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Direct Observation of Exceptional Points in Photonic Crystal by Cross-Polarization Imaging in Momentum Space

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This study explores exceptional points (EPs) in photonic crystals (PhCs) and introduces an experimental technique for their single-shot observation. Exceptional points are spectral singularities found in non-Hermitian systems, such as leaky PhC slabs. However, directly observing EPs in PhC systems using regular reflectivity spectroscopy is a considerable challenge due to interference between guided resonances and background signals. In this work, we present a simple, nondestructive technique that employs crossed polarizations to directly observe EPs in momentum-resolved resonant scattering. This approach effectively suppresses the background signal, enabling exclusive probing of the guided resonances where EPs manifest. Our results demonstrate the formation of EPs in both energy-momentum mapping and isofrequency imaging. All experimental findings align seamlessly with numerical simulations and analytical models. Our approach holds great potential as a robust tool for studying non-Hermitian physics in PhC platform.

Over the past decade, there have been remarkable developments in non-Hermitian physics within photonic platforms, encompassing both purely lossy systems and parity-time symmetric systems that exhibit a balance between optical gain and losses^{1–4}. These advancements have been possible due to the expertise in crafting synthetic optical materials at the subwavelength scale, which allows for precise modulation/tailoring of their gain and loss properties. The physics of these open systems can be depicted using effective non-Hermitian Hamiltonians⁵, characterized by complex eigenvalues and non-orthogonal eigenvectors. Intriguingly, the degeneracies in a non-Hermitian Hamiltonian are termed Exceptional Points (EPs), where both the eigenvectors and eigenvalues converge. EPs represent singularities in non-Hermitian topology^{4, 6–8}, and their unique attributes have been instrumental in illustrating numerous non-trivial physical phenomena, including coupled waveguides^{9–12}, unidirectional reflectionless light propagation^{13, 14}, loss-gain microresonators for lasers^{15–20}, improved sensing sensitivity^{21–23}, enhanced local density of optical states and spontaneous emission^{24, 25}.

Leaky resonances in Photonic Crystals (PhCs) serve as a versatile platform for studying non-Hermitian band structures and EP physics^{26–28}. By engineering energy-momentum dispersion and radiative losses in PhC systems, it becomes possible to generate EPs even in the absence of gain medium and material losses^{25, 29–35}. The emergence of EPs within passive

photonic crystals has created unexplored paths for investigating the captivating physics concealed within these structures. These include EP rings^{29, 30}, bulk Fermi arc²⁹, non-Hermitian band inversion^{25, 33, 34}, non-Hermitian coupling between orthogonal polarization modes^{32, 35}, and non-trivial propagation decay³⁵. However, demonstrating EPs in such PhCs is not straightforward, as shown by the detailed analysis required in angle-resolved reflectivity measurements²⁹. This complexity stems from the Fano-shaped profile in the reflectivity spectra, which arises from the coupling between guided resonances primarily confined within the PhC slab and background optical signals emanating from the substrate and the interfaces between layers. As a result, the presence of EPs tends to be obscured in angle-resolved reflectivity spectra, necessitating advanced analytical techniques. As a potential solution, coupling the EPs to a luminescent layer allows the detection of EPs through an enhancement of fluorescence in the energy-momentum spectra²⁵. While this approach facilitates direct observation of EPs, the inclusion of an active material modifies the optical environment. It also introduces nonradiative losses, thereby significantly altering the properties of EPs from the pristine sample. Another tactic involves using a dielectric or metallic mirror as the sample substrate^{25, 34}. This leads to a Lorentz-shaped profile, as opposed to Fano ones, enabling direct observation of EPs. Nevertheless, requiring a mirror substrate imposes significant constraints and limits the design flexibility of the PhC. Therefore, it is imperative to develop a method for the direct observation of EPs in passive PhCs that maintains their inherent characteristics and imposes no restrictions on the stack design. The capacity to directly observe evidence of EPs in real-time measurements would streamline experimental procedures, eliminating the need for

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intricate treatments and model comparisons. Moreover, it will offer efficient means to verify the presence of EPs within the PhC structure, enabling the advancement of various applications.

