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A COMPARISON OF LANGUAGE PRACTICES DURING GEOMETRY CLASSES IN PRIMARY SCHOOLS IN QUÉBEC AND FRANCE

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This research focuses on the introduction of explicit teaching of symmetry, within the context of primary schools in France and in Québec, Canada (pupils are 7-8 years old). The originality of our work lies in the articulation of different theoretical approaches (mathematical, linguistic, and metaphorical-conceptual) which we describe briefly in the first part of this article. The second part of this contribution is devoted to the comparison of the same mathematical situation in two different contexts (France and Québec). We focus on the analysis of teachers' activities – language and gestures – with regards to three components (acting, talking, thinking) in order to stress the specificity of social context, and in particular the use of language in the process of negotiating meaning.

ANALYZING THE SOCIOCULTURAL DIMENSION OF LANGUAGE PRACTICES

The social dimension of learning processes in mathematics education has long been an integral part of many English-language studies (Sfard 2012; Moschkovich 2010; Morgan 2013). These studies have adopted an approach that is sensitive to social, historical, and cultural contexts and assigns a predominant role to language in the process of building knowledge. Our approach is somewhat different, however, because in the context of learning and teaching of geometry we put the adaptive and social dimensions on the same footing as they operate in the process of encountering and constructing knowledge (Bulf, Mathé, & Mithalal, 2014).

An extended approach to analysis that highlights didactics, discourse, semantics and metaphors

We implement an *a priori* analysis of the interactions between mathematical objects and teacher-students potential activities in terms of culturally determined acting-talking-thinking manifestations (Bernié, 2002; Jaubert & Rebière, 2012). The *Frequentation Modes* (ModF) tool (Bulf, Mathe & Mithalal 2014) allows us to conduct an *a posteriori* analysis of the actual experiences of acting-talking-thinking – manifestations – of mathematical objects during a learning situation occurring at school. In order to take an in depth look at these manifestations within the mathematical context of geometry, we develop a theoretical approach which considers, first, a semantic (Jacquet, Venant, & Victorri, 2005) and discursive (Kerbrat-Orecchioni, 2005) analysis and, secondly, an epistemological analysis of mathematical objects in terms of metaphores (Nuñez & Marguetis, 2014) by seeking to establish relationships between different mathematical and no-mathematical objects. According to this original approach, symmetry is stressed within its epistemological, logical (Barrier, Hache & Mathé 2013) and semantic (Victorri & Fuchs, 1996) complexity.

Extending the analysis of symmetry to consider *acting-talking and thinking* as a whole

The concept of a symmetrical figure can be considered through a variety of relationships: unary (recognizing a figure in its entirety), binary (for instance when sub-elements of the same figure are compared using criteria such as shape or size) or ternary relations (one element of a figure is associated with another element in relation to a bordering zone). We can also analyze them from the perspective of acting-talking-thinking as a whole unit of analysis, which implies an exploration of how and why these relationships can be recognized. Conducting an extended analysis helps us grasp the specific dynamics between symmetry and the manifestations of acting-talking and thinking in two different school learning situations of each of the cultural contexts we have studied. We will describe the results of this analysis in the following section. Our research is guided by some fundamental questions while focusing primarily on an analysis of the teachers' activity: how do teachers use language to engage their students in a problem-solving activity? How is knowledge mobilized through language practices? How can we describe the way in which acting, talking and thinking evolve throughout the learning process?

NEGOTIATING THE MEANING OF SYMMETRY IN TWO DIFFERENT CULTURAL CONTEXTS

The experimental problem-solving exercise¹ was tested in a class of second year primary school students (ages 7-8) in Bordeaux, France, and in an equivalent class in Saint Hyacinthe, Quebec, Canada. The first two phases of the exercise were carried out in both classes. The general goal of the problem is to find a way to recognize a *symmetrical figure* (butterfly) by superposition across the axis of symmetry. In this paper, we choose to focus our analysis on the data collected during the first discussion group conducted after the individual exploration of symmetrical figures (phase 1) in both classes in France and in Québec. The task given to students consists of matching two halves of a butterfly in order to form a pair of symmetrical figures (one part is identified by a letter and the other by a number). This particular moment in the study was selected for analysis as it reveals how this situation was managed in very different ways in both classes. For instance, while the French teacher relied on language in order to make a collective and shared *Frequentation Mode* (ModF) of the symmetrical figure, the Québécois teacher left the students to work more individually on the figure while encouraging students to share and to test the validity of different strategies.

