



**HAL**  
open science

# Sensitivity Analysis of Tension Distribution Algorithms for Cable-Driven Parallel Robots

Vincenzo Di Paola, Stéphane Caro, Matteo Zoppi

► **To cite this version:**

Vincenzo Di Paola, Stéphane Caro, Matteo Zoppi. Sensitivity Analysis of Tension Distribution Algorithms for Cable-Driven Parallel Robots. The ASME 2023 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (IDETC/CIE 2023), ASME, Aug 2023, Boston (Massachusetts), United States. hal-04303107

**HAL Id: hal-04303107**

**<https://hal.science/hal-04303107>**

Submitted on 23 Nov 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## SENSITIVITY ANALYSIS OF TENSION DISTRIBUTION ALGORITHMS FOR CABLE-DRIVEN PARALLEL ROBOTS

Vincenzo Di Paola<sup>1,2,\*</sup>, Stéphane Caro<sup>2</sup> and Matteo Zoppi<sup>1</sup>

<sup>1</sup>University of Genova, Genova, Italy

<sup>2</sup>Nantes Université, École Centrale Nantes, CNRS, LS2N, UMR 6004, F-44000 Nantes, France

### ABSTRACT

*Cable-Driven Parallel Robots (CDPRs) constitute a class of robots which displace a load by means of cables. In particular, the use of cables to guide the end-effector continues to be a challenging task. Indeed, several Tension Distribution Algorithms (TDAs) have been developed so far to properly steer the load. Although comparisons between them have been made, no performance index is established to evaluate numerically such methods. Therefore, this paper aims at defining a performance index establishing the sensitivity of a TDA by looking at the maximum tension variation that stems from a change in the external wrench. This would help both comparing and choosing the proper TDA among the existing ones. The definition of the index pops out naturally from the linearization of the equations defining the optimization problems. Commonly used TDAs are compared using the defined index. In order to show an application of it, a planar CDPR is considered and its Wrench-Feasible Workspace (WFW) is characterized by the index defined.*

**Keywords:** Sensitivity, Tension Distribution, Cable-Driven Robot

### NOMENCLATURE

$\mathbf{u}_i$	Cable direction vector $\in \mathbb{R}^3$
$\boldsymbol{\tau}$	Tension vector $\in \mathbb{R}^m$
$\mathbf{w}_e$	External wrench $\in \mathbb{R}^n$
$\mathbf{W}$	Wrench matrix $\in \mathbb{R}^{m \times n}$
$\psi$	Objective function
$\mathbf{h}$	Equality constraints
$\mathbf{g}$	Inequality constraints
$\boldsymbol{\mu}, \boldsymbol{\lambda}$	Lagrangian multipliers
$\mathcal{L}$	Lagrangian
$\sigma$	Sensitivity index

\*Corresponding author: vincenzo.dipaola@edu.unige.it  
 Documentation for asmeconf.cIs: Version 1.34, March 13, 2023.

### 1. INTRODUCTION

Nowadays, Cable-Driven Parallel Robots (CDPRs) constitute a widely known class of robots where rigid links are replaced by flexible cables [1]. This design allows reducing costs and inertia of the system thus guaranteeing the ability to displace a load quickly and within a large workspace [2–5]. These peculiarities make them suitable for many applications ranging from common pick-and-place [6] to search and rescue in natural disasters [4], rehabilitation and collaborative purposes [7].

In particular, the load is steered through the use of cables and winches driven by electric motors; a simplified scheme of these robots is proposed in Fig.1. The tensions exerted along the cables must be positive and bounded above to avoid controlling issues while accomplishing a task. Thus, choosing tension becomes a problem of crucial importance for addressing a given task. Several Tension Distribution Algorithms (TDAs) have been developed so far to solve mentioned problems [8–13]. Most of the time, because of different requirements dictated by the task, the solution to this problem relies on formulating an optimization problem. It is then clear the relevance of this topic for this class of robots and therefore why big efforts have been made hitherto in this direction. The main peculiarities of existing methods were summarized and compared in [10, 13]. However, none of the previous works investigated the sensitivity of these approaches to external disturbances or uncertainties. This question arises naturally since it is of interest assessing *how much* changes in the parameters of an optimization problem affect the optimal solution [14].

In general, sensitivity analysis was already employed for CDPRs. The purposes were somehow different w.r.t. what this paper discusses. Indeed, most of the presented works, regard the evaluation of kinematic performances [15]. Generally speaking, the first indices defined were the *manipulability* index [16] and the *dexterity index* [17] well-known in kinematics. Today, many other works have been done in the kinematic context [5, 18, 19]. In particular, understanding how much cable errors (or joint position

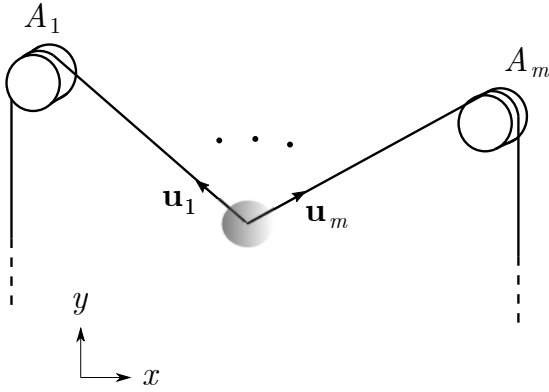


FIGURE 1: PLANAR CDPR: GENERIC PLANAR ARCHITECTURE OF A CDPR WITH A POINT-MASS LOAD.

errors) affect the pose of the end-effector was the key aspect of mentioned contributions. In fact, the definition of different indices makes possible the comparison of various architectures under different perspectives. Thus, these indices help engineers during the design process of the robots.

