SMM}$, Eq. (1)) that can be calculated as in Eq. (2).

$$\Delta E_{M}^{SMM} = 0 \quad \text{(1)}$$

$$E_{M}^{SMM} = E_{Kext}^{SMM} + E_{P}^{SMM} + E_{E}^{SMM} \quad \text{(2)}$$

With $E_{Kext}^{SMM}$ the kinetic energy of the CoM according to Duboy et al. [5], $E_{P}^{SMM}$ the potential energy due to gravity, and $E_{E}^{SMM}$ the elastic energy which is dependent of a global constant stiffness $k$. This constant stiffness $k$ is computed from different ways [6–8]. The SMM predicts the displacement of the whole body CoM only and takes into account an elastic component. Although it highlights the basic mechanisms of the locomotion and reduces the mechanical parameters taken into account, the CoM trajectory is depending upon the segment masses and locations.

On the other hand, the human body is modeled as a Poly-Articular Model (PAM, Fig. 1), i.e. as a poly-articulated system of $n$ rigid segments $S_i (i \in [1,n])$, with each a center of mass $G_i$ and a mass $m_i$ (Fig. 1). The PAM predicts the displacement of all the segments of the human body. Although the elastic component of the musculoskeletal system is involved and taken into account in the force and work production [9], the PAM does not take into account any elastic energy. The interest of this model is to simplify the model of the human body in
order to measure relevant mechanical parameters for locomotion studies, like the variation of mechanical energies of each body segment [10], the mechanical cost of movement [11], the joint torques with inverse dynamics [12–14] and the kinematic parameters (e.g. 3D body segment orientation and translation). From a gait cycle to another, the mechanical energy is the same whereas it varies throughout the movement because of internal forces work [10]. According to the mechanical energy theorem (Eq. (3)), the mechanical energy variation of the PAM results from the work of non conservative forces \( W_{Fnc}^{PAM} \). Assuming that the liaison between the foot and the ground is not dissipative and the weight is conservative, the \( W_{Fnc}^{PAM} \) represents the internal forces work \( W_{Fint}^{PAM} \).

\[
\Delta E_M^{PAM} = W_{Fnc}^{PAM} = W_{Fint}^{PAM} \tag{3}
\]

The mechanical energy of the PAM can be computed as in Eq. (4).

\[
E_M^{PAM} = E_{Kext}^{PAM} + E_p^{PAM} + E_{Kint}^{PAM} \tag{4}
\]

With \( E_{Kint}^{PAM} \) the kinetic energy of the body segments in the barycentric coordinate system according to Duboy et al. [5], decomposed from the 2nd König’s theorem.

Both of these models have their own advantages (Fig. 1): (i) the SMM predicts the displacement of the body CoM and considers elastic energy without taking into consideration the translational and rotational energies of the body segments, and (ii) the more complex PAM takes into account the body segment energies without considering elastic energy. However, as explained, both are aware to model gait [3,10].

These two models are generally presented as independent in the literature, and the objective of this paper is to establish a link between them. By applying the theorem of the mechanical energy to the SMM and the PAM for the same movement, we obtain:

\[
\Delta \left( E_{Kext}^{SMM} + E_p^{SMM} + E_E^{SMM} \right) = 0
\]

\[
\Delta \left( E_{Kext}^{PAM} + E_p^{PAM} + E_{Kint}^{PAM} \right) = W_{Fint}^{PAM} \tag{5}
\]
Two terms are common to both the equations: $\Delta E_{K_{ext}}$ and $\Delta E_p$. By assuming their equality, the link between the SMM and the PAM can be established as follows:

$$\Delta E_{E_{SMM}} = \Delta E_{K_{int}^{PAM}} - W_{Fint}^{PAM} \quad (6)$$

Finally, the variation of the elastic energy ($\Delta E_{E_{SMM}}$) represents the variation of the kinetic energy of the body segment in the barycentric coordinate system ($\Delta E_{K_{int}^{PAM}}$) minus the work of the internal forces ($W_{Fint}^{PAM}$). The energy balance being considered during one gait cycle, the comparison of SMM and PAM supposes that the mechanical work of the internal forces balances the dissipation/storage and generation/restitution of the energy to zero over the whole gait cycle.