In the French classroom: a collective encounter using oral language to negotiate meaning

From the beginning of the lesson, the teacher listened to students' ideas and helped them to reformulate and to develop their ways of talking about and working with symmetrical figures, until, through collective negotiation, they created a more precise ModF shared by the entire class. This ModF of a symmetrical figure is not the only one to have emerged during the phases of individual research and group sharing. However, we have chosen to describe it because it has been negotiated and shared with the other students and will be used as a starting point for the rest of the work. The

¹ Fénichel et Al., 2004, 139-142

emergence of this ModF was initiated by the teacher when she identified, in the students' discourse, a problematic situation that illustrated the necessity to reconsider actions:

89 Teacher: you found the C, well...So, apparently (*she gestures towards the board*), now you generally agree on all those letters, OK, however here (*she points to the number five that is associated with the figures B and C*) you don't really know...So, well, how did you do it, how can you be certain that it is in fact, the right half of the butterfly...[...] Because I could have done that too (*she puts the figure number five next to another half*) and said that it's the right, um, I could have said that it's good um, it's good because it looks like a... It's a butterfly, isn't it? (*students say yes*). I put the number five figure with figure A, and, in fact, it's one, it's the right one, I can say that it's good, it's a butterfly, right?

We observed how the teacher took up, reformulated, and encouraged students to refine their ideas (they should have been associating the two halves of the butterfly), even slipping clues about the relationship of comparison into the discussion. The teacher asked for students' responses and quickly set up the problem to be solved once students had associated the different pairs of wings. Suggesting what students should be doing through her own speech, she refined students' definitions by referring when necessary to the nature of the objects in question, for instance *the half butterflies* and not the letters and numbers which up until this point had been used to indicate them metonymically. The teacher helped students reconsider their point of view, going from a superficial vision (*"It looks like... a butterfly, doesn't it?"*) to a more localized vision focusing on surface contours and lines. The use of the verb "to do" (*"How did you do it?"*) centered the discussion on the action and aimed to involve the students in a verbalization of the procedures and criteria they have been using. The criteria of *similar shape* emerged during the class (teacher: *"What has the same shape?"*). The teacher places an emphasis on the term *shape* by using it in a question. The goal of the teacher is to get the students talking about the criteria for symmetry, not only about the size of the figures, but also about their shape. In particular, we would like to highlight the fact that the teacher introduced more specialized mathematical vocabulary, *the same shape*, which corresponds to a criteria for comparison that the teacher wished to produce and to share as a first ModF. A student then came up to the board. He developed the idea that working on the general appearance of the butterfly was insufficient.

112 Teacher: lift up your hand...Yann ?

113 Yann: well, in fact, I, well, it's that things that have the same shape, it's like, imagine (*he gets up and comes to the board*) in fact, it's like if we put for instance this butterfly, this half of butterfly with the figure A (*he puts the half of figure 5 with another figure while talking*) I agree that it makes a butterfly (*teacher says yes*). But the only problem, is that in fact, **it's not at all the same drawing**

116 Teacher: it's **not the same drawing**. You mean that...

This student's intervention is very interesting because we observe a kind of refinement in the way of discussing the butterfly halves that allows him to be using more precise vocabulary, in contrast with other students that came and shared their results on the board. The negotiation of « new » criteria for comparison is based upon a ModF that was previously accepted, considering the size and the shape as criteria. Yann brings up, for example, the size comparison (*it's bigger*) to further the discussion and refine this criteria by observing that the size of two figures must be *exactly the same*. This refinement of the mathematical vocabulary also represents a change in actions, because

students will, from this moment on, compare *every single element* of the figure considered (in this example, the wings and the antennas):

