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# Topology Optimization of Magnetic Actuator based on Reluctance Network Modeling and Adjoint Variable Method

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Topology Optimization (TO) has great significance in primarily the concept design of a product. In the electrical engineering community, conventional topology optimization is usually based on a spatial discretization that also is used as the mesh for Finite Element Analysis (FEA). In this paper, we proposed to use a local equivalent circuit modeling method - Reluctance Network Analysis (RNA). Afterward, a gradient-based optimization algorithm, line search method, was chosen as the optimizer where the sensitivity information was calculated by Adjoint Variable Method (AVM). Finally, the feasibility of this model for topology optimization was verified through a case study of a magnetic actuator.

*Index Terms*—Topology optimization, magnetic actuator, reluctance network analysis, adjoint variable method

## I. INTRODUCTION

TOPOLOGY OPTIMIZATION has received wide attention from electrical engineers since the seminal work by Bendsoe and Kikuchi [1]. Contrary to parametric optimization methods, the optimization has no initial constraint, so, it may result in an entirely novel topology that can't be imagined beforehand.

Many different approaches, such as Homogenization Method, Density Method, ON/OFF Method, Level-set Method and etc., were developed for TO [2]. In the electromagnetic field, the Density Method was the most widely adopted for TO. A typical case is the so-called Solid Isotropic Material with Penalization (SIMP) which introduces an interpolation function that interprets the material properties as continuous variables. Design space is discretized into elements, then, different predefined materials are allocated into them.

To process TO, a numerical model is required for characterizing the performances. There are several approaches for modeling in electrical engineering, and the two popular used are the finite element analysis (FEA) and the reluctance network analysis (RNA). Their different characteristics were analyzed in [3]. This paper presents an effort to implement reluctance network modeling. In order to verify the feasibility of this model in TO, a classical problem of magnetic actuator, as shown in Fig.2, is chosen as the case study due to its simple structure and magnetic path. It consists of three parts, the core (with 'C' shape), the coil (in the yellow regions), and the armature (the moving part). The chosen design region is marked out with a dashed line box as shown in Fig. 2 and the optimization objective is to maximize the magnetic force imposed on the armature. The density method was adopted and the material in the design space was updated toward the steepest descent direction. To speed up the solving, the sensitivity information usually is obtained through AVM. Many applications of this method are based on FEA, in this paper, the AVM will be developed for RNA cases.

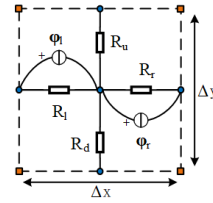


Fig. 1. The mesh element and its associated magnetic equivalent circuit

## II. RELUCTANCE NETWORK MODEL

The physical electromagnetic domain is decomposed into elements via a mesh as shown by the dashed line with four vertices in Fig.1. In this case, each element can be represented by a magnetic equivalent circuit consisting of four lumped reluctances ( $R_l$ ,  $R_r$ ,  $R_u$ ,  $R_d$ ) and two flux sources ( $\varphi_l$ ,  $\varphi_r$ ) with a structure similar to Fig.1. Every reluctance is expressed by (1) or (2), while the flux sources are expressed by (3). The reluctance values depend on the discretization of the region to be studied and actuator dimensions. The coefficient  $N$  is chosen to respect Ampère's theorem in each loop of the circuit.

$$R_l = R_r = \frac{1}{\mu_0 \mu_r} \frac{1}{z} \frac{\Delta x / 2}{\Delta y} \quad (1)$$

$$R_u = R_d = \frac{1}{\mu_0 \mu_r} \frac{1}{z} \frac{\Delta y / 2}{\Delta x} \quad (2)$$

$$\varphi_l = \varphi_r = \frac{NI}{R_l} = \frac{NI}{R_r} \quad (3)$$

Where  $\mu_0$  is the vacuum permeability,  $\mu_r$  is the relative permeability of the physical material,  $z$  is the depth of the device, and  $\Delta x$ ,  $\Delta y$  are the step size of the mesh.

Therefore, the whole system can be represented by an equivalent network. We apply nodal analysis to every node of the circuit. This equation for the whole system can be expressed by (4), where the  $[\Lambda]$  is the matrix that contains the reluctances of the system, the  $\{\varepsilon\}$  is the magnetic potential vector and the  $\{\varphi\}$  is the flux sources vector due to current in the winding.

