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Production Planning with Flexible Manufacturing Systems under Demand Uncertainty

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ABSTRACT
This paper delves into the impacts of an ongoing global crisis on the resilience of supply chains. Furthermore, it proposes measures to address and mitigate the disruptions caused by the prevailing uncertainties. For example, while the economy has started to recover after the pandemic and demand has increased, companies have not fully returned to their pre-pandemic levels. To enhance their supply chain resilience and effectively manage disruptions, one viable strategy is the implementation of flexible/hybrid manufacturing systems. This research is motivated by the specific requirements of Vestel Electronics, a household appliances company, which seeks a flexible/hybrid manufacturing production setup involving dedicated machinery to meet regular demand and the utilization of flexible manufacturing systems to handle surges in demand. We employ a scenario-based approach to model demand uncertainty, enabling the company to make immediate and adaptive decisions that take advantage of the cost-effectiveness of standard production and the responsiveness of flexible manufacturing systems. To solve the problem, we propose a heuristic algorithm based on column generation. The numerical results demonstrate that our optimization model provides solutions with an average optimality gap of less than 6% while also reducing the average cost of standard production schemes without FMS by over 12%.

KEYWORDS
inventory control; production planning; scheduling; flexible manufacturing systems; stochastic optimization

1. Introduction

As transportation and communication technology improved in recent decades, countries have grown increasingly interconnected, leading to more intense global competition in the industry. This has resulted in a shift in market dynamics, where supply has grown at a faster pace than demand. Traditional manufacturing systems, which were built to produce only one specific product, were no longer able to keep up with these new and dynamic market conditions, which were characterized by frequent product launches, fluctuating demand, changing government regulations, and rapid technological advancements. To address the challenges posed by these changing market conditions, flexible manufacturing systems (FMSs) were developed as a potential so-
lution. These systems are typically made up of computer numerical control (CNC) machines, which have a built-in ability to produce a wide range of products, providing manufacturers with the flexibility they need to adapt to changing market demands (Yelles-Chaouche et al. 2021).

There are three significant categories of flexibility: component or basic flexibility, system flexibility, and aggregate flexibility (Sethi and Sethi 1990). These three categories and their components are presented in Figure 1. Machine flexibility, as a component of basic flexibility, plays a crucial role in modern manufacturing processes. It refers to the ability of machines and equipment within a manufacturing system to adapt to different tasks and product variations without requiring significant changes or reconfiguration (Chandra and Tombak 1992). This flexibility empowers manufacturers to produce a wide range of products efficiently and cost-effectively, making it a fundamental aspect of modern production systems. In this study, we consider two types of machines on the same production floor: typical manufacturing machines (TMMs) and FMSs. Both TMMs and FMSs can be used to produce a product, but they have different production costs and output capacities. However, TMMs are characterized by their relatively lower flexibility, as they are designed to adhere to a fixed production sequence established at the start of the planning horizon. They operate accordingly throughout the entire manufacturing process. On the other hand, FMSs consist of versatile machines that can be adjusted for different settings during each production period based on real-time feedback. These systems can be quickly reconfigured to accommodate changes in drum types or other requirements, allowing for efficient and agile production processes. This study is motivated by the current practices observed at a Vestel Electronics drum production facility. It explores various forms of adaptability but excludes routing, expansion, and market flexibility.

![Figure 1. Linkage between various flexibility taken from Sethi and Sethi (1990) and post-processed](image-url)

Different factors, such as pandemics, can contribute to fluctuations in demand, which affect various sectors, including healthcare, manufacturing, and tourism (Don-thu and Gustafsson 2020; He and Harris 2020; Wen et al. 2021). These uncertainties
have led to significant changes in the behaviour of businesses, employees, and consumers (Seyfi et al. 2022). Some of these changes have had positive effects on specific industries and individuals, such as the growth of e-commerce and improved customer satisfaction (Mofokeng 2021), the rise of remote working opportunities (Vyas and Butakhieo 2021), the advancement of online education offered by universities and other institutions (Tang et al. 2023), and a reduction in global greenhouse gas emissions due to decreased petroleum usage and lower oil prices (Le Billon et al. 2021). On the other hand, some industries have faced negative consequences, with businesses struggling or facing closure due to demand fluctuations and supply chain disruptions (Ivanov 2021). Shortages of raw materials have caused input prices to rise, impacting the supply side (Priya, Cuce, and Sudhakar 2021). Additionally, lockdowns and restrictions have led to job losses and furloughs for many individuals (Kaushik and Guleria 2020).

Lockdowns have had a massive impact on Vestel Electronics, a leading global electronics and appliances manufacturer established in 1984 in Manisa, Turkey. Vestel is recognized as part of the Zorlu Group since 1994 for its innovative products, including 352 washing machine models with 55 unique drum designs, all components made in-house. Due to the pandemic, the company was forced to suspend production from March 30 to April 5, 2020. Afterward, Vestel’s plants operated at reduced capacity due to containment measures, as reported on the Istanbul public disclosure platform. The pandemic disrupted Vestel’s extensive supply chains to East Asia. The outbreak in China caused immediate supply chain disruptions, leading to global demand fluctuations for Vestel products. The pandemic affected all supply chain aspects, including transportation (Mogaji 2020), storage (Priya, Cuce, and Sudhakar 2021), and currency exchange rates (Li et al. 2022). Vestel has been adapting to the "new normal" by employing strategies to counter supply chain disruptions, with multisourcing being a key approach.

