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On the improvement of the sensitivity levels of VLBI solutions from a combination with GNSS

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Abstract

Very Long Baseline Interferometry (VLBI) is the only technique that connects the celestial and the terrestrial reference frames. Because of its essential role, it is important to investigate the quality of VLBI observations and their effect on the estimated parameters. The fundamental goal of the quality assessment using the sensitivity is to find out to what level VLBI can detect station displacements in the adjusted coordinates. However, the global VLBI station network is worse in terms of the number and global distribution of stations when compared to the GNSS station network. In this study, we aim to improve the capacity of VLBI to detect station displacements through an inter-technique combination with GNSS. The effect of the selection of co-location sites in the inter-technique combination is investigated with four scenarios and we investigate how the sensitivity is affected by the assumed standard deviations of the local ties. The lower the standard deviations of the local ties, the higher the obtained sensitivity level. When larger standard deviations are assumed, the sensitivity levels of the VLBI stations are obtained up to 3 mm. When a smaller deviation is assumed for a local tie having various standard deviations obtained from different local tie observations, the sensitivity levels change too. If local ties with standard deviations of 1 cm and more are excluded, the detection capacities of the VLBI stations based on the gross errors improve by around 1 mm. When smaller standard deviations are assumed among the local ties, which are used in the inter-technique combination, the sensitivity levels drop below 2 mm. Hence, this study quantifies how important the selection of the co-location sites and the standard deviations of the local ties are for the accuracy of geodetic products obtained by inter-technique combination.

Keywords: VLBI, GNSS, Inter-technique combination, Sensitivity.

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1 Introduction

Space geodetic techniques such as Very Long Baseline Interferometry (VLBI), Satellite Laser Ranging (SLR), Global Navigation Satellite Systems (GNSS), Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS), are combined on the inter-technique level to determine Global Geodetic Reference Frames (GGRF). For the determination of the International Terrestrial Reference Frame 2008 (ITRF2008) in Altamimi et al. (2011) observations from these four space geodetic techniques were used that span between 12 and 30 years. Time series of intra-technique pre-combined station positions and Earth Orientation Parameters (EOP) were used as input data (Altamimi et al., 2011). Also, for its successor, the International Terrestrial Reference Frame 2014 (ITRF2014) in Altamimi et al. (2016) except for the early years of SLR data, the entire observational history of the space geodetic techniques at that time was used. Furthermore, ITRF2014 was realized with enhanced modeling of non-linear station motions and annual harmonic signals. The co-seismic and post-seismic displacements and deformations caused by major earth-quakes were modeled in ITRF2014 (Altamimi et al., 2016) through exponential or logarithmic functions or by combinations of both. Although the ITRF2020 uses time series of station positions, Earth Orientation Parameters (EOPs), and local ties at colocation sites like the previous ITRF solutions, there are some novelties for the processing (<https://itrf.ign.fr/en/solutions/ITRF2020>). Besides, ITRF2020 is realized with an extended mathematical model to improve the quality. The processing of the ITRF2020 involves (i) clustering the time series of the space techniques by adding local ties and equating station velocities and seasonal signals at colocation sites, (ii) estimating annual and semi-annual terms for the stations of the four techniques with adequate time spans, and (iii) applying Post-Seismic Deformation (PSD) models determined with GNSS data to the time series of the other techniques at the earthquake colocation sites (Altamimi et al., 2021; <https://itrf.ign.fr/en/solutions/ITRF2020>).

VLBI is one of the space geodetic techniques realizing the TRFs. VLBI measures the difference of arrival time of noise-like signals that propagate from extragalactic radio sources to the radio telescopes on Earth. From the observables, the baseline vectors between the radio telescopes can be estimated with geocentric parameterization at millimeter precision. It is the only technique for the determination of the kinematic celestial reference frame and it contributes significantly to the terrestrial reference frame (Schuh and Behrend, 2009). The second realization of the International Celestial Reference System (ICRF2) Fey et al. (2015) was calculated using 30 years of VLBI observations. It consists of positions of 3414 compact extra galactic objects (Fey et al., 2015). For the third realization of the ICRS, ICRF3, 40 years of VLBI observations at the standard geodetic and astrometric radio frequencies (IEEE designation: S- and X-band corresponding to central frequencies at 2.3 GHz and 8.4 GHz,

respectively) and 15 years of observations obtained at higher radio frequencies (K-band 24 GHz since 2002 and X- /Ka-band 8.4 GHz and 32 GHz since 2005, respectively) were involved to create a multi-wavelength frame (Charlot et al., 2020). For the standard geodetic / astrometric reference frame at 2.3/8.4 GHz, 4536 extragalactic radio sources are included¹. A major task for the frame determination is the selection of high-quality reference stations and reference radio sources (Heinkelmann, 2012). Therefore, it is substantial to investigate the effects of the a priori coordinates on the positions of the VLBI radio telescopes and radio sources. One suitable method is the sensitivity analysis (Even-Tzur, 2006; Küreç Nehbit et al., 2020).

The importance of the VLBI measurements for TRF is that it determines the scale parameter. Due to the uniqueness of the VLBI technique for the realization of both the ICRF and the ITRF, accurate VLBI solutions are essential. To obtain most accurate geodetic products, the separately evaluated space geodetic techniques are typically combined applying local ties (e.g., Altamimi et al., 2016) and global ties (e.g., Seitz et al., 2012; Glaser et al., 2015). Developments towards space ties (e.g., Thaller et al., 2011; Zoulida et al., 2016; Bruni et al., 2018), tropospheric ties (Krügel et al., 2007; Pollet et al., 2014; Kitpracha et al., 2022; Wang et al., 2022) and clock ties (Kodet et al., 2022) are under investigation. The local ties at co-located sites play an essential role in the combination of different terrestrial reference frames (Altamimi et al., 2012; Kwak et al., 2016). The accuracy of geodetic products obtained with the combination of the space geodetic techniques strongly depends on the accuracy of the local tie components and the locations of the corresponding co-location sites. The effects on the TRF through changing the standard deviations of the local ties were investigated in Glaser et al. (2019).

