



HAL
open science

Design of a variable sampling interval exponentially weighted moving average median control chart in presence of measurement errors

Kim Duc Tran, Huu Du Nguyen, Thi Hien Nguyen, Kim Phuc Tran

► **To cite this version:**

Kim Duc Tran, Huu Du Nguyen, Thi Hien Nguyen, Kim Phuc Tran. Design of a variable sampling interval exponentially weighted moving average median control chart in presence of measurement errors. *Quality and Reliability Engineering International*, 2020, 37, pp.374 - 390. 10.1002/qre.2726 . hal-04315122

HAL Id: hal-04315122



<https://hal.science/hal-04315122v1>

Submitted on 8 Dec 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Design of a variable sampling interval exponentially weighted moving average median control chart in presence of measurement errors

Kim Duc Tran¹ | Huu Du Nguyen^{2,4}  | Thi Hien Nguyen^{2,3} | Kim Phuc Tran⁵ 

¹Institute of Research and Development, Duy Tan University, Danang, 550000, Vietnam

²Institute of Artificial Intelligence and Data Science, Dong A University, Danang, Vietnam

³Departement of Mathematics, CY TECH, CY Cergy Paris University, Paris, France

⁴Departement of Mathematics, Faculty of Information Technology, Vietnam National University of Agriculture, Hanoi, Vietnam

⁵Ecole Nationale Supérieure des Arts et Industries Textiles, GEMTEX Laboratory, Roubaix, France

Correspondence

Huu Du Nguyen, Institute of Artificial Intelligence and Data Science, Dong A University, Danang, Vietnam
 Departement of Mathematics, Faculty of Information Technology, Vietnam National University of Agriculture, Hanoi, Vietnam.
 Email: dunh@donga.edu.vn

Abstract

In the literature, many control charts monitoring the median is designed under a perfect condition that there is no measurement error. This may make the practitioners confusing to apply these control charts because the measurement error is the true problem in practice. In this paper, we consider the effect of measurement error on the performance of the exponentially weighted moving average (EWMA) control chart combining with the variable sampling interval (VSI) strategy. A linear covariate error model is supposed to model the measurement error. The performance of the VSI EWMA median control chart is evaluated through the average time to signal. The numerical simulation shows that the measurement errors have a negative influence on the proposed chart.

KEYWORDS

control chart, EWMA, measurement errors, median, VSI

1 | INTRODUCTION

Statistical process control (SPC) is widely used in monitoring industrial manufacturing processes. Its goal is to improve product quality and to be sure that a process is in control. Among several techniques of SPC, a control chart is a useful tool to achieve this goal by helping manufacturers in detecting timely the out-of-control conditions. A control chart includes an upper control limit (UCL) and/or lower control limit (LCL) that allow defining whether or not a process is in control. The sample characteristics of interest are plotted on the control chart, and the number of samples plotted above the upper limit and/or under the lower limit is counted. Each type of control chart has its own rule to alarm a signal when the process is out of control.

In the literature, several control charts have been introduced, and the related references are abundant. One can name the Shewhart control chart as the first control chart designed by Dr. Shewhart.¹ This chart is very popular in SPC, and its wide application over a long period of time has motivated researchers to introduce many other control charts with higher performance in detecting process shifts. There are three main kind techniques in these new control charts, including new rules to alarm a signal such as the synthetic control chart (Hu et al.² and Tran et al.³) and the run rules control chart (Tran⁴); the adaptive strategies by changing parameters of the process such as the variable

sampling interval (VSI) chart (Reynolds et al.⁵) and the variable sample size chart (Reynolds and Arnold⁶); and the advanced charts by considering the memories of the chart like the cumulative sum (CUSUM) control chart (Brook and Evans,⁷ Wu et al.,⁸ and Maravelakis⁹) and the exponentially weighted moving average (EWMA) control chart (Maravelakis et al.¹⁰). In addition, researchers recently suggest combining the adaptive strategies with the advanced control charts to provide new kinds of control charts like the VSI CUSUM control chart (Nguyen et al.¹¹) and the VSI EWMA control chart (Nguyen et al.¹²). In general, these new hybrid control charts significantly improve the performance of the original charts.

The usual assumption used in many control charts is that there is no measurement error. However, this assumption seems to be not realistic in practice because the measurement error does exist for every measurement system no matter how good they are. Under the presence of measurement error, the performance of the control chart differs from the performance of the corresponding chart without considering the measurement error due to the change of the parameters of the model. This problem has been pointed out in various control charts by a larger number; see, for example, Nguyen et al.,¹³⁻¹⁶ Linna et al.,¹⁷ Costa and Castagliola,¹⁸ Noorossana and Zerehsaz,¹⁹ and Tran et al.^{20,21}

Among several characteristics of interest from a process, the median is quite an important characteristic and the median (denoted by \tilde{X}) chart is also a popular chart for monitoring the mean of a process. According to the discussion in Castagliola et al.,²² the \tilde{X} chart is interested in monitoring a process running for a long time because it is robust against outliers or small deviations from normality. This property makes it more advantageous compared to the \bar{X} chart in such a case. The study in Khoo²³ also points out that a sample median chart is suitable for detecting permanent shifts in the process. In the literature, the EWMA \tilde{X} chart is introduced by Castagliola.²⁴ This study is then extended in Tran et al.²⁵ by combining the EWMA chart with the VSI strategy. The Shewhart- \tilde{X} control chart is studied in Khoo.²³ The median control charts with estimated process parameters are also a major concern in several studies, such as Castagliola and Figueiredo²⁶ with the Shewhart-type chart, Castagliola et al.²² with the EWMA-type chart, Saha et al.²⁷ with the run sum-type chart, and Khoo et al.²⁸ with VSI EWMA-type chart. Tran et al.²⁰ presents the performance of the Shewhart chart for a normal process under a linear covariate error model. The synthetic \tilde{X} control chart for monitoring the process mean with measurement errors is studied in Tran et al.³

The goal of this paper is to investigate the performance of the VSI EWMA- \tilde{X} control chart in the presence of measurement errors. As cited above, the VSI EWMA- \tilde{X} control chart has been designed in Tran et al.,²⁵ and it is shown that by combining with the VSI strategy, the VSI EWMA- \tilde{X} leads to impressive performance in detecting process mean shifts, and it improves significantly the performance of the EWMA- \tilde{X} control chart. However, in this study, the authors assumed that the characteristic of interest is measured exactly without error. To make the application of the VSI EWMA- \tilde{X} control chart more practical, it is necessary to consider the effect of measurement error on the chart's performance. Similar to previous studies, we apply the linear covariate error model introduced in Linna and Woodall²⁹ for the measurement error.

The rest of the paper is organized as follows. In Section 2, we provide briefly the linear covariate error model and the distribution of the sample median \tilde{X} . Sections 3 and 4 present the implementation and the design of the VSI EWMA- \tilde{X} control chart. The effect of measurement error on the proposed control chart's performance is investigated in Section 5. An illustrative example based on the data from the production process of milk bottles is given in Section 6. Finally, Section 7 is for some concluding remarks.

2 | A BRIEF REVIEW OF THE LINEAR COVARIATE ERROR MODEL AND SAMPLE MEDIAN DISTRIBUTION

A brief review of the linear covariate error model (suggested in Linna and Woodall²⁹) and the sample median distribution is presented in this section. To monitor the quality of characteristic X , suppose that a set of samples $\{X_{i,1}, X_{i,2}, \dots, X_{i,n}\}$ of size n is taken at the sampling period $i = 1, \dots$. We assume that $X_{i,j}$ are independent and identically distributed from a normal distribution, $X_{i,j} \sim N(\mu_0 + \delta\sigma_0, \sigma_0^2)$, where δ represents the standardized shift size. The process is considered to be in control when $\delta = 0$; otherwise, it is considered as out of control. The in-control μ_0 and σ_0 are assumed to be known. Due to the measurement error problem, the true value $X_{i,j}$ is impossible to obtain. Instead, we can only obtain the observed value $\{X_{i,j,1}^*, X_{i,j,2}^*, \dots, X_{i,j,m}^*\}$ where $X_{i,j,k}^*$ is the k th measurement of the item j at the sampling time i and $m \geq 1$ is the number of repeated measurements per item. As suggested in Linna and Woodall,²⁹ we have a linear covariate model as

$$X_{ij,k}^* = A + BX_{ij} + \varepsilon_{ij,k},$$

where A and B are two constants estimated from phase I data and $\varepsilon_{ij,k} \sim N(0, \sigma_M^2)$ is a normal random error, which is independent of X_{ij} .

