



HAL
open science

Collective rationality and functional wisdom of the crowd in far-from-rational institutional investors

Kevin Primicerio, Damien Challet, Stanislao Gualdi

► **To cite this version:**

Kevin Primicerio, Damien Challet, Stanislao Gualdi. Collective rationality and functional wisdom of the crowd in far-from-rational institutional investors. *Journal of Economic Interaction and Coordination*, 2020, 16 (1), pp.153-171. 10.1007/s11403-020-00288-0 . hal-04317258

HAL Id: hal-04317258

<https://hal.science/hal-04317258v1>

Submitted on 1 Dec 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Wisdom of the institutional crowd

Kevin Primicerio ^{1,*}, Damien Challet ^{1,†}, Stanislao Gualdi ^{1,2,‡}

¹ Laboratory of Mathematics in Interaction with Computer Science, CentraleSupélec, Grande Voie des Vignes, 92290 Châtenay-Malabry, France

² Capital Fund Management, 23 rue de l'Université, 75007, Paris, France

Abstract

The average portfolio structure of institutional investors is shown to have properties which account for transaction costs in an optimal way. This implies that financial institutions unknowingly display collective rationality, or Wisdom of the Crowd. Individual deviations from the rational benchmark are ample, which illustrates that system-wide rationality does not need nearly rational individuals. Finally we discuss the importance of accounting for constraints when assessing the presence of Wisdom of the Crowd.

1 Introduction

The collective ability of a crowd to accurately estimate an unknown quantity is known as the “Wisdom of the Crowd” [1] (WoC thereafter). In many situations, the median estimate of a group of unrelated individuals is surprisingly close to the true value, sometimes significantly better than those of experts [2, 3, 4, 5]. WoC may only hold under some conditions [1, 6]: for example social imitation is detrimental as herding may significantly bias the collective estimate [7, 8]. WoC is a reminiscent of collective rationality without explicit individual rationality: when it applies, it is a consistent aggregation of possibly inconsistent individual estimates [9]. This is to be contrasted with the mainstream economic paradigm which takes a short-cut by assuming that collective rationality reflects individual rationality, where only a “typical” decision maker – the representative agent – is considered [10] or team reasoning where the individual agents explicitly optimize the collective welfare [11]. Aggregation of quite diverse individual actions, especially in a dynamic context where expectations are continuously revised, is still an open problem [12].

Although almost all known examples of WoC are about a single number or coordinate, there is no reason why WoC could not be found for whole functional relationships between several quantities. For example, Haerdle and Kirman analyse the prices and volume of many transactions in Marseille fish market: while the relationship between these two quantities is rather noisy, the market self-organises so that when more fish are sold, prices are lower, as revealed by a local average [13]. More generically, many simple relationships found in Economics textbooks may only hold on average, but not for each agent or each transaction.

*kevin.primicerio@centralesupelec.fr

†damien.challet@centralesupelec.fr

‡stanislao.gualdi@gmail.com

Asset price efficiency is an obvious instance of WoC in Finance: it states that current prices, determined by the actions of many traders, are the best possible estimates and fully reflect all available information [14, 15, 16]. Another WoC candidate is portfolios. While many market participants, especially investment funds, strive to build optimal portfolios, each following its own criteria and constraints (performance objective, risk, tracking error, etc.), the question here is whether their collective behaviour may be related to a rational benchmark. Fortunately, this implies that we do not need to understand the minute details of all the portfolios and can focus on average quantities instead.

2 Wisdom of crowd

Let us define some necessary quantities to be more precise. At time t , fund i has capital $W_i(t)$ which is invested into $n_i(t)$ securities among $M(t)$ existing ones. As a result, each security α , whose capitalization is denoted by $C_\alpha(t)$, is found in $m_\alpha(t)$ portfolios. The explicit time dependence is dropped hereafter.

The only quantity defined above which depends on asset allocation strategies of fund i is n_i , the number of securities it chooses to invest in. Our main hypothesis is thus that WoC is found in the average relationship between n_i and W_i . A simple rational benchmark is proposed by [17] : when a fund with capital W_i is able to invest the same amount in each of the n_i chosen securities and if the transaction cost does not depend on the security, then the optimal n_i is such that

$$W_i \propto n_i^\mu. \tag{1}$$

where the exponent μ is determined by the transaction costs fee structure; for example, proportional transaction costs lead to $\mu = 1$, while a fixed cost per transaction corresponds to $\mu = 2$ (see [17] for more details). Allowing for individual fluctuations, Eq. (1) becomes $\log W_i = \mu \log n_i + \epsilon_i$, where ϵ_i has zero average. Denoting local average of x_i by x , the local average of Eq. (1) yields

$$W \propto n^\mu. \tag{2}$$

Flat fee per transaction ($\mu = 2$) is a popular request of large clients of broker. [17] find indeed that for wealthy individual investors and asset managers, exponent $\mu = 2$ within statistical uncertainty. We will thus test the occurrence of WoC from the value of exponent μ . More precisely, our hypothesis is that if (i) the effective transaction cost per transaction is the same for all assets and (ii) funds are able to build equally weighted portfolios, then Eq. (2) holds and that $\mu = 2$, which is a sign of WoC.