Until recently, sensitivity analysis has been applied to small scale geodetic networks mostly based on measurements of the Global Positioning System (GPS). For instance, it was used for the investigation of the datum definition (Even-Tzur, 2006), of monitoring crustal deformation (Hsu and Hsiao, 2002), of a priori sensitivity levels (Küreç and Konak, 2014), and of the determination of the experimental sensitivity levels (Konak et al., 2019). Küreç Nehbit et al. (2020) performed quality assessments, such as the robustness and sensitivity analyses, for the first time based on measurements of the VLBI technique. Therein, the sensitivity and robustness levels of the VLBI stations, and of the radio sources have been determined. Although the best sensitivity level is obtained with maximal eigenvalue, the eigenvalues in the z-direction were selected to obtain the sensitivity levels of a VLBI station, and the eigenvalues in declination direction were selected for the sensitivity level of a radio source during the numerical application in that study (Küreç Nehbit et al., 2020). In contrast to that, in the present study the axis with the maximal eigenvalue among the respective coordinate directions was used.

¹ <http://hpiers.obspm.fr/icrs-pc/newwww/icrf/icrf3-ReadMe.txt>

The goal of the study is to improve the detection capacity of the VLBI stations based on the undetectable gross errors through the inter-technique combination with GNSS using different scenarios. Besides, it is also aimed to determine the effects of the standard deviations of the local ties on network quality. This study shows us the importance of applying the statistical constraint to the local ties used for the inter-technique combination to improve network quality. Also, it can be seen that a local tie having larger standard deviations than the other local ties affects the associated VLBI station more and other stations in the network less. We organize our manuscript in the following way: section 2 presents the methods used for the statistical analysis. Information about the data is given in sub-section 2.1. Then, the sensitivity analysis is described in sub-section 2.2. In sub-section 2.3, the computational procedure of the sensitivity levels for inter-technique combinations is introduced. The TRF solutions obtained with different mathematical models and the local ties used for the realization of the ITRF 2014 are investigated based on their effects on the network quality. According to this, the effects of (i) the a priori coordinates (Table 1), (ii) the inter-technique combination on VLBI stations, and (iii) the standard deviations of the local ties at co-location sites are investigated based on the sensitivity analyses. The results are presented in section 3. In sub-section 3.1, the effects of the different selections of datum stations on the sensitivity levels are shown. The effects of the inter-technique combination on the sensitivity levels of the VLBI telescopes are demonstrated in sub-section 3.2. It is examined the impact of the standard deviations of the local ties on the sensitivity levels using four scenarios in sub-section 3.3. Finally, in section 4, the results are discussed and the recommendations for further research are given.

2 Method

2.1 Data

The continuous VLBI Campaign 2014 (CONT14) observed from 2014-May-6, 00:00 UT to 2014-May-20, 23:59 UT is selected as the data set to compare VLBI stations in terms of the sensitivity with the same conditions e.g., scheduling, the locations of stations, and the selection of radio sources. The CONT14 VLBI campaign consists of 15 VLBI sessions, which demonstrate the current capabilities of the VLBI technique (<https://ivscc.gsfc.nasa.gov/program/cont14>). For this study we use the solution prepared by the GFZ VLBI group. They were evaluated using the software VieVS@GFZ (GFZ, Potsdam, Germany, see Nilsson et al. (2015)), a fork from the Vienna VLBI Software developed at the Vienna University of Technology, Austria (Böhm et al., 2012). Specifically, we modified and extended the least-squares adjustment module of the VieVS@GFZ software to perform the quality assessments. For the inter-technique combination, GNSS observations during the same time period of CONT14 were evaluated using the GFZ GNSS Analysis Software EPOS8 (Gendt et al., 1997; Männel et

al., 2020). Two different GNSS solutions were used in this study (i) the IGS Rapid Products-IGR and (ii) the IGS Final Products-IGS. The IGS Final Products with a latency of 12-18 days are designed for applications which require high consistency and quality. The IGS Rapid Products with a latency of 17-41 hours have a slightly worse quality than the Final Products (<https://igs.org/products/>) but are available within shorter time.

The normal equation (NEQ) matrices obtained from both, VieVS@GFZ and EPOS8, were accumulated and combined with co-location sites using MatLab[®]. In this study, the issues have been considered in the selection of six local ties: a local tie (i) should have two different local tie observations (i.e. ONSA-ONSALA60), (ii) should have large standard deviations (i.e. ZECK-ZELENCHK), (iii) should have smaller standard deviations than (ii) (i.e. TSKB-TSUKUB32), and (iv) contains each one of three local ties i.e. NYA1-NYALES20, MATE-MATERA, and HOB2-HOBART26. For the inter-technique combination, local ties at five of six co-location sites have been selected in the scenarios to clearly understand the effect of the local ties to the other stations (Figure 1). Whereas there are more than six co-location sites, to test the effect of the standard deviations on the sensitivity levels we used six co-location sites. In each scenario, the local ties at five out of six co-location sites (ONSA-ONSALA60, NYA1-NYALES20, ZECK-ZELENCHK, MATE-MATERA, HOB2-HOBART26, and TSKB-TSUKUB32) were selected to keep the same number of local ties for the inter-technique combination. Furthermore, the local tie at the ONSA-ONSALA60 co-location site is applied with different standard deviations in some scenarios (Table 2). To understand how effective the standard deviation at a co-location site for the sensitivity levels is, ONSA-ONSALA60 having two different standard deviation values was selected. In other words, the impact of the standard deviations was tested if a co-location site has more than one local measurement. Analyzing the optimal number and location of the co-location sites should be considered as a future study.