Compared to relying solely on single suppliers, multisourcing in conjunction with hybrid and flexible manufacturing systems can significantly bolster supply chain resilience by mitigating risks and diversifying sourcing options (Sodhi, Tang, and Willenson 2023). Hybrid and flexible manufacturing systems offer a way to increase supply chain resilience and handle disruptions, compared to alternative solutions such as higher labor costs in local production or increased inventory holding costs. Manufacturing practices are evolving to meet the growing demand for personalized products in mass production, so companies must design flexible manufacturing systems to produce a range of goods within a specific category while maintaining high quality and low cost. Vestel also uses outsourcing to fulfill customer demand and avoid lost sales in a competitive market. This paper focuses on the drum production of washing machines at Vestel using a hybrid manufacturing system that combines traditional mass manufacturing and flexible manufacturing systems. FMS installations, which are wholly owned and operated by Vestel, serving as an essential component of their manufacturing capabilities, offer significant advantages in manufacturing due to their versatility in handling sudden changes.

This paper contributes to the existing literature in three key ways. Firstly, it suggests strategies to enhance supply chain resilience under uncertainty, emphasizing the importance of hybrid manufacturing systems in making adjustable decisions according to the revealed portion of demand. Secondly, it introduces a scenario-based modeling approach applicable in practice for adaptive decision-making in response to demand uncertainty and proposes a novel column generation-based heuristic solution approach that can tackle realistically sized real-world instances. Lastly, the numerical results demonstrate the effectiveness of the proposed model and the heuristic approach, with
solutions showing an average optimality gap of less than 6% and a reduction in the average cost of standard production schemes without FMS by over 12%. This contribution provides a practical tool for companies like Vestel Electronics to optimize manufacturing operations and reduce costs.

The remainder of the paper is organized as follows. Section 2 provides a literature review of flexible manufacturing systems. The mathematical model for the proposed problem is presented in Section 3, followed by a description of the solution approaches in Section 4. In Section 5, we present numerical experiments. Finally, the conclusion and the managerial implications of the study are presented in Section 6.

2. Literature Review

The initial works on FMSs date back to the 1970s (Hutchinson 1977). Later, Carrier et al. (1984) provide the formal introduction of FMS. Main research topics investigated by the researchers since then are (i) general analytical models to exploit the potential of FMSs (Buzacott and Yao 1986), (ii) scheduling models and rules to be used in FMSs (Gupta, Gupta, and Bector 1989; Sawik 1999; Sabuncuoglu 1998; Sawik 2011), and (iii) layout models to better manage the product flow in FMSs (Moslemipour, Lee, and Rilling 2012; Kouvelis, Chiang, and Kiran 1992; Kimemia and Gershwin 1985).

Charles H. Fine (1990) analyze the tradeoff between cost and flexibility when determining the manufacturing capacity in an FMS. They formulate the problem as a two-stage stochastic program. They first, make the investment decision under demand uncertainty, and then the production decisions are made after the demand is realized, given the investment decisions made in the first stage. Schweitzer and Seidmann (1991) use a mean value analysis queueing network model to determine the minimum cost processing rates for a given the FMS throughput target, the work-in-process level, part routes, transporter delays and the variable capacity cost function for each machine. Wang, Wang, and Zhang (2011) provides a recent review of studies on flexible operations mainly focusing on models with independent and dependent capacity allocation decisions over multiple periods.

Yang and Peters (1998) analyze FMS layout problems under unequal machine sizes and fixed loading/unloading points. Different from the quadratic assignment problem-based enumeration approach commonly used in the literature, the authors develop a construction-type heuristic algorithm to determine the best layout. Hottenrott, Schiffer, and Grunow (2022) analyze the implementation of FMS in the automotive industry and determine the best layout by solving a chance-constrained formulation of the problem using a tailored branch-and-price algorithm. Pourvaziri et al. (2022) propose a robust layout approach that considers the changes in the product demand and mix by changing the product routes instead of rearranging the layout. They decompose the problem into two subproblems. In the first one, they determine a robust layout, and in the second one, they decide on the routes of the products. The first subproblem is solved by combining the design of experiments and a hybrid genetic-tabu search algorithm. The routes of the products in each period are determined by a branch-and-cut algorithm.

Lee, Shin, and Dong-HoLee (2020) analyze operations scheduling problem for an advanced FMS where multi-fixture pallets are used to load multiple parts simultaneously. Their objective is to minimize makespan and total tardiness. They decompose the problem into four subproblems: palletizing, pallet input sequencing, pallet routing, and pallet process sequencing, and solve each subproblem using the variants of
well-known scheduling heuristics. Zhang et al. (2020) consider a flexible job shop environment with assembling operations and determine the production schedule under multiple objectives. A distributed ant colony system is proposed to find Pareto optimal solutions for the objectives of minimizing makespan, total tardiness, and total workload. Canadan and Yazgan (2015) develop a genetic algorithm to minimize the makespan in a flexible job shop scheduling problem. They used a Taguchi orthogonal array method to determine the parameters of the proposed genetic algorithm. Moslehi and Mahnam (2011) analyze a multi-objective flexible job shop scheduling with different release times and under multiple objectives. They hybridize particle swarm optimization and local search to address the problem. Pitts Jr and Ventura (2009) study the scheduling of production jobs within a type of FMS called a flexible manufacturing cell. They propose a two-stage tabu search algorithm to minimize the manufacturing makespan for medium-to-large-scale problems. The algorithm combines two heuristics in the first stage and a tabu search metaheuristic in the second stage to provide improved manufacturing makespan solutions with significant savings in computational time. Several studies on FMS assume that demand is uncertain and develop models to address demand uncertainty. However, unlike our study, these studies assume a single-period problem and determine the configuration of each line/machine only once. Some recent studies on FMS with demand uncertainty are summarized in Table 1.