Other works focus, for instance, on the effects of the assembly errors and dimensional tolerances on the performance of the robots [20].

A recent work that aims to evaluate how a tension error in one cable affects the overall distribution of tensions in the other cables has been presented in [21]. However, this methodology is developed regardless of the existing TDAs.

Hence, a preliminary analysis of the TDAs' sensitivity is hereby proposed. Indeed, the aim of this work consists in introducing an index measuring the maximum cable tension variation within the Wrench Feasible Workspace (WFW) [22] when the external wrench changes. In other words, this index helps estimate the maximum tension variation due to external disturbances. Moreover, it enables comparisons between TDAs. Therefore, as with kinematic indices, this study can guide the choice of the TDA to be used to fulfil a given task.

This paper is structured as follows. Section 2 recalls the main equations governing the CDPRs. Section 3 briefly describes the TDAs used for this work. In Section 4 the methodology used to define the index is described. Section 5 employs a planar CDPR to investigate the sensitivity of the TDAs. Finally, the results are summarized in Section 6.

## 2. SYSTEM MODELING

Let's consider CDPRs with  $m$  cables and a load with  $n$  Degree of Freedom ( $DoF$ ). Moreover, assume  $m > n$  thus, redundant CDPRs are considered. Generally, the Degree of Redundancy ( $DoR$ ) is defined to quickly characterize these classes of CDPRs as  $DoR = m - n \geq 1$ . Thus,  $DoR$  returns the number of redundant cables w.r.t. the  $DoF$ .

Now, the equilibrium of the load is given by

$$\mathbf{W}\boldsymbol{\tau} + \mathbf{w}_e = \mathbf{0}, \quad (1)$$

where  $\mathbf{w}_e$  is the external wrench,  $\boldsymbol{\tau}$  is the cable tension vector and

$\mathbf{W}$  is the wrench matrix<sup>1</sup> which is defined as

$$\mathbf{W} = (\mathbf{u}_1 \quad \dots \quad \mathbf{u}_m), \quad (2)$$

here  $\mathbf{u}_i$  represents the  $i^{th}$  cable direction unitary vector. Thus, if  $DoR \geq 1$ , there exist infinite solutions of Eq.(1) grouped in the following set

$$\Sigma = \{\boldsymbol{\tau} \mid \mathbf{W}\boldsymbol{\tau} + \mathbf{w}_e = \mathbf{0}\}. \quad (3)$$

However, to maintain the equilibrium of the platform the cable tension limits have to be taken into account. Hence, the  $m$ -dimensional convex hypercube  $\Pi$  that defines the domain of the *feasible* tensions is

$$\Pi = \{\boldsymbol{\tau} \mid \mathbf{0} < \underline{\boldsymbol{\tau}} \leq \boldsymbol{\tau} \leq \bar{\boldsymbol{\tau}}\}, \quad (4)$$

where  $\underline{\boldsymbol{\tau}}, \bar{\boldsymbol{\tau}} \in \mathbb{R}^{m,+}$  are positive tension vectors limits containing the  $i^{th}$  lower and upper cable tension limits. Consequently, the set of feasible solutions  $\Gamma$  satisfying both Eq.(1) and Eq.(4), is

$$\Gamma = \Sigma \cap \Pi. \quad (5)$$

## 3. TENSION DISTRIBUTION ALGORITHMS

As anticipated, the problem of choosing a set of cable tensions among infinite solutions was extensively studied. Consequently, several TDAs exist, each with its own peculiarities. As expressed in [23], comparing them is not straightforward as there are many aspects, regarding the algorithms, to be considered. Generally, existing approaches are compared based on [23]: real-time capability, workspace coverage, continuity of the solution, degree of robustness<sup>2</sup> and generality of the approach based on the achievable  $DoR$ .

However, many other aspects should be considered and this motivates the necessity to characterise each method succinctly and effectively. Although not all the features can be compared at once, some of them can be evaluated by means of an index; introduced in the following Section 4.