In this letter, we demonstrate that EPs in a passive PhC structure can be directly observed in angle-resolved reflectivity using a crossed-polarization setup. This technique, termed resonant scattering, effectively suppresses the specular reflection, thereby eliminating Fano interference and allowing for the exclusive probing of the PhC's resonant modes. EPs are discerned in single-shot imaging without any post-processing in both energy-momentum dispersion measurements and isofrequency contour measurements. The experimental findings align well with the results from numerical simulations. Our work introduces a straightforward and non-destructive method to monitor EPs in momentum space, further broadening the potential for resonance-enhanced scattering in photonic systems.

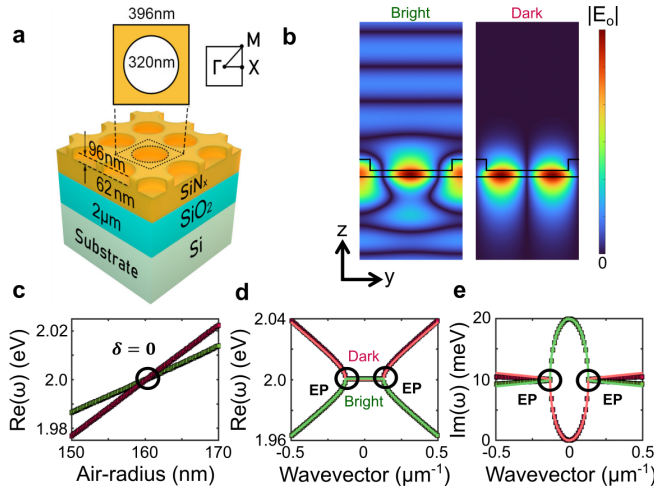


FIG. 1. Sample design and photonic band structure. a) Sketch of the sample design. The inset shows the Brillouin zone of the reciprocal lattice. b) The field intensity of the bright and dark modes at $k = 0$. c) The energy of bright and dark modes at $k = 0$ for different hole radii. The black open circle marks the position where the detuning $\delta = 0$. d) Real and e) imaginary parts of the energy eigenvalues along ΓM direction for structure with $r = r_{EP} = 160$ nm. The green color corresponds to the bright mode while the red represents the dark modes. Analytical calculations are shown in solid line (pale red and pale green) and the numerical simulations are presented with square symbols (dark red and dark green). The refractive indices of SiN_x and SiO_2 used in the FEM simulations are 2.02 and 1.46, respectively. Analytical fits are obtained with $E_0 = 2.0$ eV, $\delta = 0.59$ meV, $\gamma = 19.9$ meV, $v = 76.6$ meV $\cdot\mu\text{m}$ and $\sigma = -35$ meV $\cdot\mu\text{m}^{-2}$.

The sample design, depicted in Fig. 1a, comprises a 158 nm thick SiN_x layer, partially etched to form a square lattice of holes (96 nm depth and radius $r = 160$ nm) with a period of 396 nm. This PhC slab is positioned on a silicon substrate with a 2 μm -thick layer of SiO_2 serving as an optical spacer. In passive PhC slabs, EPs arise when two resonances of differing radiative losses coalesce^{25,29,32}. In our design, these resonances encompass a leaky mode, termed a bright mode, and a non-radiating mode, named the dark mode^{25,29,30}. The

bright (dark) mode is permitted (forbidden) to radiate into the far field due to its symmetric (anti-symmetric) near-field pattern. Figure 1b displays the electric field distribution of these modes, derived from Finite Element Method (FEM) simulations using Comsol Multiphysics. It reveals that the electric field of the bright mode radiates into free space in the far field, whereas the field distribution of the dark mode is predominantly confined to the SiN_x layer, not radiating into the far field. Notably, these modes resemble those previously documented for engineering exceptional rings^{29,30}. At an oblique angle (i.e., nonzero momentum k), mirror symmetry is disrupted, causing the modes to hybridize. The effective Hamiltonian of the band structure near the Γ point is given by²⁵:

$$H(k) = \begin{pmatrix} E_0 + \frac{\delta + \sigma k^2}{2} & v \cdot k \\ v \cdot k & E_0 - \frac{\delta + \sigma k^2}{2} \end{pmatrix} + \begin{pmatrix} i\gamma & 0 \\ 0 & 0 \end{pmatrix} \quad (1)$$

In the Hermitian term (i.e. first term) of (1), E_0 is the offset energy of the bands, δ is the frequency detuning between the bright mode and the dark mode at $k = 0$; and v , σ are coefficients of second-order $k \cdot p$ perturbation theory. In the non-Hermitian term (i.e. second term) of (1), γ corresponds to the radiative losses of the bright mode, given by:

$$E_{\pm}(k) = E_0 + i\frac{\gamma}{2} \pm \sqrt{\left(\frac{\delta + \sigma k^2 + i\gamma}{2}\right)^2 + v^2 k^2} \quad (2)$$