Yadav and Jayswal (2018) provide a recent review of modelling FMS and highlight the importance of a flexible system, in case of changing demand, that is capable of (i) producing different part types, (ii) performing satisfactorily in case of a schedule change, (iii) recovering from a breakdown and (iv) adding new part designs into the production line. In a common production environment, a single floor is used to satisfy the demands of multiple products from multiple companies. This floor contains two different types of machines: TMM and a limited number of FMSs. Depending on the type of machine, it is possible to produce a product using either a TMM or an FMS, and their production costs and capacities vary. Under stationary demand patterns, this two-mode setup is as efficient as traditional mass manufacturing, although the cost per unit of production may be lower. However, under highly volatile demand patterns, the cost per unit of production may be higher, but the output can be flexible. The key takeaways of this study can be outlined in three aspects. Firstly, it is novel in the field of flexible manufacturing systems as it presents a novel optimization-under-uncertainty framework. The proposed stochastic production planning problem is the first to incorporate both flexible and traditional manufacturing systems and offers robust solutions to address demand uncertainty. Secondly, the authors propose a column generation-based heuristic method that can solve large-scale problems in a case study of Vestel’s drum production, whereas a commercial solver could not. The linear programming relaxation of the mixed-integer programming models provides an efficient lower-bound mechanism for evaluating the proposed algorithm. On average, the proposed column generation-based approach has an optimality gap of 5.2% when both FMS and TMM are implemented for realistically sized instances. Finally, the study successfully develops cost-effective schedules that minimize (on average) 12.6% of production costs compared to standard methods without flexible manufacturing systems, despite the challenges posed by uncertain demand.
The recent papers that consider demand uncertainty in FMS are presented in this table.

Table 1. Recent papers considering demand uncertainty in FMS

<table>
<thead>
<tr>
<th>Article</th>
<th>Assumptions</th>
<th>Objective</th>
<th>Solution Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perraudat, Dauzere-Peres, and Vialletelle (2022)</td>
<td>Machines are capacitated, demand uncertainty characterized by product cannibalization, lead time to qualify a product for a machine</td>
<td>Minimize the discounted cost of (product, machine) qualifications for a robust model against the uncertainty on the product mix</td>
<td>A robust reformulation where demand uncertainty is described by product cannibalization and a binary search approach to characterize the robustness of a set of qualifications</td>
</tr>
<tr>
<td>Wang and Tang (2018)</td>
<td>Workers with different skill levels, arriving time of order, limit on the number of assembly lines and workers in each line</td>
<td>Multiobjective: Minimize labour cost and maximize service level (percentage of orders served within an appropriate delivery time)</td>
<td>Non-dominated Sorting Genetic Algorithm II (NSGA-II) combined with sample average approximation</td>
</tr>
<tr>
<td>Fujita et al. (2022)</td>
<td>Several assembly lines to produce multiple products, workers switching between assembly lines before demand realization, constant yield per worker</td>
<td>Maximize total profit (revenue minus training cost, salary, production and shortage cost)</td>
<td>Formulated as a bi-level mixed-integer linear programming problem. First, a worker allocation problem is solved before demand realization. Then, the production quantity allocation is determined based on the realized demand.</td>
</tr>
</tbody>
</table>
3. Problem Definition

This section provides the mathematical formulation of the *stochastic production planning* (SPP) problem. The formulation explains the goal, assumptions, and mathematical optimization model of the problem. In the context of SPP, Vestel aims to satisfy the demand for multiple washing machine drums \((J = 1, \ldots, \Gamma)\) over a specific planning horizon \(T\). A scenario tree approach is adopted for modelling to capture the uncertainty in demand. A scenario tree is a graphical representation of different possible scenarios that may unfold over the planning horizon. Each scenario corresponds to a specific sequence of demand realizations, reflecting the uncertainty in the demand.

The scenario tree comprises nodes, which are essential decision points where different actions can be taken based on the evolving demand. In our context, the term “node” refers to a decision point representing a particular system state within the planning horizon. Each node in the scenario tree corresponds to a specific time period and reflects the available information up to that point. At each node, a decision regarding production planning can be made based on the demand realization up to that period.

The decision variables are categorized into two types: here-and-now decisions and wait-and-see decisions. Here-and-now decisions are made at the beginning of the planning horizon using TMMs. Wait-and-see decisions, on the other hand, offer more flexibility and are fulfilled using FMSs, based on the actual demand observed during the planning horizon. Both TMMs and FMSs are assumed to possess the capability to produce all types of products, with setup times between productions being dependent on the production sequence. The parameters \(a_{tj}\) and \(a'_{tj}\) represent the production capacities of TMMs and FMSs, respectively, for product \(j\). Additionally, \(d_{ej}\) represents the demand for product \(j\) at node \(e\) within the scenario tree. Variable costs that are incurred in addition to fixed costs are also considered in this model. In the case of TMMs, the fixed production cost for product \(j\) during period \(t\) is represented by \(A_{tj}\) and the variable production cost is represented by \(v_{tj}\). Both variable and fixed costs are incurred in FMSs and are indicated by \(A'_{tj}\) and \(v'_{tj}\), respectively. Inventory holding costs are also present, represented by \(h_{tj}\), which represents the cost of keeping a unit of product \(j\) in inventory per period. The production capacity of each period is represented by \(C_{tj}\), while the inventory capacity is limited by \(S_j\), which is the maximum amount of space that can be used for storage after each period. Finally, the outsourcing cost of product \(j\) in each period is indicated by \(\beta_j\).

The sets, parameters, and decision variables used in the mathematical optimization model and throughout the study are presented below.