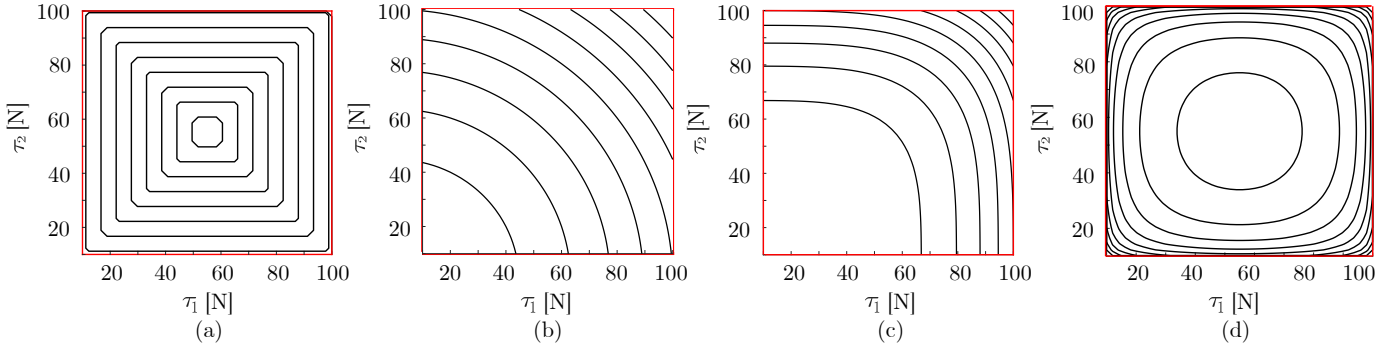
The aim of this section is to recall a few methods that will be investigated in terms of sensitivity. Among the existing approaches to compute cable tension solving an optimization problem, the choice lies in the Linear Programming [8], Quadratic Programming [11], 4-norm Programming [12] and the Analytic Centre method [13]. These methods have distinct features in terms of robustness and continuity of the solutions that make them interesting to examine.

### 3.1 Linear Programming (LP)

The Linear Programming method is employed to ensure a robust solution and because of its computational speed in solving the tension distribution problem with generic (any  $DoR$ ) CDPRs. In other words, the aim is to find a tension vector inside  $\Gamma$  as far as possible from the boundary of the hypercube  $\Pi$ . To achieve this, the objective function is built such that the distance  $d(\boldsymbol{\tau})$  of the

<sup>1</sup>Only planar CDPRs with point-mass load are considered in this work.

<sup>2</sup>By definition, a TDA is said to be *robust* if the provided solution is far from the boundary of  $\Pi$ .



**FIGURE 2: LEVEL SET: THE CONTOURS SHOW THE LEVEL-SETS INSIDE THE 2D BOX  $\Pi$  (RED LINES). IN PARTICULAR, FIGURE (A) DEPICTS THE LP METHOD, (B) THE QP METHOD, (C) THE 4-NORM AND (D) THE ANALYTIC CENTRE. OBSERVE THAT THEIR SHAPE INFLUENCES THE SOLUTION OF THE OPTIMIZATION PROBLEM WHEN INTERSECTED WITH THE AFFINE SPACE EQ. (1).**

tension vector  $\tau$ , from the facets of  $\Pi$ , is maximized. Formally, the optimization problem reads as

$$\begin{aligned} \operatorname{argmax}_{\tau \in \Pi} \quad & d(\tau), \\ \mathbf{W}\tau + \mathbf{w}_e = \mathbf{0} \end{aligned} \quad (6)$$

where the objective function is

$$d(\tau) = \min\{\bar{\tau} - \tau_1, \tau_1 - \underline{\tau}, \dots, \bar{\tau} - \tau_m, \tau_m - \underline{\tau}\}. \quad (7)$$

This formulation ensures reaching the maximum, among the existing techniques, robustness of the tension profiles. However, tension profiles are prone to discontinuities when a small change in the end-effector position occurs.

### 3.2 Quadratic Programming (QP)

A widespread approach solving the problem of tension discontinuity [8] while allowing reducing the energy consumption amounts to minimize the 2-norm of the cable tension vector

$$\begin{aligned} \operatorname{argmin}_{\tau \in \Pi} \quad & \|\tau\|_2^2, \\ \mathbf{W}\tau + \mathbf{w}_e = \mathbf{0} \end{aligned} \quad (8)$$

Despite the energy consumption is reduced, the system inherits low stiffness and therefore, the accuracy of the task can be affected by vibrations and oscillations of the platform. Moreover, the robustness of this method is the lowest possible as the solution always lies on the boundary of  $\Pi$ .

### 3.3 4-Norm Programming

An optimisation problem that turns out to be a suitable candidate for clarifying some of the characteristics of the index that will be defined in what follows, consists in minimising the 4-norm instead of the 2-norm (QP). The mathematical formulation takes the following form

$$\begin{aligned} \operatorname{argmin}_{\tau \in \Pi} \quad & \|\tau\|_4^4, \\ \mathbf{W}\tau + \mathbf{w}_e = \mathbf{0} \end{aligned} \quad (9)$$

which is quite similar to the previous one (QP). In other words, one could generically formulate all the above problems by using the  $p$ -norm  $1 \leq p \leq \infty$ . However, for sake of clarity, the methods were synthetically, although explicitly, reported.