In Table 2, we investigate the effect of the local ties, that were assumed having different standard deviations. One of two local tie measurements with large and small standard deviations in ONSA-ONSALA60 was selected for scenario 1 and scenario 2 respectively. In scenario 3, the effect of the local ties having smaller standard deviations between two different local ties was investigated. Therefore, TSKB-TSUKUB32 with smaller standard deviations was selected instead of ZECK-ZELENCHK. For the first three scenarios, only one local tie was altered one by one, whereas, for scenario 4, the local tie stations having the smallest standard deviations were selected among six co-location sites. While testing the scenarios, it is aimed to understand the effects of the local ties used to combine space geodetic techniques. Although a co-location site exclusion is tested in this study, the co-location sites having weak standard deviations can be re-weighted using the robust estimation. Better sensitivity levels will be obtained by increasing the number of co-location sites to be used.

2.2 Sensitivity Analyses

Sensitivity is the minimum value of the undetectable gross errors in the adjusted coordinate differences of a station at a network that is evaluated in multi-sessions. The network can be observed in several sessions ('global solution') or just in a single session. The sensitivity levels tell us whether the network stations are weak or powerful based on the detection capacity of the gross errors in either single or multi-session analysis. Furthermore, the effects of the locations of stations relative to another and the effects of total observation weights at a station can be quantified (Küreç and Konak, 2014; Konak et al., 2019).

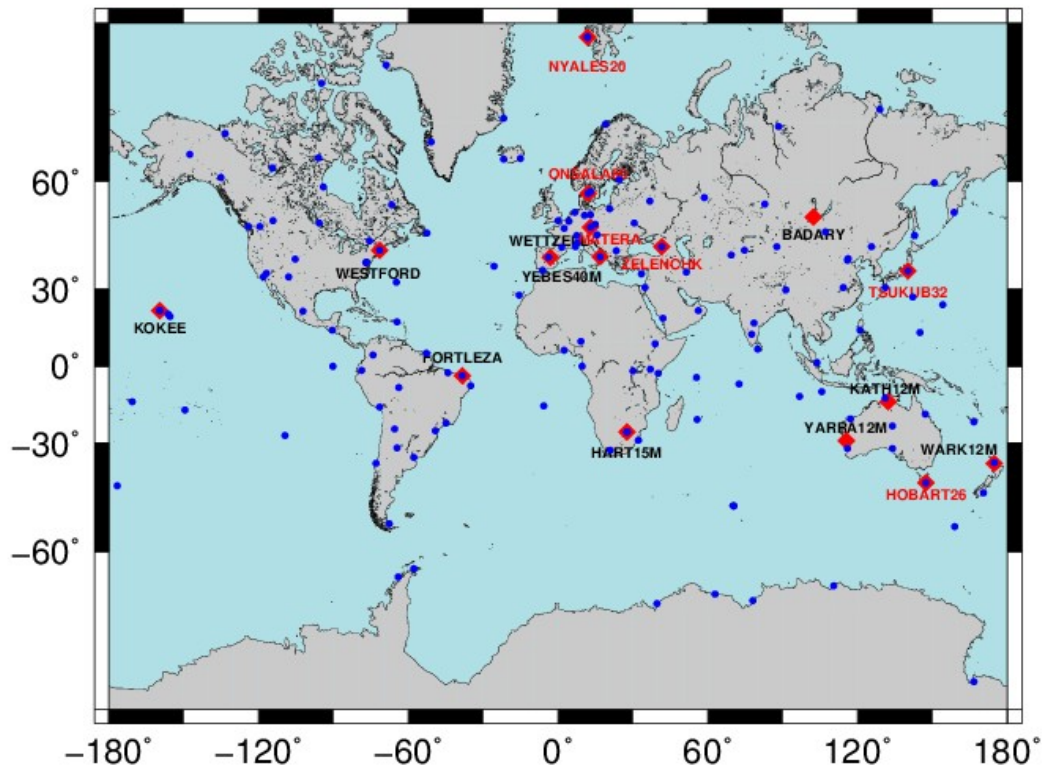


Figure 1. 17 VLBI (red diamonds) and about 150 GNSS stations (blue dots) used in this study. Stations for the combination are given in red letters.

To obtain sensitivity levels of network stations, we use the cofactor matrix of the displacement vector Q_{dd} and the cofactor matrix of the adjusted coordinates Q_{xx} in multi-session and single-session analyses, respectively. In the multi-session solution, the displacement vector is obtained from the adjusted coordinate differences represented by different sessions. According to deformation analyses, the cofactor matrices will be constant for each session in the multi-session solution if the observation weights, the measurement schedule, and the station distribution are identical (Konak et al., 2019; Küreç Nehbit et al., 2020). However, this condition cannot be

realized in reality for any space geodetic technique because the total observation weights at a station cannot be controlled.

The cofactor matrix of the adjusted coordinates of single sessions can be used to compute the sensitivity levels of VLBI stations. In a single 24-hour VLBI session, usually antenna and radio source coordinates, clock parameters, pole coordinates, and Universal Time 1 (UT1) minus Coordinated Universal Time (UTC), celestial pole offsets, and atmospheric parameters are the unknown parameters of the adjustment model (Schuh and Behrend, 2012; Nothnagel, 2018). The determination of the unknown parameters with the extended mathematical model of the least-squares adjustment was defined in Teke (2011). Based on the cofactor matrix of unknown parameters Q_{xx} that represents the full set of unknown parameters of an individual VLBI session, the cofactor matrix of the displacement vector $Q_{d_i d_i} = Q_{x_i x_i}$ can be obtained for each VLBI station. The details of determining the cofactor matrix of the displacement vector for each station and each radio source were described in Küreç Nehbit et al. (2020).