**Sets:**

- \(J\): set of products \((\{1, 2, \ldots, \Gamma\})\) – indexed by \(j\)
- \(K\): set of TMMs \((\{1, 2, \ldots, M\})\) – indexed by \(k\)
- \(K'\): set of FMSs \((\{1, 2, \ldots, M'\})\) – indexed by \(k'\)
- \(\hat{K}\): set of machines \((\hat{K} = K \cup K')\) – indexed by \(\hat{k}\)
- \(T\): set of time periods in the planning horizon \((\{1, 2, \ldots, L\})\) – indexed by \(t\)
- \(E\): set of demand scenario or node indexes \((\{1, 2, \ldots, S\})\) – indexed by \(e\)
- \(N_t\): set of nodes in period \(t \in T\)
- \(N(e, l)\): set of children nodes of \(e\) in the next \(l\) periods \(\cup \{e\}\)

**Parameters:**

- \(a_{tj}\): units of product \(j\) that each TMM produces
\(d_{e}ji\): units of product \(j\) that each FMS produces
\(s_{e}ji\): setup time between product \(j\) and \(j'\) on TMM
\(s_{e}ji'\): setup time between product \(j\) and \(j'\) on FMS
\(d_{e}j\): demand for product \(j\) in node \(e\)
\(v_{ej}\): variable cost of production of product \(j\) in period \(t\) by a TMM
\(v_{e}j'\): variable cost of production of product \(j\) in period \(t\) by an FMS
\(A_{ej}\): fixed cost of production of product \(j\) in period \(t\) by a TMM
\(A_{e}j'\): fixed cost of production of product \(j\) in period \(t\) by an FMS
\(h_{e}j\): unit holding cost of product \(j\)
\(\beta_{ej}\): unit outsourcing cost of product \(j\)
\(C\): production capacity of TMM during the planning horizon
\(C'\): production capacity of FMS during the planning horizon
\(S\): inventory capacity per period
\(R\): a relatively large constant
\(pd(e,1)\): immediate predecessor of node \(e\)
\(pd(e)\): nodes in the path from the source node to \(e\)
\(pr(e)\): probability of scenarios in path \(pd(e)\) being realized
\(t(e)\): period of node \(e\)

Decision Variables:

\[y_{ektj} = \begin{cases} 
1, & \text{if there is a production of TMM for product } j \text{ on machine } k \text{ in period } t \\
0, & \text{otherwise} 
\end{cases}\]

\[x_{k'ej} = \begin{cases} 
1, & \text{1 if there is a production of FMS for product } j \text{ on machine } k' \text{ in node } e \\
0, & \text{otherwise} 
\end{cases}\]

\(I_{ej}\): inventory of product \(j\) at the end of node \(e\)

\(B_{ej}\): amount of product \(j\) outsourced at the end of node \(e\)

A sample scenario tree is presented in Figure 2 using SPP notations. The figure displays the demand tree structure, where node \(e_6\) follows node \(e_2 = pd(e_6,1)\). The set of child nodes of \(e_1\) in the next period is represented by \(\bar{N}(e_1,1)\) and \(\bar{N}(e_1,2)\). In the scenario tree, nodes \(e \in N\) represent demand outcomes at \(t(e)\) with a probability of \(pr(e)\) and every path from the root node to a leaf node \(e \in N\) is referred to as a scenario. For example, \(pd(e_{14}) = \{0, e_2, e_6, e_{14}\}\), with \(\bar{N}_T\) denoting the set of nodes in the last period of the planning horizon.

The SPP problem can be formulated as a mixed-integer programming (MIP) model that integrates both the considerations of TMMs and FMSs. Within this MIP model, production decisions can be classified into two categories: 1) Here-and-now decisions \((y)\): These decisions pertain to TMMs and are made at the start of the planning horizon. Once determined, they remain fixed and cannot be adjusted based on the scenarios that unfold. Notably, TMM production plans are established at the beginning of the planning horizon due to the necessity of special initial setups required for different production types. These setups must be addressed before the facilities commence operations. 2) Wait-and-see decisions \((x)\): These decisions relate to FMSs and offer greater adaptability. Unlike TMMs, FMSs do not require dedicated initial setups and are more flexible in accommodating various modes of manufacturing. Consequently, they can be adjusted in response to different scenarios that are realized over time. It must be noted that in the mathematical optimization models presented in this
study, an ellipsis indicates that the expression continues onto the next line.

The MIP model incorporating the aforementioned here-and-now and wait-and-see decisions is presented as follows:

**MIP:**

\[
\begin{align*}
\min & \quad \sum_{e \in N_e} \text{pr}(e) \sum_{e' \in \text{pd}(e)} \left( \sum_{j} \sum_{k'} v'_{t(e')j} a'_{j} x_{k'e'j} + \cdots \right. \\
& \quad \left. \cdots \sum_{j} \sum_{k'} A'_{t(e')j} x_{k'e'j} + \sum_{j} \left( h_{j} I_{e'j} + \beta_{j} B_{e'j} \right) \right) + \cdots \\
& \quad \left. \cdots \sum_{t} \sum_{j} \sum_{k} \left( v_{tj} a_{j} y_{ktj} + A_{tj} y_{ktj} \right) \right) \\
\text{s.t.} & \quad (R-1)x_{k'ej} + \sum_{j'} e' \in N_{e,s_{j'j'}-1} x_{k'e'j'} \leq R \quad \forall e, j, k' \quad (2) \\
& \quad (R-1)y_{ktj} + \sum_{j'} \sum_{t'=t}^{t+s_{j'j'}-1} y_{ktj'} \leq R \quad \forall j, k, t \quad (3)
\end{align*}
\]

