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## Safe Peeling for L0-Penalized Problems

Théo Guyard<sup>1,2</sup> Gilles Monnoyer<sup>3</sup> Cédric Herzet<sup>1</sup> Clément Elvira<sup>4</sup> Ayşe-Nur Arslan<sup>5</sup>

<sup>1</sup>Inria, Centre de l'Université de Rennes, France

<sup>2</sup>Insa Rennes, IRMAR UMR CNRS 6625, France

<sup>3</sup>UCLouvain, ELEN ICTEAM, Belgique

<sup>4</sup>CentraleSupélec Rennes, IETR UMR CNRS 6164, France

<sup>5</sup>Inria, Centre de l'Université de Bordeaux, France

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In many application fields such as machine learning, signal processing or inverse problems, it is of paramount interest to solve optimization problems of the form

$$\min_{\mathbf{x} \in \mathbb{R}^n} f(\mathbf{x}) + \lambda \|\mathbf{x}\|_0 \quad (1)$$

where  $f(\cdot)$  is some loss function,  $\|\cdot\|_0$  is the so-called  $\ell_0$ -norm that promotes sparsity in the optimizers by counting the number of non-zeros in its argument and  $\lambda > 0$  is an hyperparameter [1]. This problem can be solved effectively via Branch-and-Bound (BnB) algorithms by adding a bound constraint  $\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$  to reformulate the  $\ell_0$ -norm using a binary variable through Big-M relations [2]. In practice, the choice of the values in the bounds  $\mathbf{l}$  and  $\mathbf{u}$  must respect two conflicting imperatives. On the one hand, the additional constraint must not modify the solutions of the initial problem. Hence, the interval  $[\mathbf{l}, \mathbf{u}] \subseteq \mathbb{R}^n$  should be chosen wide enough to contain all the optimizers of (1). On the other hand, the quality of the relaxations constructed across the BnB tree degrades with the spread of  $[\mathbf{l}, \mathbf{u}]$ . Hence, setting a too-wide interval is likely to increase the number of nodes explored during the BnB algorithm and penalize the solution time.

We propose a new strategy, dubbed *peeling*, to address this issue. By leveraging a duality property linking consecutive nodes, we locally tighten the interval  $[\mathbf{l}, \mathbf{u}]$  which results in a strengthening of the relaxations constructed. This procedure is carried out in a *safe* manner, *i.e.*, we guarantee the validity of the BnB procedure and the optimality of the solutions obtained. Peeling can be seen as a generalization of the *screening* methodology recently introduced for  $\ell_0$ -penalized problems [3]. Through different numerical experiments, we show that peeling allows significant gains in the number of nodes explored and in the solution time.

## References

- [1] Bertsimas, D. et al. (2016). Best subset selection via a modern optimization lens. *The Annals of Statistics*, 44(2), 813-852.
- [2] Ben Mhenni, R. et al. (2022). Global optimization for sparse solution of least squares problems. *Optimization Methods and Software*, 37(5), 1740-1769.
- [3] Atamturk, A. et al. (2020). Safe screening rules for L0-regression from perspective relaxations. *International Conference on Machine Learning*, PMLR, 421-430.