By solving the matrix system the values of magnetic potential in each node are obtained. Other information such as magnetic potential difference and magnetic flux can also be deduced.

$$[\Lambda]\{\varepsilon\} = \{\varphi\} \quad (4)$$

### III. SENSITIVITY COMPUTATION

The magnetic force (F) applied on the armature is computed through Maxwell stress tensor method. For the density method, the permeability of the predefined material is defined as (5) where  $\rho$  is the normalized density and the exponential term  $n$  of this mapping function is named penalization coefficient. By setting the different values of  $n$ , we would get different interpolation functions and its influence on the optimization result was investigated in the paper [4]. In this paper, the line search was chosen as the optimization algorithm and the element's material was updated toward the steepest descent direction. Therefore, gradient information is needed. Usually, the sensitivity can be obtained through the finite difference method or AVM. The main drawback of the finite difference method is that it is computationally expensive for TO which gets large amounts of design variables [5]. In comparison, the AVM needs only an extra calculation to get the sensitivities of all the variables.

Firstly, differentiate a normalized density in the design region,  $\rho_i$ , on both sides of the system equation. Then we would get an expression of (6), where the derivative of  $\{\varphi\}$  to  $\rho$  equals 0, cause the flux source is independent of the design variables. The objective function, F, can be seen as an implicit function,  $\tilde{F}(\varepsilon, \rho)$ , about  $\varepsilon$  and  $\rho$ . A similar differential equation of the objective function to each variable  $i$  gets the form (7). Combine the two formulas, (6) and (7), finally we get a new expression (8). The introduced variable  $\lambda$ , is the adjoint variable. We could get the sensitivity of all the design variables by determining the value of adjoint variable once time.

$$\mu = \mu_0 (1 + (\mu_r - 1)\rho^n) \quad (5)$$

$$\frac{\partial[\Lambda]}{\partial\rho_i}\{\varepsilon\} + [\Lambda]\frac{\partial\{\varepsilon\}}{\partial\rho_i} = 0 \quad (6)$$

$$\frac{dF}{d\rho_i} = \frac{\partial\tilde{F}(\varepsilon, \rho)}{\partial\rho_i} + \left(\frac{\partial\tilde{F}(\varepsilon, \rho)}{\partial\varepsilon}\right)^T \frac{\partial\varepsilon}{\partial\rho_i} \quad (7)$$

$$\frac{dF}{d\rho_i} = \frac{\partial\tilde{F}(\varepsilon, \rho)}{\partial\rho_i} - \underbrace{\left(\frac{\partial\tilde{F}(\varepsilon, \rho)}{\partial\varepsilon}\right)^T}_{\lambda^T} [\Lambda]^{-1} \frac{\partial[\Lambda]}{\partial\rho_i}\{\varepsilon\} \quad (8)$$

### IV. RESULTS

The system in this case study is assumed to be linear. The optimized topology as shown in Fig. 2 seems logical because it reduces the flux leakage and links more flux to the armature. In order to remove the intermediate materials ( $0 < \rho < 1$ ), a filter is required. Among all different filtering functions, the feasibility factor (FF) is adopted in this paper. Fig. 3 depicts the

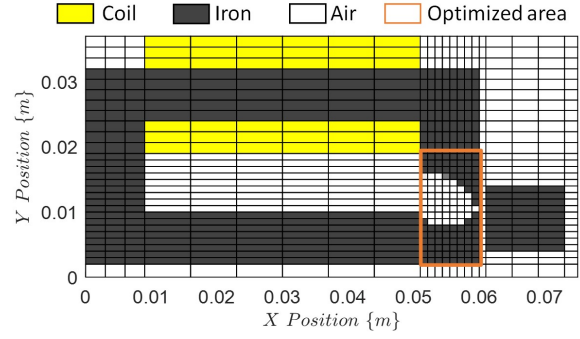


Fig. 2. Optimal design and the used mesh

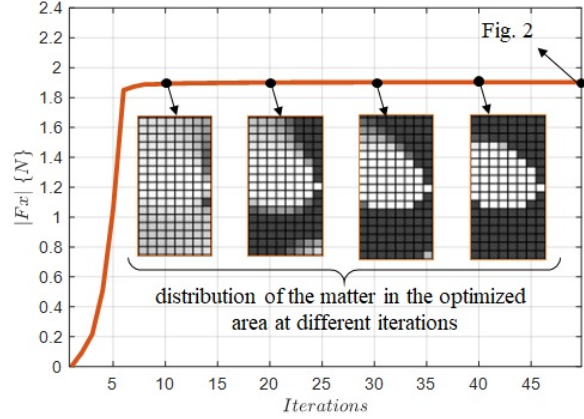


Fig. 3. Force evolution

evolution of magnetic force with iterations. The force reaches its relatively high value around the 8th iteration, but a clear boundary of the topology in the design space appears until around the 35th iteration. Regarding of time consumption, the optimization based on AVM gets considerable time saving compared with the finite difference method. As a whole, the topology optimization based on the RNA model is feasible in this case and it seems that it could be applied to more complex cases such as electrical machines, etc. Magnetic saturation effects and volume constraints would be discussed in the final paper.

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