Figure 2. Structure of the scenarios of the problem
\[
\sum_{j} \sum_{k} \sum_{t} a_{j} y_{ktj} \leq C \\
\sum_{j} \sum_{e} \sum_{k'} a'_{j} x_{k'ej} \leq C' \\
\sum_{e \in N_i} I_{ej} \leq S \quad \forall t \\
\sum_{e \in N_i} B_{ej} \leq \mu_j \quad \forall j, t \\
B_{pd(e,1)j} - B_{ej} + I_{ej} - I_{pd(e,1)j} + d_{ej} = \cdots \\
\cdots \sum_{k} a_{j} y_{kt(e)j} + \sum_{k'} a'_{j} x_{k'ej} \quad \forall e, j \\
x_{k'ej} \in \{0, 1\}; \quad y_{ktj} \in \{0, 1\} \quad \forall e, j, k, k', t \\
I_{ej}, B_{ej} \geq 0 \quad \forall e, j
\]

In MIP, the objective function (1) minimizes the total expected cost, which encompasses production, holding, and outsourcing costs. Constraints (2) and (3) regulate the setup times between the production of FMSs and TMMs, respectively. The total production of TMMs and FMSs during the planning horizon is limited and constraints (4) and (5) enforce this restriction. Constraints (6) ensure inventory constraints for each period. The amount of product \( j \) that can be outsourced in period \( t \) is limited by \( \mu_j \), which is upheld by constraints (7). Constraints (8) are inventory balance constraints, while constraints (9) and (10) specify the domains of the variables.

4. Column Generation Approach

Solving the MIP model for large instances is computationally intensive. To address this challenge, this section proposes a heuristic algorithm based on column generation, which implements Dantzig-Wolfe decomposition to decompose the mathematical model (MIP) into the master problem and two subproblems. The algorithm starts by solving the linear programming (LP) relaxation of the reduced master problem (RMP) with a feasible set of initial solutions. This set includes decisions for both TMMs and FMSs. Dual variables for RMP are obtained, and subproblems are solved to identify profitable columns added to RMP. This process continues until all decision variables in RMP are integers or a time limit is reached. Unlike the branch and price approach, which branches on decision variables, the proposed method solves the master problem with integer decision variables using all generated columns. The proposed column generation-based heuristic follows a specific sequence of steps in each iteration. Initially, a column (\( \lambda \)) associated with a wait-and-see production schedule of an FMS (\( x \)) is introduced to RMP. Then, another column (\( \theta \)) associated with a here-and-now production schedule of a TMM (\( y \)) is added to the RMP-WS. This process continues until the objective function values for subproblems PP-x and PP-y become non-negative, indicating the generation of useful columns for the RMP or reaching the time limit. Finally, the master problem with binary and integer decision variables is solved. The steps of the column generation-based heuristic algorithm are presented in Figure 3.

This algorithm differs from conventional branch-and-price algorithms as it solves the
The steps of the column generation-based heuristic are presented in this figure.

**Figure 3.** Steps of column generation-based heuristic algorithm

RMP using integer decision variables instead of branching on them, reducing computational time. Hence, it is referred to as a heuristic algorithm. For further information on the column generation algorithm, refer to Lübecke and Desrosiers (2005), Desaulniers, Desrosiers, and Solomon (2005), and Lübecke (2010). The total production cost of TMM schedule $m$ is denoted by $\tilde{c}_m$, while $\hat{c}_m'$ represents the total production cost of FMS schedule $m'$. The mathematical optimization of RMP is given as the following.
RMP:

\[
\min \sum_{e \in \mathbb{N}_L} \text{pr}(e) \left( \sum_{e' \in \text{pd}(e)} \sum_j h_{j} I_{e'j} + \beta_j B_{e'j} \right) + \cdots
\]

\[
\cdots \sum_{m'} \tilde{c}_{m'} x_{m'} + \sum_{m} \tilde{c}_m y_m
\tag{11}
\]

s.t. \[
\sum_{e \in \mathbb{N}_t} \sum_j I_{ej} \leq S \quad \forall t \tag{12}
\]

\[
\sum_{e \in \mathbb{N}_t} B_{ej} \leq \mu_j \quad \forall j, t \tag{13}
\]

\[
B_{\text{pd}(e,1)j} - B_{ej} + I_{ej} - I_{\text{pd}(e,1)j} + d_{ej} = \cdots
\]

\[
\cdots \sum_{m'} \lambda_{ej} x_{m'} + \sum_{m} \theta_{t(e)j}^m y_m
\quad \forall e, j \tag{14}
\]

\[
\sum_{j} \sum_{m} \sum_{t} \theta_{t(e)j}^m y_m \leq C \tag{15}
\]

\[
\sum_{j} \sum_{m'} \sum_{e} \lambda_{ej}^{m'} x_{m'} \leq C' \tag{16}
\]

\[
\sum_{m} y_m \leq M \tag{17}
\]

\[
\sum_{m'} x_{m'} \leq M' \tag{18}
\]

\[
y_m \geq 0, \quad x_{m'} \geq 0 \quad \forall m, m' \tag{19}
\]

\[
I_{ej}, B_{ej} \geq 0 \quad \forall e, j \tag{20}
\]

The RMP model minimizes the total expected cost for the TMM and FMS production schedules, holding inventory, and outsourcing costs through its objective function (11). The capacities for inventory and outsourcing are ensured by constraints (12) and (13), while inventory balance is maintained through constraints (14). Constraints (15) and (16) enforce the production capacities for TMMs and FMS, respectively. The total number of production schedules for TMMs and FMSs must be within the available resources and it is ensured by constraints (17) and (18). The dual variables obtained from (12) to (18) are represented by \( \delta_t, \gamma_{tj}, \eta_{ej}, \iota, \) and \( \zeta. \)