### 3.4 Analytic Centre Solution

Guaranteeing robust, continuous and differentiable tension profiles for a system with generic  $DoR$  is, generally, the purpose intended to be achieved by the various existing TDAs. However, they do not succeed in satisfying them all contemporary. This motivated the introduction of the Analytic Centre. Indeed, it was introduced as an attempt to take the strengths of the various methods presented in the literature. Essentially, to achieve mentioned goal, a new objective function was chosen, it exploits the properties of logarithms (or barrier functions), as explained in [13]. Formally, the objective function has the form

$$\phi(\tau) = \sum_{i=1}^m -\log(\bar{\tau} - \tau_i) - \log(\tau_i - \underline{\tau}), \quad (10)$$

which is one particular case of the general one while the problem can be formulated as

$$\begin{aligned} \operatorname{argmin} \quad & \phi(\tau), \\ \mathbf{W}\tau + \mathbf{w}_e = \mathbf{0} \end{aligned} \quad (11)$$

Observe that, apart from ensuring continuity, differentiability and robustness of the tension profiles, it eliminates tension inequalities simplifying the problem formulation and numerical resolution.

The main differences among the reported methods can be visualized in Fig. 2 where the level-sets generated by the different objective functions are explicitly depicted. Their shape is fundamental to understand how the tension solution is selected when intersected with Eq. (1).

#### 4. PERFORMANCE INDEX

The introduction of an index measuring the sensitivity of the solution provided by the TDAs is a necessary condition to make comparisons among them.

To reach this aim, it becomes of primary importance to estimate the variation of the tension vector  $\delta\boldsymbol{\tau}$  w.r.t. a variation of the external wrench  $\delta\mathbf{w}_e$ . Therefore, taking advantage of the Karush–Kuhn–Tucker (KKT) theorem [25], it becomes possible to transform a generic optimization problem into a system of equations of the form

$$f(\boldsymbol{\tau}, \mathbf{W}, \mathbf{w}_e) = \begin{cases} \frac{\partial \mathcal{L}}{\partial \boldsymbol{\tau}} = 0 \\ \mathbf{h}(\boldsymbol{\tau}) = \mathbf{0} \\ \mathbf{g}(\boldsymbol{\tau}) \leq \mathbf{0}, \end{cases} \quad (12)$$

where  $h(\boldsymbol{\tau})$  and  $g(\boldsymbol{\tau})$  represent the equality and inequality constraints, respectively while  $\mathcal{L}$  is the Lagrangian function, namely

$$\mathcal{L} = \psi(\boldsymbol{\tau}) + \boldsymbol{\mu}^T \mathbf{g}(\boldsymbol{\tau}) + \boldsymbol{\lambda}^T \mathbf{h}(\boldsymbol{\tau}), \quad (13)$$

with  $\psi(\boldsymbol{\tau}) : \mathbb{R}^m \rightarrow \mathbb{R}$  objective function or cost function to be minimized (or maximized). It is generally related to the optimization problem considered and can assume different forms, e.g. expression of  $\phi(\boldsymbol{\tau})$  Eq. (10) used in Eq. (11). Remaining symbols  $\boldsymbol{\mu}$  and  $\boldsymbol{\lambda}$  are known as the KKT multipliers.

Now, linearizing Eq.(12) gives

$$\frac{\partial f}{\partial \boldsymbol{\tau}} \delta\boldsymbol{\tau} + \frac{\partial f}{\partial \mathbf{w}_e} \delta\mathbf{w}_e = \mathbf{0}. \quad (14)$$

Rearranging this equation, the link between  $\delta\boldsymbol{\tau}$  and  $\delta\mathbf{w}_e$  is found, in compact form

$$\delta\boldsymbol{\tau} = -\left(\frac{\partial f}{\partial \boldsymbol{\tau}}\right)^\dagger \frac{\partial f}{\partial \mathbf{w}_e} \delta\mathbf{w}_e, \quad (15)$$

then, calling

$$\mathbf{S} = \left(\frac{\partial f}{\partial \boldsymbol{\tau}}\right)^\dagger \frac{\partial f}{\partial \mathbf{w}_e} \quad (16)$$

one has

$$\delta\boldsymbol{\tau} = -\mathbf{S} \delta\mathbf{w}_e, \quad (17)$$

where  $\mathbf{S}$  is the sensitivity matrix, formally

$$\mathbf{S} = \begin{pmatrix} \frac{\partial \tau_1}{\partial w_{e,1}} & \cdots & \frac{\partial \tau_1}{\partial w_{e,n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \tau_m}{\partial w_{e,1}} & \cdots & \frac{\partial \tau_m}{\partial w_{e,n}} \end{pmatrix} \quad i = 1, \dots, m \quad j = 1, \dots, n \quad (18)$$

where the indices  $i, j$  identify the components of  $\mathbf{S}$  as well as those of the vectors  $\boldsymbol{\tau}$  and  $\mathbf{w}_e$ .

At this point, all the elements necessary for the definition of the performance index have been collected. Consequently, borrowing the idea from [18] and considering  $\delta\boldsymbol{\tau}$  as if it were the variable of the joint space while  $\delta\mathbf{w}_e$  represents the term in the Cartesian space, respectively, the sensitivity index can be introduced as

$$\sigma = \max_{\|\delta\mathbf{w}_e\|_2=1} \|\delta\boldsymbol{\tau}\|_\infty. \quad (19)$$

This, in complete analogy with the kinematic indices [18, 19, 26]. It was proven that optimization problem Eq.(19) can be directly solved substituting Eq.(17) into Eq.(19) to obtain

$$\sigma = \max_{\|\delta\mathbf{w}_e\|_2=1} \|\mathbf{S} \delta\mathbf{w}_e\|_\infty = \|\mathbf{S}\|_{2,\infty}. \quad (20)$$

where the matrix norm at RHS is known as *mixed Hölder norm* [27].