Both conditions must cease to hold for larger investment funds. Indeed, condition (i) cannot be true for them since large trades (even when split into meta-orders) have a price impact which grows with their size and depends on volatility and average turnover [18]. Condition (ii) ceases to hold for large funds which spread their investments on many securities: because the capitalization of assets and their average daily turnover are very heterogeneous, large funds cannot invest enough money in assets with a small capitalization so as to build an equally-weighted portfolio. As a result, on average, the local average W is expected to increase more slowly as a function of n in the large n region; equivalently, the exponent μ is expected to be smaller than 2. In summary, two different regimes should emerge: one with $\mu_{<} = 2$ for small enough n and $\mu_{>} < 2$ for larger n .

Figure 1 plots W_i versus n_i in logarithmic scale: a cloud of point emerges, with a roughly increasing trend. The large amount of noise confirms the great diversity of fund allocation strategies.

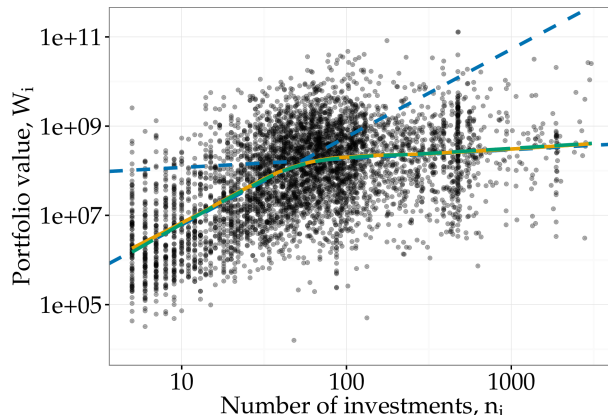


Figure 1: Total mark-to-market value W_i as a function of the number of investments n_i , with a robust locally weighted regression fit (yellow line) and two linear fits (blue dashed lines) for two different ranges of n . Robust locally weighted regression fit for the simulated data (in green).

WoC may only appear in some average behaviour. This is why we computed a locally weighted polynomial regression [19]. As expected, two distinct regions appear. In each of them, the local regression follows a roughly linear behaviour.

The cross-over point n^* between the two regions is algorithmically determined for each quarterly snapshot (see S.I.); it is stable as time goes on (see Fig. 11 in S.I.). The two exponents $\mu_<$ and $\mu_>$ are quite stable as a function of time as well (see Fig. 11 in S.I.); their time-averages $\overline{\mu_<} \simeq 2.1 \pm 0.2$ and $\overline{\mu_>} \simeq 0.3 \pm 0.1$ are markedly different, which points to distinct collective ways of building portfolios in these two regions.

So far, $\overline{\mu_<}$ is compatible with the WoC hypothesis. Let us check the validity of conditions (i) and (ii) above. When condition (ii) is not satisfied, then condition (i) also must cease to hold, thus we can focus on the former. Condition (ii) says that the diversity of investment fractions $p_{i\alpha} = W_{i\alpha}/W_i$ for $W_{i\alpha} > 0$ must be very small among α . This may be summarized in a single number by the scaled Shannon Entropy $S_i = -\frac{1}{\log_2 n_i} \sum_{\alpha} p_{i\alpha} \log_2 p_{i\alpha}$, which equals 1 and is maximal when all the non-null $p_{i\alpha}$ are equal. Figure. 2 reports the scaled entropy S_i of all the funds for a given time snapshot, together with the local average S . The latter increases up to about $n \simeq n^*$ and then decreases. The fact that $S < 1$ is due in part to price fluctuations: even if fund i builds an equally weighted portfolio at time t (thus $S_{i,t} = 1$), $S_{i,t+1} < 1$ at a later date. The importance of this mechanism is confirmed by Monte-Carlo simulations: the red line of Fig. 2 shows the effect of natural asset price evolution on perfectly equally weighted portfolios after three months, using asset price volatility measured in our dataset between the time of the snapshot and the three previous months: the resulting scaled entropy S_{MC} increases as a function of n , mirroring the local average of S_i in the same figure for $n < n^*$. Thus, the decrease for $n > n^*$ is due to impossibility for larger funds to build equally-weighted portfolios. A further argument supporting our claim that investment funds strive to build equally weighted portfolios (on average) is provided by the entropy measured on the set of common positions between two consecutive snapshots multiplied by $S_{MC}(n_i)/S_{MC}(n_{i,restricted})$ in order to account for the dependence of S on n ; the local average of the resulting entropy corresponds to the dashed blue line: it is clearly smaller than the entropy of

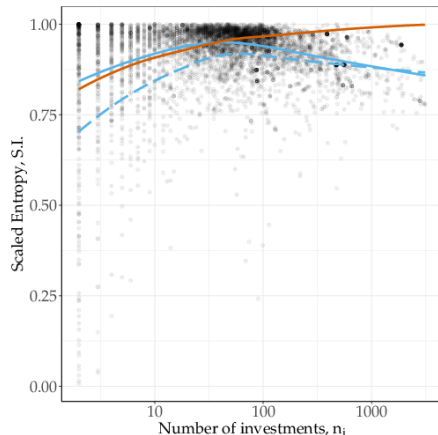


Figure 2: Scaled Shannon entropy S_i as a function of the number of investments n_i for all the funds on 2013-03-31 (circles) and robust local weighted regression fit, for all positions (blue line), numerical simulation of the effect of price fluctuations on the entropy on initially equally weighted portfolios (red line), where a volatility similar to the observed volatility in the real data, is applied; robust local weighted regression fit restricted to the unchanged portfolio positions from the previous time step, multiplied by the ratio between the simulated entropy for the full portfolio and the restricted portfolio (dashed blue line);

the new portfolio, hence new positions purposefully bring S_i closer to equally weighted portfolios. Therefore, condition (ii) is valid when $\mu = 2$; conversely, $\mu \neq 2$ when condition (ii) ceases to hold.