The subproblems for FMS and TMM production schedules are PP-x and PP-y, respectively. The PP-x subproblem is as follows:

PP-x:

\[
\min \sum_e \text{pr}(e) \sum_j \left( z_{ej} \left( a'_j t_{t(e)j} + A'_t(e)j \right) \right) - \sum_e \sum_j a'_j z_{ej} \eta_{ej} - \sum_t \delta_t - t \tag{21}
\]

s.t. \[
(R - 1) z_{ej} + \sum_{j'} \sum_{e' \in \mathbb{N}(e,e',j')} z_{e'j'} \leq R \quad \forall e, j \tag{22}
\]

\[
z_{ej} \in \{0, 1\} \quad \forall e, j. \tag{23}
\]
In PP-x, the objective function (21) minimizes the cost of an FMS production schedule. Constraints (22) ensure compliance with setup times between consecutive FMS productions. After solving PP-x, values are calculated for $\lambda_{e_j}^{m'}$ by $\lambda_{e_j}^{m'} = a_j' z_{j e}$ and for $\tilde{c}^{m'}$ by $\tilde{c}^{m'} = \sum_{e} pr(e) \sum_{j} \tilde{z}_{e j} (a_j' v_{t(e) j} t) + A'_t(e)j$.

Next, the formulation for the PP-y subproblem is as follows:

**PP-y:**

$$\min \sum_{t} \sum_{j} q_{jt} \left( a_j v_{t j} + A_t - \sum_{e \in N_t} a_j \eta_{e j} \right) - \sum_{t} \delta_t - \ell$$  

s.t.  

$$(R - 1) q_{jt} + \sum_{j'} \sum_{t'=t}^{t+s_j'} q_{j't'} \leq R$$  

$\forall j, t$  

$q_{jt} \in \{0, 1\}$  

$\forall j, t$.  

(24)

(25)

In the subproblem PP-y, the goal is to find the most cost-efficient production schedule for a TMM. The constraints (25) ensure the set-up times between consecutive TMM productions. Upon solving PP-y and finding the optimal values of $q_{jt}$, the values of $\theta_{t,j}^{0}$ and $\tilde{c}_m$ are calculated as $\theta_{t,j}^{0} = a_j q_{jt}$ and $\tilde{c}_m = \sum_{j} q_{jt} (a_j v_{t j} + A_t)$.

5. **Numerical Results**

In this section, we evaluate the proposed column generation-based heuristic algorithm in a real-life setting. Vestel designates 20 TMMs and 6 FMSs for the production of different types of drums for washing machines in its production line. A schedule is established for a period of two weeks, and data is gathered over a period of 12 months (24 fortnights) with a separate instance created for 14 days ($L = 14$). The collected data includes 50 different drum types to be produced on one of the TMMs or FMSs, with the demand for the drums in each period potentially reaching a value of up to 2000 units.

We take into consideration three different demand nodes, namely low, nominal, and high, at each period, resulting in a total of 314 leaf nodes in the last period. The nominal demand of each product for the first period is set equal to the data gathered from Vestel. A constant value (denoted by $\rho$) calculates the low and high demand values in each period. Specifically, the low demand is calculated as $(1 - \rho) \times$ nominal demand and the high demand is calculated as $(1 + \rho) \times$ nominal demand. In this study, we consider $\rho = 0.2$. In the context of this study, it is important to note that the amount of time required to transition between the drums (setup times) in Vestel varies for TMMs and FMSs due to the machine specifications. Specifically, the setup time for TMMs ranges between 5 and 10 minutes, while for FMSs, it ranges between 1 and 5 minutes, depending on the current and next product. Additionally, the inventory capacity of Vestel for each period ($S$) is 50,000 units. While a TMM can produce up to 70,000 units during the planning horizon, each FMS can produce up to 100,000 units. It is also worth mentioning that Vestel does not currently impose an upper limit on the number of drums that can be outsourced. However, the mathematical model provided in this study is capable of accommodating the upper limit for it. Furthermore, the number of products that each TMM can produce on a daily basis ranges between

13
100 and 150, while an FMS can produce between 120 and 300 units per day. In terms of production capacity per period, an FMS is capable of producing between 150 and 300 units of each type of drum, while a TMM can produce between 50 and 100 units of each type of drum. The unit variable production cost of a TMM, depending on the machine specifications, is between 10 and 15 units for each product type, and the production cost of an FMS is between 2 and 8. However, while the fixed cost of a TMM is between 50 and 60, the fixed cost of an FMS is between 100 and 150. Moreover, the holding cost of each drum type is a value between 0.8 and 1.4, and the outsourcing cost of each product type is between 4 and 12 depending on the size of the products. Currently, Vestel uses a scheduling module integrated with their enterprise resource planning (ERP) system. This module receives orders from the ERP software, presents a visual representation of the orders, and allows the user to manually assign items to machines, taking setup times and other factors into account. The user’s expertise plays a significant role in determining the final schedules, making them dependent on the user. In considering the manufacturing process, we examine two distinct cases: Case I, where only TMMs are utilized, with all 26 machines being of the TMM type, and Case II, where a combination of 20 TMMs and 6 FMSs is employed as in the actual system.