Interestingly, when the detuning δ is tuned to $\delta_{EP} = -\frac{\sigma\gamma^2}{4v^2} \approx 0$, the two eigenvalues coalesce to $E_{EP} = E_0 + i\frac{\gamma}{2}$ at $k_{EP} = \pm\frac{\gamma}{2v}$ and EPs are formed. This configuration can be achieved in our design since δ can be finely tuned by sweeping the air-radius of the hole lattice. Indeed, Figure 1c presents FEM results of the energy of the two modes at $k = 0$ as functions of the air radius r . It shows that the zero-detuning configuration is achieved for radius $r_{EP} = 160$ nm. Figures 1d,e presents the real (Fig. 1d) and imaginary (Fig. 1e) parts of two complex eigenvalues obtained by FEM simulations along ΓM direction of the Brillouin zone when $r = r_{EP}$. These numerical results are perfectly fitted by the analytical model (2). They clearly evidence the formation of EPs on ΓM path at $k_{EP} = \pm 0.13 \mu\text{m}^{-1}$. The dark and bright modes can also be inferred in Fig. 1e in which at $k = 0$, the imaginary of the dark mode reduces to zero whereas the bright mode exhibits the highest loss. The calculated band structures in both directions ΓM and ΓX can be found in Supplementary Material (SM, Fig. S1).

The sample is fabricated using Plasma Enhanced Chemical Vapor Deposition to deposit SiN_x . This is followed by laser interference lithography and dry etching to pattern the SiN_x layer into PhC structures. Further fabrication details are provided in Section II of the SM. A scanning electron microscopy (SEM) image of the final sample is presented in the inset of Fig. 2.

To solely probe the guided resonances in our PhC structure, we employ the resonant scattering measurement technique that involves reflectivity/transmission spectroscopy in a cross-polarization setup³⁶⁻⁴¹. While no previous studies have utilized this method to unveil EPs, it has been effectively

used to discern bare photonic modes for localized states in PhC cavities^{36,37,42,43} and the band structures of guided mode resonances^{39,44,45}. The underlying principles of the resonant scattering technique, used to determine the spectral position of guided resonances in PhC, are discussed extensively in earlier reports^{39–41}. In essence, the cross-polarization arrangement in resonant scattering spectroscopy results in the suppression of specular reflection/transmission and its interference with light scattered from the guided resonances. Consequently, the spectra exhibit a zero baseline and symmetric profile peaks, diverging from the Fano profile. This facilitates direct insight into the spectral position and line-width of the guided resonance.

The optical setup for the resonant scattering measurement in momentum space is illustrated in Fig. 2. Broadband light (Thorlabs SLS202C) is focused onto the sample through a microscope objective (OptoSigma 50X, NA = 0.55). Polarization is controlled using a linear polarizer (LP1) and a half-wave plate (HWP1). Reflected light is collected by the same objective and separated by using a beam splitter (BS). A Fourier lens (L1) images the objective's back focal plane. For resonant scattering, background signals are filtered with another polarization set (LP2 and HWP2), while regular reflectivity omits this step. For momentum-resolved spectroscopy, light is focused via lens (L2) onto a monochromator (Princeton Instrument HRS-300) slit, then relayed to a 2D spectroscopic camera (Kuro model). This setup maps the energy-momentum band structure in one shot, with the direction determined by the monochromator slit. In isofrequency Fourier imaging, a band-pass filter (BF, 640 nm \pm 10 nm) is implemented. Its transmitted wavelength is tuned to match that of the EP (i.e., 620 nm) by operating at a non-zero angle of incidence. Finally, a flipped mirror (FM) projects the spectrally-filtered back-focal plane onto a CMOS camera (Caltex). This maps photonic modes in momentum space (k_x, k_y) at EP energy with a single-shot measurement.