Let \bar{X}_{ij}^* denote mean of m quantities $\{X_{ij,1}^*, X_{ij,2}^*, \dots, X_{ij,m}^*\}$, that is,

$$\begin{aligned} \bar{X}_{ij}^* &= \frac{1}{m} \sum_{k=1}^m X_{ij,k}^* \\ &= \frac{1}{m} \sum_{k=1}^m (A + BX_{ij} + \varepsilon_{ij,k}) \\ &= A + BX_{ij} + \frac{1}{m} \sum_{k=1}^m \varepsilon_{ij,k}. \end{aligned} \tag{1}$$

Then, \bar{X}_{ij}^* follows a normal distribution with mean

$$\mu^* = E(\bar{X}_{ij}^*) = A + B(\mu_0 + \delta\sigma_0), \tag{2}$$

and standard variation

$$\sigma^* = \sigma(\bar{X}_{ij}^*) = \sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}. \tag{3}$$

We now define the sample median of the mean values $\{\bar{X}_{i,1}^*, \bar{X}_{i,2}^*, \dots, \bar{X}_{i,n}^*\}$ corresponding to subgroup $i = 1, 2, \dots$, as

$$\tilde{X}_i^* := \begin{cases} \bar{X}_{i,((n+1)/2)}^* & \text{if } n \text{ is odd,} \\ \frac{\bar{X}_{i,(n/2)}^* + \bar{X}_{i,(n/2+1)}^*}{2} & \text{if } n \text{ is even,} \end{cases} \tag{4}$$

where $\{\bar{X}_{i,(1)}^*, \bar{X}_{i,(2)}^*, \dots, \bar{X}_{i,(n)}^*\}$ is the ordered sample of the mean values for each subgroup $i = 1, 2, \dots$. According to the discussion in Tran et al.,²⁰ the sample size n is usual to assume to be an odd value when working with the sample median. In the rest of the paper, we will consider the case where n is odd. For this case, the cumulative distribution function (c.d.f.) $F_{\tilde{X}_i^*}(x|n)$ of the sample median \tilde{X}_i^* is defined as (see Castagliola and Figueiredo²⁶)

$$\begin{aligned} F_{\tilde{X}_i^*}(x|n) &= F_\beta \left(\Phi \left(\frac{x - \mu^*}{\sigma^*} \right) \middle| \frac{n+1}{2}, \frac{n+1}{2} \right) \\ &= F_\beta \left(\Phi \left(\frac{x - A - B(\mu_0 + \delta\sigma_0)}{\sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}} \right) \middle| \frac{n+1}{2}, \frac{n+1}{2} \right), \end{aligned} \tag{5}$$

where $\Phi(x)$ is the c.d.f. of the standard normal distribution and $F_\beta(x|a, b)$ is the c.d.f. of the beta distribution with parameters $(a, b) = (\frac{n+1}{2}, \frac{n+1}{2})$.

3 | IMPLEMENTATION OF THE VSI EWMA- \tilde{X} CONTROL CHARTS WITH MEASUREMENT ERRORS

As in the EWMA-type chart, the observed characteristics of interest \tilde{X}_i^* are not monitored directly. Instead, we monitor its EWMA sequence

$$Z_i^* = (1 - \lambda)Z_{i-1}^* + \lambda \tilde{X}_i^*, i = 1, 2, \dots, \tag{6}$$

where λ is a smoothing parameter that can be assigned a value in $[0,1]$ and the initial value $Z_0^* = \mu_0^* = A + B\mu_0$. Adjusting the value of λ helps to change the memory of the EWMA charts to handle certain sizes of faults. The VSI EWMA- \tilde{X} control chart consists of the UCL, the LCL, the upper warning limit (UWL), and the lower warning limit (LWL), which are defined as follows.

$$LCL = A + B\mu_0 - K\sqrt{\frac{\lambda}{2-\lambda}\left(B^2\sigma_0^2 + \frac{\sigma_M^2}{m}\right)}, \tag{7}$$

$$UCL = A + B\mu_0 + K\sqrt{\frac{\lambda}{2-\lambda}\left(B^2\sigma_0^2 + \frac{\sigma_M^2}{m}\right)}, \tag{8}$$

$$LWL = A + B\mu_0 - W\sqrt{\frac{\lambda}{2-\lambda}\left(B^2\sigma_0^2 + \frac{\sigma_M^2}{m}\right)}, \tag{9}$$

$$UWL = A + B\mu_0 + W\sqrt{\frac{\lambda}{2-\lambda}\left(B^2\sigma_0^2 + \frac{\sigma_M^2}{m}\right)}, \tag{10}$$

where $K > 0$ is a control coefficient used to specify the width of the control limits and $W(0 < W < K)$ is a warning coefficient used to determine the width of the warning limits in the VSI-type chart. The closer to 0 is the warning limit coefficient W , the smaller is the warning region.

The proposed control chart is adaptive with respect to the time interval in which a sample is taken. The sampling interval varies depending on the information from the most recent sample collected. In particular, random samples are collected in intervals of variable length according to the function:

$$h(i) = \begin{cases} h_S & \text{if } Z_{i-1}^* \in R_S, \\ h_L & \text{if } Z_{i-1}^* \in R_L, \end{cases} \tag{11}$$

where $h(i)$ is the time practiced to take the i th sample and h_S and h_L represent the “short time” and “long time” interval, with the corresponding regions R_S and R_L . Figure 1 shows a graphical view of the VSI EWMA- \tilde{X} chart. It is divided into three regions:

- The central region, that is, $[LWL, UWL]$. When the control statistic Z_i^* falls within this region, the next sample should be taken after a “long time” interval h_L , because there is no evidence to say that the process is being shifted.

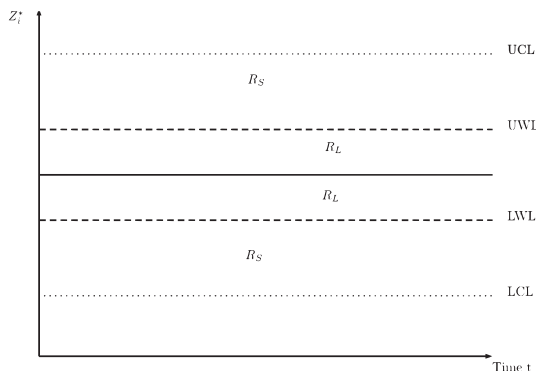


FIGURE 1 Three regions of the variable sampling interval (VSI) exponentially weighted moving average (EWMA)- \tilde{X} control chart. LCL, lower control limit; LWL, lower warning limit; UCL, upper control limit; UWL, upper warning limit

- The warning region, that is, $[LCL, LWL) \cup (UWL, UCL]$. If the control statistic falls into this region, the next sample should be taken faster, after a “short time” h_S , because the process seems to be shifting into out-of-control conditions.
- Out-of-control regions, that is, $(-\infty, LCL) \cup (UCL, +\infty)$. A signal from the chart is given to alarm that the process is out of control.

4 | DESIGN OF OPTIMAL VSI EWMA- \tilde{X} CONTROL CHARTS WITH MEASUREMENT ERRORS

The performance of the VSI EWMA- \tilde{X} control chart is evaluated by the measure of average time to signal (ATS). It is the expected value of the time after the process starts until the first signal. When the process is in control, it is denoted by ATS_0 . Conversely, when the process is out of control, it is denoted by ATS_1 . For any VSI-type chart, the smaller the value of the ATS_1 , the better the performance of the control chart. In this study, the ATS is calculated by using the Markov chain method which is proposed by Brook and Evans⁷ (see Appendix A1).

When samples are taken at fixed intervals of time (FSI), the ATS is simply a multiple of the ARL . The sampling interval h_F is fixed, and we have

$$ATS^{FSI} = h_F \times ARL^{FSI}. \quad (12)$$

Because of the variation of the interval sampling in the VSI-type chart, the ATS is defined as

$$ATS^{VSI} = E(h) \times ARL^{VSI}, \quad (13)$$

where $E(h)$ is the expected sampling interval value.

The design of the proposed control chart is defined by finding related parameters that minimize the ATS_1 . To make a fair comparison with the FSI charts, we suppose that the in-control values of both the average time to signal ATS_0 and the average sampling interval $E_0(h)$ are the same as ARL_0 and h_F . Without loss of generality, we assume that $h_F = 1$ time unit in the FSI-type chart. Moreover, according to the discussion in Nguyen et al.,¹² the short sampling interval h_S and the warning coefficient W are fixed. Thus, the design of the VSI EWMA- \tilde{X} control chart is now to find an optimal combination of parameters λ^* , K^* , and h_L^* which minimize the out-of-control ATS for predefined values of ATS_0 , W , and h_S and specific values of n , δ , m , B , and η . In particular, the optimization problem consists in finding the optimal parameters λ^* , K^* , and h_L^* such that

$$(\lambda^*, K^*, h_L^*) = \underset{(\lambda, K, h_L)}{\operatorname{argmin}} ATS(n, \lambda, K, W, h_L, h_S, \delta, B, m, \eta), \quad (14)$$

subject to the constraint

$$\begin{aligned} E_0(h) &= 1, \\ ATS(n, \lambda, K, W, h_L, h_S, B, m, \eta, \delta = 0) &= ATS_0. \end{aligned} \quad (15)$$

5 | NUMERICAL RESULTS

In this section, we present the optimal parameters λ^* , h_L^* , and K^* of the VSI EWMA- \tilde{X} control chart and the corresponding ATS_1 for some scenarios of other parameters. The following specific values are considered in our simulation:

- The shift size $\delta \in \{0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5, 2.0\}$.

- The sample size $n \in \{3, 5, 15\}$.
- The warning limit coefficient $W \in \{0.2, 0.6\}$.
- The precision error ratio $\eta \in \{0, 0.1, 0.3\}$.
- The number of repeated measurements per item $m \in \{1, 5, 10\}$.
- The slope in the linear covariate model $B \in \{0.8, 1, 1.2\}$.

The optimal parameters (λ^* , K^* , and h_L^*) and the corresponding ATS_1 of the VSI EWMA- \tilde{X} control chart in the presence of the measurement error are given in Tables 1–3. Some conclusions can be drawn from these results as follows.