117 Yann: well, (*he shows the parts of the drawing*) this one already is bigger, you see and they don't have the same antennas. So I crossed it out (*he makes the gesture of crossing out the half of figure number 5, teacher says yes*) each time that I try. And at a point in time I noticed that the 5 (*he places together the halves A and 5*). I did it and I saw that it was exactly the same antennas and exactly the same wings (*he shows the class at the same time*) and I said that's it

120 Teacher: OK, it's the same antennas and the same wings or?

121 Yann: um, here for example, it's the same antennas (*he passes his finger over each antenna*) and there they are the same wings because there is this (*he passes his finger over a part of the wing*) but also at the moment, I think that some students may have fallen into a trap and chosen number 2

Near the end of these discussions, numerous deictic expressions are linked with this new way of thinking, as well as gestures regarding the symmetry near the zone where the two halves of the butterfly are joined. These gestures describe a new way of acting that takes into consideration elements that are in very close proximity and must correspond exactly on one side and the other of the butterfly. Consequently, the ternary relationship is considered: the axis of symmetry is designated by the students' gestures because it marks the zone that separates the two sides but the axis is not yet expressed in oral language. The student's way of acting led to an enrichment of vocabulary ("*precisely*") taken up by the teacher, showing a new way to think about symmetrical figures. In fact, we have already taken note of the significance of deictic expressions in the exchange among students and teachers which seem to have contributed to the use of the vocabulary that was less precise. According to our hypothesis this type of vocabulary may be the result of how the teacher managed the idea of sharing by often inviting students to come to the board with their butterfly halves. Students next moved on to the relationship between two objects of 1D² or 0D dimensions with respect to a defined bordering zone. Thus, we observed a highly intricate relationship between the dimensions of acting, speaking, and thinking, leading to the emergence and negotiation of a new frequentation mode, mediated by different forms of language associated with physical, bodily, perceptual, or metaphorical structures of experience. This frequentation mode can be describe in the following manner: two figures separated and juxtaposed by the action of cutting or folding along the edge of rectangles and aligned side by side. This is in fact a visual comparison of sub elements of the surfaces (1D or 0D) that correspond to criteria regarding shape and size that are compared (binary or ternary relationships are included) when considering a zone (on one side or the other of the axis). The teacher thus ensured that this new frequentation mode was accepted and shared by the class.

In the Canadian classroom: the free expression of individuality during a discussion group

The teacher presented the first phase of the activity, the exploration phase, while specifying on many occasions that students were to develop individual strategies. Before the group sharing of findings phase, she suggested that the students validate their answers and compare their strategies with another student. The teacher also proposed that students might identify the butterfly halves that

² These dimensions refer to the nature of the objects being considered – point (0D), surface contours (1D), sub-figure (2D), etc. (Duval, 2005)

go together with the same color, and she allowed the students to cut out the butterflies from the rectangles. The degree of freedom provided to the students – and the choice to not necessarily associate the butterfly halves in a table – already serves as evidence of the ways in which the actions of the two teachers differ. As soon as students began working on the problem, the teacher repeated their ideas. She did not concentrate on the link between the *two elements* or figures (symmetry). Despite the fact that this phase lasted longer in the Canadian class, the teacher did not attempt to refine or redirect the students' ideas, and allowed them to freely express themselves. We observed that during the initial exploration phase, the students' procedures were more varied than those of their French counterparts. This can be partly explained by the fact that both halves of the butterflies were printed on the same piece of paper this time, and the students were not asked to create a table of their results, as was the case in France. However, the discussion that followed with the whole class was not very productive in terms of negotiating meaning.

105 Teacher: It's alright, it's you who decides how you want to arrange them. It's up to you to choose a strategy, you choose the strategy that you want. You have the right to start like that and the others using another strategy [...]

During the group sharing of findings, the teacher tries to elicit different methods of arranging the butterfly halves in order to allow each student to justify their choice. In contrast with the teacher from France who frequently sought to bring to light contradictions among the students' comments and choices and to encourage the use of a more specific vocabulary when describing the relationship between the butterfly halves, such as the same shape or size, the Québécoise teacher seems to be content with the students' identification of different criteria for local validation – based upon the students' actions – such as when the two halves go together. Thus, the classroom discourse is not managed in the same manner...