Quite tellingly, the same exponent was found for large private investors and asset managers (with much smaller amounts of money under management). Thus the collective behaviour of large investment funds is essentially the same one. Since one finds the same exponent μ over many decades of portfolio values for a wide spectrum of market participants, and since $\mu = 2$ corresponds to a realistic transaction cost per transaction, we argue that WoC is a plausible explanation of the average portfolio structure. Note that $\mu = 2$ does not imply that funds really face constant transaction cost per transaction, only that their population acts as if it does. Finally, we stress that WoC holds for a whole functional relationship over many decades of n and W , not only for a single number, which considerably extends its reach.

3 Asset selection model

So far, bringing to light WoC in the $\mu = 2$ region only required to focus on the number of securities in a portfolio, not on how funds select securities. This implicitly assumed that funds could invest in all securities they wished, which is clearly not the case in the large diversification region: the fact that the exponent μ is much smaller in this region implies that funds need on average to split their investments into many more securities. This is most likely due to liquidity constraints: large funds cannot invest as much as they wish in some assets because there are simply not enough shares to build a position larger than a certain size without impacting too much their prices. Each fund has its own way to determine the maximal amount to invest in a given security α ; a common criterion

is to limit the fraction $W_{i\alpha}/C_\alpha$. Fig. 8 in S.I. strongly suggests that each fund fixes its upper bound

$$f_i^{(\max)} \geq \max_\alpha f_{i\alpha} \quad \text{where } f_{i\alpha} = \frac{W_{i\alpha}}{C_\alpha}. \quad (3)$$

It turns out that $f_i^{(\max)}$ is highly heterogeneous among funds $\log_{10}(f_i^{(\max)}) \simeq -3.0 \pm 1.0$ (see Fig. 9), which reflects both the heterogeneous ways of portfolio construction and also the confidence of a fund in its abilities to execute large trades without too much price impact. The existence of such limits implies that portfolios are less likely to be equally weighted in the large diversification region, as seen indeed in the decrease of the average portfolio weights scaled entropy for $n \geq 70$ (blue line in Fig. 2).

Funds, however, do not invest in a randomly chosen security, even in the low diversification region. Figure 3 displays a scatter plot of the capitalization C_α of each security α versus m_α , the number of funds which have invested in this security, together with a local non-linear fit. Similarly to W vs n , one finds a power-law relationship

$$\log C_\alpha = \gamma \log m_\alpha + \epsilon_\alpha \quad (4)$$

for large enough m (see S.I.). Hence in local average notations, $C \propto m^\gamma$. Exponent γ is stable during the period 2007-2014 (see Fig. 11 in S.I.) and its average $\bar{\gamma} \simeq 2.2 \pm 0.1$.

In short, one needs to introduce a model of how funds choose to invest in securities to reproduce the average behaviour of both Eqs (4) and (1). Since one sees a cross-over between two types of behaviour rather than an abrupt change, we create logarithmic bins of the axis n_i and denote the bin number of fund i by $[n_i]$. Two mechanisms must be specified: how a fund selects security α and how much it invests in it. The latter point is dictated by Fig. 8 in the large n_i region where fund i invests $W_{i\alpha} = f_i^{(\max)} C_\alpha$; for the sake of simplicity, we approximate $f_i^{(\max)}$ by the median value of $f_i^{(\max)}$ in the bin $[n_i]$, denoted by $f_{[n_i]}^{(\max)}$. In the small diversification region, we assume that $n_i = n_i^{\text{opt}}$, thus $W_{i\alpha} = W_i/n_i^{\text{opt}}$ to be consistent with our previous results. We choose a security selection mechanism that rests on the market capitalization C_α of a security α (see S.I.) which is a good proxy of the liquidity (Fig. 10). We perform Monte-Carlo simulations from the empirical selection probabilities and $f_{[n_i]}^{(\max)}$ and display the resulting W vs n and C vs m in Figs 1 and 3 (continuous green lines), in good agreement with the local averages (continuous orange lines). One notices a discrepancy in the relationship C vs m for large n , which mainly comes from funds in the large diversification region. (See Fig 12 S.I).

The large diversification region illustrates how constraints may considerably modify the rational benchmark. While the above mechanism of security selection is able to reproduce adequately the behaviour of well diversified funds, we could not find a rational benchmark for the dependence of f^{\max} and n_i . Thus, the case for WoC in the large diversification region is not entirely closed.

