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# A Life-Cycle Model with health spending and endogenous life time

Laurent Denis\*      Xavier Fairise<sup>†</sup>      Ze Zhong Shang<sup>‡</sup> §

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## Abstract

In this article, we study a life-cycle model of health stock and health spending. We use a simplified version of the model Grossman (1972). Contrary to Laporte (2015) and Strulik (2015), we suppose that the depreciation rate of health stock increases with age. Lifespan is thus an endogenous variable. We also suppose that the multiplicative productivity term of the income function (depending on the health of the agent) displays a hump shaped profile over time. Our contribution is to investigate the Grossman (1972)'s model in a non stationary environment with an endogenous finite lifetime. After derivation of the optimality conditions the model is solved numerically and we simulate the path followed by the main variables over the agent's life-cycle. Calibrated on the US economy with realistic values of parameters, the model is able to replicate a growing profile of health spending and a significant share of health spending at the end of life roughly in accordance with the data.

**Keywords:** Health stock, health spending, lifespan, life-cycle

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# 1 Introduction

Health spending holds a growing place in economic debate. At the macroeconomic level, health spending represents a growing share of the GDP. At the microeconomic level, health spending is a key variable in agent choices over the life-cycle. Moreover, it tends to represent a growing share in agents' spending over their life cycle. The issue of health spending and its growing trend may stem from several factors: ageing, scientific innovations or new medical treatments, etc.. Ageing leads to an increase in demand and scientific innovations lead to an increase in supply (new treatments, quality improvement,...).

In this article, we are interested in studying health spending decisions over the life-cycle. What are the main features of health spending over the life-cycle? French, McCauley, Aragon, et al. (2017) provide a lot of data concerning end-of-life medical spending in a large set of countries. They focus particularly on the last three years (and the last year) of life. In most countries, medical spending made at the last year of life represents 8 to 11% of the aggregate medical spending (8.5% in the US). Concerning the last three years of life, the medical spending made during this period represents between 16.7 and 24.5% of the aggregate medical spending made over the life-cycle (16.7% in the US). Even if we consider the heterogeneity between countries, it is clear that medical spending in the last three years of life represents a significant share of the aggregate medical spending. Using US data, Halliday, He, Ning, and Zhang (2017) and Fonseca, Michaud, Galama, and Kapteyn (2008) report an increasing profile of medical spending over the life-cycle, medical spending being approximatively zero around 20-25 years old. Jung and Tran (2013) reach similar conclusions. They also estimate the evolution of the share of health spending. At 20-25 years old, health spending is zero while at 85, it roughly represents more than 50% of income. The profile is increasing and convex. Finally, we underline that at the aggregate level, medical spending represent around 17% of the GDP in the US.

From a theoretical point of view, Grossman (1972) develops a framework including a modeling of health spending. Grossman (1972)'s model is a life-cycle one and he considers the health level may be represented by a stock variable, that is the health stock or health capital. The health stock has a positive impact on the agent's welfare. It depreciates at a rate possibly increasing with age. The health stock can also be improved or maintained by continuously health investments. Finally, the income of the agent is an increasing function of his health. If the health stock becomes less than a threshold, the agent dies, lifespan is an endogenous variable. The health capital concept is in the continuity of the human capital concept introduced among others by Ben-Porath (1967). The agent has also access to a financial market on which a risk-free asset is exchanged. Grossman (1972)'s model includes several other features we will not detail.

Grossman (1972)'s model is at the base of a vast literature. Halliday, He, Ning, and Zhang (2017) and Fonseca, Michaud, Galama, and Kapteyn (2008) use life-cycle models including an health stock variable, an endogenous health spending and a financial asset. They evaluate the ability of their models to quantitatively replicate some stylized facts. Fonseca, Michaud, Galama, and Kapteyn (2008) are interested by retirement choices in an uncertain environment. They simulate the path of some key variables and discuss

their adequacy with their empirical counterpart. Halliday, He, Ning, and Zhang (2017) focus on the growing profile of health spending over the life-cycle. Their model, with endogenous labor supply, is calibrated, numerically solved and simulated. They discuss the ability of their model to replicate the profile of some variables such as consumption or medical spending.

Some authors theoretically study the Grossman (1972)'s model. In order to obtain analytical results and to isolate the key mechanism, they use a simplified version (often in continuous time) with only a health stock variable and without a financial asset. Laporte (2015) derives some results concerning the dynamic of the model (uses of phases diagrams). Strulik (2015) obtains a closed-form solution and performs exercises of comparative dynamics. Finally, Laporte and Ferguson (2017) introduce uncertainty into the model.

In the contributions of Laporte (2015), Laporte and Ferguson (2017) and Strulik (2015), the depreciation rate of health stock is constant over time, which allows the agent to potentially live forever. Indeed, in the Grossman's model, the agent stays alive as long as the health stock is above the threshold. If the health stock depreciation rate is constant, the agent may have the possibility to indefinitely maintain his health stock above the threshold. In this article, we consider a more realistic assumption and suppose that the health stock depreciation rate increases as the agent ages. Lifespan is now an endogenous variable. We also depart from Strulik (2015) by supposing that the multiplicative productivity term of the income function (depending of the health stock) is not constant over time. We guess it displays a hump shaped profile over time. This allows to replicate an hump shaped profile of income and to assess its consequences on the agent choices. In order to avoid utility being negative, we use an utility function with a strong and positive enough constant, which is in line with that of Rosen (1988) and Hall and Jones (2007). This model, written in continuous time, is solved numerically. A realistic calibration with a life span of 85 years shows that our model is able to replicate a growing path of the health spending over the life-cycle and a significant share of health spending especially at the end of life. We also perform a sensitivity analysis, which allows us to evaluate how the numerical results are modified if the value of some key parameters are changed.