In Table 2, the optimality gaps of the solutions for Case II and the improvement obtained by comparing it with Case I for real-life instances are presented. Although there are solutions implemented in practice, instead of using them as a benchmark, we chose to use a lower bound for evaluating the performance of the proposed approaches. The optimality gap is determined by using the optimal objective function value of the linear programming (LP) relaxation of MIP using equation (27). Therefore, the numbers provided are safe approximations of the actual optimality gap.

\[
\text{Optimality Gap} = \frac{\text{OFV} - \text{LB}}{\text{LB}} \times 100\%
\]

where OFV stands for the objective function value obtained by the CG-based heuristic algorithm and LB is that of the LP relaxation. Table 2 presents the optimality gap and OFV value improvement yielded by the flexible system that utilizes 20 TMMs and 6 FMSs (Case II) for a system solely based on 26 TMMs (Case I). Rows one, four, and seven present the fortnight IDs of the instances gathered from Vestel, where “Ins.” denotes the “instance”, “Gap” denotes the “optimality gap”, and “Imp.” denotes the “improvement” obtained in Case II comparing with Case I. To calculate the objective function of the model for Case I, we set the number of FMSs equal to zero and solve the same model. By doing so, there will be no need for solving PP-x as a subproblem anymore; therefore, only PP-y is utilized in the column generation-based algorithm. The proposed column generation-based algorithm is implemented using Python, utilizing Gurobi 9.5 as the optimization solver. All experimentation is conducted on a system equipped with an Intel Xeon CPU operating at 2.4 GHz and 32 GB of RAM.

The results presented in Table 2 demonstrate the effectiveness of incorporating FMSs in conjunction TMMs to improve overall performance. Using FMSs in Vestel’s production processes has shown significant potential for enhancing production efficiency. The average improvement of 12.63% across all instances indicates a consistently positive impact on the performance of the proposed algorithm. This improvement highlights the importance of adopting flexible manufacturing approaches and hence integrating FMSs with TMMs to address real-world manufacturing complexities faced by Vestel effectively.
Optimality gaps and the improvements obtained in Case II with respect to Case I are presented in this table.

**Table 2.** Optimality gaps and improvements obtained in Case II

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap</td>
<td>3.00%</td>
<td>3.70%</td>
<td>4.30%</td>
<td>5.39%</td>
<td>3.21%</td>
<td>4.41%</td>
<td>4.34%</td>
<td>6.57%</td>
</tr>
<tr>
<td>Imp.</td>
<td>12.89%</td>
<td>15.01%</td>
<td>16.23%</td>
<td>14.02%</td>
<td>15.93%</td>
<td>18.73%</td>
<td>16.84%</td>
<td>14.03%</td>
</tr>
<tr>
<td>Ins.</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>Gap</td>
<td>4.92%</td>
<td>3.60%</td>
<td>3.38%</td>
<td>2.04%</td>
<td>1.88%</td>
<td>8.87%</td>
<td>7.65%</td>
<td>5.55%</td>
</tr>
<tr>
<td>Imp.</td>
<td>18.09%</td>
<td>15.95%</td>
<td>13.23%</td>
<td>8.27%</td>
<td>5.35%</td>
<td>7.68%</td>
<td>8.65%</td>
<td>11.30%</td>
</tr>
<tr>
<td>Ins.</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>Gap</td>
<td>4.92%</td>
<td>6.63%</td>
<td>9.86%</td>
<td>11.06%</td>
<td>4.19%</td>
<td>3.36%</td>
<td>7.96%</td>
<td>5.19%</td>
</tr>
<tr>
<td>Imp.</td>
<td>10.73%</td>
<td>10.44%</td>
<td>9.61%</td>
<td>12.32%</td>
<td>11.12%</td>
<td>13.23%</td>
<td>12.40%</td>
<td>11.10%</td>
</tr>
</tbody>
</table>

The summary of the results in Table 2 is presented in Table 3. The average optimality gap is recorded as 5.25%; the worst-case optimality gap observed in this study is 11.06%. This means that significant optimality performance is achieved even in the instance with the most challenging characteristics. Moreover, on the other end of the spectrum, the best-case optimality gap is as impressive as 1.88%. Next, the improvement made by Case II over I across different instances, i.e., ranging from 5.35% to 18.73%, is on average, 12.64%. Therefore, incorporating FMSs with TMMs demonstrates the ability of the algorithm to deliver notable enhancements in production cost consistently.

**Table 3.** Summary of optimality gaps and improvements obtained by Case II

<table>
<thead>
<tr>
<th>Measure</th>
<th>Average</th>
<th>Worst</th>
<th>Best</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap</td>
<td>5.25%</td>
<td>11.06%</td>
<td>1.88%</td>
</tr>
<tr>
<td>Improvement</td>
<td>12.63%</td>
<td>5.35%</td>
<td>18.73%</td>
</tr>
</tbody>
</table>

In order to scrutinize the characteristics of instances with the highest improvement (HI), average improvement (AI), and lowest improvement (LI), the cost components of Cases I and II are presented in Table 4.

**Table 4.** Cost components of important instances in Case I and Case II

<table>
<thead>
<tr>
<th>Ins.</th>
<th>Case I</th>
<th>Case II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TMM</td>
<td>Inv.</td>
</tr>
<tr>
<td>HI (6)</td>
<td>62.55%</td>
<td>6.23%</td>
</tr>
<tr>
<td>AI (23)</td>
<td>76.38%</td>
<td>5.11%</td>
</tr>
<tr>
<td>LI (13)</td>
<td>85.54%</td>
<td>7.23%</td>
</tr>
</tbody>
</table>

In this table, the first column represents the instances, and columns 2 through 4 represent the production costs of TMMs, inventory and outsourcing costs in Case I, respectively. Columns 5 through 8 represent the same cost components for Case II. In
Case I, instances exhibit a consistent trend where the production cost for TMMs is significant, accounting for the largest proportion of total costs. The absence of FMSs in this scenario leads to relatively high outsourcing costs, indicating the reliance on external services for specific manufacturing processes. Inventory costs are also notable, contributing to the overall expenses. The cost distribution for instances LI, AI, and AI demonstrates variations based on their individual improvement levels. However, the absence of FMSs remains a common factor affecting cost dynamics in Case I.