Data

Our dataset consists of an aggregation of the following publicly available reports (in order of reliability): the SEC Form 13F, the SEC's EDGAR system forms N-Q and N-CSR and (occasionally) the form 485BPOS. Our work focuses on the period starting from the first quarter of 2005 to the last quarter of 2013.

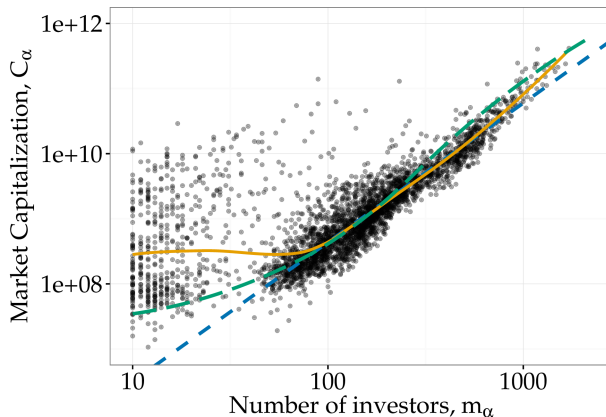


Figure 3: Market capitalization of securities as a function of the number of investors in logarithmic scale. From the local non-linear robust fit (yellow line) we observe a linear relationship for assets with more than about 100 investors. The blue dashed line corresponds to a linear fit on that group of asset. Hence $W_\alpha \propto m_\alpha^\gamma$, with $\gamma \simeq 2.1$. Robust locally weighted regression fit for the simulated data (in green).

These forms are filled manually and are thus error prone. We partially solve this issue by cross-checking different sources (which often contains overlapping information) and by filtering data before processing (see details in S.I.).

The main limitation of this dataset is that it provides accurate figures for long positions only. The other positions (short, bonds, ...) are most of the time only partially known. The frequency of the dataset is also inhomogeneous: data for most of the funds are quarterly updated (depending on regulations), hence we decided to restrict ourselves to 4 points in a year only. Such frequency is probably too low for investigating the dynamics of individual behaviour but is not a problem for we focus on an aggregate and static representation of the investment structure.

Discussion and conclusion

While WoC is commonly applied to a population collectively guessing a single number, we investigate here a fundamentally different situation and provide evidence for a collective functional optimization of the asset ownership structure. What the reference function should be is dictated by optimality arguments. In the case of financial markets, the rational benchmark was not related to the efficient market hypothesis, but to the way a large population of professional fund managers build their portfolios. Whereas each fund has its own benchmark with respect to which the fund performance may be assessed, this, fortunately, has no discernible influence on the average structure of their portfolio. In addition, WoC is often meant as a collective guessing of non-experts; one thus may conclude that the population investigated here has decidedly more expertise than the subjects of other WoC studies. What kind of expertise the typical fund manager has is not obvious, at least when one looks at their pure performance (see e.g. [20]). In addition, the optimal relationship between the number of assets in a portfolio and the value of the latter is clearly not broadly known in these circles, as shown by the very large deviations from the ideal case in Fig. 1, and the collective

expertise only appears when their decisions are suitably averaged. The presence of WoC when the subjects face strong constraints, as those of highly diversified funds, is more conjectural, and more work will be needed in that respect.

At a higher level, our results suggest that, while individuals may deviate much from the rational expectation theory, standard economic theory may hold at a collective level, without need for micro-founded individual decisions: the average decision may in some cases be approximated by a rational, representative agent. Our results however only hold on a snapshot of the system, for which individual fluctuations may be averaged out. In a dynamic setting, the very large deviations from the rational benchmark may not be neglected in the presence of feedback loops [21]. In other words, the dynamics of these fluctuations are worth investigating in their own right.

Acknowledgements

S. Gualdi acknowledges support of Labex Louis Bachelier (project number ANR 11-LABX-0019)

References

- [1] Surowiecki J. *The wisdom of crowds*. Anchor; 2005.
- [2] Galton F. *Vox populi (The wisdom of crowds)*. *Nature*. 1907;75:450–51.
- [3] Hill S, Ready-Campbell N. *Expert stock picker: the wisdom of (experts in) crowds*. *International Journal of Electronic Commerce*. 2011;15(3):73–102.
- [4] Landemore HE. *Why the many are smarter than the few and why it matters*. *Journal of public deliberation*. 2012;8(1).
- [5] Nofer M, Hinz O. *Are crowds on the internet wiser than experts? The case of a stock prediction community*. *Journal of Business Economics*. 2014;84(3):303–338.
- [6] Davis-Stober CP, Budescu DV, Dana J, Broomell SB. *When is a crowd wise?* *Decision*. 2014;1(2):79.
- [7] Lorenz J, Rauhut H, Schweitzer F, Helbing D. *How social influence can undermine the wisdom of crowd effect*. *Proceedings of the National Academy of Sciences*. 2011;108(22):9020–9025.
- [8] Muchnik L, Aral S, Taylor SJ. *Social influence bias: A randomized experiment*. *Science*. 2013;341(6146):647–651.
- [9] Hogarth RM. *A note on aggregating opinions*. *Organizational Behavior and Human Performance*. 1978;21(1):40–46.
- [10] Hartley JE, Hartley JE. *The representative agent in macroeconomics*. Routledge; 2002.
- [11] Colman AM, Pulford BD, Rose J. *Collective rationality in interactive decisions: Evidence for team reasoning*. *Acta psychologica*. 2008;128(2):387–397.
- [12] Kirman AP. *Whom or what does the representative individual represent?* *The Journal of Economic Perspectives*. 1992;6(2):117–136.