By this way, we should experimentally verify Eq. (6) and then determine the link between SMM & PAM, which are generally used separately in the literature. The goal of the study is therefore to experimentally investigate both side of Eq. (6) and verify the equality over a gait cycle.

2. Methods

2.1. Population

Nineteen healthy men volunteered (23±5 y; 1.79±0.07 m; 80.7±11 kg) for this experimentation. They were equipped with 42 reflective markers recorded by twelve optoelectronic cameras sampled at 200 Hz (VICON, Oxford’s metrics, Oxford, UK). The participants performed barefoot walking and running tests on a treadmill (PF 500 CX, PRO FORM, Villepreux, France) embed on a large force platform recording at 1 kHz (AMTI, Watertown, MA, USA). The kinematic and kinetic data were filtered with 4th order zero lag Butterworth filters with a cut off frequency of 6 Hz and 10 Hz [15], respectively
2.2. Experimentation

To induce dynamic similarity between the participants, the speed and frequency were determined according to Froude (\( Fr = \frac{v^2}{gl} \); with \( v \) the speed, \( g \) the gravity and \( l \) the CoM height) and Strouhal (\( Str = \frac{fl}{v} \); with \( f \) the step frequency) combination as suggested by Villeger et al. [2] for walking and Villeger et al. [16] for running.

Firstly, the participants were asked to walk and run with their preferred step frequency at 0.56, 1.11, 1.67, 2.22 m.s\(^{-1}\) and 1.67, 2.22, 2.78, 3.33, 3.89, 4.44 m.s\(^{-1}\), respectively. From these tests, a mean of \( Fr (\overline{Fr}) \) and \( Str (\overline{Str}) \) was computed for each speed stage.

Secondly, similar speed and similar step frequency were imposed to each subject \( j \) at each speed stage (Eq. 7 and Eq. 8).

\[
\begin{align*}
v_{\text{sim}_j} &= \sqrt{\overline{Fr} \cdot g \cdot l_j} \quad (7) \\
f_{\text{sim}_j} &= \frac{\overline{Str} \cdot v}{l_j} \quad (8)
\end{align*}
\]

Only these similar conditions (\( v_{\text{sim}}, f_{\text{sim}} \)) were treated in the present study.

2.3. Assessed parameters

In this study, the human body was considered as a whole of 16 rigid body segments [17]. The functional centers of rotation of the hips and the shoulders were determined with the SCoRE method [18].

The walking and running trials were performed on a treadmill. While the belt induces a foot translation during the stance phase, we have to consider that the horizontal component of the GRF is working in the global Galilean reference. Both equalities of Eq. (5) and Eq. (11; 14) consider it but the Eq. (6) remains unchanged after simplification. Finally, the three
variables appearing in the theoretical link (Eq. (6)), $E_E$, $E_{Kint}$ and $W_{Fint}$ were computed as follows.

2.3.1. Elastic energy

$$E_E^{SMM} = \frac{1}{2}k\Delta l^2$$

Where $\Delta l$ is the three-dimensional length variation between the CoM and the Center of Pressure (CoP), and $k$ is the spring stiffness computed according to Farley & Gonzalez [7] as:

$$k = \frac{\|\text{GRF}\|_{max}}{\Delta l_{max}}$$

(with $\|\text{GRF}\|_{max}$ the maximal value of the GRF norm and $\Delta l_{max}$ the maximal variation of the three dimensional distance between the CoM and the CoP).