The paper is organized as follows. In section 2 we present the model and derive the optimality conditions, in section 3, we discuss the calibration strategy and the numerical results.

## **2 A life-cycle model with health capital**

### **2.1 Presentation of the model**

Our model is based on Grossman (1972). We use a simplified version similar to the one developed by Strulik (2015), Laporte (2015) and Laporte and Ferguson (2017) while keeping the main ingredients of Grossman's model. Agent's health is characterized by its health capital level and agent's utility depends on consumption and health. The income of the agent is an increasing function of the health capital. The agent must allocate its

income between consumption and investment in health capital. Finally, we assume that health capital depreciates at a rate increasing with age. The agent ceases to live if the health capital fall below a threshold.

**The intertemporal utility function** The representative agent maximizes its intertemporal discounted utility:

$$\int_0^T e^{-\rho t} \left( \frac{(C_t^\gamma H_t^{1-\gamma})^{1-\sigma}}{1-\sigma} + b \right) dt \quad (2.1)$$

with  $C_t$  the consumption and  $H_t$  the health capital.  $T$  is the agent lifespan which is an endogenous variable. More precisely, we consider  $t = 0$  corresponds to the beginning of the active life (that is 25 years) and  $T$  is the lifespan in adulthood (that is  $T+25$  years).  $\rho > 0$  is the discount rate,  $\sigma > 1$  is the intertemporal elasticity of substitution and  $\gamma \in ]0, 1[$  is the weight of consumption in utility. It is also possible to interpret  $C_t^\gamma H_t^{1-\gamma}$  as an aggregate good produced with private consumption and health capital through a constant return to scales technology.

$b$  is a constant positive term. The key role of this parameter is widely discussed by Rosen (1988) and Hall and Jones (2007). In particular, the value of  $b$  must be sufficiently high to ensure a positive flow of utility. We'll discuss this point in appendix A.

**The law of motion of health capital** Health capital evolution is described by the following equation:

$$\dot{H}_t = AI_t - \delta_t H_t \quad (2.2)$$

$I_t$  represents the investment in health capital. Health capital is produced through a linear technology,  $A$  being the efficiency of this technology. Thus, at each date, the health capital is increased by an amount  $AI_t$ .

Health capital depreciates at a rate  $\delta_t$  increasing with the age of the agent. Following Fonseca, Michaud, Galama, and Kapteyn (2008), we guess that  $\delta_t$  could be written as follows:

$$\delta_t = \delta_1 \exp(\delta_2 t)$$

with  $\delta_1 > 0$  and  $\delta_2 > 0$ .  $\delta_1$  represents the health depreciation at the beginning of the adulthood.

Death occurs when health capital falls below the threshold  $\underline{H} > 0$ . The agent is still alive at time  $t$  if  $H_t \geq \underline{H}$ .

**The aggregate resource constraint** There is a unique good produced by the household which is consumed and invested. The good is produced by the household through a decreasing return to scales technology. Health capital  $H_t$  is the unique input. The household's production can also be interpreted as an income in real terms. The agent can improve its health by reducing consumption and increasing investment in health capital.

At each period  $t$ , one has:

$$\theta_t H_t^\alpha - I_t - C_t = 0 \quad (2.3)$$

where  $\theta_t > 0$  denotes the efficiency of the production technology. Having assumed a decreasing return to scales technology, one has  $\alpha \in ]0, 1[$ . In order to generate an inverted-U profile of income, we will suppose that productivity does the same. One has  $\theta_t = \theta \exp(v_0 + v_1 t + v_2 t^2)$  with  $\theta > 0$  and the polynomial parameters  $v_0, v_1$  and  $v_2$  such that the agent's productivity has an inverted-U profile.

Finally, we impose the following positivity condition on investment,  $I_t \geq 0, \forall t \in [0, T]$ . This condition is natural since health capital can not be “uninstalled” and sold.

**Household's programme** The initial health capital of the agent is  $H_0 > \underline{H}$ . The agent chooses the optimal path of consumption and investment maximizing his lifetime utility subjected to resource constraints. First, let us assume a given lifespan  $T$ . The household's program is therefore written as:

$$\max \int_0^T e^{-\rho t} \left( \frac{(C_t^\gamma H_t^{1-\gamma})^{1-\sigma}}{1-\sigma} + b \right) dt \quad (2.4)$$

subject to

$$\begin{cases} \dot{H}_t = AI_t - \delta_t H_t & (q_t) \\ \theta_t H_t^\alpha - C_t - I_t = 0 & (\mu_t) \\ I_t \geq 0 & (\eta_t) \\ H_t - \underline{H} \geq 0 & (\xi_t) \\ H_T - \underline{H} \geq 0 \end{cases} \quad (2.5)$$

**The optimality conditions** The Hamiltonian and Lagrangian associated to the above program are written as:

$$\begin{aligned} \mathcal{H}_t &= \frac{(C_t^\gamma H_t^{1-\gamma})^{1-\sigma}}{1-\sigma} + b + q_t(AI_t - \delta_t H_t) \\ \mathcal{L}_t &= \mathcal{H}_t + \mu(\theta_t H_t^\alpha - C_t - I_t) + \eta I_t + \xi_t(H_t - \underline{H}) \end{aligned}$$