The associated weight matrix ($P_{d_i} = (Q_{d_i d_i})^{-1}$) belonging to the i -th station is decomposed into eigenvalues and eigenvectors. Thereafter, the three eigenvalues in the x -, y -, and z -directions are determined for each station. Although it is not under guarantee to obtain the eigenvalues strictly to these three directions, these are demonstrated as follows:

$$A_{x,y,z} = \begin{bmatrix} \lambda_x & 0 & 0 \\ 0 & \lambda_y & 0 \\ 0 & 0 & \lambda_z \end{bmatrix} \quad (1)$$

(Cai 2004).

The sensitivity levels can be computed based on the eigenvalues in the selected direction. Besides, if the maximum or minimum eigenvalues for each antenna are selected – the coordinate direction will be different for each antenna – the best and worst sensitivity levels of the station will be:

$$\|d\|_{min} = \frac{\delta_0 \sigma}{\sqrt{\lambda_{max}}}, \quad (2)$$

$$\|d\|_{max} = \frac{\delta_0 \sigma}{\sqrt{\lambda_{min}}}, \quad (3)$$

where δ_0 is the threshold value of the non-centrality parameter and σ is the standard deviation, the square root of σ^2 , the a priori variance of the average error of the unit weight (Hsu and Hsiao, 2002). The threshold value of the non-centrality parameter $\delta_0^2 = 10.903$ is used (Aydin and Demirel, 2004) assuming the power of the test (

$\gamma_0=80\%$), the significance level ($\alpha_0=5\%$), and the dimension of the network ($h=3$). The standard deviation of the adjustment can be used as an empirical approximation for the a priori variance of the average error of the unit weight.

2.3 Sensitivity Levels of VLBI and GNSS Combined at Normal Equation Level

At co-location sites occupied with several space geodetic techniques, local measurements are done between the markers of the different stations to realize the local tie (Altamimi et al., 2016). Applying the local ties, the observed coordinate differences at co-location sites are inserted as pseudo-observations (Thaller, 2008). GNSS and VLBI at the normal equation level are accumulated as follows:

$$\begin{bmatrix} N_{VLBI} & 0 \\ 0 & N_{GNSS} \end{bmatrix} \begin{bmatrix} x_{VLBI} \\ x_{GNSS} \end{bmatrix} = \begin{bmatrix} n_{VLBI} \\ n_{GNSS} \end{bmatrix} \quad (4)$$

where N_{GNSS} and N_{VLBI} are the single-technique normal equations of GNSS and VLBI, n_{GNSS} and n_{VLBI} are the right hand side vectors of GNSS and VLBI, respectively. Accordingly, only the positions of the VLBI and the GNSS stations remain in the NEQs for combination and the subsequent sensitivity analysis. The weight factors obtained for GNSS and VLBI were applied for the combination of the NEQ matrices.

To account for the different number and accuracy of the individual single-technique observations and to reduce the weight differences between VLBI and GNSS observations, empirical weight factors (w_{GNSS}, w_{VLBI}) have been used. The weighting factors of the VLBI and the GNSS observations can be computed as:

$$w_1 = w_{VLBI} = \frac{\frac{[\text{trace}(N_{GNSS}) + \text{trace}(N_{VLBI})]}{2}}{\text{trace}(N_{VLBI})} \quad (5)$$

$$w_2 = w_{GNSS} = \frac{na_{GNSS}}{na_{VLBI}} \cdot \frac{\frac{[\text{trace}(N_{GNSS}) + \text{trace}(N_{VLBI})]}{2}}{\text{trace}(N_{GNSS})} \quad (6)$$

where na_{GNSS} and na_{VLBI} are the number of GNSS and VLBI stations, respectively. Applying these weight factors, eq. (4) can be restated as

$$\begin{bmatrix} w_1 \cdot N_{VLBI} & 0 \\ 0 & w_2 \cdot N_{GNSS} \end{bmatrix} \cdot \hat{x} = \begin{bmatrix} w_1 \cdot n_{VLBI} \\ w_2 \cdot n_{GNSS} \end{bmatrix} \quad (7)$$

$$N_w \cdot \hat{x} = n_w \quad (8)$$

where N_w and n_w are the respective re-weighted normal equation system and right hand side vector of the multi-technique solution.

At co-location sites, the linear form of the functional model of the local ties is as follows:

$$l = L - L_0 = \Delta X^{ij} - \Delta X_0^{ij} \quad (9)$$

where l is the reduced observation vector, L is the observation vector, L_0 is the vector with the approximate values. ΔX^{ij} consists of the measured coordinate differences between co-located stations i and j and ΔX_0^{ij} stands for the computed coordinate differences obtained from a priori coordinates of space geodetic reference points (Glaser et al., 2019). Equation (9) can be written in matrix form as:

$$\Delta X^{ij} = X^j - X^i = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{tie} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{VLBI} - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GNSS} \pm \begin{bmatrix} \sigma_{\Delta x} \\ \sigma_{\Delta y} \\ \sigma_{\Delta z} \end{bmatrix}. \quad (10)$$

The formal errors of the local tie ($\sigma_{\Delta x}, \sigma_{\Delta y}, \sigma_{\Delta z}$) are computed using error propagation: and P_{tie} the weight matrix of the local ties is computed with the following equation:

$$P_{tie} = \begin{bmatrix} \frac{1}{(\sigma_{\Delta x})^2} & 0 & 0 \\ 0 & \frac{1}{(\sigma_{\Delta y})^2} & 0 \\ 0 & 0 & \frac{1}{(\sigma_{\Delta z})^2} \end{bmatrix} \quad (11)$$