- [13] Härdle W, Kirman A. Nonclassical demand: A model-free examination of price-quantity relations in the Marseille fish market. *Journal of Econometrics*. 1995;67:227–257.
- [14] Malkiel BG, Fama EF. Efficient capital markets: A review of theory and empirical work. *The journal of Finance*. 1970;25(2):383–417.
- [15] Malkiel BG. The efficient market hypothesis and its critics. *The Journal of Economic Perspectives*. 2003;17(1):59–82.
- [16] Fama EF. Market efficiency, long-term returns, and behavioral finance. *Journal of financial economics*. 1998;49(3):283–306.
- [17] de Lachapelle DM, Challet D. Turnover, account value and diversification of real traders: evidence of collective portfolio optimizing behavior. *New Journal of Physics*. 2010;12(7):075039.
- [18] Bouchaud JP. Price impact. *Encyclopedia of quantitative finance*. 2010;.
- [19] Cleveland WS, Grosse E, Shyu WM. Local regression models. *Statistical models in S*. 1992;2:309–376.
- [20] Barras L, Scaillet O, Wermers R. False discoveries in mutual fund performance: Measuring luck in estimated alphas. *The Journal of Finance*. 2010;65(1):179–216.
- [21] Gualdi S, Tarzia M, Zamponi F, Bouchaud JP. Tipping points in macroeconomic agent-based models. *Journal of Economic Dynamics and Control*. 2015;50:29–61.
- [22] Muggeo VM. Estimating regression models with unknown break-points. *Statistics in medicine*. 2003;22(19):3055–3071.

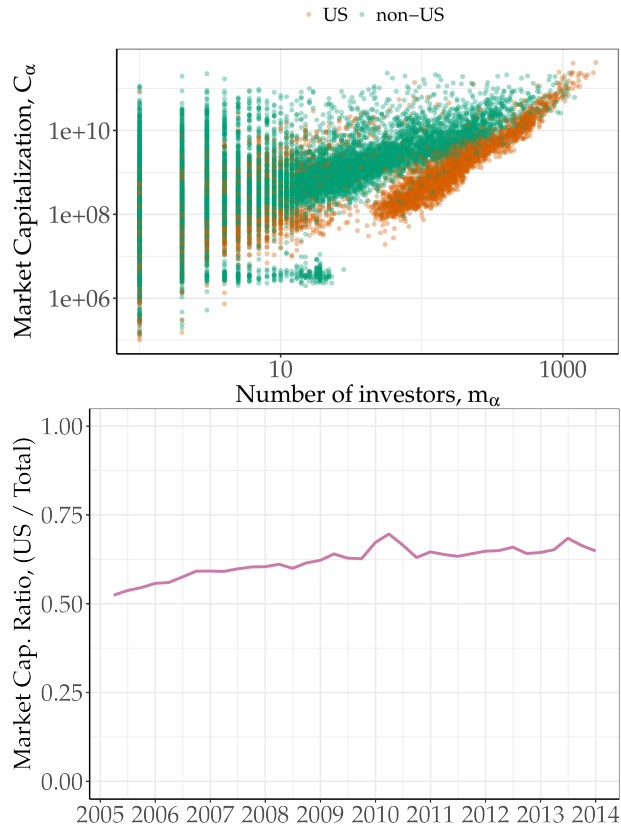


Figure 4: Top: Market capitalization as a function of the number of investors for all securities. Bottom: Temporal evolution of the aggregated market capitalization of US over the total market capitalization.

Supporting Information (SI)

4 Filtering

In order to remove inconsistencies in the dataset, we applied the following filters

4.1 Country of origin

Our dataset is sparse and heterogeneous. Indeed, the quality of the sources of data is directly related to each country’s disclosure regulations. For these reasons we decided to keep only the entities which use an US based mail address.

About 60% of the total market capitalization of the dataset is concentrated in US based securities. Figure 4 shows two large clouds of dots, each of them corresponds to a different region of origin: green (resp. orange) cloud corresponds to non-US (resp. US) based securities. The origin of this large difference between these two regions are not clear: it could for example come from

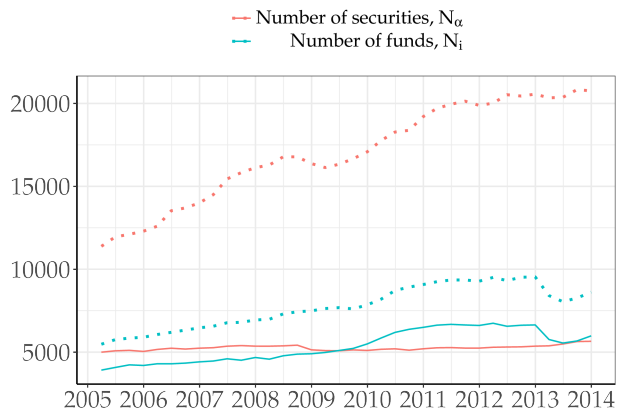


Figure 5: Temporal evolution of the number of funds N_i and securities N_α in the database. Un-filtered in dashed lines and US based only in solid lines.

differences in regulations in non-US countries. It turns out that the ratio of the investment values in US and non-US assets varies little as a function of time (see Fig. 4), which does not affect the exponent μ in Eq. 1. As a consequence we focused on US securities.