111 Teacher: ok, I would like to know with your butterflies, **how did you find the butterflies that went together**, and we will write, which butterfly... Which letter goes with which number. If we begin with number 1. Which letter goes with number 1 (*Many students lift up their hand*) Olivier, which letter [...]

115 Olivier: there are sides that are almost equal

116 Teacher: **there are sides that are almost equal...** What do you mean?

117 Olivier: the antennas are crooked...

118 Teacher: ah! **The antennas are crooked... I see**

119 Olivier: the bottoms too

120 Teacher: the bottoms too, the bottom here you mean that it is crooked? Here and on top? [...] Perfect, did anyone else find another way to show that 1 goes with letter D. Karl (*the teacher writes 1-D on the Smartboard*)

125 Karl: I saw that they had the same, they had the same, they had the same antennas [incomprehensible explanations] that had the same wings on the top, **I verified, that had the same wings on the bottom, I verified again, I told myself that the two will go together, I cut them and I put them together**

128 Teacher: when you say that you checked, you checked with your eyes? You looked at them, is that what you meant?

129 Karl: well, yes, because I already had the crooked wings, the two have a ball on the top that's the same, and a wing on the bottom that's the same

130 Teacher: perfect, super, now, number two (*writes 2 on the Smartboard*) which one goes with number two

The students seem to be aware of the notion of strategy, so they formulate one with confidence. However, the negotiation of a ModF that articulates these strategies does not seem to emerge easily during exchanges among the teacher and students. The approaches generally remain individual, despite the phase of group negotiation. While, the French teacher had been successful in engaging, via verbal language, a negotiation with her class, this was not the case with the Canadian teacher. We realize that the interventions of the Québécoise teacher value the richness and the multitude of actions, without seeking – at least not explicitly – to produce a common language, we could say, without institutionalizing a ModF of symmetrical figures that may be characterized as having the same shape and the same size. These differences lead us towards hypotheses linked to the predominant cultural approaches in each of the contexts, i.e. in France, a cultural approach of *effective, but guided negotiation* and in Québec, *the management of debates* that leaves much room for individual expression, but where we rarely achieve a consensus and finally, the conclusions may be given by the people who manage the debate. If we go back to the above transcription, the student Karl used the word « verify », (*I verified, that had the same wings on the bottom, I verified again...*) this is in fact an important change that occurred during the discussions because it is insufficient to simply share a strategy, one must also justify it. The teacher seems to realize the importance of this specific information given by the student, but her questioning does not enable the student to develop and to explain his comparison strategy to the class. However, following other exchanges, the students have little by little taken a closer and more precise look at the components of the butterfly halves, as the gestures and the visual language used by the teacher and by the students leads to the description of local shapes of the butterfly halves. The halves are generally replaced by the elements that designate them (1D), but a return towards the characteristics of the real butterflies occurred when the idea of resemblance emerged between the *wings* or the *antennas* of the butterfly drawings. Finally, the teacher included, as the centrepiece of her conclusion, a metaphor for the flight of a real butterfly through which she tried to justify the importance of having wings of the same shape and size. Hence, she makes a departure from the context of mathematical figures and seeks to create links with a real world situation in an attempt to articulate the two contexts and to provide support for the students' reasoning. The flight metaphor was also used by the French teacher during phase 2 of the activity, however the goal of presenting this metaphor was to introduce the notions of superposition and turning around. In fact, different components of this situation are supported by the use of the same metaphor. The Québécoise teacher tries to relate the notions of same shape and size with the butterfly's ability to fly, while the French teacher invokes this metaphor to extend the negotiation beyond local criteria. While local criteria may be useful in a binary vision of symmetry, in order to lead the students towards a more global recognition and ternary conception of symmetry, the French teacher used this metaphor. However, the metaphor seemed more accessible in the Québécois context, but in the French context, the metaphor was not at all understood. We observe here, how metaphors cross over disciplinary frontiers and the effects that they may or may not provide on the mathematical activity of students.

170 Teacher: (*in response to Ayleen who said 4 goes with E because it has a pointy part on the bottom*) the E has a pointed part on the bottom, but this one does too (*she shows the 3 on the board*) it has a pointy part on the bottom...