Practically, since the columns of the matrix  $\mathbf{S}$  identify the variation in the tension of each cable w.r.t. a variation of the wrench in a specific direction, this index estimates the maximum (upper bound) tension variation induced by a change in the external wrench. This can help quantify *how much* can be the variation in the tension along the cables. Observe that measuring cable tension experimentally is a hard task. Therefore, the possibility to gain some information (upper bound on  $\delta\boldsymbol{\tau}$ ) based on knowledge of some disturbances can be useful when measuring cable tension with sensors.

**Remark 4.1** *Taking a point-mass as end-effector only requires taking into account the resultant force acting on the load. Hence no dimensional unit issue arises. In case the load is a rigid body, then two separate indices can be used [26].*

**Remark 4.2** *The linearized Equation (17) pops out to be similar to the closed-form solution  $\delta\boldsymbol{\tau} = -\mathbf{W}^\dagger \delta\mathbf{w}_e$ . However, the procedure here developed can be considered a generalization of it in the context of sensitivity. Indeed, taking the derivatives of Eq.(12) appearing in the definition of matrix  $\mathbf{S}$  Eq.(16), one can find the explicit form*

$$\mathbf{S} = \begin{pmatrix} \nabla_{\boldsymbol{\tau}}^2 \psi \\ \mathbf{W} \\ \mathbb{I} \\ -\mathbb{I} \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} \\ \mathbb{I} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \quad (21)$$

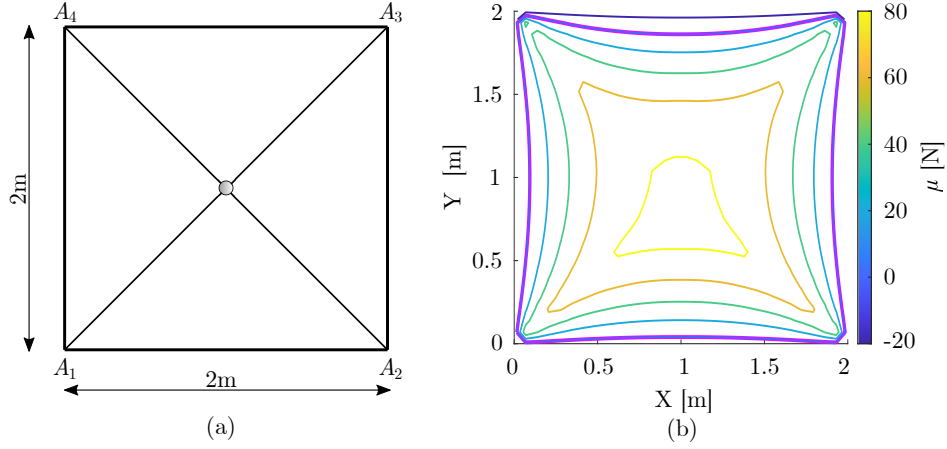
where  $\nabla_{\boldsymbol{\tau}}^2 \psi$  is the second-order gradient<sup>3</sup> of the objective function  $\psi(\boldsymbol{\tau})$  w.r.t. the tension vector  $\boldsymbol{\tau}$  while  $\mathbb{I}$  identifies the identity matrix. Hence, from Eq.(21), one can observe that  $\mathbf{S}$  reduces to  $\mathbf{W}^\dagger$  when no optimization problem is considered, i.e. when  $\psi(\boldsymbol{\tau})$  and  $g(\boldsymbol{\tau})$  are not considered. Observing this analogy stresses the familiar relationship between the solution provided by the closed-form (pseudo-inverse) and the QP approach. Indeed, the solution with pseudo-inverse leads to minimize the distance from the origin  $\boldsymbol{\tau} = \mathbf{0}$ . The main difference with QP, then is the absence of tension limits  $\underline{\boldsymbol{\tau}}, \bar{\boldsymbol{\tau}}$ .

#### 5. NUMERICAL EXAMPLE

In this section, some simulations are performed to compare the TDAs recalled in Section 3. First of all, to evaluate the index,

<sup>3</sup>Observe that computing the derivative of the Lagrangian function w.r.t.  $\boldsymbol{\tau}$  resembles in computing the second-order gradient of the objective directly.





**FIGURE 3: CASE STUDY: (A) PLANAR CDPR WITH FOUR CABLES AND A POINT-MASS LOAD WHILE (B) SHOWS THE WFW OF THE CDPR. OBSERVE THAT THE WFW IS FOUND BY USING THE CAPACITY MARGIN  $\mu$  [24], THE PURPLE CURVE  $\mu = 0$  DELIMITS ITS BOUNDARY.**

the workspace of the robot is discretized and only the points inside the WFW are considered. Subsequently, for every feasible point, the index is computed. The value assumed by the index  $\sigma$  is assigned to the point considered and a map is generated within the workspace. This process is repeated for each TDA reported above.