4.2 Frequency

Large funds are requested to report their positions at a frequency which depends on the applicable regulation. As a result, reporting frequency ranges from monthly to yearly, most funds filing quarterly reports. We therefore focused of the latter.

4.3 Penny Stocks

The “penny stocks”, i.e., usually securities which trade below \$5 per share in the USA, are not listed on a national exchange. Since they are considered highly speculative investments and are subject to different regulations, we filtered them out.

4.4 Size

We also filtered out small founds and securities and applied the following filters: $fW_i > 10^5$ USD, $C_\alpha > 10^5$ USD, $n_i \geq 5$, $m_\alpha \geq 10$.

4.5 Output

We restricted our study to 36 quarterly snapshots starting from the first quarter of 2005 and ending with the last quarter of 2013. Figure 5 reports the evolution of the number of securities and funds in the database before and after filtering.

5 Asset selection modelling

The framework we introduce in this paper follows a series of a few elementary steps described below. The aim is for the model to be sensitive to the different constraints which dominates the portfolio selection of a fund.

5.1 Finding n^*

For date t , we define the cross-over point n^* between the two regions which appear in the local polynomial regression. We determine this point value with a likelihood maximization of the model

$$W = \mu_{<}n + (\mu_{>} - \mu_{<})(n - n^*)\theta(n - n^*), \quad (5)$$

where $\theta(x)$ is the Heaviside function. We use a recursive method to find parameters $\mu_{<}$, $\mu_{>}$ and n^* [22]. Figure 11 shows that n^* is stable as a function of time.

5.2 Asset selection in the small diversification region $n_i < n^*$

In this region, we consider the equally weighted portfolio hypothesis to be true. Each position has a size $\frac{W_i}{n_i^{\text{opt}}}$, where n_i^{opt} is the optimal number of position computed with eq 1. The funds select their asset randomly with a probability proportional to C_α . Also, in order to construct an equally-weighted portfolio, a position is valid only if it is of size $\frac{W_i}{n_i^{\text{opt}}}$.

5.3 Asset selection in the large diversification region $n_i \geq n^*$

In this region, the liquidity constraints make it harder for funds to keep an equally weighted portfolio and portfolio values are thus spread on a larger number of assets. We propose here a stochastic model of asset selection based on two main ingredients: first that the selection probability of asset α by fund i depends on the diversification of a fund n_i and on the scaled rank of the capitalization of asset α , and that the investment is bounded by an hard constraint on the fraction of market capitalization of asset α .

We chose a security selection mechanism which rests on the scaled rank of capitalization of security α , defined as $\rho_\alpha = \frac{r_\alpha}{M}$ where r_α is the rank of capitalization C_α and M the number of securities at a given time. The selection probability $P(W_{i\alpha} > 0 | \rho_\alpha)$ is then obtained by parametric fit to a beta distribution in each logarithmic bin. Note that we do not use the same rank-based selection mechanism in the low-diversification region because in this case it is harder to have a good fit with the beta distribution. This is however only a minor point since the capitalization is approximately power-law distributed and the two selection mechanisms are basically equivalent (the rank is proportional to a power of the capitalization) and indeed results are very similar in both cases.

Figure 6 shows that the distribution of the ranks in which a fund is invested is sensitive to its diversification n_i for 2013-03-31. The Beta distribution, which is limited to a $[0, 1]$ interval, is flexible enough to describe the asset selection mechanism of a fund.

$$f(x; a, b) = \frac{1}{B(a, b)} x^{a-1} (1-x)^{b-1}, \quad (6)$$

where a and b are the shape parameters of the distribution, and B is a normalization constant.