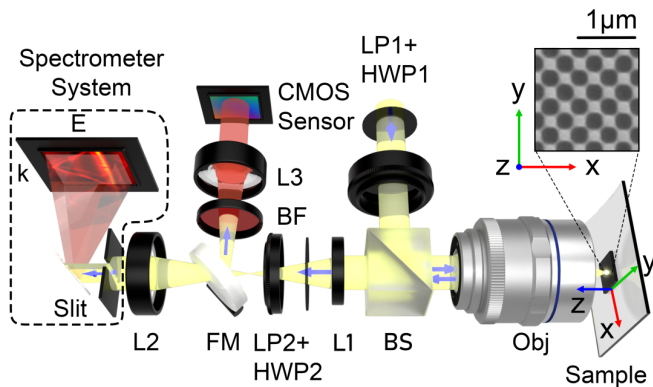


FIG. 2. **Experimental setup.** Illustration of the optical measurement system. OL: objective lens. BS: beamsplitter. LP: linear polarizer. HWP: half-wave plate. L1, L2, L3: lens system. FM: flippable mirror. BF: band-pass filter (640nm \pm 10nm). Inset: SEM image of the sample's surface.

Four linear polarizations are defined: horizontal (\leftrightarrow), vertical (\updownarrow), diagonal (\nearrow), and anti-diagonal (\nwarrow). These polariza-

tions correspond to angles of 0°, 90°, 45°, and -45°, respectively, between the axes of the polarizers (LP1, LP2) and the x-axis of the sample. Resonant scattering experiments employ two configurations: the crossed polarizations of [$\updownarrow, \leftrightarrow$] and [\nwarrow, \nearrow]. In the [$\updownarrow, \leftrightarrow$] setup, light polarized in \updownarrow is incident on the sample, and the reflected light is analyzed in \leftrightarrow polarization. Conversely, in the [\nwarrow, \nearrow] configuration, the sample is illuminated with \nwarrow -polarized light and the reflection is analyzed in \nearrow polarization.

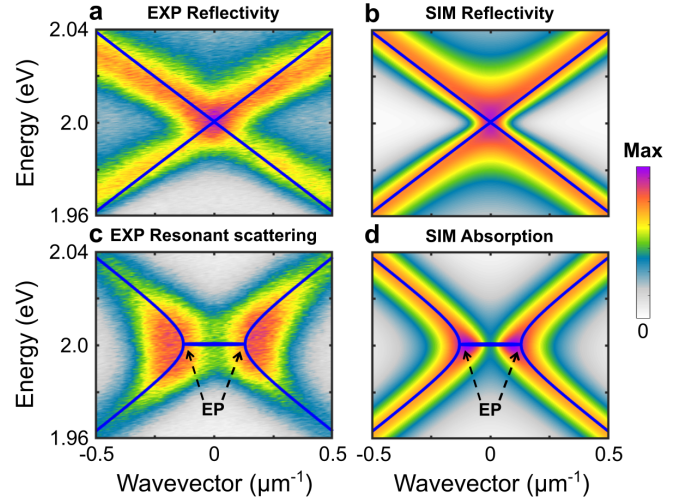


FIG. 3. **Angle-resolved reflectivity measurement.** a) Experimental and b) FEM simulation results of regular reflectivity with \nearrow -polarized excitation. c) Experimental results of resonant scattering measurement in the [$\updownarrow, \leftrightarrow$] configuration. d) FEM simulation results of absorption with \nearrow -polarized excitation. The blue solid line indicates analytical calculation of the band structure with $\gamma = 0$ in (a,b), and $\gamma = 19.9$ meV in (c,d). Each single shot of the measured spectrum is integrated for 100 ms and the monochromator slit is set at 100 μ m. For the absorption simulation, an imaginary component of 0.005 has been added to the refractive index of SiN_x .

We first perform a regular energy-momentum-resolved reflectivity experiment. The sample is oriented so that its x-axis is 45° tilted with respect to the slit of the monochromator, so the collection plane corresponds to the Γ M direction. Since the two modes of interest are of s-polarized light^{29,30}, their polarization for this collection plane is purely \nearrow . Fig. 3a presents the experimental result of regular reflectivity for \nearrow excitation. The corresponding numerical result by FEM is presented in Fig. 3b. Following the reflectivity peak, both experimental and numerical results show Dirac cone degeneracy at $k = 0$ instead of branch degeneracy of EPs. Indeed, they are nicely fitted by the band structure of the Hermitian term of (1), given by $E_0 \pm v \cdot k$ at zero-detuning and in the vicinity of $k = 0$ (see solid blue curves in Figs. 3a,b). Therefore, regular reflectivity cannot provide a direct observation of non-Hermitian degeneracy. These findings are corroborated by the studies of Zhen et al.²⁹. In their earlier research, reflectivity mapping only shows Hermitian degeneracy (i.e. Dirac cones) and EPs only came to light after tedious analysis of reflectivity spectra and fitting with a theoretical model.