(Hobiger and Otsubo, 2014; Glaser et al., 2019). The normal equation matrix of the local tie $N_{local\ tie}$ is computed as follows:

$$N_{local\ tie} = A_{tie}^T P_{tie} A_{tie} \quad (12)$$

where A_{tie} the design matrix of the local tie consists of “-1” and “1” for the tie vector linking the i^{th} and j^{th} stations, respectively. The local tie information at the co-location sites is used as pseudo-observations of the normal equation matrix:

$$(N_w + N_{local\ tie}) \cdot x - (n_w + n_{local\ tie}) = 0 \quad (13)$$

$$N_{total} \cdot x - n_{total} = 0 \quad (14)$$

By applying the equations (1) to (3) and using the reweighted NEQ matrix, we compute the sensitivity level of each VLBI station as

$$m_0^{total} = \sqrt{\frac{(v^T P v)_{VLBI} + (v^T P v)_{GNSS}}{f_{VLBI} + f_{GNSS}}} \quad (15)$$

$$\|d\|_{min} = \frac{\delta_0 m_0^{total}}{\sqrt{\lambda_{max}}} \quad (16)$$

where m_0^{total} is the standard deviation of the combined VLBI and GNSS observations using an inter-technique combination, and, f_{VLBI} , f_{GNSS} are degree of freedom of the VLBI and the GNSS observations, respectively.

3 Results

3.1 The Effects of A priori Coordinates

First, we investigate the effects on the sensitivity level of the different selection of datum stations on the VLBI-only TRF determination. The datum and non-datum stations of the two TRFs, ITRF2008 and ITRF2014, are applied in this study as provided in Tab. 1. The results indicate that the sensitivity levels are smaller if applying the ITRF2014 datum than the ITRF2008 datum (Figures 2 and 3) on average. As expected, taking the a priori values of celestial coordinates either from ICRF2 or from ICRF3 has no significant effect on the sensitivity of terrestrial stations. However, most of the stations, such as WARK12M, HOBART26, KOKEE, HOBART12, have smaller sensitivity levels in case of the ITRF2014 (Figure 2). In general, the results show that when using more datum stations, the obtained sensitivity levels are smaller. Focusing on the stations, which are used as datum stations of ITRF2014 only (e.g., WARK12M), the sensitivity levels are significantly smaller (2 mm) compared to ITRF2008 (stations without stars in Figure 2) because these stations have a larger degree of freedom than the datum stations in the least-squares solution. Besides that, considering that the ITRF2014 datum consists of more sites, and most of them are on stable parts of the tectonic plates (Altamimi et al., 2016), the sensitivity levels can be obtained more reliable. After the release of the ITRF2008, the earthquakes that happened time span from 2009.0 to 2015.0 have been investigated, and larger than a magnitude of 6.0 MW (<https://earthquake.usgs.gov/>). Since on the 10th of

March 2011, a magnitude of 9.0 MW earthquake happened on the northeast coast of Honshu that is far from about 340 km from the city of Tsukuba, the VLBI telescope TSUKUB32 located in Tsukuba was removed from the IVS observing plan for a month (MacMillan et al., 2012). Therefore, TSUKUB32 was not selected as a datum station as the stations used as datum stations in ITRF2008 were not affected by earthquakes. Nevertheless, excluding the weak stations based on statistics from the list of the datum stations is important for the network quality. If the stations having large sensitivity levels, e.g., FORTLEZA and HART15M, are excluded from the datum list, their sensitivity levels get larger and the sensitivity levels of the other stations get smaller.

As a result of this research, in all sessions of CONT14, the sensitivity levels of the VLBI stations were determined with the modified least-squares adjustment module of VieVS@GFZ by using ITRF2014 and ICRF3 for datum definition.

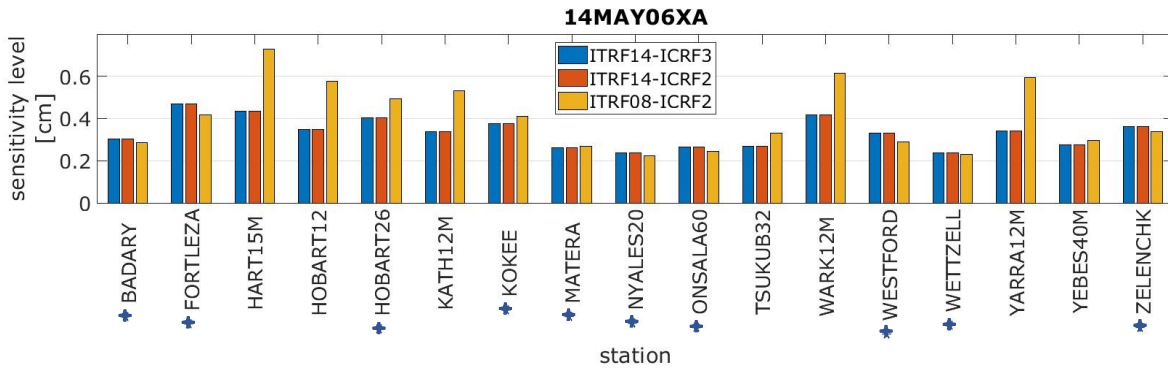


Figure 2. Sensitivity levels of the VLBI stations depending on the different reference frames obtained for session 14MAY06XA. Blue stars indicate the datum stations in common for both datums, ITRF2014 and ITRF2008 (see Tab. 1).