- Given the values of other parameters, the optimal values (λ^* , K^* , and h_L^*) depend on η . However, when $\delta \leq 0.2$, this dependence is insignificant as the values of λ^* , K^* , and h_L^* are almost the same as $\delta \leq 0.2$. If $\delta > 0.2$, the values of (λ^* , K^*) decrease whereas the values of h_L^* increase when η increases. For example, with $n = 5$, $B = 1$, $m = 1$, $h_S = 0.1$, $W = 0.2$, and $\delta = 0.3$ in Table 2, we have $(\lambda^*, K^*) = (0.0837, 1.4212)$ and $h_L^* = 2.9729$ when $\eta = 0$, whereas $(\lambda^*, K^*) = (0.0783, 1.4108)$ and $h_L^* = 2.9845$ when $\eta = 0.3$.
- In general, the precision error ratio η has a negative effect on the chart's performance. The value of the ATS_1 decreases when η increases. For example, given $n = 3$, $\eta = 0.28$, $m = 1$, $\delta = 0.1$, $W = 0.6$, and $h_S = 0.1$ in Table 1, we have $ATS_1 = 128.6$ when $\eta = 0.1$ and $ATS_1 = 135.5$ when $\eta = 0.3$.
- The choice of the short sampling interval h_S effects the value of ATS depends on h_S . In particular, the smaller value of h_S leads to smaller values of ATS_1 . For example, when $\delta = 0.2$, $n = 15$, $W = 0.6$, $\eta = 0.28$, $m = 1$, and $B = 1$ in Table 3, we have $ATS_1 = 16.4$ for $h_S = 0.1$ and $ATS_1 = 18.1$ for $h_S = 0.5$. However, when $\delta \geq 1.5$, the increase of h_S leads to the decrease of the ATS_1 .
- The influence of the warning coefficient W on the performance of the proposed chart depends on the shift size δ . In general, when $\delta \leq 0.3$, the smaller value of W leads to the smaller value of the ATS_1 , whereas the opposite is true when $\delta > 0.3$. For example, we have $ATS_1 = 103.3$ when $W = 0.2$ and $ATS_1 = 107.1$ when $W = 0.6$ given that $n = 5$, $B = 1$, $m = 1$, $h_S = 0.5$, $\eta = 0.28$, and $\delta = 0.1$ in Table 2.
- The value of the ATS_1 reduces as B increases. For example, when $n = 15$, $\eta = 0.28$, $m = 1$, $\delta = 0.1$, $W = 0.6$, and $h_S = 0.5$, we have $ATS_1 = 54.3$ for $B = 0.8$ and $ATS_1 = 51.7$ for $B = 1.2$; see Table 3.
- When m increases, the ATS_1 decreases significantly, especially when δ is small. For example, with the same value of $n = 3$, $W = 0.2$, $h_S = 0.1$, $B = 1$, $\eta = 0.28$, and $\delta = 0.1$, we have $ATS_1 = 130.6$ when $m = 1$ and $ATS_1 = 124.6$ when $m = 10$. That is to say, taking multiple measurements per item, in each sample, is an efficient strategy to compensate for the effect of measurement errors in the VSI EWMA- \tilde{X} control chart.
- The sample size n has a strong influence on the chart performance: the larger the sample size n , the smaller the value of ATS_1 . For example, in Tables 1 and 2, with $h_S = 0.1$, $m = 1$, $B = 1$, $W = 0.2$, $\delta = 0.1$, and $\eta = 0.1$, we have $ATS_1 = 124.8$ when $n = 3$ whereas $ATS_1 = 88.3$ when $n = 5$.

We also make a comparison between the performance of the VSI EWMA- \tilde{X} control chart in the presence of measurement errors with the performance of other control charts in the same conditions in the literature, involving the Shewhart- \tilde{X} control chart (Tran et al.²⁰) and the synthetic \tilde{X} control chart (Tran et al.³). The compared results show that regardless of the presence of measurement errors, the VSI EWMA- \tilde{X} control chart leads to the best performance. For example, given the value of $n = 3$, $B = 1$, $m = 1$, $\eta = 0.1$, and $\delta = 0.1$, we have $ARL_1 = 335$ in the Shewhart- \tilde{X} control chart (tab. 1 in Tran et al.²⁰), $ARL_1 = 319.8$ in the synthetic \tilde{X} control chart (tab. 3 in Tran et al.³) compared with $ATS_1 = 12.48$ (for the case $h_S = 0.1$, $W = 0.2$ in Table 1 in this study).

6 | ILLUSTRATIVE EXAMPLE

In this section, we illustrate the implementation of the VSI EWMA- \tilde{X} control chart under the presence of measurement errors. We consider a real context from the production process of milk bottles introduced in Castagliola et al.²² This context is also widely applied in the literature; see, for example, Tran et al.²⁵ According to Castagliola et al.,²² the weight Y (in milliliters) of each bottle in the production process is the quality characteristic of interest. The weight Y is supposed to follow a normal distribution with parameters (μ_0, σ_0^2) . From phase I data, it is estimated that $\hat{\mu}_0 = 500.023$

TABLE 1 Optimal couples (λ^* , K^*) (first row of each block) and values of (h_s^*, ATS) (second row of each block) of the variable sampling interval (VSI) exponentially weighted moving average (EWMA) control chart in the presence of measurement error for $n = 3$

δ	$h_s = 0.1 \quad m = 1 \quad B = 1$																								
	$W = 0.2$				$W = 0.6$				$W = 0.2$				$W = 0.6$												
	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$									
0.10	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)									
	(3.5157,123.9)	(3.5157,124.8)	(3.5157,131.6)	(1.4386,128.6)	(1.4386,127.7)	(1.4386,128.6)	(1.4386,135.5)	(2.3976,133.8)	(2.3976,134.6)	(2.3976,134.6)	(2.3976,141.4)	(1.2437,135.9)	(1.2437,136.8)	(1.2437,136.8)	(1.2437,143.5)	(1.2437,143.5)									
0.20	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)									
	(3.5157,38.5)	(3.5157,38.8)	(3.5157,41.6)	(1.4386,40.4)	(1.4386,40.0)	(1.4386,40.4)	(1.4386,43.3)	(2.3976,46.4)	(2.3976,46.8)	(2.3976,46.8)	(2.3976,49.8)	(1.2437,47.3)	(1.2437,47.7)	(1.2437,47.7)	(1.2437,50.8)	(1.2437,50.8)									
0.30	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)									
	(3.5094,19.7)	(3.5094,19.9)	(3.5094,21.1)	(1.4642,19.4)	(1.4642,19.4)	(1.4642,19.6)	(1.4642,20.9)	(2.3941,24.8)	(2.3941,24.8)	(2.3941,24.8)	(2.3976,26.4)	(1.2579,24.6)	(1.2579,24.8)	(1.2579,24.8)	(1.2579,26.3)	(1.2579,26.3)									
0.50	(0.1325,1.8589)	(0.1315,1.8577)	(0.1097,1.8273)	(0.1140,1.8339)	(0.1140,1.8339)	(0.1131,1.8325)	(0.1097,1.8273)	(0.1089,1.8259)	(0.1089,1.8259)	(0.1081,1.8247)	(0.1024,1.8151)	(0.1097,1.8273)	(0.1097,1.8273)	(0.1097,1.8273)	(0.1097,1.8273)	(0.1097,1.8273)									
	(3.9019,9.6)	(3.9034,9.7)	(3.6220,9.9)	(1.5014,8.3)	(1.5014,8.3)	(1.5020,8.3)	(1.5039,8.9)	(2.4575,11.8)	(2.4575,11.8)	(2.4583,11.9)	(2.4646,12.6)	(1.2799,11.2)	(1.2799,11.2)	(1.2799,11.2)	(1.2799,11.2)	(1.2799,11.2)									
0.70	(0.2142,1.9273)	(0.2127,1.9265)	(0.2018,1.9198)	(0.1952,1.9154)	(0.1952,1.9154)	(0.1938,1.9145)	(0.1828,1.9067)	(0.1742,1.9000)	(0.1742,1.9000)	(0.1731,1.8991)	(0.1643,1.8917)	(0.1690,1.8957)	(0.1690,1.8957)	(0.1690,1.8957)	(0.1690,1.8957)	(0.1690,1.8957)									
	(3.8114,6.5)	(3.8126,6.5)	(3.8215,6.8)	(1.5037,4.8)	(1.5037,4.8)	(1.5041,4.8)	(1.5071,5.1)	(2.5821,7.6)	(2.5821,7.6)	(2.5828,7.7)	(2.5882,8.1)	(1.2841,6.7)	(1.2841,6.7)	(1.2841,6.7)	(1.2860,7.1)	(1.2860,7.1)									
1.00	(0.3415,1.9769)	(0.3394,1.9764)	(0.3236,1.9721)	(0.3239,1.9721)	(0.3239,1.9721)	(0.3218,1.9715)	(0.3062,1.9668)	(0.2783,1.9572)	(0.2783,1.9572)	(0.2766,1.9566)	(0.2634,1.9514)	(0.2773,1.9569)	(0.2773,1.9562)	(0.2773,1.9562)	(0.2773,1.9562)	(0.2773,1.9562)									
	(3.7442,4.8)	(3.7450,4.8)	(3.7509,4.9)	(1.5157,2.9)	(1.5157,2.9)	(1.5159,2.9)	(1.5178,3.0)	(2.5395,5.0)	(2.5395,5.0)	(2.5400,5.0)	(2.5439,5.2)	(1.2899,3.9)	(1.2899,3.9)	(1.2899,3.9)	(1.2912,4.2)	(1.2912,4.2)									
1.50	(0.5498,2.0100)	(0.5498,2.0100)	(0.5287,2.0079)	(0.5218,2.0072)	(0.5218,2.0072)	(0.5190,2.0069)	(0.4977,2.0045)	(0.4745,2.0016)	(0.4745,2.0016)	(0.4711,2.0012)	(0.4465,1.9977)	(0.4681,2.0008)	(0.4681,2.0004)	(0.4681,2.0004)	(0.4681,2.0004)	(0.4681,2.0004)									
	(3.6982,4.0)	(3.6982,4.0)	(3.7011,4.0)	(1.5017,1.9)	(1.5017,1.9)	(1.5028,1.9)	(1.5028,1.9)	(2.5055,3.5)	(2.5055,3.5)	(2.5059,3.5)	(2.5086,3.6)	(1.2801,2.3)	(1.2801,2.3)	(1.2801,2.3)	(1.2810,2.4)	(1.2810,2.4)									
2.00	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.7137,2.0207)	(0.7137,2.0207)	(0.7100,2.0205)	(0.6818,2.0192)	(0.5498,2.0100)	(0.5498,2.0100)	(0.5498,2.0100)	(0.5498,2.0100)	(0.6849,2.0196)	(0.6849,2.0196)	(0.6849,2.0196)	(0.6849,2.0196)	(0.6849,2.0196)									
	(3.5094,3.8)	(3.5094,3.8)	(3.5094,3.8)	(1.5318,1.6)	(1.5318,1.6)	(1.5325,1.7)	(2.4990,2.9)	(2.4990,2.9)	(2.4990,2.9)	(2.4990,2.9)	(2.4990,3.0)	(1.2957,1.7)	(1.2957,1.7)	(1.2957,1.7)	(1.2962,1.8)	(1.2962,1.8)									
$h_s = 0.1 \quad \eta = 0.28 \quad m = 1$																									
δ	$W = 0.2$												$W = 0.6$												
	$B = 0.8$				$B = 1.2$				$B = 0.8$				$B = 1.2$				$B = 0.8$				$B = 1.2$				
	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	
0.10	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
	(3.5157,134.3)	(3.5157,130.6)	(3.5157,128.6)	(1.4386,138.1)	(1.4386,138.1)	(1.4386,134.5)	(1.4386,132.5)	(2.3976,144.0)	(2.3976,140.4)	(2.3976,140.4)	(2.3976,138.4)	(1.2437,146.1)	(1.2437,146.1)	(1.2437,146.1)	(1.2437,142.6)	(1.2437,140.6)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
0.20	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
	(3.5157,42.7)	(3.5157,41.2)	(3.5157,40.4)	(1.4386,44.4)	(1.4386,44.4)	(1.4386,42.8)	(1.4386,42.0)	(2.3976,51.0)	(2.3976,49.4)	(2.3976,49.4)	(2.3976,48.5)	(1.2437,52.0)	(1.2437,52.0)	(1.2437,52.0)	(1.2437,49.4)	(1.2437,49.4)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
0.30	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)
	(3.5094,21.5)	(3.5094,20.9)	(3.5094,20.5)	(1.4642,21.4)	(1.4642,21.4)	(1.4642,20.7)	(1.4642,20.3)	(2.3976,27.0)	(2.3976,26.2)	(2.3976,26.2)	(2.3976,25.7)	(1.2579,26.9)	(1.2579,26.9)	(1.2579,26.9)	(1.2579,26.1)	(1.2579,25.6)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
0.50	(0.1097,1.8273)	(0.1253,1.8498)	(0.1274,1.8525)	(0.1097,1.8273)	(0.1097,1.8273)	(0.1097,1.8273)	(0.1097,1.8273)	(0.1002,1.8113)	(0.1032,1.8164)	(0.1032,1.8164)	(0.1048,1.8193)	(0.0926,1.7967)	(0.0926,1.7967)	(0.0926,1.7967)	(0.0926,1.7967)	(0.0926,1.7967)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
	(3.6220,10.1)	(3.9137,10.1)	(3.9101,10.0)	(1.5039,9.1)	(1.5039,8.8)	(1.5039,8.6)	(1.5039,8.2)	(2.4670,12.9)	(2.4670,12.9)	(2.4670,12.9)	(2.4618,12.3)	(1.2684,12.3)	(1.2684,12.3)	(1.2684,12.3)	(1.2673,11.9)	(1.2673,11.7)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
0.70	(0.1977,1.9171)	(0.2033,1.9207)	(0.2065,1.9227)	(0.1787,1.9036)	(0.1843,1.9078)	(0.1875,1.9101)	(0.1875,1.9101)	(0.1611,1.8888)	(0.1655,1.8927)	(0.1655,1.8927)	(0.1681,1.8949)	(0.1563,1.8844)	(0.1563,1.8844)	(0.1563,1.8844)	(0.1599,1.8877)	(0.1599,1.8877)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
	(3.8251,6.9)	(3.8203,6.7)	(3.8176,6.6)	(1.5083,5.2)	(1.5083,5.2)	(1.5067,5.1)	(1.5068,5.0)	(2.5904,8.2)	(2.5875,8.0)	(2.5875,8.0)	(2.5859,7.9)	(1.2865,7.3)	(1.2865,7.3)	(1.2865,7.3)	(1.2853,7.0)	(1.2853,7.0)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
1.00	(0.3177,1.9703)	(0.3258,1.9727)	(0.3304,1.9740)	(0.3003,1.9649)	(0.3083,1.9675)	(0.3129,1.9689)	(0.3129,1.9689)	(0.2585,1.9493)	(0.2652,1.9521)	(0.2652,1.9521)	(0.2690,1.9537)	(0.2490,1.9452)	(0.2490,1.9452)	(0.2490,1.9452)	(0.2678,1.9532)	(0.2678,1.9532)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
	(3.7532,5.0)	(3.7800,4.9)	(3.7483,4.9)	(1.5186,3.1)	(1.5175,3.0)	(1.5170,3.0)	(1.5170,3.0)	(2.5454,5.3)	(2.5433,5.2)	(2.5433,5.2)	(2.5422,5.1)	(1.2735,4.3)	(1.2735,4.3)	(1.2735,4.3)	(1.2907,4.1)	(1.2907,4.1)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
1.50	(0.5195,2.0070)	(0.5321,2.0083)	(0.5392,2.0090)	(0.4896,2.0036)	(0.5006,2.0049)	(0.5069,2.0056)	(0.5069,2.0056)	(0.4375,1.9963)	(0.4498,1.9982)	(0.4498,1.9982)	(0.4570,1.9993)	(0.4331,1.9956)	(0.4331,1.9956)	(0.4331,1.9956)	(0.4516,1.9985)	(0.4516,1.9985)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
	(3.7025,4.1)	(3.7066,4.0)	(3.6996,4.0)	(1.5031,2.0)	(1.5026,1.9)	(1.5023,1.9)	(1.5023,1.9)	(2.5097,5.6)	(2.5082,5.6)	(2.5082,5.6)	(2.5074,5.6)	(1.2813,2.5)</													