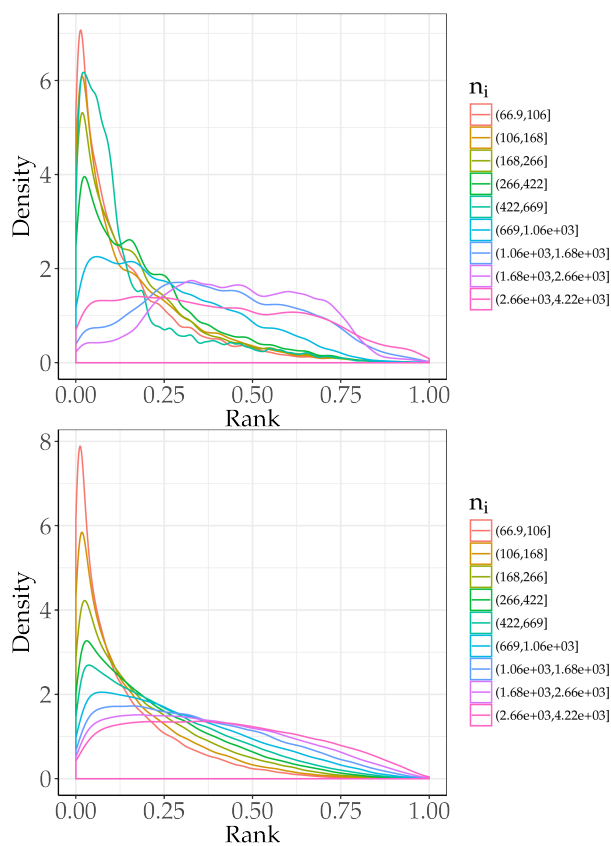


Figure 6: Top: Empirical probability density function of investing in a security of scaled capitalization rank ρ given the diversification n_i of the fund. Bottom: Probability density function of investing in a security of scaled capitalization rank ρ given the diversification n_i of the fund, given by the model.

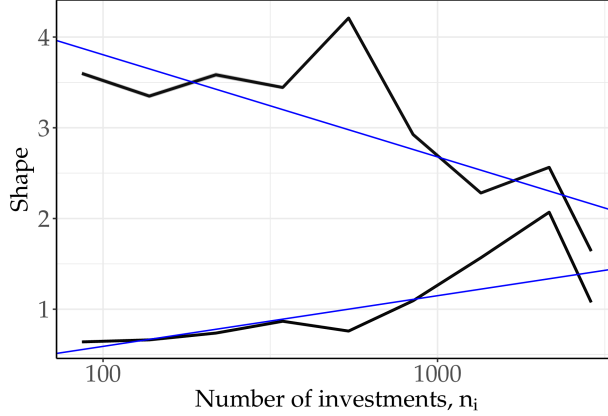


Figure 7: Coefficients a and b of the Beta Distribution 6 as a function of n_i . Linear fits are for eye-guidance only.

Maximum investment ratio

The funds limit their investment in a given asset. They seem to follow a simple rule: defining the investment ratio $f_{i,\alpha} = \frac{W_{i\alpha}}{C_\alpha}$, one easily sees in Fig. 8 that each fund has a maximum investment ratio

$$f_i^{\max} = \max_\alpha \left(\frac{W_{i\alpha}}{C_\alpha} \right) \quad (7)$$

Since the average exchanged dollar-volume of an asset is proportional to its capitalization (Fig. 10), the existence of f_i^{\max} is a way to account for the available liquidity.

Although that limit is clear for an individual fund, there is a large range of empirical values f_i^{\max} Fig. 9.

6 Simulation

The simulation is done in a few simple steps:

1. Compute n^* using the segmented model Eq. 5.
2. Select a fund i , with a number of assets n_i .
3. If $n_i < n^*$:
 - (a) Compute its optimal portfolio value using Eq. 1. The fund will invest $\frac{W_i^{\text{opt}}}{n_i}$ for every position.
 - (b) Select assets randomly with a probability proportional to C_α .
4. Else if $n_i \geq n^*$:
 - (a) Compute its f_i^{\max} , so that the fund i will invest f_i^{\max} in n_i assets.

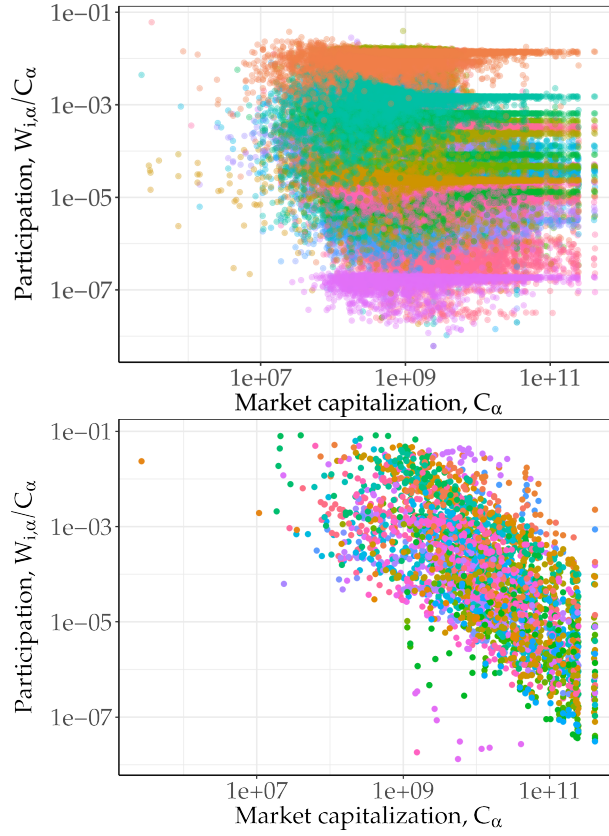


Figure 8: Fraction of the market capitalization of a security held by a fund. Each color represent a different fund. Top: Funds with a large diversification ($n_i > 800$). We can clearly see a delimitation for most of the funds, which correspond to the maximum fraction f_i^{\max} . The value of f_i^{\max} widely differs from one fund to another. Bottom: Funds with a low diversification ($n_i < 60$), f_i^{\max} doesn't appear.