The following optimality conditions are obtained:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial C_t} &= \gamma \frac{(C_t^\gamma H_t^{1-\gamma})^{1-\sigma}}{C_t} - \mu_t = 0, \quad t \in [0, T] \\ \frac{\partial \mathcal{L}_t}{\partial I_t} &= q_t A - \mu_t + \eta_t = 0, \quad t \in [0, T] \\ \dot{q}_t &= \rho q_t - \frac{\partial \mathcal{L}_t}{\partial H_t} = (\rho + \delta_t) q_t - (1-\gamma) \frac{(C_t^\gamma H_t^{1-\gamma})^{1-\sigma}}{H_t} - \mu_t \theta \alpha H_t^{\alpha-1} - \xi_t \\ & \quad t \in [0, T] \end{aligned}$$

Otherwise, one has the following exclusion conditions:

$$\begin{aligned} \eta_t I_t &= 0, t \in [0, T], \text{ with } \eta_t \geq 0 \\ \xi_t (H_t - \underline{H}) &= 0, t \in [0, T[, \text{ with } \xi_t \geq 0 \\ q_T (H_T - \underline{H}) &= 0, \text{ with } q_T \geq 0 \end{aligned}$$

**Optimal lifetime** We previously considered that the lifetime  $T$  was given. In our setting,  $T$  must be chosen in such a way it maximizes the agent's intertemporal utility along the optimal path, that is:

$$\int_0^T e^{-\rho t} \left( \frac{(C_t^\gamma H_t^{1-\gamma})^{1-\sigma}}{1-\sigma} + b \right) dt$$

The optimal lifetime  $T^*$  satisfies the following optimality condition<sup>1</sup>:

$$\mathcal{H}(T^*) = \left( \frac{(C_{T^*}^\gamma H_{T^*}^{1-\gamma})^{1-\sigma}}{1-\sigma} + b \right) + q_{T^*}(AI_{T^*} - \delta_{T^*}H_{T^*})$$

The agent is alive as long as  $H_t \geq \underline{H}$ . If at date  $T$ , one has  $H_T > \underline{H}$  and  $H_T - \delta_T H_T dt \geq \underline{H}$ , the agent can live an additional time interval  $dt$  without investing in health capital. Clearly,  $T$  is thus not an optimal life time. We conclude at the optimal time  $T^*$ , one has  $H_{T^*} = \underline{H}$ .

### 3 Numerical investigations

The system of equations can not be solved analytically. The time interval  $[0, T]$  is discretized and the set of equations given the solution of the model is solved numerically using a standard Newton algorithm<sup>2</sup>.

#### 3.1 Calibration strategy

The model is calibrated in order to match US data. The discount rate  $\rho$  is set to 0.04, which correspond to an annual interest rate of 4%. The risk aversion coefficient take the traditional value of 1.5. As previously discussed,  $\sigma$  must be greater than 1. Parameter  $\alpha$  is the elasticity of income with respect to the health stock. We choose  $\alpha = 0.15$ , which is of the same order as the value of 0.146 estimated by Fonseca, Michaud, Galama, and Kapteyn (2008). Parameter  $\gamma$  (resp.  $1 - \gamma$ ) corresponds to the weight of private consumption (resp. health stock) in utility. In a static model, this parameter, together with  $\alpha$ , determines the share of consumption (and health spending) in income<sup>3</sup>. In USA, this share is roughly of 17% and in France, it is of 11%. Given the retained value of  $\alpha$ , a value of  $\gamma = 0.95$  would be necessary to obtain (in the static model) a health spending

<sup>1</sup>This optimality condition giving the optimal time is obtained by deriving the intertemporal utility with respect to  $T$  and applying the envelop theorem. This condition can be found in any standard optimal control book.

<sup>2</sup>The system of equations to be solved is shown in appendix B.

<sup>3</sup>Consider the simplified model:

$$\begin{aligned} & \max_{C, I, H} \frac{(C^\gamma H^{1-\gamma})^{1-\sigma}}{1-\sigma} \\ \text{s.t. } & \begin{cases} \theta H^\alpha - C - I = 0 & (\lambda) \\ AI - H = 0 & (\mu) \end{cases} \end{aligned}$$

share of 19%. There is no clear evidences in the literature concerning the numerical value of this parameter. For instance, Fonseca, Michaud, Galama, and Kapteyn (2008) set  $\gamma = 1$  whereas Halliday, He, Ning, and Zhang (2017) chose  $\gamma = 0.64$ . Parameter  $\gamma$  is set at 0.98, which allows to obtain a health spending share close to 15 – 16%. The parameters  $A$  and  $\theta$  are normalized to 1.