TABLE 1 (Continued)

δ	$h_S = 0.5 \quad \eta = 0.28 \quad B = 1$											
	$W = 0.6$				$W = 0.2$				$W = 0.6$			
	$m = 1$	$m = 5$	$m = 10$	$m = 10$	$m = 1$	$m = 5$	$m = 10$	$m = 10$	$m = 1$	$m = 5$	$m = 10$	$m = 10$
0.10	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
	(3.5157,130.6)	(3.5157,125.3)	(3.5157,124.6)	(1.4386,134.5)	(1.4386,129.1)	(1.4386,128.4)	(2.3976,135.1)	(2.3976,140.4)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
0.20	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
	(3.5157,41.2)	(3.5157,39.0)	(3.5157,38.8)	(1.4386,42.8)	(1.4386,40.6)	(1.4386,40.3)	(2.3976,49.4)	(2.3976,47.0)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)	(0.0500,1.6686)
0.30	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6738)	(0.0511,1.6738)	(0.0509,1.6724)	(0.0511,1.6738)	(0.0514,1.6750)	(0.0514,1.6750)
	(3.5094,20.9)	(3.5094,20.0)	(3.5094,19.9)	(1.4642,20.7)	(1.4642,19.7)	(1.4642,19.6)	(2.3976,26.2)	(2.3955,25.0)	(0.1077,1.8240)	(2.3948,24.9)	(1.2579,24.9)	(1.2579,24.7)
0.50	(0.1253,1.8498)	(0.1310,1.8570)	(0.1317,1.8579)	(0.1097,1.8273)	(0.1126,1.8317)	(0.1133,1.8328)	(0.1032,1.8164)	(0.1032,1.8164)	(0.1077,1.8240)	(0.1083,1.8250)	(0.0955,1.8024)	(0.1006,1.8119)
	(3.9137,10.1)	(3.9043,9.7)	(3.9031,9.7)	(1.5039,8.8)	(1.5022,8.4)	(1.5018,8.3)	(2.4637,12.5)	(2.4588,12.0)	(2.4588,12.0)	(2.4582,11.9)	(1.2673,11.4)	(1.2655,11.3)
0.70	(0.2033,1.9207)	(0.2119,1.9260)	(0.2130,1.9266)	(0.1843,1.9078)	(0.1929,1.9139)	(0.1941,1.9147)	(0.1655,1.8927)	(0.1724,1.8985)	(0.1724,1.8985)	(0.1733,1.8993)	(0.1599,1.8877)	(0.1681,1.8949)
	(3.8203,6.7)	(3.8132,6.5)	(3.8123,6.5)	(1.5067,5.1)	(1.5043,4.9)	(1.5040,4.8)	(2.5875,8.0)	(2.5832,7.7)	(2.5832,7.7)	(2.5827,7.7)	(1.2858,7.1)	(1.2844,6.8)
1.00	(0.3258,1.9727)	(0.3382,1.9761)	(0.3399,1.9765)	(0.3083,1.9675)	(0.3206,1.9712)	(0.3223,1.9717)	(0.2652,1.9521)	(0.2756,1.9562)	(0.2756,1.9562)	(0.2769,1.9567)	(0.2639,1.9516)	(0.2759,1.9558)
	(3.7500,4.9)	(3.7454,4.8)	(3.7448,4.8)	(1.5175,3.0)	(1.5161,2.9)	(1.5159,2.9)	(2.5433,5.2)	(2.5403,5.0)	(2.5403,5.0)	(2.5399,5.0)	(1.2911,4.2)	(1.2901,4.0)
1.50	(0.5321,2.0083)	(0.5498,2.0100)	(0.5498,2.0100)	(0.5006,2.0049)	(0.5174,2.0068)	(0.5196,2.0070)	(0.4498,1.9982)	(0.4692,2.0010)	(0.4692,2.0010)	(0.4718,2.0013)	(0.4448,1.9975)	(0.4632,2.0001)
	(3.7006,4.0)	(3.6982,4.0)	(3.6982,4.0)	(1.5026,1.9)	(1.5019,1.9)	(1.5018,1.9)	(2.5082,3.6)	(2.5061,3.5)	(2.5061,3.5)	(2.5058,3.5)	(1.2809,2.4)	(1.2803,2.3)
2.00	(0.0514,1.6750)	(0.0514,1.6750)	(0.0514,1.6750)	(0.6857,2.0194)	(0.7079,2.0204)	(0.7108,2.0206)	(0.5498,2.0100)	(0.5498,2.0100)	(0.5498,2.0100)	(0.5498,2.0100)	(0.6580,2.0179)	(0.6826,2.0192)
	(3.5094,3.8)	(3.5094,3.8)	(3.5094,3.8)	(1.5324,1.7)	(1.5319,1.6)	(1.5319,1.6)	(2.4990,3.0)	(2.4990,2.9)	(2.4990,2.9)	(2.4990,2.9)	(1.2961,1.8)	(1.2958,1.7)