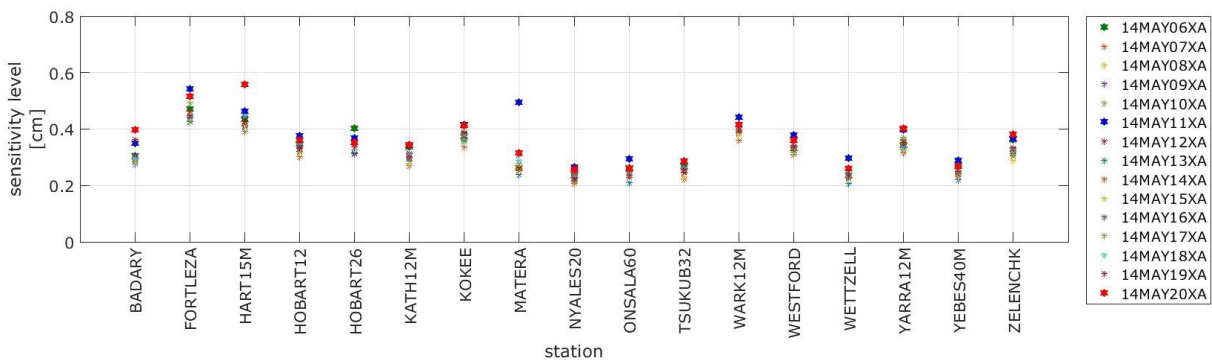


Figure 3. Comparison of sensitivity levels of all sessions in CONT14 applying ITRF2014 and ICRF3 (“ITRF14-ICRF3”).

Table 1. Datum stations selected in the VLBI-only TRF analysis

station	ITRF2008	ITRF2014
WARK12M		X
WESTFORD	X	X
HOBART12		X
KOKEE	X	X
HOBART26	X	X
NYALES20	X	X
TSUKUB32		X
BADARY	X	X
MATERA	X	X
ONSALA60	X	X
WETTZELL	X	X
ZELENCHK	X	X
KATH12M		X
YEBES40M		X
YARRA12M		X
HART15M		X
FORTLEZA	X	X

In Figure 3, in session 14MAY11XA represented with the blue star, MATERA had data during the first 8 hours only for unknown reasons. In session 14MAY20XA represented with the red star, there was a clock jump at HART15M (<https://ivscc.gsfc.nasa.gov/sessions/2014/c1415/>). As a result of these cases, MATERA and HART15M have sensitivity differences in the particular CONT14 sessions. In Figure 3, it can be seen the stations such as NYALES20, TSUKUB32, ONSALA60, WETTZELL and YEBES40M have sensitivity levels below 4 mm in all sessions of CONT14 better than the other stations. FORTLEZA has the worst sensitivity levels compared to the other antennas. The possible reasons could be its rather remote location in the network and the effects of the atmosphere changing rapidly with time.

3.2 The Effects of the Inter-Technique Combination

For this study, we use the single-technique solutions as input for the combination of the different space geodetic techniques. As usual, we combine GNSS and VLBI on the normal equation level applying local ties according to equation (13). No combination of Earth Orientation Parameters (EOP) as global ties was done. Using inter-technique combination, it can be expected that the quality of the station coordinates increases (Tanır, 2007). In other words, the quality of the estimated parameters improves due to the working principle of the least squares method. The importance of this study is to control the detection capacity of the stations based on the undetectable gross errors in the network. Also, the novelty of the study is to investigate the quality assessment of the VLBI and GNSS combined at the normal equation level. For this reason, we investigate how the sensitivity levels of the VLBI stations change through the inter-technique combination. Besides, the covariance matrices generated

assuming statistical white noise in the VLBI and GNSS data are used to examine the theoretical improvements that might be expected in a combined solution.

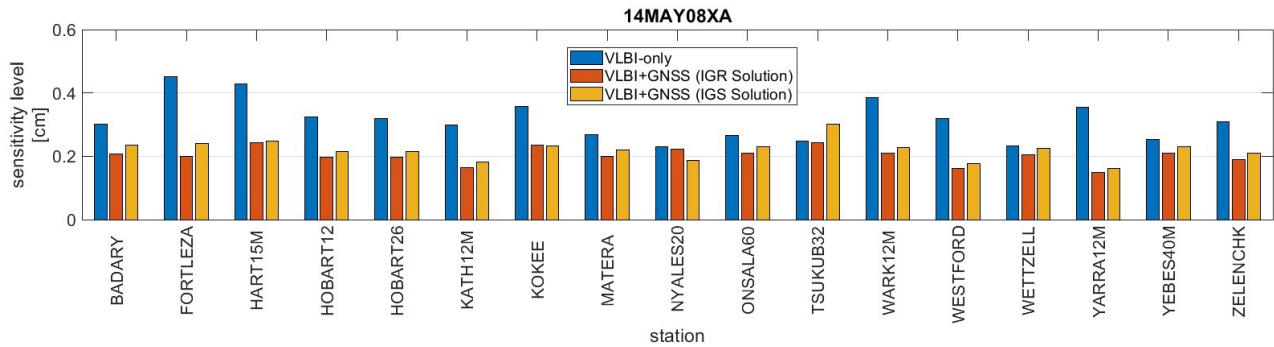


Figure 4. Sensitivity levels of the VLBI telescopes for the VLBI-only solution and for the combined VLBI and GNSS solutions (IGR and IGS) of the session 14MAY08XA of CONT14.

Figure 4 shows the sensitivity levels exemplarily for session 14MAY08XA. It can be seen the sensitivity levels of all stations decrease to around 2 mm or less through the combination except TSUKUB32. Besides, when the IGR and IGS solutions are compared, it is seen that the sensitivity levels of many stations have changed very little. To resolve the reference frame, in the IGR solution, the finite constraints are applied to the station coordinates (Ray et al., 2004) while in the IGS solution, the free network solution with the No-Net-Rotation (NNR) and No-Net-Translation (NNT) constraints is applied. Due to the finite constraints in the IGR solution, the sensitivity levels of most VLBI stations have better results than the IGS solutions. We use two different GNSS solutions to determine the effects of the solutions on the network quality. The differences in the total observation weights at a station can be the reason for the sensitivity differences based on the solutions. The smaller the sensitivity levels, the better the identification of the gross errors within the network. In the VLBI-only solution, FORTLEZA has the worst sensitivity levels in all sessions of CONT14. Through the combination of VLBI and GNSS at NEQ level, the sensitivity level of FORTLEZA shows the largest improvement of more than 2 mm. In all sessions, almost similar patterns to Figure 4.