The profile of  $\theta_t$  over the life cycle determines the income profile. According to the Lee, Lee, and Mason (2008), labor income attains a peak around 50 years old. Between 25 and 50 years old, labor income increases by around 70% (see Lee, Lee, and Mason (2008), figure 5). Using PSID data, Halliday, He, Ning, and Zhang (2017) report a humped shaped profile of labor income over the life cycle. A peak occurs between the age of 45 and the age of 50. Between the age of 25 and the age of 50, labor income roughly increase of 90%. Finally, at age 60, labor income decreased by about 30%. The calibration of the parameters of the productivity function  $\theta_t$  seeks to reproduce the same orders of magnitude concerning the income profile. To determine the values of  $v_0$ ,  $v_1$  and  $v_2$ , we proceed as follows. During a period of 25 years (from 0 to  $t^* = 25$ ), we guess that the individual productivity grows from  $\theta_0$  to  $\theta^*$  (at age  $t^*$ ). We impose the normalization  $\theta_0 = \theta$  and we set the following growth rate  $z_1 = \frac{\theta^*}{\theta_0}$  with  $z_1 > 1$ . We then deduce the values of  $v_0$ ,  $v_1$  and  $v_2$ . One has  $\log(\theta_t/\theta) = v_0 + v_1 t + v_2 t^2$ . Knowing that  $\theta_0 = \theta$ , it is immediate that  $v_0 = 0$ . Now consider  $\log(\theta^*/\theta) = v_1 t^* + v_2 t^{*2}$ .  $\theta_t$  being maximum at time  $t^*$ , this latter satisfies  $t^* = -\frac{v_1}{2v_2}$ . We deduce the following equation giving the value of  $v_2$ :  $\log(z_1) = v_1 t^* + v_2 t^{*2} = -v_2 t^{*2}$ . Setting  $z_1 = 1.8$  and  $t^* = 25$ , the following values are obtained  $v_0 = 0$ ,  $v_1 = 0.0470$  and  $v_2 = -0.0009$  and the resulting income profile is close to its empirical counterpart. Labor income increases by around 70% between the ages of 25 and 50.

Fonseca, Michaud, Galama, and Kapteyn (2008) assume the health stock is between 0 and 100, in other words they choose a 0-100 scale. They also report that the health stock decreases from 73.3 at age 26 to 39.7 at age 85. We calibrate the model in order to satisfied the same proportions and we impose  $\frac{H}{H_0} = \frac{39.7}{73.3}$ . The level of these variables is set in order to have an health investment equal to 0 during the first periods (as it is the case in Halliday, He, Ning, and Zhang (2017) and Fonseca, Michaud, Galama, and Kapteyn (2008)). One has  $H_0 = 5.1558$  and  $H = 2.7545$ . Finally, following Fonseca, Michaud, Galama, and Kapteyn (2008), the parameters describing the depreciation of health capital take the values  $\delta_1 = 0.035$  and  $\delta_2 = 0.025$ . We impose  $b = 4$ , so that the agent lifetime is of around 85 years.

The benchmark calibration is summarized in tables 1 and 2.

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It is easy to determine that:

$$\frac{I}{C+I} = 1 - \gamma + \gamma\alpha$$



Table 1: Benchmark calibration

$\rho$	$b$	$\sigma$	$\gamma$	$\alpha$	$A$	$\delta_1$	$\delta_2$	$H_0$	$\underline{H}$
0.04	4	1.5	0.98	0.15	1	0.035	0.025	5.1558	2.7545

Table 2: Benchmark calibration

$\theta$	$x_0$	$v_1$	$v_2$
1	0	0.0470	-0.0009

### 3.2 Numerical results

To begin, we briefly discuss how the benchmark calibration allows to adjust the targeted values. We obtain an optimal  $T$  of 60, which corresponds to a lifespan of 85 years. The share of health spending ( $\frac{I}{C+I}$ ), that is its average value calculated over the life-cycle, is of 17.17%. Figure 1 presents the evolution of health spending, consumption, health stock and income over the life-cycle. We observe that health spending is zero around from the age of 25 to the age of 35. Income displays a hump shaped profile and it reaches a peak around 50 years. Between 25 and 50 years old, income increases of 70%, next, it decrease of about 8.1% until the age of 60, which is consistent with the data (Halliday, He, Ning, and Zhang (2017)).

We now discuss how the model fit the data. French, McCauley, Aragon, et al. (2017) report that the medical spending of the last three years represents 16.70% of the US aggregate medical spending. In the other countries they study, this value is around 20%. As we consider a representative agent over his life-cycle, we calculate the share of the last three years health spending in total health spending. We find a value of about 7.14%. This value is less than the one reported by French et al. (2017). However, health spending during the last three years of life have a significant level.

The profile of health expenditures (figure 1) is globally increasing, which is roughly consistent with the data (see Halliday, He, Ning, and Zhang (2017) and Fonseca, Michaud, Galama, and Kapteyn (2008)). However, we underline that the health expenditures profil is concave and during the last years of life, it is slightly decreasing. The decrease of the health stock is not monotonous<sup>4</sup>. Health stock displays a slight bump around the age of 60 and it attains its critical threshold  $\underline{H}$  at the age of 85. The bump of the health stock is generated by the hump shaped income profile. The profile of health expenditures is concave, which means that the agent significantly invests in health during the income growth phase. It follows a phase of health capital accumulation from 40 to 60 years old.

Income and consumption displays an hump shaped profile. Consumption grows by 48% between the ages of 25 and 50. It then decreases by 9% between 50 and 60 years old and by 81% between 60 and 85 years old<sup>5</sup>. During the working life, the consumption

<sup>4</sup>Following Halliday, He, Ning, and Zhang (2017) and Fonseca, Michaud, Galama, and Kapteyn (2008), the health index is monotonically decreasing over the life-cycle.

<sup>5</sup>Halliday, He, Ning, and Zhang (2017) report an hump shaped profile of consumption with an increase of 60 % between 25 and 50 years old and successive decreases of 30% between 50 and 60 years old and of 28% between 60 and 80 years old.

profile is slightly smoother than that of income. Indeed, in this class of models, agents wish to smooth their consumption over their life-cycle. However, after age 60, consumption decreases dramatically, which does not fit the data.