TABLE 2 Optimal couples (λ^* , K^*) (first row of each block) and values of (h_s^* , ATS) (second row of each block) of the variable sampling interval (VSI) exponentially weighted moving average (EWMA) control chart in the presence of measurement error for $n = 5$

δ	$h_s = 0.1 \quad m = 1 \quad B = 1$															
	$W = 0.2$				$W = 0.6$				$W = 0.2$				$W = 0.6$			
	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$
0.10	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)	(0.0500,1.3341)
0.20	(2.8875,87.6)	(2.8875,88.3)	(2.8875,89.0)	(2.8875,89.7)	(1.2549,94.3)	(1.2549,95.0)	(1.2549,95.7)	(1.2549,96.4)	(0.0516,1.3400)	(0.0516,1.3400)	(0.0516,1.3400)	(0.0516,1.3400)	(0.0516,1.3400)	(0.0516,1.3400)	(0.0516,1.3400)	(0.0516,1.3400)
0.30	(2.8875,25.8)	(2.8875,26.0)	(2.8875,27.7)	(2.8875,27.7)	(1.2692,28.2)	(1.2692,28.4)	(1.2692,28.4)	(1.2692,28.4)	(0.0644,1.3788)	(0.0644,1.3788)	(0.0644,1.3788)	(0.0644,1.3788)	(0.0644,1.3788)	(0.0644,1.3788)	(0.0644,1.3788)	(0.0644,1.3788)
0.50	(2.9729,13.5)	(2.9742,13.6)	(2.9845,14.5)	(2.9845,14.5)	(1.2754,14.2)	(1.2754,14.3)	(1.2754,14.3)	(1.2754,14.3)	(0.1576,1.5075)	(0.1576,1.5075)	(0.1576,1.5075)	(0.1576,1.5075)	(0.1576,1.5075)	(0.1576,1.5075)	(0.1576,1.5075)	(0.1576,1.5075)
0.70	(3.0466,6.6)	(3.0475,6.6)	(3.0553,6.9)	(3.0553,6.9)	(1.3074,6.0)	(1.3074,6.0)	(1.3074,6.0)	(1.3074,6.0)	(0.2629,1.5598)	(0.2629,1.5598)	(0.2629,1.5598)	(0.2629,1.5598)	(0.2629,1.5598)	(0.2629,1.5598)	(0.2629,1.5598)	(0.2629,1.5598)
1.00	(3.0162,4.6)	(3.0162,4.6)	(3.0162,4.7)	(3.0162,4.7)	(1.3094,3.5)	(1.3094,3.5)	(1.3094,3.5)	(1.3094,3.5)	(0.4240,1.5932)	(0.4240,1.5932)	(0.4240,1.5932)	(0.4240,1.5932)	(0.4240,1.5932)	(0.4240,1.5932)	(0.4240,1.5932)	(0.4240,1.5932)
1.50	(3.1654,3.7)	(3.1658,3.7)	(3.1694,3.8)	(3.1694,3.8)	(1.3177,2.1)	(1.3177,2.1)	(1.3177,2.1)	(1.3177,2.1)	(0.6621,1.6129)	(0.6621,1.6129)	(0.6621,1.6129)	(0.6621,1.6129)	(0.6621,1.6129)	(0.6621,1.6129)	(0.6621,1.6129)	(0.6621,1.6129)
2.00	(3.1387,3.3)	(3.1389,3.3)	(3.1405,3.3)	(3.1405,3.3)	(1.3112,1.5)	(1.3112,1.5)	(1.3112,1.5)	(1.3112,1.5)	(0.8569,1.6184)	(0.8569,1.6184)	(0.8569,1.6184)	(0.8569,1.6184)	(0.8569,1.6184)	(0.8569,1.6184)	(0.8569,1.6184)	(0.8569,1.6184)
2.50	(3.0516,1.3400)	(3.0516,1.3400)	(3.0516,1.3400)	(3.0516,1.3400)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
3.00	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
3.50	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
4.00	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
4.50	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
5.00	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
5.50	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
6.00	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
6.50	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
7.00	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
7.50	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
8.00	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
8.50	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
9.00	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
9.50	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)
10.00	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(2.8817,3.1)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(1.3094,1.3)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)	(0.0613,1.3705)

(Continues)

TABLE 3 Optimal couples (λ^* , K^*) (first row of each block) and values of (h_s^* , ATS) (second row of each block) of the variable sampling interval (VSI) exponentially weighted moving average (EWMA) control chart in the presence of measurement error for $n = 15$

δ	$h_s = 0.5 \quad m = 1 \quad B = 1$											
	$W = 0.1$				$W = 0.2$				$W = 0.6$			
	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.3$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.3$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.3$
0.10	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)
	(1.8808,35.0)	(1.8808,35.3)	(1.8808,37.9)	(1.0318,47.8)	(1.0318,47.4)	(1.0318,50.8)	(1.4893,42.7)	(1.4893,43.1)	(1.4893,45.9)	(1.4893,45.9)	(1.0176,50.0)	(1.0176,53.1)
0.20	(0.0931,0.8556)	(0.0924,0.8549)	(0.0869,0.8495)	(0.0867,0.8493)	(0.0867,0.8493)	(0.0764,0.8375)	(0.0812,0.8432)	(0.0805,0.8425)	(0.0757,0.8366)	(0.0757,0.8366)	(0.0764,0.8375)	(0.0764,0.8375)
	(1.9431,10.8)	(1.9439,10.9)	(1.9501,11.6)	(1.0413,15.4)	(1.0413,15.4)	(1.0394,16.5)	(1.5322,14.7)	(1.5322,14.7)	(1.4993,15.5)	(1.4993,15.5)	(1.0219,17.2)	(1.0219,18.3)
0.30	(0.1748,0.9041)	(0.1736,0.9036)	(0.1640,0.8998)	(0.1365,0.8868)	(0.1365,0.8868)	(0.1365,0.8868)	(0.1483,0.8928)	(0.1473,0.8923)	(0.1394,0.8883)	(0.1394,0.8883)	(0.1365,0.8868)	(0.1190,0.8762)
	(1.9554,5.7)	(1.9560,5.7)	(1.9606,6.1)	(1.0469,7.8)	(1.0469,7.7)	(1.0469,8.3)	(1.5384,7.9)	(1.5388,7.9)	(1.5415,8.4)	(1.5415,8.4)	(1.0261,9.0)	(1.0249,9.6)
0.50	(0.3485,0.9403)	(0.3464,0.9401)	(0.3307,0.9382)	(0.2990,0.9338)	(0.2990,0.9338)	(0.2785,0.9306)	(0.2956,0.9333)	(0.2938,0.9331)	(0.2799,0.9308)	(0.2799,0.9308)	(0.2832,0.9313)	(0.2652,0.9282)
	(1.9838,3.0)	(1.9841,3.0)	(1.9865,3.1)	(1.0515,3.3)	(1.0515,3.3)	(1.0519,3.5)	(1.5515,3.8)	(1.5517,3.9)	(1.5533,4.1)	(1.5533,4.1)	(1.0288,4.0)	(1.0291,4.3)
0.70	(0.5118,0.9524)	(0.5089,0.9523)	(0.4879,0.9512)	(0.4982,0.9518)	(0.4949,0.9516)	(0.4700,0.9502)	(0.4573,0.9495)	(0.4541,0.9493)	(0.4310,0.9477)	(0.4310,0.9477)	(0.4668,0.9501)	(0.4388,0.9483)
	(1.9688,2.3)	(1.9689,2.3)	(1.9720,2.4)	(1.0529,1.9)	(1.0529,1.9)	(1.0531,2.1)	(1.5402,2.6)	(1.5403,2.6)	(1.5414,2.7)	(1.5414,2.7)	(1.0295,2.4)	(1.0296,2.5)
1.00	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.7396,0.9588)	(0.7363,0.9587)	(0.7109,0.9583)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.7325,0.9587)	(0.7287,0.9586)
	(1.9633,2.0)	(1.9633,2.0)	(1.9633,2.1)	(1.0562,1.2)	(1.0562,1.2)	(1.0562,1.3)	(1.5352,1.9)	(1.5352,1.9)	(1.5352,1.9)	(1.5352,1.9)	(1.0312,1.4)	(1.0312,1.5)
1.50	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.9638,0.9604)	(0.9621,0.9604)	(0.9476,0.9604)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.9649,0.9604)	(0.9500,0.9604)
	(1.9633,2.0)	(1.9633,2.0)	(1.9633,2.0)	(1.0560,1.1)	(1.0560,1.1)	(1.0560,1.1)	(1.5352,1.6)	(1.5352,1.6)	(1.5352,1.6)	(1.5352,1.6)	(1.0311,1.1)	(1.0311,1.1)
2.00	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)
	(1.9633,2.0)	(1.9633,2.0)	(1.9633,2.0)	(1.0523,1.1)	(1.0523,1.1)	(1.0523,1.1)	(1.5352,1.5)	(1.5352,1.5)	(1.5352,1.5)	(1.5352,1.5)	(1.0290,1.0)	(1.0290,1.0)
$h_s = 0.5 \quad \eta = 0.28 \quad m = 1$												
δ	$W = 0.2$				$W = 0.6$							
	$B = 0.8$	$B = 1$	$B = 1.2$	$B = 1.2$	$B = 0.8$	$B = 1$	$B = 1.2$	$B = 1.2$				
	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.3$	$\eta = 0$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.3$				
0.10	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)	(0.05000,0.7943)				
	(1.8808,38.9)	(1.8808,37.5)	(1.8808,36.7)	(1.0318,50.4)	(1.0318,52.1)	(1.0318,49.5)	(1.4893,47.0)	(1.4893,45.5)				
0.20	(0.0849,0.8474)	(0.0877,0.8503)	(0.0893,0.8519)	(0.0764,0.8375)	(0.0867,0.8493)	(0.0867,0.8493)	(0.0740,0.8345)	(0.0763,0.8374)				
	(1.9525,11.9)	(1.9492,11.5)	(1.9474,11.3)	(1.0394,16.9)	(1.0413,16.4)	(1.0413,16.1)	(1.5006,15.9)	(1.4989,15.4)				
0.30	(0.1605,0.8983)	(0.1653,0.9004)	(0.1681,0.9015)	(0.1365,0.8868)	(0.1365,0.8868)	(0.1365,0.8868)	(0.1365,0.8867)	(0.1405,0.8889)				
	(1.9625,6.2)	(1.9600,6.0)	(1.9586,5.9)	(1.0469,8.5)	(1.0469,8.2)	(1.0469,8.1)	(1.5425,8.6)	(1.5411,8.3)				
0.50	(0.3248,0.9375)	(0.3329,0.9385)	(0.3375,0.9390)	(0.2719,0.9294)	(0.2810,0.9310)	(0.2862,0.9318)	(0.2747,0.9299)	(0.2818,0.9311)				
	(1.9875,3.2)	(1.9862,3.1)	(1.9853,3.1)	(1.0521,3.6)	(1.0519,3.5)	(1.0517,3.5)	(1.5540,4.1)	(1.5531,4.0)				
0.70	(0.4800,0.9508)	(0.4908,0.9514)	(0.4969,0.9517)	(0.4605,0.9497)	(0.4735,0.9504)	(0.4808,0.9509)	(0.4225,0.9471)	(0.4341,0.9480)				
	(1.9707,2.4)	(1.9700,2.4)	(1.9696,2.3)	(1.0532,2.1)	(1.0531,2.1)	(1.0530,2.0)	(1.5418,2.7)	(1.5409,2.7)				
1.00	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.7010,0.9582)	(0.7145,0.9584)	(0.7220,0.9585)	(0.6511,0.9571)	(0.6511,0.9571)				
	(1.9633,2.1)	(1.9633,2.1)	(1.9633,2.1)	(1.0562,1.3)	(1.0562,1.3)	(1.0562,1.3)	(1.5352,2.0)	(1.5352,1.9)				
1.50	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.9414,0.9604)	(0.9498,0.9604)	(0.9542,0.9604)	(0.6511,0.9571)	(0.6511,0.9571)				
	(1.9633,2.0)	(1.9633,2.0)	(1.9633,2.0)	(1.0560,1.1)	(1.0560,1.1)	(1.0560,1.1)	(1.5352,1.6)	(1.5352,1.6)				
2.00	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)				
	(1.9633,2.0)	(1.9633,2.0)	(1.9633,2.0)	(1.0523,1.1)	(1.0523,1.1)	(1.0523,1.1)	(1.5352,1.5)	(1.5352,1.5)				