2.3.2. Internal kinetic energy

$$E_{Kint}^{PAM} = \frac{1}{2} \sum_{i=1}^{16} \left( m_i \nu_i^G + m_i K_i^2 \cdot \omega_i^S \right)$$

Where $\nu_i^G$ is the linear velocity of the $i^{th}$ segment CoM in the barycentric coordinate system, $K_i$ is the radius of gyration of the $i^{th}$ segment around its CoM and $\omega_i^S$ is the angular velocity of the $i^{th}$ segment in the barycentric coordinate system.

2.3.3. Internal forces work

$$W_{Fint}^{PAM} = \Delta(E_{Kint}^{PAM} + E_{Kext}^{PAM}) + \Delta E_p^{PAM} - W_{GRF}$$

Where $E_{Kext}$ is the external kinetic energy, $E_p$ is the potential energy due to gravity and $W_{GRF}$ the work of the GRF. These variables were calculated as:

$$E_{Kext}^{PAM} = \frac{1}{2}m \nu_{G/0}^2$$

With $\nu_{G/0}$ the velocity of the CoM in the global coordinate system.

$$E_p^{PAM} = m g \cdot r_{G/0}^g$$

with $g = \begin{bmatrix} 0 \\ 0 \\ 9.81 \end{bmatrix}$
With \( \mathbf{g} \) the gravity and \( \mathbf{r}^{C0} \) the position of the CoM in the global coordinate system,

\[
W_{GRF} = \int (\mathbf{GRF}/_0 \cdot \mathbf{v}^{C0}_/0) \, dt
\]  

(14)

With \( \mathbf{GRF}/_0 \) and \( \mathbf{v}^{C0}_/0 \) the GRF and the velocity of the CoP expressed in the global coordinate system, respectively. During walking, the GRF and the CoP under each foot during the double support phase were estimated from transition functions [19].

2.3.4. Dimensionless parameters

The parameters were expressed in a dimensionless form [2,16,20] to avoid both anthropometric and time effects. Energies and works depend on length \((L)\), mass \((M)\), and time \((T)\) physical dimensions: \([W] = [E] = ML^2 T^{-2}\). Thus, to express them in a dimensionless form, they were normalized by parameters of the same dimension combining the CoM height \((l)\), the body mass \((m)\), and the step time \((t)\).

These parameters were averaged in percentage of the gait cycle (100 frames) across 5 cycles. The cycle started at the first contact of the left foot on the ground, which was detected by the force platform (vertical force threshold = 10 N).

2.4. Statistical analysis

At each speed stage, the terms of Eq. (6) were averaged for all participants and compared with a sliding window ANOVA \((p<0.05)\) for each 20% of gait cycle with a sliding step of 10% \((0-20\%, 10-30\%, 20-40\%, 30-50\%, 40-60\%, 50-70\%, 60-80\%, 70-90\%\) and \(80-100\%)\), once the normality had been checked for each 20% using Kolmogorov-Smirnov tests.

Determination coefficients \((R^2, \ p<0.05)\) were computed to compare \(\Delta E_E^{SMM}\) and \(\Delta E_{Kint}^{PAM} - W_{Fint}^{PAM}\) throughout the gait cycle.
3. Results

The variation of the elastic energy ($\Delta E_E^{SMM}$) and the variation of the internal kinetic energy minus the work of the internal forces ($\Delta E_{Kint}^{PAM} - W_{Fint}^{PAM}$) are presented in Fig. 2 for walking and in Fig. 3 for running.

During walking, the determination coefficients between both terms were high for the first two speeds. The ANOVA revealed statistically significant differences between both terms during the double support phase at 0.56 m.s$^{-1}$ and from the midstance to the contralateral heel strike at 1.67 and 2.22 m.s$^{-1}$. At the beginning of the double support phase, the elastic energy made a gap because of an estimated initial value of the elastic energy of the spring at heel strike (Fig. 2).

During running, the determination coefficients between both terms were high for all speeds ($R^2 \geq 0.9$) despite a non significant slight decrease as speed increased (from 0.98 to 0.90). For all speeds except the lowest, the ANOVA showed statistically significant differences during the aerial phase. A temporal lag between both terms was observed and increased with running speed.