Our model is able to reproduce an increasing profile of health spending. It also reproduce a hump shaped profile of consumption. We note that consumption decreases very strongly after 60 years old. The hump shaped profile of the productivity  $\theta_t$  generates the hump shape profile of income which in turn determines the profile of consumption. However, the consumption profile is less smooth than in the data. This may be explained by the absence of financial market. The agent cannot transfer income over time and the smoothness of consumption is not enough.

We point out that way to obtain a hump shaped profile of income. Our model looks like life-cycle models with human capital and endogenous wage (see Ben-Porath (1967)). This class of model is able to generate a hump shaped income profile. In our model, the health stock variable plays a similar role to human capital. We underline two differences. Firstly, we guess the depreciation rate increases with age. Secondly, the elasticity of income with respect to health capital is, in our benchmark calibration, equal to 0.15. This value is too small to generate a hump shaped profile of income. A value of around 0.8 would be necessary to obtain it, which is not an admissible value. Income is less sensitive to health than to education.

### 3.3 Sensitivity analysis

In this subsection, we examine how the numerical results are modified if we change some key parameters.

**The elasticity  $\sigma$**  We simulated the model for two other value of  $\sigma$ , that is  $\sigma \in \{1.25; 1.5; 2\}$  (Figure 2). Recall that  $\frac{1}{\sigma}$  is the intertemporal elasticity of substitution. This variable expresses the will of the agent to substitute intertemporally consumption (or more precisely, the “composite good”  $C^\gamma H^{1-\gamma}$ ). Thus, increasing  $\sigma$  means a reduction of the willingness of the agent to make intertemporal substitutions. In other word, the agent prefers a smoother consumption profile. As we want to evaluate the impact of a perturbation of the elasticity  $\sigma$  on the trade-off made by the agent, the numerator of the utility function  $1 - \sigma$  is always parametrized using the benchmark value of 1.5, otherwise, the utility level and the lifespan would be impacted by an other channel that consumption and health stock. In other words, a perturbation of  $\sigma$  would have effects similar to a change in  $b$ .

Table 3: Elasticity  $\sigma$

$\sigma$	1.25	1.5	2
Lifespan	89	85	81
Health spending share	21.38%	17.17%	14.91%

Increasing parameter  $\sigma$  leads to a decrease in the lifespan and in the health spending share (table 3). If the elasticity  $\sigma$  is low, the intertemporal elasticity of substitution is

high and the agent willingness to smooth consumption is low. As the depreciation rate of health increases with age, the necessary health expenditures are continuously growing. Consequently, the agent should accept a strong and increasing reduction of consumption in order to stay alive. He is willing to do so if the intertemporal elasticity of substitution is low.

Figure 2 presents the paths followed by health spending, consumption, health stock and income for the three values of  $\sigma$ . We can see that the paths followed by consumption and income coincide. We also observe there is now upward shift of consumption as  $\sigma$  increases.

**The parameter  $b$**  We consider two alternative values for parameter  $b$ , that is  $b \in \{2; \mathbf{3.8}; 6\}$  (Figure 3). As discussed in A, this parameter plays a key role in the determination of the lifespan. He represents the utility of being alive. The results shown in table 4 point that the lifespan is increasing relative to  $b$ . We also observe an increase in the health spending share. This is obviously related to the lifespan increase and the increase in the health stock depreciation rate. Consequently, to live longer, the agent must increase his health spending share as he gets holder. It follows an increase in the heath spending share.

$b$	2	3.8	6
Lifespan	2	85	88
Health spending share	9.48%	17.17%	21%

Figure 3 presents the paths followed by health spending, consumption, health stock and income for the three values of  $b$ . The income and consumption paths are roughly superimposed. In other words, the path of a given value of  $b$  approximatively extends the one obtained with a smaller value. An increase in  $b$  corresponds to an increase in the utility of being alive. Consequently, the agent is willing to accept a reduction in consumption to finance health spending in order to stay alive. After 55 years old, the paths of health spending and health stock move upwards as  $b$  increases. This reflect the increase of the health spending share.

**The parameter  $\gamma$**  We suppose that  $\gamma \in \{0.9; \mathbf{0.98}; 1\}$ . Recall this parameter has a direct impact on the behaviour of the agent in regards to health spending.

The more interesting case is  $\gamma = 0.9$ . Under this value of  $\gamma$ , the agent chooses to devote a greater share of his expenses to health spending (Table 5). The health spending is then strictly positive very early (Figure 4). Recall that with higher values of  $\gamma$ , there is no health spending until 35 years old. The paths followed by the health stock and the health spending move upwards as  $\gamma$  decreases. The income profile does the same.

Table 5: Parameter  $\gamma$ 

$\gamma$	0.9	0.98	1
Lifespan	86	85	84
Health spending share	21.57%	17.17%	15.46%

## 4 Conclusion

In this article, we study a simplified version of the Grossman (1972)’s model. We mainly follow Laporte (2015) and Strulik (2015). The agent can allocate his income between consumption and health spending. The health of the agent is characterized by his health stock or health capital. The depreciation of the health stock may be compensated by health spending. The agent stays alive as long as the health stock is above a threshold. Our main contribution is to investigate the Grossman (1972)’s model in a non stationary environment with an endogenous finite lifetime. One key assumption concerns the depreciation rate of the health stock. Contrarily to Laporte (2015) and Strulik (2015), we suppose the depreciation rate increases with age. This last assumption avoids the problem of potential immortality of the agent. We also suppose the productivity term of the income function displays a hump shaped profile over the life cycle. Finally, we point out that there is no financial market.