3.3 The Impact of the Standard Deviations of the Local Ties

The accuracy of the geodetic products obtained from the combination of the space geodetic techniques depends - among others - on the accuracy of the local ties and the geographical locations of the co-location sites. We investigate the effect on the sensitivity levels by a selection of the co-location sites using four different scenarios, see section 2.1 and Table 2. To investigate the effect of the accuracy of the local ties, four scenarios were applied with six co-location sites. In each scenario, five of six co-location sites were used. Nevertheless, one

of the six co-location sites has two different standard deviations named maximal and minimal standard deviation. In the first scenario, from the co-located sites with different standard deviation values, one with the maximal standard deviation values was selected. Furthermore, one of the rest co-located sites has the largest values of standard deviations. In the second scenario, the co-location sites having minimal standard deviations were selected. In the next scenario, one of the five co-locations mentioned in the first scenario sites having the largest standard deviations was altered. In the last scenario, five co-location sites having smaller standard deviations among six co-location sites were selected. In other words, to determine the effect of the selections, in the first three of four scenarios, only one local tie was altered at a time. In each scenario, five co-location sites were used to keep the number of local ties large enough. The sixth co-location sites were used to alter local tie in the scenarios. Explicitly, the local ties at five of six GNSS-VLBI co-location sites (ONSA-ONSALA60, NYA1-NYALES20, ZECK-ZELENCHK, MATE-MATERA, HOB2-HOBART26, and TSKB-TSUKUB32) as given in e.g., ITRF2014, were applied and the scenarios were compared assessing the sensitivity levels of the VLBI stations.

Table 2. The standard deviations of the local ties used in the four scenarios (<https://itrf.ign.fr/>). Local ties shown with the red numbers in scenario 1 are altered in the other scenarios and represented with the blue numbers.

Local tie	Scenario 1			Scenario 2			Scenario 3			Scenario 4		
	$\sigma_{\Delta x}$ [mm]	$\sigma_{\Delta y}$ [mm]	$\sigma_{\Delta z}$ [mm]	$\sigma_{\Delta x}$ [mm]	$\sigma_{\Delta y}$ [mm]	$\sigma_{\Delta z}$ [mm]	$\sigma_{\Delta x}$ [mm]	$\sigma_{\Delta y}$ [mm]	$\sigma_{\Delta z}$ [mm]	$\sigma_{\Delta x}$ [mm]	$\sigma_{\Delta y}$ [mm]	$\sigma_{\Delta z}$ [mm]
ONSA-ONSALA60	1.94	1.63	2.11	1.55	1.65	1.65	1.94	1.63	2.11	1.55	1.65	1.65
NYA1-NYALES20	2.42	2.42	2.42	2.42	2.42	2.42	2.42	2.42	2.42	2.42	2.42	2.42
ZECK-ZELENCHK	10.15	10.15	10.15	10.15	10.15	10.15	-	-	-	-	-	-
MATE-MATERA	1.40	0.61	1.7	1.40	0.61	1.70	1.40	0.60	1.70	1.40	0.61	1.70
HOB2-HOBART26	1.30	0.51	1.40	1.30	0.51	1.40	1.30	0.51	1.40	1.30	0.51	1.40
TSKB-TSUKUB32	-	-	-	-	-	-	0.86	0.98	1.12	0.86	0.98	1.12

Figure 5 shows that the sensitivity levels of the VLBI antennas improve slightly when the local ties are selected for co-location sites, that have smaller standard deviations, such as for scenario 2 vs. scenario 1. Between scenario 1 and scenario 3, the sensitivity changes by about 1 mm depending on the sites when the local ties are applied at sites that have larger standard deviations among two different local ties, e.g. when ZECK-ZELENCHK is used instead of TSKB-TSUKUB32 (Table 2). It is the effect of the worse local tie at ZELENCHK. In the sessions of CONT14, similar patterns to Figure 5 are obtained except for the session 14MAY20XA. In the session 14MAY20XA, when scenario 1 and scenario 2 were applied, it was seen that the geometrical condition

deteriorated by the reason of the selection of the co-location sites and their numbers. Besides, after the combination, the coordinate unknowns of the VLBI and the GNSS stations were computed. The datums of the VLBI and the GNSS networks are different from each other. On the other hand, station coordinates are accurate enough to be considered as approximate coordinates in the combination. When the estimated coordinates based on scenario 4 are compared to the coordinates determined before the combination, it is seen that the x-, y-, and z- directions have differences up to 2 cm depending on the stations in most of the sessions. The differences between the coordinate unknowns are the effects of the local ties. The statistical analyses will not give realistic results if the mathematical model of the adjustment model is invalid. In this case, it could be considered that there could be outliers in the observations. WRMS was computed for the differences of the coordinate unknowns for individual sessions. Obtained WRMS values spread between 0.001 cm and 0.007 cm. Accordingly, it could be said that before and after the combination obtained coordinates are compatible. To determine whether the changes are significant, the S-transformation is applied to both adjusted coordinate differences and their cofactor matrices to solve the datum problem according to the different barycenter (Teunissen, 1984). After the S- transformation, the x-, y-, and z- directions have differences below 1 cm depending on the stations in most sessions. Then the hypothesis tests are used. If the null hypothesis is accepted, it is thought that approximate coordinates are obtained sufficiently. As it is mentioned, using S-Transformation GPS and VLBI coordinates were obtained at the same barycenter. Adjusted coordinate differences were tested to analyze whether it is significant. The test values are obtained between 0.09 and 0.13. The values are compared with the threshold value determined from the Fisher distribution $(F_{h,f,1-\frac{\alpha}{2}})$, and it is obtained that there are no significant differences according to the statistical analyses. It does not give us information about the accuracy of the adjusted coordinates. This information could be obtained from the sensitivity levels of the stations or the Helmert position error values computed from the cofactor matrix of the combined VLBI and GNSS (Figure 6).