(Continues)

TABLE 3 (Continued)

δ	$h_S = 0.5 \quad \eta = 0.28 \quad B = 1$														
	$W = 0.6$					$W = 0.2$					$W = 0.6$				
	$m = 1$	$m = 5$	$m = 10$	$m = 5$	$m = 1$	$m = 1$	$m = 5$	$m = 10$	$m = 5$	$m = 1$	$m = 1$	$m = 5$	$m = 10$	$m = 5$	$m = 1$
0.10	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)	(0.0500,0.7943)
0.20	(1.8808,37.5)	(1.8808,35.5)	(1.8808,35.2)	(1.8808,35.2)	(1.0318,50.4)	(1.0318,48.0)	(1.0318,47.7)	(1.0318,47.7)	(1.4893,45.5)	(1.4893,43.3)	(1.4893,43.0)	(1.4893,43.0)	(1.4893,43.0)	(1.0176,50.2)	(1.0176,49.9)
0.30	(0.0877,0.8503)	(0.0920,0.8545)	(0.0925,0.8551)	(0.0925,0.8551)	(0.0867,0.8493)	(0.0867,0.8493)	(0.0867,0.8493)	(0.0867,0.8493)	(0.0763,0.8374)	(0.0802,0.8421)	(0.0807,0.8426)	(0.0807,0.8426)	(0.0807,0.8426)	(0.0764,0.8375)	(0.0764,0.8375)
0.50	(1.9492,11.5)	(1.9444,10.9)	(1.9438,10.9)	(1.9438,10.9)	(1.0413,16.4)	(1.0413,15.6)	(1.0413,15.5)	(1.0413,15.5)	(1.4989,15.4)	(1.5324,14.7)	(1.5321,14.6)	(1.5321,14.6)	(1.5321,14.6)	(1.0219,17.3)	(1.0219,17.2)
0.70	(0.1653,0.9004)	(0.1728,0.9033)	(0.1738,0.9037)	(0.1738,0.9037)	(0.1365,0.8868)	(0.1365,0.8868)	(0.1365,0.8868)	(0.1365,0.8868)	(0.1405,0.8889)	(0.1467,0.8920)	(0.1475,0.8924)	(0.1475,0.8924)	(0.1475,0.8924)	(0.1365,0.8868)	(0.1365,0.8868)
1.00	(1.9600,6.0)	(1.9564,5.8)	(1.9559,5.7)	(1.9559,5.7)	(1.0469,8.2)	(1.0469,7.8)	(1.0469,7.8)	(1.0469,7.8)	(1.5411,8.3)	(1.5390,8.0)	(1.5387,7.9)	(1.5387,7.9)	(1.5387,7.9)	(1.0261,9.1)	(1.0261,9.0)
1.50	(0.3329,0.9385)	(0.3453,0.9399)	(0.3469,0.9401)	(0.3469,0.9401)	(0.2810,0.9310)	(0.2952,0.9333)	(0.2971,0.9336)	(0.2971,0.9336)	(0.2818,0.9311)	(0.2927,0.9329)	(0.2942,0.9331)	(0.2942,0.9331)	(0.2942,0.9331)	(0.2799,0.9308)	(0.2815,0.9311)
2.00	(1.9862,3.1)	(1.9843,3.0)	(1.9841,3.0)	(1.9841,3.0)	(1.0519,3.5)	(1.0515,3.4)	(1.0515,3.3)	(1.0515,3.3)	(1.5531,4.0)	(1.5518,3.9)	(1.5517,3.9)	(1.5517,3.9)	(1.5517,3.9)	(1.0288,4.0)	(1.0288,4.0)
0.10	(0.4908,0.9514)	(0.5074,0.9522)	(0.5096,0.9523)	(0.5096,0.9523)	(0.4735,0.9504)	(0.4930,0.9515)	(0.4956,0.9516)	(0.4956,0.9516)	(0.4341,0.9480)	(0.4524,0.9492)	(0.4548,0.9493)	(0.4548,0.9493)	(0.4548,0.9493)	(0.4617,0.9498)	(0.4642,0.9499)
0.20	(1.9700,2.4)	(1.9690,2.3)	(1.9689,2.3)	(1.9689,2.3)	(1.0531,2.1)	(1.0529,2.0)	(1.0529,2.0)	(1.0529,2.0)	(1.5412,2.7)	(1.5404,2.6)	(1.5403,2.6)	(1.5403,2.6)	(1.5403,2.6)	(1.0295,2.4)	(1.0295,2.4)
0.30	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.7145,0.9584)	(0.7345,0.9587)	(0.7370,0.9587)	(0.7370,0.9587)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.7266,0.9586)	(0.7295,0.9586)
0.50	(1.9633,2.1)	(1.9633,2.0)	(1.9633,2.0)	(1.9633,2.0)	(1.0562,1.3)	(1.0562,1.3)	(1.0562,1.3)	(1.0562,1.3)	(1.5352,1.9)	(1.5352,1.9)	(1.5352,1.9)	(1.5352,1.9)	(1.5352,1.9)	(1.0312,1.4)	(1.0312,1.4)
0.70	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.9498,0.9604)	(0.9611,0.9604)	(0.9624,0.9604)	(0.9624,0.9604)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.9624,0.9604)	(0.9636,0.9604)
1.00	(1.9633,2.0)	(1.9633,2.0)	(1.9633,2.0)	(1.9633,2.0)	(1.0560,1.1)	(1.0560,1.1)	(1.0560,1.1)	(1.0560,1.1)	(1.5352,1.6)	(1.5352,1.6)	(1.5352,1.6)	(1.5352,1.6)	(1.5352,1.6)	(1.0311,1.1)	(1.0311,1.1)
1.50	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)	(0.6511,0.9571)
2.00	(1.9633,2.0)	(1.9633,2.0)	(1.9633,2.0)	(1.9633,2.0)	(1.0523,1.1)	(1.0523,1.1)	(1.0523,1.1)	(1.0523,1.1)	(1.5352,1.5)	(1.5352,1.5)	(1.5352,1.5)	(1.5352,1.5)	(1.5352,1.5)	(1.0290,1.0)	(1.0290,1.0)

and $\hat{\sigma}_0 = 0.9616$. In the implementation of the proposed chart, we suppose a sample of the size $n = 5$ is taken for each subgroup, and we choose the value of ATS_0 equal to 370.4. Similar to Tran et al.,²⁰ we assume $\eta = 0.28$, $A = 0$, $B = 1$, and $m = 1$ for the parameters of the linear covariate error model. We also consider a shift of $0.5\sigma_0$ in the mean as an alarm that the process is out of control. Then, with the sampling interval $h_S = 0.5$ and $W = 0.2$, the optimal parameters (λ^*, K^*) are $\lambda^* = 0.1467$ and $K^* = 1.4989$. From the formulas (7)–(10), we obtain

$$\begin{aligned}
 LCL &= 500.0230 - 1.4989 \sqrt{\frac{0.1467}{2 - 0.1467} (1 + 0.28^2)} \times 0.9616 = 499.6019, \\
 UCL &= 500.0230 + 1.4989 \sqrt{\frac{0.1467}{2 - 0.1467} (1 + 0.28^2)} \times 0.9616 = 500.4441, \\
 LWL &= 500.023 - 0.3 \sqrt{\frac{0.1467}{2 - 0.1467} (1 + 0.28^2)} \times 0.9616 = 499.9387, \\
 UWL &= 500.023 + 0.3 \sqrt{\frac{0.1467}{2 - 0.1467} (1 + 0.28^2)} \times 0.9616 = 500.1073.
 \end{aligned}$$