We derive the optimality conditions that allow to characterize the solution. The optimal control problem presents some difficulties. There are two occasionally binding constraints (health spending is positive or zero and the health stock is greater than or equal to the critical threshold) and lifespan is endogenous.

We characterize numerically the solution of the model. The model is calibrated on the US economy. The model is able to reproduce an increasing profile of health spending over the life-cycle and a significant share of health spending especially at the end of life. The model also reproduces a hump shaped profile of consumption. However, consumption at older ages is too low compared to the data. This may be due to the assumptions of this simplified model. For instance, there is no financial market. The agent can not transfer income through time and cannot smooth its consumption path.

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## A The constant term in the utility function

Here we discuss the meaning of the term  $b$  in the utility function. Rosen (1988) is interested by the question of the value of life and develops theoretical frameworks allowing to highlight its determinants. Two classes of models are considered, models with a survival rate and deterministic life-cycle models. Note that these two classes of models are closely related and have very similar results. In the line of Rosen (1988), we use a deterministic life-cycle model in which the lifetime is endogenously determined. In models like ours, the constant term  $b$  of the utility function plays an important and “non-standard” role because it has implications on the decisions of the agents. This point is discussed in detail by Rosen (1988) and Hall and Jones (2007). In standard microeconomic theory, it is always possible to consider an increasing transformation of a utility function. This transformation does not impact the marginal utility ratios and thus the optimal decisions. Our model works very differently than standard ones. Indeed, in our framework, as in Rosen (1988)’s, the choice problem is a very particular one. The agent has to determine the optimal consumption and health investment plans. These decisions only depend on the marginal utility and the constant  $b$  does not impact the optimality conditions. Our

model is also a life-cycle one and the agent has to determine his optimal lifetime  $T^*$ . The associated optimality condition (see subsection 2.1) significantly differs from the other optimality conditions and the level of utility now matters. It is thus easy to understand that the value of  $b$  will significantly alter agent decisions (through the value of the optimal lifetime). Furthermore, the optimization problem determining the optimal lifetime has a solution if  $b$  is positive and sufficiently large.

**The life-cycle utility** We follow Rosen (1988) who explains how this utility function may be set and how the constant  $b$  may be understood.

The lifetime is  $T$ . The instantaneous utility of the agent (during his lifetime) is:

$$u(C_t, H_t) = \frac{(C_t^\gamma H_t^{1-\gamma})^{1-\sigma}}{1-\sigma}$$

The agent utility over its life-cycle is given by:

$$\int_0^T e^{-\rho t} u(C_t, H_t) dt$$

However, the above expression does not provide a complete evaluation of the agent utility over his life-cycle. Indeed, we suppose that the agent incurs a disutility of being death, the instantaneous disutility being  $-b$ . Evaluated at  $t = 0$ , the disutility of being death at  $t > T$  is given by  $\beta^t(-b)$ . To sum up, the life-cycle utility of the agent is written as:

$$\int_0^T e^{-\rho t} u(C_t, H_t) dt + \int_T^\infty e^{-\rho t} \beta^t(-b) dt$$

Rearranging the above expression, one gets:

$$\int_{t=0}^T e^{-\rho t} u(C_t, H_t) + \int_T^\infty e^{-\rho t} (-b) dt = \int_{t=0}^T e^{-\rho t} (u(C_t, H_t) + b) dt - \frac{b}{\rho}$$

The term  $\frac{b}{\rho}$  being independent of  $T$  may be dropped. We thus obtain the intertemporal utility function of subsection 2.1, that is:

$$\int_{t=0}^T e^{-\rho t} (u(C_t, H_t) + b) dt$$

**Optimal longevity** The optimality condition giving the optimal lifetime  $T^*$  is given in Subsection 2.1. One has:

$$\mathcal{H}(T^*) = \frac{(C_{T^*}^\gamma H_{T^*}^{1-\gamma})^{1-\sigma}}{1-\sigma} + b + q_{T^*}(I_{T^*} - \delta_{T^*} H_{T^*-1}) = 0$$

Evaluated at  $t = 0$ , the above condition can be rewritten as follows:

$$e^{-\rho T^*} \mathcal{H}(T^*) = e^{-\rho T^*} [u(C_{T^*}, H_{T^*}) + b] + e^{-\rho T^*} q_{T^*}(I_{T^*} - \delta_{T^*} H_{T^*-1}) = 0$$

or

$$\begin{aligned}
e^{-\rho T^*} \mathcal{H}(T^*) &= e^{-\rho T^*} [u(C_{T^*}, H_{T^*}) + b] + e^{-\rho T^*} q_{T^*} \dot{H}_{T^*} \\
&= e^{-\rho T^*} [u(C_{T^*}, H_{T^*}) + b] + \beta^{T^*} u_C(C_{T^*}, H_{T^*}) \frac{1}{A} \dot{H}_{T^*} \\
&= 0
\end{aligned} \tag{A.1}$$

It is possible to show that  $e^{-\rho T^*} \mathcal{H}(T^*)$  is the marginal net gain of being alive one more period<sup>6</sup>. If the agent stays alive one period more, his gain in utility is  $e^{-\rho T^*} [u(C_{T^*}, H_{T^*}) + b]$ , the sum of the utility derived from consumption and health capital and of the utility of being alive ( $b$ ). However, in order to ensure the supplementary consumption (and maintain the health capital above or equal to the threshold  $\underline{H}$ ) induced by the increase of the lifespan, the agent must reduce its consumption over his life-cycle. The cost, in term of utility, is given by  $e^{-\rho T^*} u_C(C_{T^*}, H_{T^*}) \frac{1}{A} \dot{H}_{T^*} = e^{-\rho T^*} q_{T^*} \dot{H}_{T^*}$ . We underline that  $\dot{H}_{T^*}$  is necessarily negative. At the end of his life, the health capital of the agent is decreasing, so that  $e^{-\rho T^*} q_{T^*} \dot{H}_{T^*} < 0$ . At the optimum, the net gain of being alive one more period is equal to zero.