In the following discussion, a planar CDPR with four cables is considered. The load is a point mass ( $DoR = 2$ ) of mass  $m = 1kg$ . Cable tension limits are set to be  $\underline{\tau} = 10N$  and  $\bar{\tau} = 100N$ , respectively. The architecture of the robot and its WFW are depicted in Figure 3 for sake of clarity. Results of the simulations and the characterization of the WFW by means of  $\sigma$  are depicted in Fig. 4.

First, from Fig. 4 one can understand that the AC method results to be more sensitive than all the other methods. This means that, around each point of the WFW, the LP, QP and 4-norm generate a  $\|\delta\tau\|$  that is lower than the one of the AC for a given  $\|\delta\mathbf{w}_e\|_2 = 1$ .

To fully grasp why this happens, one should focus on both *how the TDAs work* and *what happens inside the sensitivity matrix*, i.e., how the index works. With this in mind, let's pair the methods by affinity in order to shed light on the results and discuss the above-mentioned aspects. Hence, consider the QP and 4-norm together and LP and AC consequently<sup>4</sup>.

Now, starting from QP and 4-norm and analysing their results, one sees that some of the cable tensions assume the lowest values possible. In particular, for this case study, two tensions  $\tau_1, \tau_2$  (attached at the bottom of the robot) take the lowest bound  $\underline{\tau}$  as they do not bear any wrench. Therefore, only the remaining components  $\tau_3, \tau_4$  vary while the load moves inside the workspace. However,  $\tau_3, \tau_4$  vary as less as possible (because of how the problem is defined), they just have to maintain the equilibrium of the load. This explains why the sensitivity  $\sigma$  assumes similar values ( $\sigma < 0.38$ ) inside the WFW. However, looking at how QP and 4-norm approaches work, is not enough to understand why maps Figs.4 (b)-(c) are different. Indeed, this is due to the values assumed by  $\nabla_{\tau}^2\psi$  inside matrix  $\mathbf{S}$ . Computing the

second-order gradient for both

$$\nabla_{\tau}^2\psi_{QP} = 2\mathbb{I}_m \quad \nabla_{\tau}^2\psi_{4-norm} = 12 \text{diag}(\tau_i)^2, \quad (22)$$

shows that  $\nabla_{\tau}^2\psi_{QP}$  is constant, i.e., it does not depend on the solution  $\tau$  while  $\nabla_{\tau}^2\psi_{4-norm}$  does. As a consequence, the  $\sigma$ -map is symmetric for the QP and it does not change if, for example, the external wrench changes.

Analogously, the same reasoning can be done for LP and AC. However, this time the values of  $\sigma$  are significantly different: the sensitivity of AC is far more higher than LP (QP and 4-norm as well). The reason why lies again in the second-order gradient (or matrix  $\mathbf{S}$ ). Indeed, in both cases the solution  $\tau$  is robust but

$$\nabla_{\tau}^2\psi_{LP} = 0 \quad (23)$$

results to be null while

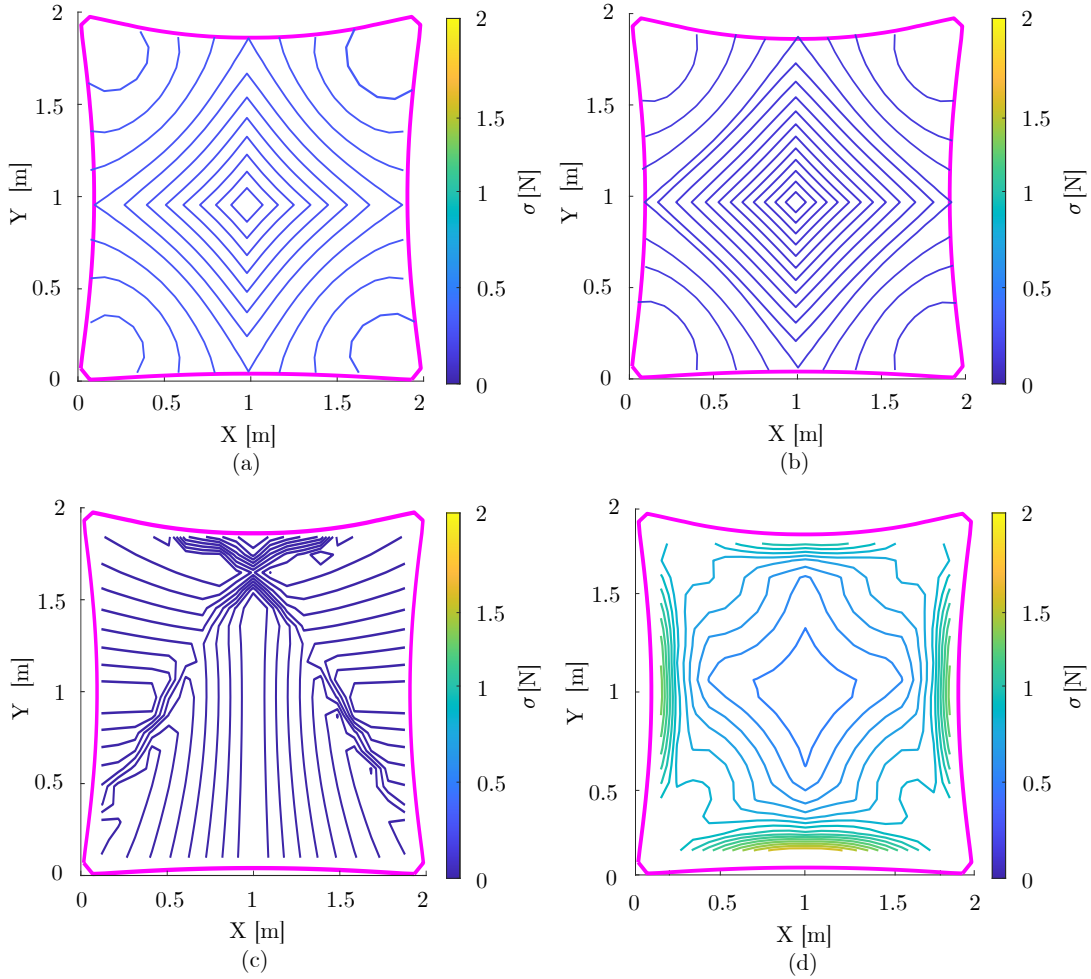
$$\nabla_{\tau}^2\psi_{AC} = \text{diag}\left(\frac{1}{(\bar{\tau} - \tau_i)^2} - \frac{1}{(\tau_i - \underline{\tau})^2}\right) \quad (24)$$