4. Discussion

The aim of this study was to experimentally investigate the relationship between the variation of elastic energy of the SMM and the variation of internal kinetic energy minus the internal forces work of the PAM. The theoretical relationship between the models of the SMM and the PAM presented in Eq. (6) is valid for overground and treadmill displacements.
Here, this relationship was studied through the dimensionless parameters. Therefore, this
equation suggests that the internal forces work is the single variable allowing the human body
to behave like a spring mass model with a constant stiffness (Eq. (15)). The internal kinetic
energy resulting from the internal and the external forces works, the stiffness $k$ follows from
an adaptation of the internal forces work.

$$
\Delta \left( \frac{1}{2} k \Delta l^2 \right) = \Delta \frac{1}{2} \sum_{i=1}^{16} \left( m_i \nu_{i,G}^2 + m_i k_i^2 \cdot \omega_{i,G}^2 \right) - W_{F_{int, PAM}}^\text{PAM}
$$  (15)

$\Delta E_{SMM}^\text{SM} \text{ and } \Delta E_{Kint, PAM} - W_{F_{int, PAM}}^\text{PAM}$ are closely linked ($R^2$ ranging from 0.79 to 0.98)
although stiffness management appears more complex for the two higher walking speeds.

Equation 6 is verified over a wide range of walking and running speeds suggesting that the
sum of the internal forces produced a work similar to the one produced by a spring of constant
stiffness. However this assumption is not verify during i) the double support [RHS to LTO,
Fig 2]at slow walking speed, ii) from the mid stance to the contralateral heel strike at high
walking speeds and iii) during the aerial phase of the running speeds [LTO to RL, Fig 3.]. In
this last case, the difference is due to the vertical excursion of the COM that exceeds the value
recorded at landing during the bounce (see blue curve) and to an estimated initial value of the
elastic energy of the spring at heel strike. Appearing during the aerial phase, this gap is of low
interest to verify our hypothesis. Concerning the few other events where the Eq 6 is not
verified, we remind here some studies that enlighten the physiological process induced and
the possible causes of the inequality.

At walking speeds close to transition to running, both terms of Eq. (6) are different
from the mid stance to the contralateral heel strike, and the internal forces work does not
produce a work similar to the one produced by a spring of constant stiffness. Several
hypotheses may explain this difference. Firstly, Hunter [21] suggested that the global stiffness
of the SMM is likely to be variable and comprises two parts: one high at the impact and one
low after this impact. This assumption is corroborated by a recent study [22] that showed a
pre-activation of lower limb muscles before and at the beginning of the stance phase. Secondly, the musculo-tendinous tissues have elastic properties allowing the storage/recoiling of elastic energy [23]. The storage of the elastic energy in the musculo-tendinous tissues is taken into account in the internal forces work of the PAM which does not consider any elastic component, but this storage is considered as an energy dissipation. The elastic energy storage and return by the musculo-tendinous tissues has a direct action on the joint movement, and then on the internal forces work. This phenomenon can explain the lag between both the terms of Eq. (6) which increases with walking speed as suggested by the use of biological tissues elastic properties [9,23]. Including prestimulation, a muscle’s time-delayed, a gained force and a reflex parameters allow simulating the muscles control of segmented legs acting as a leg spring [24]. All these parameters are sensitive to time and walking or running speeds and may explain the time lag observed in the present study. Note, here, that the same regulation processes appear finely regulated to reach the good stiffness at the good time and over the whole joints in all other conditions.

Indeed, the formal relationship (Eq. (6)) suggests that the stiffness of the SMM is linked to the internal forces work. The internal forces work is the sum of the mechanical works of each joint which confirm that the global stiffness in the SMM is the sum of all joint stiffness in the PAM [25]. Then, it can be concluded that each joint’s work was coordinated to produce a global stiffness. For constant or variable stiffness, different joint works combination may be considered: (i) joints stiffness are constant and added in a constant global stiffness, (ii) joints stiffness are variable and coordinated in a constant global stiffness, or (iii) joints stiffness are variable and combined to produce a variable global stiffness.