Next, we undertake resonant scattering experiments in the

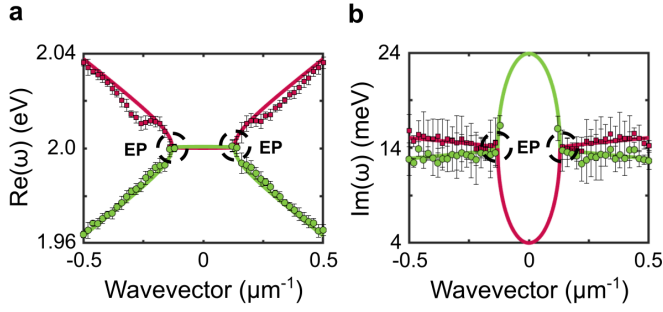


FIG. 4. **Quantitative analysis of EP formation.** a) Real and b) imaginary part of the eigenmodes. Symbols correspond to experimental data extracted from the fitting of resonant scattering spectra. Solid lines correspond to analytical calculation with inhomogeneous broadening $\gamma_i = 4$ meV included. The green color corresponds to the bright mode while the red represents the dark modes.

$[\uparrow, \leftrightarrow]$ configuration. This configuration is chosen since it enables the probing of our photonic modes of pure \nearrow polarization (i.e., \nearrow has a non-zero projection in both \uparrow and \leftrightarrow). As mentioned above, the crossed polarization setup suppresses specular signals and Fano features that hindered the EPs observation in the reflectivity measurements of Figs.3a,b. Therefore, EPs can be directly revealed in the mapping of the resonant scattering experiment. Indeed, Fig.3c presents the experimental results of the energy-momentum-resolved resonant scattering measurement. The real part of eigenmodes from (2) is plotted over the experimental data. It shows that the non-Hermitian bands are revealed in this single-shot experiment by simply monitoring the intensity peaks. In particular, the branch points and momentum gap, hallmarks of EPs, are clearly evidenced, without using any post-treatment. We note that the dominant scattering signal at EPs (Fig.3c) is due to the enhancement of local density of states at these spectral singularities^{24,25}. These features are in agreement with the simulation results of absorption when the structure is excited with \nearrow -polarized light (Fig.3d), a comprehensive comparison between reflectivity and absorption for varied radii is shown in Fig. S3 of the SM. It confirms that resonant scattering techniques can be used to probe EPs in PhC in a similar fashion as active measurements (i.e., photoluminescence or absorption). The broadening of experimental data (Fig.3c) with respect to the FEM results (Fig.3d) can be attributed to an inhomogeneous broadening effect, as discussed in the following.

To analyze quantitatively the formation of EPs in Fig.3c, we extract the real and imaginary parts of the eigenvalues from resonant scattering measurements. This is achieved by fitting the resonances in the spectrum for each wavevector. It is important to note the presence of an inhomogeneous broadening in the measured spectra. This broadening is due to spatial variation of the hole radius, an undesired feature that is typical of laser interference lithography⁴⁶. To account for this inhomogeneous broadening, we used Gaussian profiles instead of Lorentzian ones to fit the resonant scattering spectra. Examples of fitted spectra are shown in Fig. S4 of the SM. Figures 4a and 4b show the real (determined by the position of the resonance) and imaginary (extracted from the linewidth of