We simulate phase II data from a normal distribution. In particular, the first 10 subgroups are simulated from $N(\hat{\mu}_0, \hat{\sigma}_0^2)$, and the last 10 subgroups are simulated from $N(\hat{\mu}_0 + 0.5 * \hat{\sigma}_0, \hat{\sigma}_0^2)$, that is, with a shift of the size $\delta = 0.5$, leading to higher milk weight. The simulated phase II data are presented in Table 4. The rightmost two columns of the Table 4 show the corresponding sample median values \tilde{T}_i^* and the EWMA sequence Z_i^* , whereas the second and the third columns present the corresponding sampling interval and the total time. Figure 2 plot the VSI EWMA- \tilde{X} control chart in the presence of measurement errors corresponding to the phase II data set in Table 4. As can be seen from this figure, from sample 13, the process is detected to be out of control (corresponding to the bold values in Table 4).

TABLE 4 Phase II dataset (SI: sampling interval)

Sample	SI	Total time	Phase II (X_{ij})				\tilde{X}_i^*	Z_i^*	
1	1.63	1.63	501.4833	500.1706	499.9020	498.8775	499.6307	499.9020	500.0052
2	1.63	3.26	500.1050	499.9799	498.6100	501.0395	498.8064	499.9799	500.0015
3	1.63	4.89	499.5145	501.4646	501.1188	501.3637	499.7332	501.1188	500.1654
4	0.5	5.39	499.5620	499.8151	500.2803	500.7398	497.1229	499.8151	500.1140
5	0.5	5.89	499.7502	499.8326	499.0275	499.8758	499.8762	499.8326	500.0727
6	1.63	7.52	498.5258	499.6318	500.6597	500.4488	499.8711	499.8711	500.0432
7	1.63	9.15	499.8433	500.9569	500.0851	499.6945	498.8933	499.8433	500.0138
8	1.63	10.78	499.4935	499.7990	501.0783	500.8580	500.4979	500.4979	500.0849
9	1.63	12.41	500.2730	500.9816	499.2352	500.5186	498.6644	500.2730	500.1125
10	0.5	12.91	500.8658	502.7562	499.4933	501.3502	499.5705	500.8658	500.2230
11	0.5	13.41	501.1827	500.6837	501.3678	500.0807	502.1853	501.1827	500.1931
12	0.5	13.91	502.1790	500.6295	500.1285	499.6191	502.1439	500.6295	500.2571
13	0.5	14.41	501.3198	503.0769	502.9001	501.9779	498.7582	501.9779	500.5096
14	0.5	14.91	501.7116	499.0937	501.5189	501.9622	501.3381	501.5189	500.6577
15	0.5	15.41	501.6141	500.1646	502.7406	501.7982	501.1249	501.6141	500.7980
16	0.5	15.91	500.9030	499.9101	498.3796	501.3495	501.8195	500.9030	500.8134
17	0.5	16.41	497.5061	500.1663	501.0336	501.0758	497.7357	500.1663	500.7184
18	0.5	16.91	499.8274	499.9869	500.5583	500.3271	500.5652	500.3271	500.6610
19	0.5	17.41	499.5205	499.9218	500.1015	502.3268	501.0534	500.1015	500.5790
20	0.5	17.91	501.1493	502.6016	498.8458	500.9987	499.9475	500.9987	500.6405

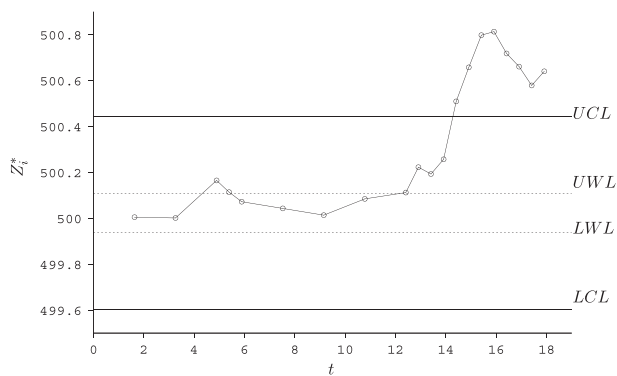


FIGURE 2 The variable sampling interval (VSI) exponentially weighted moving average (EWMA)- \tilde{X} control chart in the presence of measurement errors corresponding to the Phase II data set in Table 4. LCL, lower control limit; LWL, lower warning limit; UCL, upper control limit; UWL, upper warning limit

7 | CONCLUDING REMARKS

In this paper, we have investigated the effect of measurement error on the VSI EWMA- \tilde{X} control chart. The obtained results show that the measurement error has a negative influence on the proposed chart performance. These negative impacts can be reduced by increasing either the sample size n or the number of multiple measurements per item. We have also compared the performance of the VSI EWMA- \tilde{X} control chart with other median charts in the literature in the same condition of the presence of measurement errors. It is shown that our proposed control chart in this study leads to better performance compared with the others.

ORCID

Huu Du Nguyen  <https://orcid.org/0000-0001-6067-6676>

Kim Phuc Tran  <https://orcid.org/0000-0002-6005-1497>

REFERENCES

- Shewhart WA. *Economic Control of Quality of Manufactured Product*. New York: Van Nostrand. 1931.
- Hu X, Castagliola P, Sun J, Kho MBC. The effect of measurement errors on the synthetic \bar{X} chart. *Qual Reliab Eng Int*. 2015;31(8):1769-1778.
- Tran PH, Tran KP, Rakitzis A. A synthetic median control chart for monitoring the process mean with measurement errors. *Qual Reliab Eng Int*. 2019;35:1100-1116.
- Tran KP. Run rules median control charts for monitoring process mean in manufacturing. *Qual Reliab Eng Int*. 2017;33(8):2437-2450.
- Reynolds M, Amin R, Arnold J, Nachlas J. \bar{X} charts with variable sampling intervals. *Technometrics*. 1988;30(2):181-192.
- Reynolds M, Arnold J. EWMA control charts with variable sample sizes and variable sampling intervals. *IIE Trans*. 2001;33(6):511-530.
- Brook D, Evans D. An approach to the probability distribution of CUSUM run length. *Biometrika*. 1972;59(3):539-549.
- Wu Z, Jiao J, Yang M, Liu Y, Wang Z. An enhanced adaptive CUSUM control chart. *IIE Trans*. 2009;41(7):642-653.
- Maravelakis P. Measurement error effect on the CUSUM control chart. *J Appl Stat*. 2012;39(2):323-336.
- Maravelakis P, Panaretos J, Psarakis S. EWMA chart and measurement error. *J Appl Stat*. 2004;31(4):445-455.
- Nguyen HD, Tran KP, Heuchenne H. CUSUM control charts with variable sampling interval for monitoring the ratio of two normal variables. *Qual Reliab Eng*. 2019;36(2):474-497.
- Nguyen HD, Tran KP, Heuchenne C. Monitoring the ratio of two normal variables using variable sampling interval exponentially weighted moving average control charts. *Qual Reliab Eng Int*. 2019;35(1):439-460.
- Nguyen HD, Nguyen QT, Tran KP, Ho DP. On the performance of VSI Shewhart control chart for monitoring the coefficient of variation in the presence of measurement errors. *Int J Adv Manuf Technol*. 2019;104:211-243.
- Nguyen HD, Tran KP, Celano G, Maravelakis PE, Castagliola P. On the effect of the measurement error on Shewhart t and EWMA t control charts. *Int J Adv Manuf Technol*. 2020;107:4317-4332.
- Tran KP, Nguyen HD, Tran PH, Heuchenne C. On the performance of CUSUM control charts for monitoring the coefficient of variation with measurement errors. *Int J Adv Manuf Technol*. 2019;104:1903-1917.
- Nguyen HD, Tran KP, Goh T. Variable sampling interval control charts for monitoring the ratio of two normal variables. *J Test Eval*. 2019;48(3):2505-2529.
- Linna K, Woodall W, Busby K. The performance of multivariate control charts in the presence of measurement error. *J Qual Technol*. 2001;33(3):349-355.
- Costa A, Castagliola P. Effect of measurement error and autocorrelation on the \bar{X} chart. *J Appl Stat*. 2011;38(4):661-673.
- Noorossana R, Zerehsaz Y. Effect of measurement error on phase II monitoring of simple linear profiles. *Int J Adv Manuf Technol*. 2015;79:2031-2040.