Joint torques are taken into account in the computation of the internal forces work. It can therefore be expected that the contribution of each joint works to produce a global stiffness may vary throughout the movement and with the locomotion mode. Hence, the
contribution of the different joints in the global stiffness could be estimated. Joint stiffness is also evoked as a key criterion in disabilities such as falls [26,27], balance management [28,29], and cerebral palsy [30]. The organization of local work or local stiffness to produce a global stiffness could be considered as a new criterion of motor behavior optimization useful for rehabilitation procedures, mechanical energy optimization, and prostheses/exoskeleton stiffness management. Furthermore, the SMM is used to control humanoid robots [31], which is corroborated by the current results suggesting it is an accurate model for gaits.

In conclusion, the equality between the variation of the elastic energy of the SMM and the variation of the internal kinetic energy minus the internal forces work of the PAM is verified over a wide range of spontaneously chosen pattern. This assumption is not verified during the double support phase at slow walking and during the second half of stance at high walking speed. Referring to previous works, two hypotheses are proposed to explain these differences: (i) the time lag between both models may result from structural properties of the musculoskeletal system and time-delayed reflex affecting the time course of the elastic energy storage and return and ultimately the internal force production at the joints of the PAM and (ii) the structural properties of the musculo-tendinous system reveal a non-linear force-length response [32] and may induce a variable joint stiffness. New models taking into account this non-linearity and the whole body segments may highlight the respective contributions of the structures characteristics and the motor control in the management of the gait efficiency.

The double support phase disappears during running. The comparison of walking (Fig 2) and running (Fig 3) performed at 1.67 and 2.22m/s reveals a more complex energy management during walking. These speeds are close to the walk to run spontaneous transition. The left to right side transfer is more fluid in running and might explain the switch in response to the optimization of the energy management.
Finally, the theoretical relationship suggests that mechanical works of each joint coordinate together to produce a global stiffness which highlights a new spontaneous optimization of the movement. Future investigations have to be conducted to discern the contribution of the elasticity of the structures to the mechanical work when the global elasticity can be imitated with a good coordination of the legs.

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References


Figure legends

Fig. 1. Spring Mass Model (SMM, with m, the mass; CoP, the center of pressure; CoM, the center of mass; k, the spring stiffness; l, the distance between CoP and CoM) and Poly-Articular Model (PAM, with CoP, the center of pressure, M, the total mass; G, the center of gravity; mi and Gi, the mass and the center of mass of the segment, respectively) including 16 segments (Si) used for walking and running gaits.

Fig. 2. Dimensionless means of the elastic energy variation (blue) and the internal kinetic energy variation minus the internal forces work (red) during a normalized walking cycle of the 19 subjects. The blue and red areas represent the standard deviation of both terms. Horizontal black bars with an asterisk show the significance differences enlightened by the sliding window ANOVA (p<0.05). The castellated bars at the bottom represent the single support phases (lower level) and the double support phases (upper level). RTO, Right Toe Off; RHS, Right Heel Strike; LTO, Left Toe Off, LHS, Left Heel Strike.

Fig. 3. Dimensionless means of the elastic energy variation (blue) and the internal kinetic energy variation minus the internal forces work (red) during a normalized running cycle of the 19 subjects. The blue and red areas represent the standard deviation of both terms. Horizontal black bars with an asterisk show the significance differences enlightened by the sliding window ANOVA (p<0.05). The castellated bars at the bottom represent the single support phases (lower level) and the aerial phases (upper level). LL, Left Landing; LTO, Left Take Off; RL, Right Landing; RTO, Right Take Off.
Fig 1
Fig 2
Fig 3