In contrast, Case II showcases a significant transformation in cost allocation by incorporating FMSs. LI, AI, and AI all experience a substantial decrease in the production cost for TMMs, as FMSs assume a more prominent role in the manufacturing process. The production cost of flexible systems becomes a substantial component, indicating the efficient utilization of this advanced manufacturing system. Notably, the outsourcing reduces remarkably in Case II, owing to the cost-effectiveness and resource optimization achieved through FMS implementation. This indicates a shift towards in-house production, reducing the reliance on costly external services. The decrease in outsourcing costs observed in Case II highlights the cost-saving potential of adopting FMS in manufacturing operations. The flexibility and automation offered by FMSs enable companies to handle a broader range of tasks internally, leading to enhanced cost management and efficiency gains. Consequently, companies can redirect resources more effectively, reduce lead times, and gain a competitive edge in the market.

6. Managerial Insights and Conclusion

This paper presents a problem that aims to minimize the total (expected) operational manufacturing, holding, and outsourcing costs by utilizing flexible manufacturing systems (FMS). The numerical experiments have shown that utilizing an optimization method for the manufacturing scheduling problem and considering uncertain demand and flexible manufacturing systems can efficiently manage production planning for Vestel Electronics, a well-known household appliances company in Türkiye.

In the remainder of this section, we discuss how Vestel and other companies that use FMSs can benefit from the presented research in four different aspects.

The utilization of FMSs is extremely valuable and can revolutionize how manufacturing is managed. However, despite the advantages of using FMSs, it also increases the number of potential manufacturing patterns, which can make the already challenging task of managing demand uncertainty even more difficult. To address this, optimization methods for viable FMSs can provide significant advantages for companies like Vestel in managing this challenge more effectively. Needless to say, we do not interfere in the machinery purchase decision, but in order for the company to decide on what rate they can substitute the machines, considering the cost-saving achieved according to the analysis, we suggest the conducted necessary economic analysis. Furthermore, solving an optimization problem becomes increasingly important in managing goods as efficiency and profitability are at stake. In other words, implementing optimization techniques in manufacturing management offers a unique opportunity to balance supply with demand in an uncertain environment and, more importantly, maximize the efficiency of this high-cost operation. The proposed optimization method aims to tackle these issues that all manufacturers may face within practical time frames while producing near-optimal solutions, yielding optimality gaps of less than 10% in the presented case study.
In a scenario where demand is volatile and fluctuates frequently, the ability to customize supply is crucial, highlighting the importance of using FMSs. This is supported by the numerical results presented in this study, which showed that using FMSs significantly reduced overall costs, with an average improvement of between 12% and 18%. This indicates that it would be beneficial for Vestel to put in significant effort to implement flexible manufacturing systems to improve service quality. Although the experiments in this study were limited in the number of flexible machines used, even more significant improvements would be possible if more FMSs were implemented. This should be a key goal for manufacturers.

Production plans made by FMSs can adapt to different demand realizations, unlike TMMs, which are predetermined and independent of the realized demand. However, in practice, the actual demand will rarely match one of the scenarios in the tree. To ensure the validity of FMS decisions in such cases, we propose two approaches for production scheduling. These methods guarantee the feasibility of a production schedule obtained from the FMS. The first approach is to choose the production schedule of the nearest larger demand node in the decision tree for the realized demand of the associated period. The second approach, referred to as folding horizon, reoptimizes the production schedules of FMS after the demand of each period is realized. The advantage of the first approach is its speed, while the second approach may yield better solutions but takes longer to compute. It is noteworthy that the second may not be practical when adaptive decisions must be made immediately after the demand has occurred due to increased CPU time requirements.

While this study has provided valuable insights into the benefits of utilizing FMSs in managing demand uncertainty for Vestel Electronics, there are certain limitations that should be acknowledged. Firstly, the research focused solely on the specific context of Vestel Electronics, and the generalizability of the findings to other industries or companies with different operational characteristics may require further investigation. Secondly, the study assumed that all parameters, such as machine substitution rates and operational costs, remained constant throughout the planning horizon. In reality, these parameters might change over time, and incorporating such dynamic aspects could enhance the model’s accuracy. Additionally, the optimization model considered only the manufacturing and outsourcing costs, but other factors like environmental impacts or social aspects could be integrated for a more comprehensive analysis. In future research, we aim to develop robust mathematical optimization models that incorporate supply and demand uncertainty elements within a unified framework.

While the practical implications for Vestel Electronics and similar companies are evident, the academic community stands to benefit from the novel methodologies and optimization techniques introduced. This work not only bridges the gap between flexible manufacturing systems and demand uncertainty but also offers a fresh perspective on how traditional and modern manufacturing paradigms can coexist and complement each other. The mathematical models and solution approaches delineated in this study can serve as a foundation for future research in this domain, encouraging scholars to delve deeper into the intricacies of manufacturing under uncertainty. Furthermore, the case study of Vestel Electronics provides a real-world context, making this research a valuable reference for both academia and industry.
Data Availability Statement

The participants of this study did not give written consent for their data to be shared publicly, so due to the sensitive nature of the research, supporting data is not available.

References