This optimality condition is written in terms of utility level, and thus, the constant term  $b$  plays a key role in the tradeoffs made by the agent. To underline the role played by  $b$  assume that  $b = 0$ . With our specification of the utility function and assuming  $\sigma > 1$  (see Hall and Jones (2007)), the condition (A.2) becomes:

$$\frac{1}{1 - \sigma} + \frac{\gamma}{C_{T^*}} \frac{1}{A} \dot{H}_{T^*} = 0$$

All the terms of the above expression are negative. It follows the optimality condition that gives the optimal lifespan works only if  $b > 0$  is sufficiently large to ensure a strictly positive utility. In other words, the parameter  $b$  is crucial to obtain a well-posed optimization problem.

**The role of parameter  $b$**  Here we discuss the role of parameter  $b$  and how it determines the agent's choices.

As previously mentioned,  $b$  is the utility of being alive. An increase in  $b$  will provide incentives to increase lifetime for a given amount of resources. Given the resources constraints, the agent should then reduce his consumptions. We consider a simplified version of Rosen (1988)'s model. The agent is endowed with wealth  $W$  and the interest rate is equal to the time preference rate. We thus use  $\rho$  to compute the sum of discounted consumption flows in the budget constraint. We also impose that consumption is constant over time. One has:

$$\begin{aligned}
\int_0^T e^{-\rho t} \left[ \frac{C^{1-\sigma}}{1-\sigma} + b \right] dt &= \frac{1 - e^{-\rho T}}{\rho} \left[ \frac{C^{1-\sigma}}{1-\sigma} + b \right] \\
W = \int_0^T e^{-\rho t} C dt &= \frac{1 - e^{-\rho T}}{\rho} C
\end{aligned}$$

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<sup>6</sup>This can be shown by differentiating the agent welfare with respect to  $T$  along an optimal path.

Determine now the optimality conditions with respect to  $C$  and  $T$ , that is:

$$\frac{1 - e^{-\rho T}}{\rho} C^{-\sigma} - \lambda \frac{1 - e^{-\rho T}}{\rho} = 0$$

$$e^{-\rho T} \left[ \frac{C^{1-\sigma}}{1-\sigma} + b \right] - \lambda e^{-\rho T} C = 0$$

$\lambda$  being the Lagrange multiplier.

The optimal consumption is given by:

$$C = \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{1}{1-\sigma}} b^{\frac{1}{1-\sigma}}$$

Note that  $C$  exists only if  $b > 0$  and  $\sigma > 1$ . Let's differentiate  $C$  with respect to  $b$ :

$$\frac{\partial C}{\partial b} = \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{1}{1-\sigma}} \frac{1}{1-\sigma} b^{\frac{1}{1-\sigma}-1} < 0$$

The optimal value of  $T$  is given by:

$$W = \frac{1 - e^{-\rho T}}{\rho} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{1}{1-\sigma}} b^{\frac{1}{1-\sigma}}$$

It is easy to show that  $\frac{\partial T}{\partial b} > 0$ . This example shows clearly the role of parameter  $b$  and what are the tradeoffs made by the agent. An increase in  $b$  correspond to an increase in the utility of being alive. If  $b$  increases, the agent chooses to live longer and, knowing that its wealth is given, reduces his instantaneous consumption.

To finish, we discuss what happens if the agent can invest in health capital. We use the model with a survival rate of Hall and Jones (2005), which allows to have analytical results. The agent's problem writes:

$$\max p(H)U(C)$$

$$\text{s.t. } Y - C - H \geq 0$$

$p(H) = H^\omega$ , with  $\omega < 1$  is the survival rate and  $Y$  is the agent's income.

We deduce the following optimality conditions:

$$p(H)U'(C) - \lambda = 0$$

$$p'(H)U(C) - \lambda = 0$$

Note that the level of utility must be positive, otherwise the second optimality condition is not consistent.

Let's define the elasticities  $\xi_H = H \frac{p'(H)}{p(H)}$  and  $\xi_C = C \frac{U'(C)}{U(C)}$ . From the optimality conditions, we deduce:

$$\frac{C}{H} = \frac{\xi_C}{\xi_H} = \frac{\xi_C}{\omega}$$



Assuming that  $\frac{\partial C}{\partial Y} > 0$ , we determine the effect of an increase in  $Y$  on the ratio  $\frac{C}{H}$ . One gets:

$$\frac{\partial}{\partial Y} \left( \frac{C}{H} \right) = \frac{1}{\omega} \frac{\partial \xi_C}{\partial C} \frac{\partial C}{\partial Y}$$

The sign of the above expression depends on the sign of the derivative of the elasticity  $\xi_C$ . If this elasticity is decreasing in consumption, then, the ratio  $\frac{C}{H}$  is decreasing in  $Y$ . It follows that the share of health expenditures increases as the income  $Y$  increases.