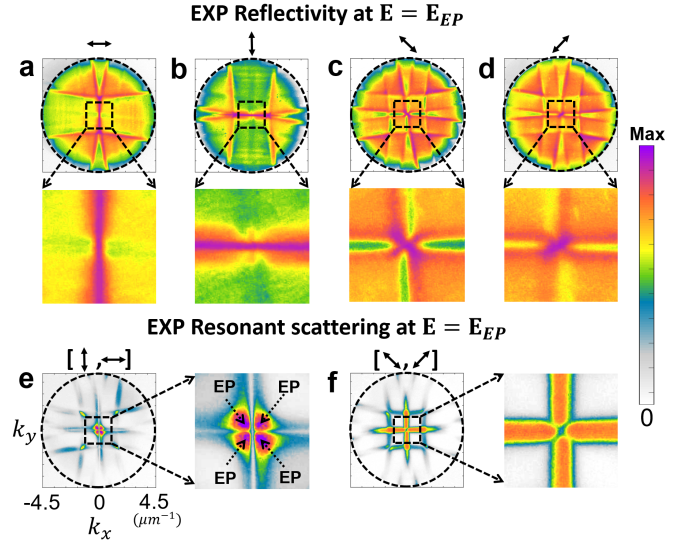


FIG. 5. **Fourier imaging at the EPs energy.** a-d) Experimental results of the regular reflectivity image for four different polarizations of excitation light. e,f) Experimental results of the resonant scattering image in two different polarization configurations. The magnified views close to the Γ point of corresponding Fourier images are shown right below (a-d) and on their right side (e,f). The black dashed circle indicates the limit of the field of view, which corresponds to the numerical aperture $\text{NA} = 0.55$ of the microscope objective. The scale in k_x is equivalent to k_y for all the images. The EPs are points indicated by the black arrow in the magnified picture of (e).

the resonance) parts of the eigenvalues of the band diagram, respectively. Importantly, a simultaneous degeneracy of real and imaginary parts is evidenced at $k = \pm 0.13 \mu\text{m}^{-1}$, thus confirming EPs formation at these wavevectors. We note that the extracted imaginary part, shown in Fig.4b, exhibits an offset of 4 meV compared to that of the numerical design (Fig.1e). This offset accounts for the inhomogeneous broadening discussed above. One may include this broadening into the analytical model by adding an offset $i\gamma_i$ to the diagonal coefficients of the non-Hermitian term in (1), with $\gamma_i = 4$ meV. Once this offset is included, the experimental values align closely with the analytical model (see Figs.4a,b). Further details of the analytical model, including inhomogeneous broadening, are discussed in Fig. S5 of the SM.

Finally, we perform Fourier imaging in the (k_x, k_y) plane at the EPs energy for different polarization configurations for regular reflectivity (Figs.5a-d) and resonant scattering (Figs.5e,f) experiments. This is achieved by adding a band-pass filter (with a finely tuned angle to gain the exact energy of EPs⁴⁷) in the collection path to extract only spectral signals in the vicinity of the EPs and imaging the back focal plane in various polarization configurations. We now focus on the zoom-in images in the vicinity of the Γ point (bottom panels of Figs.5a-d, and right panels of Figs.5e,f). These results clearly show that regular reflectivity mappings exhibit non-zero background from specular reflection, while the background is completely suppressed in resonant scattering mappings. Moreover, all isofrequency maxima curves in Figs.5a-d

go through the Γ point, suggesting a degeneracy at $k = 0$. This is consistent with the results from the energy-momentum measurements of Fig. 3a previously discussed: regular reflectivity only reveals the Hermitian version of the band structures. On the other hand, four local maxima corresponding to four EPs along ΓM directions ($\pm 45^\circ$ planes) are prominently observed in the resonant scattering mapping of the $[\uparrow, \leftrightarrow]$ configuration (Fig. 5e). Such an observation is possible thanks to the local enhancement of the scattering signal at EPs. We also note that horizontal and vertical isofrequency segments are evident in the resonant scattering mapping of the $[\searrow, \swarrow]$ configuration (Fig. 5f). These segments correspond to the slow-light upper-band along the ΓX direction (more details of the band structure along ΓX are provided in Fig. S1 of the SM).

In conclusion, we have introduced an experimental approach for directly observing EPs in PhC structures of hole lattice. By leveraging the crossed polarization technique in momentum-resolved reflectivity, we effectively circumvented the challenges that predominantly stem from the complex interplay between guided resonances and background signals. Our experimental results of momentum-resolved resonant scattering and Fourier images validate the formation of EPs in a patterned SiN_x slab, perfectly aligning with both theoretical predictions and simulations. This streamlined technique, adaptable across various PhC stacks and geometries, promises a powerful asset for diving deeper into non-Hermitian photonics and topology, particularly expanding the horizon of discoveries and applications in advanced photonic platforms.

SUPPLEMENTARY MATERIAL

The supplemental information provides details of the fabrication procedure, calculation of photonic band structures in ΓX and ΓM directions. It also discusses the simulated reflectivity and absorbance with varied hole radii, Gaussian fittings of measured reflectivity, and the broadening of the photonic modes due to the inhomogeneous hole radii.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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