Figure 6 demonstrates the Helmert position errors ($m_p = \sqrt{m_{x_i}^2 + m_{y_i}^2 + m_{z_i}^2}$) generated from the standard deviations of the estimated parameters for VLBI-only and four scenarios. It can be seen the Helmert position error of each VLBI station has improved with combined VLBI and GNSS solutions.

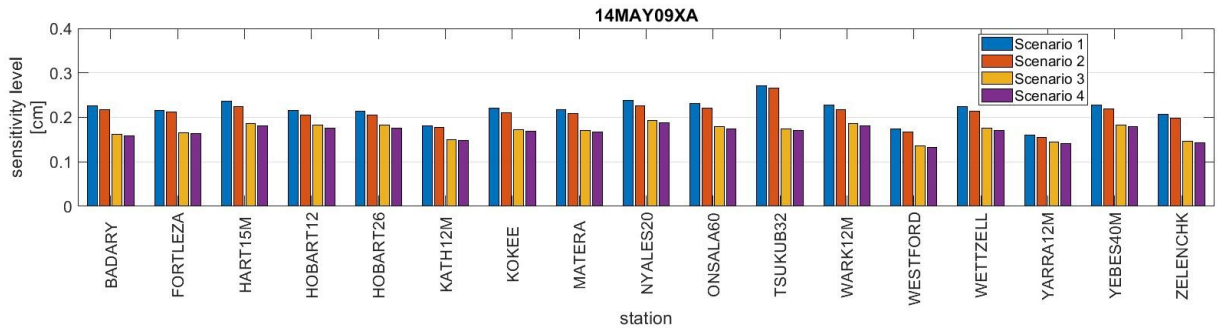


Figure 5. Comparing the sensitivity levels of the VLBI stations for the combined VLBI and GNSS solution based on four scenarios (Tab. 2) in session 14MAY09XA.

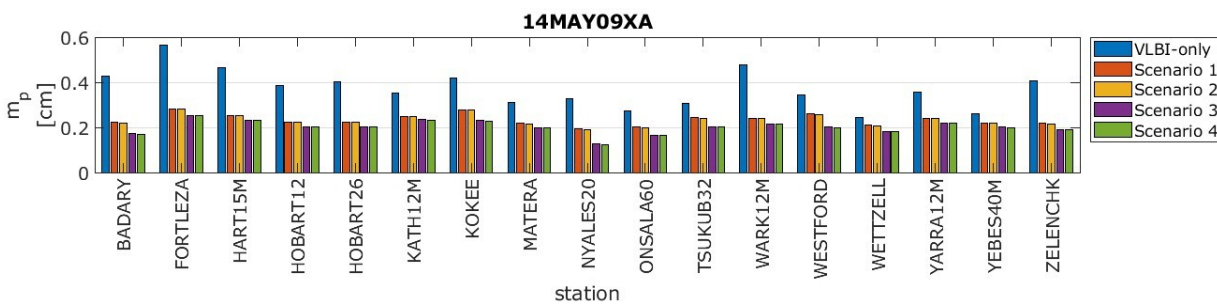


Figure 6. Comparing the Helmert position errors of VLBI stations for solutions of VLBI-only and four scenarios in session 14MAY09XA.

4 Discussion and Conclusions

At the beginning of the quality assessment, the effects of the datum stations have been investigated in terms of sensitivity quantities to determine the importance of the initial conditions. The choice of datum stations has a large impact on the sensitivity levels. When more datum stations are used, more accurate solutions can be obtained according to sensitivity because the residuals get minimized at the datum stations in the least-squares solution. Sensitivity is related to the selection of the datum stations and their locations in a network. As a result of this finding, attention should be paid to select the datum stations among the most precise stations.

According to the sensitivity levels of the VLBI stations in all sessions of CONT14, it can be seen FORTLEZA and HART15M have sensitivity levels between 4 mm to 6 mm compared to 2 mm to 4 mm for the other stations. Possible reasons for this finding could be the remote locations of these stations w.r.t. to the center of the network and for FORTLEZA we know that observations are additionally affected with time-varying ionospheric and neutral atmospheric errors. The latter is due to large and variable amount of water vapor in the atmosphere above the observing site. The stations are located at remote spots of the network, i.e. a bit isolated from

the other stations. As a consequence, the partial derivatives of coordinates in the design matrix of remote station differ substantially in comparison to stations that are close to other stations. Hence, the controllability of the remote stations through the other stations of the network is not significant. In addition, most observations at FORTLEZA station are done at the remote position in the network and consequently, the common view requires relative low elevation angles and consequently larger tropospheric errors.

The combination of VLBI with GNSS results in smaller sensitivity levels of the VLBI stations due to more stations within the network that involve more observations. To get a more accurate network quality, the selection of co-location sites for the combination was investigated by us. It is expected that good results will be obtained by using more co-location sites for the combination. Furthermore, the selection of the co-location sites and their standard deviations are also important. For this reason, the weight matrix of the co-location sites having large standard deviations can be re-weighted using a robust estimation method. In general, it can be concluded that the combined VLBI plus GNSS solution results in improved VLBI station coordinates since the sensitivity levels get smaller and so the identification of the gross errors is improved. For VLBI networks, in particular stations that obtain weak sensitivity levels because of remote network geometry could be strengthened for instance with a regional GNSS network consisting of neighboring stations with co-location sites. Furthermore, this finding is very likely to be applicable for the combination of other space geodetic techniques as well. Although this study involves daily sessions, it can be transferred to the long-term solutions and their effects on ITRFs. Based on this conclusion, future ITRFs could be realized more accurately.

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