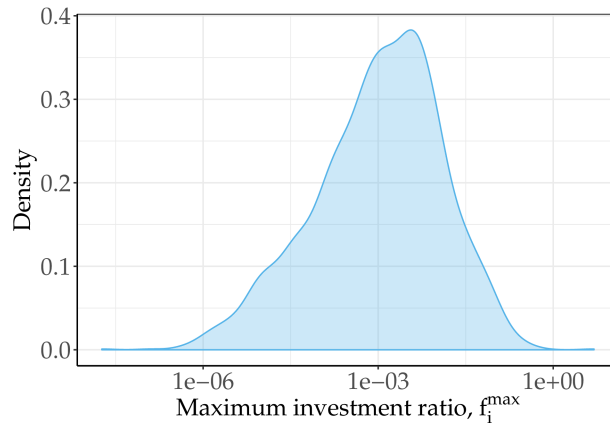


Figure 9: Empirical probability density function of f_i^{\max} for all the funds.

- (b) Select assets randomly following a Beta probability distribution Fig. 6 with the parameters found in Fig. 7.

By iterating those steps we obtain Fig. 1

Since the simulation outputs a portfolio for every fund, we can directly infer the number of investors m_α of every security.

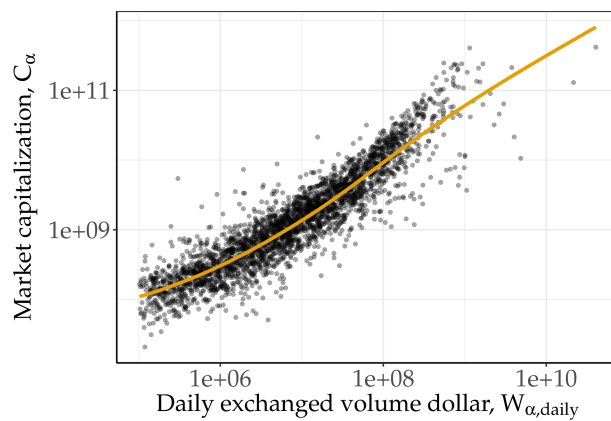


Figure 10: Market capitalization as a function of the daily exchanged volume dollar. We find a slope close to 1 for all the dates in our database, confirming the hypothesis that the daily exchanged volume dollar of an asset is proportional to its market capitalization.

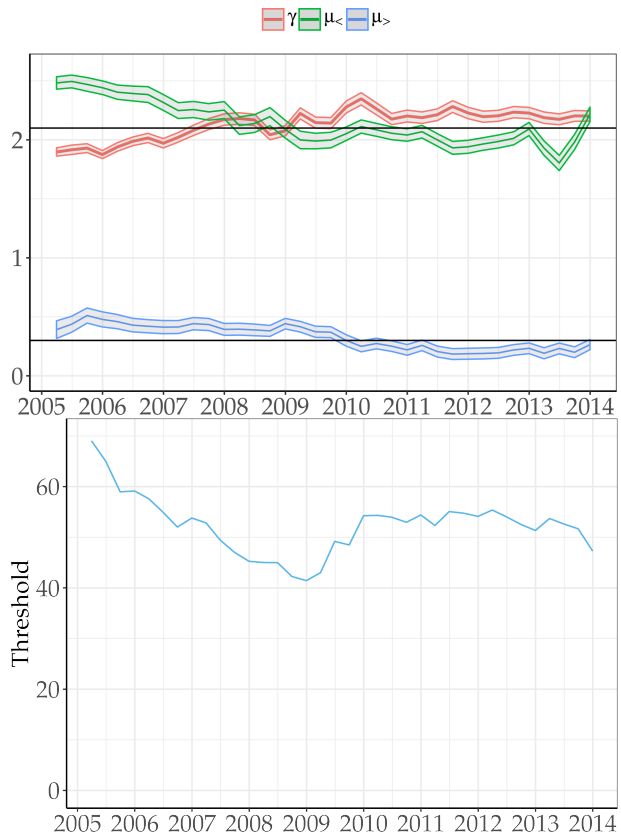


Figure 11: Top: Temporal evolution of the coefficients $\mu_{<}$, $\mu_{>}$ and γ . Bottom: Temporal evolution of the value of the cross-over point n^* between the two regions as a function of time. It reaches a global minimum after the 2008 crisis.

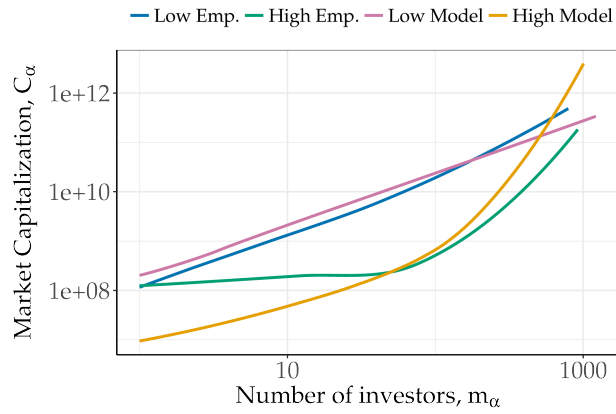


Figure 12: We separate the contribution from the low and highly diversified region. The origin of the discrepancy observed in Fig. 3 appears to be mainly due to the highly diversified region (Green dots for the empirical data, and orange dots for the model).