Suppose now that  $U(C) = \frac{C^{1-\sigma}}{1-\sigma} + b$ . One has:

$$\frac{\partial \xi_C}{\partial C} = \frac{(1-\sigma)bC^{1-\sigma}}{\left(\frac{C^{1-\sigma}}{1-\sigma} + b\right)^2}$$

Knowing that  $\sigma > 1$ , the above derivative is negative. It is also easy to check that if  $b = 0$ ,  $\xi_C = 1 - \sigma$ . It follows that  $\frac{C}{H} = \frac{1-\sigma}{\omega} < 0$  and  $\frac{\partial \xi_C}{\partial C} = 0$ . Once again, we can underline that the importance of parameter  $b$  which must be large enough to ensure the existence of a solution to the agent's problem<sup>7</sup>.

To conclude this paragraph about parameter  $b$ . Assuming that  $\sigma > 1$ , we saw, through several examples, that  $b$  must be positive and sufficiently large to ensure the consistency of a life-cycle model with endogenous life-time (or endogenous survival rate). Furthermore,  $b > 0$  ensures that the consumption-elasticity of utility is decreasing. It follows that the share of health expenditures in income increases as income increases. This is consistent with the data providing evidences that the share of income devoted to health expenditures increases as income increases.

## B The discretized system of equations

The initial health capital  $H_0$  is supposed to be given. The system to solved is defined by the following set of equations:

$$\begin{aligned} \gamma \frac{(C_t^\gamma H_t^{1-\gamma})^{1-\sigma}}{C_t} - \mu_t &= 0, \quad t \in [0, T] \\ q_t A - \mu_t + \eta_t &= 0, \quad t \in [0, T] \\ -\dot{q}_t + (\rho + \delta_t)q_t - (1-\gamma) \frac{(C_t^\gamma H_t^{1-\gamma})^{1-\sigma}}{H_t} - \mu_t \theta \alpha H_t^{\alpha-1} - \xi_t &= 0, \quad t \in [0, T] \\ \eta_t I_t &= 0, t \in [0, T], \text{ with } \eta_t \geq 0 \\ \xi_t (H_t - \underline{H}) &= 0, t \in [0, T], \text{ with } \xi_t \geq 0 \\ -\dot{H}_t + A I_t - \delta_t H_t &= 0, \quad t \in [0, T] \\ \theta_{t_i} H_{t_i}^\alpha - C_{t_i} - I_{t_i} &= 0, \quad t \in [0, T] \\ q_T (H_T - \underline{H}) &= 0, \quad \text{with } q_T \geq 0 \end{aligned}$$

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<sup>7</sup>If  $b = 0$ , the problem has a solution only if  $\sigma \in ]0, 1[$  and the ratio is constant. The health spending share is then independent of income.

The time interval  $[0, T]$  is discretized in  $N$  points, that is:  $\{t_1, \dots, t_N\}$  with  $t_1 = 0$  and  $t_N = T$  and  $dt = t_i - t_{i-1}$ .

The unknown variables are:

$$\begin{aligned}
C_{t_i}, \quad t_i &= t_1, \dots, t_N \\
H_{t_i}, \quad t_i &= t_1, \dots, t_N \\
I_{t_i}, \quad t_i &= t_1, \dots, t_N \\
q_{t_i}, \quad t_i &= t_1, \dots, t_N \\
\mu_{t_i}, \quad t_i &= t_1, \dots, t_N \\
\eta_{t_i}, \quad t_i &= t_1, \dots, t_N \\
\xi_{t_i}, \quad t_i &= t_1, \dots, t_{N-1}
\end{aligned}$$

The discretized system writes as follows:

$$\begin{aligned}
\gamma \frac{(C_{t_i}^\gamma H_{t_i}^{1-\gamma})^{1-\sigma}}{C_{t_i}} - \mu_{t_i} &= 0, \quad t_i = t_1, \dots, t_N \\
q_{t_i} A - \mu_{t_i} + \eta_{t_i} &= 0, \quad t_i = t_1, \dots, t_N \\
\left[ -(\rho + \delta_{t_{i+1}})q_{t_{i+1}} + (1 - \gamma) \frac{(C_{t_{i+1}}^\gamma H_{t_{i+1}}^{1-\gamma})^{1-\sigma}}{H_{t_{i+1}}} + \mu_{t_{i+1}} \theta \alpha H_{t_{i+1}}^{\alpha-1} + \xi_{t_{i+1}} \right] &dt \\
+ q_{t_{i+1}} - q_t &= 0, \quad t_i = t_1, \dots, t_{N-1} \\
\eta_{t_i} I_{t_i} &= 0, \quad t_i = t_1, \dots, t_N \text{ with } \eta_{t_i} \geq 0 \\
\xi_{t_i} (H_{t_i} - \underline{H}) &= 0, \quad t_i = t_1, \dots, t_{N-1} \text{ with } \xi_{t_i} \geq 0 \\
-H_{t_{i+1}} + H_{t_i} + (AI_t - \delta_{t_i} H_{t_i})dt &= 0, \quad t_i = t_1, \dots, t_N \\
\theta_{t_i} H_{t_i}^\alpha - C_{t_i} - I_{t_i} &= 0, \quad t_i = t_1, \dots, t_N \\
q_{t_N} (H_{t_{N+1}} - \underline{H}) &= 0, \quad \text{with } q_{t_N} \geq 0
\end{aligned}$$

There are  $N \times 7 - 1$  equations and  $N \times 7 - 1$  unknown variables. The system of equations is thus “well-posed” and we can undertake to solve it numerically using a Newton algorithm.

## C Graphics