171 Ayleen: but because, yes, but (*she makes a gesture with her finger from the top to the bottom to show the way that the wing is pointed, and with her two hands she draws in the air the shape of the wing*)

172 Teacher: ah yes, it is smaller (*she draws the bottom of the wing*) it is straight like that. Ok, and the other was longer (*she draws again the bottom of the wing of figure 3*) like this. [...]. The 5 goes with which figure?

173 Ève: the S [...] There is an antenna [inaudible] (*makes a gesture to show the shape of the antenna, from its head to the right in a line that is pretty straight*)

178 Teacher: ok, there is an antenna that is placed diagonally? (*makes a similar gesture that is more diagonal than Ève's*) is it like that, is that what you mean? (*Ève repeats the gesture*)

180 Teacher: a straight antenna? Ah! Almost horizontal (*they both make the gesture*), its' good, OK, Karl

181 Karl: *inaudible* the thorax is the longest *inaudible* and besides being the longest it is the biggest

182 Teacher: you are right the thorax and his body is bigger... Bravo! Super...

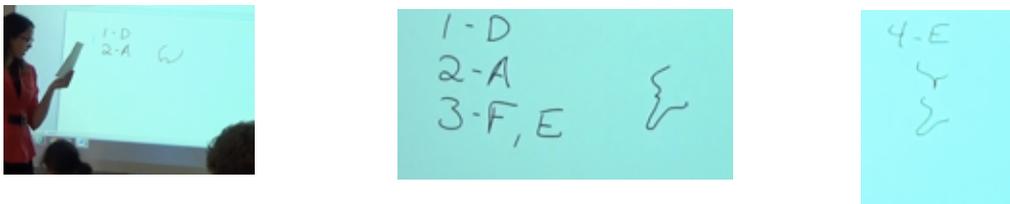


Figure 1

In conclusion, the teacher asked questions, encouraged students in the formulation of their answers and, most of the time, let them express themselves freely as they explained their procedures. She did not take up their formulation in order to improve their ways of designating the objects (herself using many deictic terms) and instead she repeated the same words and physical gestures. There was no decontextualization of students' arguments or criteria. The dimensions of acting and speaking were not clearly linked together. However, their acting was very rich and diverse and mostly linked to visual language. During this phase of the exchange, the teacher did not transform the students' observations into a clear problem for students to solve that they could have agreed on. The relationship of comparison, which emerged implicitly, did not enhance the *talking* between the students and the teacher. For example, the teacher settled on having the students indicate a length without comparing it to any other reference. Consequently, when the teacher wanted to conclude this first collective summary, she had difficulty decontextualizing the different criteria formulated by her students.

Teacher: the thorax, the size of the butterfly is it bigger or smaller...So you cut them and after you put them together, side by side to see if it's the same thing, super, so what I want you to remember...Ryan. Is that the butterflies, each side of the butterflies, **they have the same shape of wing, and pay attention, they also must have the same size of wing.** There should not be a giant wing (*big gesture*) and a wing that is small on the other side, it wouldn't work well to fly, they will fly all crooked, so the wings have to be proportionate, and they have to be the same height, the same size. Is that alright.

Despite her efforts, she was unable to highlight the fact that some students spontaneously refined their criteria of comparison. The teacher was thus compelled to summarize the first phase in a forced, explicit way.

Discussion

At this stage of our analyses, we cannot clearly state that the differences in *acting in*, *with*, and *through* language between the two teachers are the product of cultural differences, or that they can be accounted for by the backgrounds of the two teachers. The French teacher was more experienced (though experience is not the only way to develop a form of expertise), while the Canadian teacher was a relative beginner. The comparative analysis remains complex and relies on the encounter with the mathematical object at hand, as well as the emergence and development of negotiation processes favoring this encounter *in*, *with* and *through* the use of different forms of language. We should consider first of all the immediate cultural contexts giving meaning to knowledge, which determines the teacher's own conception of the power of language practices, and also look at the linking of these practices to taking ownership – by students and teachers alike – of the mathematical knowledge at hand, always complex and rich in meaning.

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