depends on  $\tau$ . Hence, because of the shape of  $\nabla_{\tau}^2\psi_{AC}$ , its values take weight inside  $\mathbf{S}$  generating the highest  $\sigma$  among LP, QP and 4-norm.

Other useful data can be acquired by computing the (numerical) gradient of  $\sigma$  namely,  $\nabla\sigma$  reported in Fig.5. This information can be relevant when the task requires precision: no vibration or oscillation of the platform. In other words, it can be useful when a small variation of  $\nabla\sigma$  is required. Indeed, in practice,  $\nabla\sigma$  quantifies the possible tension variation between two points of the workspace. Not surprisingly, the AC method has smaller  $\nabla\sigma$  than the others (smallest and homogeneous arrows inside the WFW). In fact, the AC homogeneity, in terms of  $\nabla\sigma$ , is probably linked to the property of smoothness of the tension profiles. Recall that smoothness of tension profiles means that there is a small variation in the cable tensions from one point of the workspace to another.

Collecting the information acquired from this case study, one can conclude  $\sigma$  allows for estimating an upper bound of  $\delta\tau$ , assessing robustness as well as observing the continuity of the

<sup>4</sup>The affinity can be understood by looking at the level-set reported in Figure 2.



**FIGURE 4: SENSITIVITY:  $\sigma$ -MAP WITHIN THE WFW FOR THE LP METHOD (A), THE QP METHOD (B), THE 4-NORM (C) AND THE ANALYTIC CENTRE ONES (D).**

solution looking at  $\nabla\sigma$ . Hence, many of the aspects listed in Sec.3 can be somehow visualized with this index.

Although the definition of  $\sigma$  is analogous to the well-established performance index for kinematics, some information is lost due to the linearization process. Indeed, with this definition, for example, it becomes impossible to assess the sensitivity of the single cables since it is absorbed by the use of the norm  $\|\delta\tau\|$ .

## 6. CONCLUSION

A first performance index estimating the sensitivity of TDAs was defined in this paper. The results of the simulations and the maps reported show that the index is able to furnish information about the variation of tension due to a change in the external wrench. Moreover, it tells that the homogeneity of the  $\sigma$ -gradient is linked to the ability of the TDA in providing smooth tension profiles. Although it seems to be an effective index that can help in choosing the TDA, some information is lost. Therefore, it will be necessary to dedicate future efforts to improve or edit the definition in order to recover information about single cable sensitivity. Other tests are also necessary to better understand the peculiarities of this index for instance one should consider the