20. Tran KP, Castagliola P, Balakrishnan N. On the performance of Shewhart median chart in the presence of measurement errors. *Qual Reliab Eng Int.* 2017;33(5):1019-1029.
21. Tran KP, Nguyen HD, Nguyen QT, Chattinnawat W. One-sided synthetic control charts for monitoring the coefficient of variation with measurement errors. In: International Conference on Industrial Engineering and Engineering Management (IEEM) Bangkok; 2019: 1667-1671. <https://doi.org/10.1109/IEEM.2018.8607320>
22. Castagliola P, Maravelakis PE, Figueiredo FO. The EWMA median chart with estimated parameters. *IIE Trans.* 2016;48(1):66-74.
23. Khoo M. A control chart based on sample median for the detection of a permanent shift in the process mean. *Qual Eng.* 2005;17(2): 243-257.
24. Castagliola P. An \bar{X}/R -EWMA control chart for monitoring the process sample median. *Int J Reliab Qual Saf Eng.* 2001;8(2):123-135.
25. Tran KP, Castagliola P, Nguyen TH, Cuzol A. Design of a variable sampling interval EWMA median control chart. *Int J Reliab Qual Saf Eng.* 2019;26(5):1950021.
26. Castagliola P, Figueiredo FO. The median chart with estimated parameters. *Europ J Indust Eng.* 2013;7(5):594-614.
27. Saha S, Khoo MB, Ng P, Chong Z. Variable sampling interval run sum median charts with known and estimated process parameters. *Comput Indust Eng.* 2019;127:571-587.
28. Khoo MB, Saha S, Lee HC, Castagliola P. Variable sampling interval exponentially weighted moving average median chart with estimated process parameters. *Qual Reliab Eng Int.* 2019;35(8):2732-2748.
29. Linna K, Woodall W. Effect of measurement error on Shewhart control chart. *J Qual Technol.* 2001;33(2):213-222.

AUTHOR BIOGRAPHIES

Kim Duc Tran is currently a researcher at the Institute of Research and Development, Duy Tan University, Danang, Vietnam. His research is focused on advanced statistical process monitoring techniques, machine learning, and anomaly detection techniques.

Dr. Huu Du Nguyen is a member of the Department of Mathematics, Faculty of Information Technology, Vietnam National University of Agriculture, Hanoi, Vietnam. He is also a member of Institute of Artificial Intelligence and Data Science, Dong A University, Danang, Vietnam. His research is related to reliability assessment, statistical process control, machine learning, and deep learning.

Thi Hien Nguyen received a Ph.D. degree in Mathematics and Applications from the University of Brest (France) in 2017. She is currently lecturer at CY Tech, Paris Cergy University. She is also a researcher of the Institute of Artificial Intelligence and Data Science, Dong A University (Vietnam). Her research interests focus mainly on statistical process control, Hawkes process, and spectral theory of diffusion in the incompressible medium.

Dr. Kim Phuc Tran is currently an associate professor in Automation and Industrial Informatics at ENSAIT & GEMTEX, Roubaix, France. He received the Engineering degree and the Master of Engineering degree in Automated Manufacturing. He obtained a PhD in Automation and Applied Informatics at the Université de Nantes, Nantes, France. His research is focused on advanced statistical process monitoring techniques, data mining, machine learning, deep learning, and condition monitoring systems.

How to cite this article: Duc Tran K, Du Nguyen H, Hien Nguyen T, Phuc Tran K. Design of a variable sampling interval exponentially weighted moving average median control chart in presence of measurement errors. *Qual Reliab Engng Int.* 2020;1-17. <https://doi.org/10.1002/qre.2726>

APPENDIX A

In order to calculate the *ATS* of the VSI EWMA- \tilde{X} control chart in the presence of measurement errors, we use a Markov chain method proposed in Brook and Evans.⁷ By this method, the control limit [*LCL*, *UCL*] is divided into $2p + 1$ equal-length subintervals $(H_j - \Delta, H_j + \Delta]$, $j \in \{-p, \dots, 0, \dots, +p\}$, centered at

$$H_j = \frac{UCL + LCL}{2} + 2j\Delta \quad \text{where} \quad \Delta = \frac{UCL - LCL}{2(2p + 1)}.$$

Using formulas (7) and (8) for *LCL* and *UCL*, we obtain

$$H_j = A + B\mu_0 + 2j \frac{K \sqrt{\frac{\lambda}{2-\lambda} \left(B^2 \sigma_0^2 + \frac{\sigma_M^2}{m} \right)}}{(2p + 1)}, \quad (A1)$$

$$\Delta = \frac{K \sqrt{\frac{\lambda}{2-\lambda} \left(B^2 \sigma_0^2 + \frac{\sigma_M^2}{m} \right)}}{(2p + 1)}. \quad (A2)$$

Each subinterval $(H_j - \Delta, H_j + \Delta]$, $j \in \{-p, \dots, 0, \dots, +p\}$, represents a transient state of a Markov chain. For each $j \in \{-p, \dots, 0, \dots, +p\}$, if $Z_i^* \in (H_j - \Delta, H_j + \Delta]$, then the Markov chain is in the transient state j for sample i . If $Z_i^* \notin (H_j - \Delta, H_j + \Delta]$, then the Markov chain reached the absorbing state $(-\infty, LCL] \cup [UCL, +\infty)$. We assume that H_j is the representative value of state j . Let \mathbf{Q} be the $(2p + 1, 2p + 1)$ submatrix of probabilities $Q_{j,k}$ corresponding to the $2p + 1$ transient states defined above, that is,

$$\mathbf{Q} = \begin{pmatrix} Q_{-p,-p} & \dots & Q_{-p,-1} & Q_{-p,0} & Q_{-p,+1} & \dots & Q_{-p,+p} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Q_{-1,-p} & \dots & Q_{-1,-1} & Q_{-1,0} & Q_{-1,+1} & \dots & Q_{-1,+p} \\ Q_{0,-p} & \dots & Q_{0,-1} & Q_{0,0} & Q_{0,+1} & \dots & Q_{0,+p} \\ Q_{+1,-p} & \dots & Q_{+1,-1} & Q_{+1,0} & Q_{+1,+1} & \dots & Q_{+1,+p} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Q_{+p,-p} & \dots & Q_{+p,-1} & Q_{+p,0} & Q_{+p,+1} & \dots & Q_{+p,+p} \end{pmatrix}. \quad (A3)$$

By definition, we have $Q_{j,k} = P(Z_i^* \in (H_k - \Delta, H_k + \Delta] | Z_{i-1}^* = H_j)$ or, equivalently, $Q_{j,k} = P(Z_i^* \leq H_k + \Delta | Z_{i-1}^* = H_j) - P(Z_i^* \leq H_k - \Delta | Z_{i-1}^* = H_j)$. Replacing $Z_i^* = (1 - \lambda)Z_{i-1}^* + \lambda\tilde{X}_i^*$, $Z_{i-1}^* = H_j$ and isolating \tilde{X}_i^* gives

$$\begin{aligned} Q_{j,k} &= P\left(\tilde{X}_i^* \leq \frac{H_k + \Delta - (1 - \lambda)H_j}{\lambda}\right) - P\left(\tilde{X}_i^* \leq \frac{H_k - \Delta - (1 - \lambda)H_j}{\lambda}\right) \\ &= F_{\tilde{X}_i^*}\left(\frac{H_k + \Delta - (1 - \lambda)H_j}{\lambda} \middle| n\right) - F_{\tilde{X}_i^*}\left(\frac{H_k - \Delta - (1 - \lambda)H_j}{\lambda} \middle| n\right). \end{aligned}$$

From (5), (A1), and (A2), it can easily be proven that

$$Q_{j,k} = F_\beta\left(\Phi\left(\frac{(2k + 1 - 2j(1 - \lambda))K \sqrt{\frac{\lambda}{2-\lambda}} - \frac{B\delta\sigma_0}{\sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}}}{\lambda(2p + 1)} \middle| \frac{n+1}{2}, \frac{n+1}{2}\right)\right)$$

$$-F_{\beta} \left(\Phi \left(\frac{(2k-1-2j(1-\lambda))K\sqrt{\frac{\lambda}{2-\lambda}}}{\lambda(2p+1)} - \frac{B\delta\sigma_0}{\sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}} \right) \middle| \frac{n+1}{2}, \frac{n+1}{2} \right).$$

Let us denote $\eta = \frac{\sigma_M}{\sigma_0}$ the precision error ratio. The probability $Q_{j,k}$ can be rewritten in the form

$$Q_{j,k} = F_{\beta} \left(\Phi \left(\frac{(2k+1-2j(1-\lambda))K\sqrt{\frac{\lambda}{2-\lambda}}}{\lambda(2p+1)} - \frac{B\delta}{\sqrt{B^2 + \frac{\eta^2}{m}}} \right) \middle| \frac{n+1}{2}, \frac{n+1}{2} \right) - F_{\beta} \left(\Phi \left(\frac{(2k-1-2j(1-\lambda))K\sqrt{\frac{\lambda}{2-\lambda}}}{\lambda(2p+1)} - \frac{B\delta}{\sqrt{B^2 + \frac{\eta^2}{m}}} \right) \middle| \frac{n+1}{2}, \frac{n+1}{2} \right). \quad (\text{A4})$$

Let $\mathbf{q} = (q_{-p}, \dots, q_0, \dots, q_p)^T$ be the $(2p+1)$ vector of initial probabilities associated with the $2p+1$ transient states, where

$$q_j = \begin{cases} 0 & \text{if } Z_0^* \notin (H_j - \Delta, H_j + \Delta], \\ 1 & \text{if } Z_0^* \in (H_j - \Delta, H_j + \Delta]. \end{cases} \quad (\text{A5})$$

Then, similar to Tran et al.,²⁵ the ATS_1 can be calculated by the following expression:

$$ATS_1 = \mathbf{q}^T (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{g}, \quad (\text{A6})$$

where \mathbf{g} is the vector of sampling intervals corresponding to the discretized states of the Markov chain and the j th element g_j of the vector \mathbf{g} is the sampling interval when the control statistic is in state j (represented by H_j), that is,

$$g_j = \begin{cases} h_L & \text{if } LWL < H_j < UWL, \\ h_S & \text{otherwise.} \end{cases} \quad (\text{A7})$$

Also, the average sampling interval of the VSI EWMA- \tilde{X} chart is given as

$$E(h) = \frac{\mathbf{q}^T (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{g}}{\mathbf{q}^T (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{1}}. \quad (\text{A8})$$