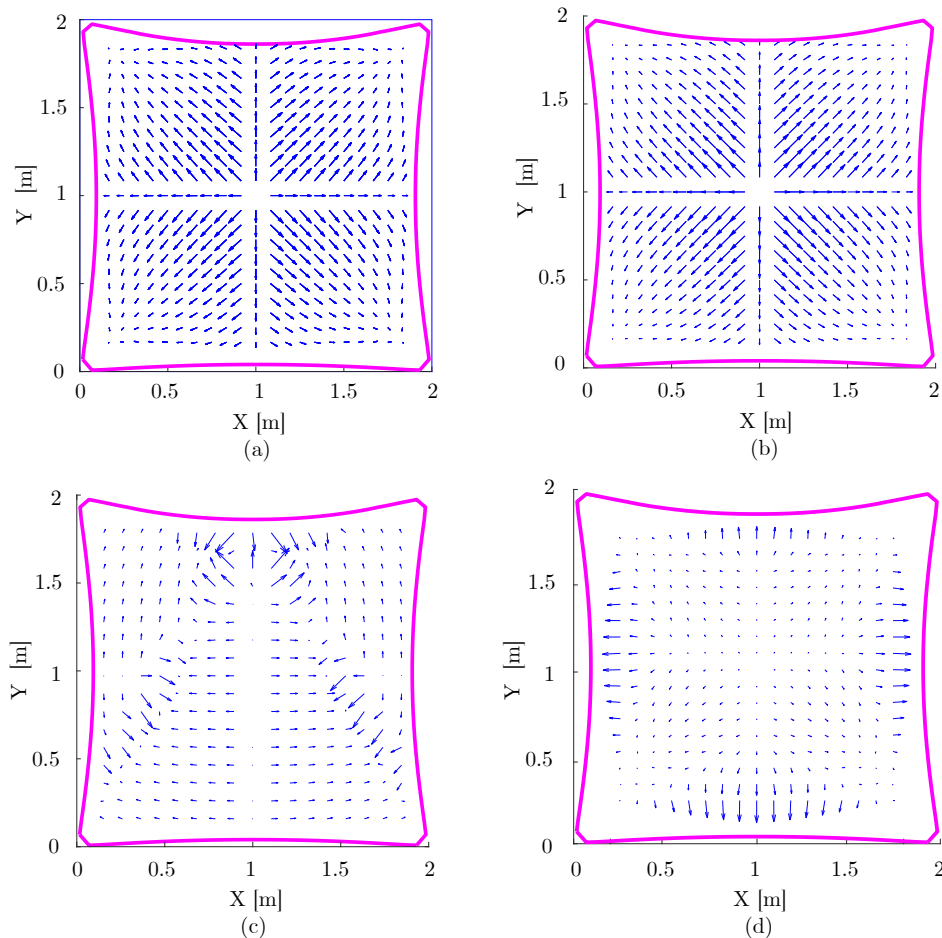
orientation of the platform and investigate a spatial case as well. Moreover, experimental tests should be undertaken to validate the proposed method.

## ACKNOWLEDGMENTS

This work has been partially supported by ROBOTEX 2.0 (Grants ROBOTEX ANR-10-EQPX-44-01 and TIRREX ANR-21-ESRE-0015).

## REFERENCES

- [1] Bruckmann, Tobias and Pott, Andreas. “Cable-Driven Parallel Robots.” *Springer Tracts in Advanced Robotics* (2018)DOI <https://doi.org/10.1007/978-3-319-76138-1>.
- [2] Albus, J., Bostelman, R. and Dagalakis, N. “The NIST ROBOCRANE.” *Journal of Robotics System* (1993).
- [3] Cone, Lawrence L. “Skycam: An Aerial Robotic Camera System.” *BYTE* (1985).
- [4] Bosscher, Paul, Williams, II, Robert L. and Tummino, Melissa. “A Concept for Rapidly-Deployable Cable Robot Search and Rescue Systems.” *International Design Engineering Technical Conferences and Computers*



**FIGURE 5: GRADIENT:  $\nabla\sigma$ -MAP WITHIN THE WFW FOR THE LP METHOD (A) THE QP METHOD (B) THE 4-NORM (C) AND THE ANALYTIC CENTRE ONES (D). OBSERVE THAT THE SIZE OF THE ARROWS IDENTIFIES THE MAGNITUDE OF THE GRADIENT.**

- and Information in Engineering Conference (2005)DOI [10.1115/DETC2005-84324](https://doi.org/10.1115/DETC2005-84324).
- [5] Merlet, J. and Daney, D. “A portable, modular parallel wire crane for rescue operations.” *IEEE International Conference on Robotics and Automation* (2010)DOI [10.1109/ROBOT.2010.5509299](https://doi.org/10.1109/ROBOT.2010.5509299).
- [6] Kawamura, Sadao, Choe, Won, Tanaka, Satoshi and Kino, Hitoshi. “Development of an Ultrahigh Speed Robot FALCON Using Parallel Wire Drive Systems.” *Journal of the Robotics Society of Japan* (1997).
- [7] Métilon, Marceau, Charron, Camilo, Subrin, Kévin and Caro, Stéphane. “Performance and interaction quality variations of a collaborative Cable-Driven Parallel Robot.” *Mechatronics* (2022)DOI <https://doi.org/10.1016/j.mechatronics.2022.102839>.
- [8] Borgstrom, Per Henrik, Jordan, Brett L., Sukhatme, Gaurav S., Batalin, Maxim A. and Kaiser, William J. “Rapid Computation of Optimally Safe Tension Distributions for Parallel Cable-Driven Robots.” *IEEE Transactions on Robotics* (2009)DOI [10.1109/TRO.2009.2032957](https://doi.org/10.1109/TRO.2009.2032957).
- [9] Gouttefarde, Marc, Lamaury, Johann, Reichert, Christopher and Bruckmann, Tobias. “A Versatile Tension Distribution Algorithm for  $n$ -DOF Parallel Robots Driven by  $n + 2$  Cables.” *IEEE Transactions on Robotics* (2015)DOI [10.1109/TRO.2015.2495005](https://doi.org/10.1109/TRO.2015.2495005).
- [10] Pott, Andreas, Bruckmann, Tobias and Mikelson, Lars. “Closed-form Force Distribution for Parallel Wire Robots.” *Kecskeméthy, A., Müller, A. Computational Kinematics. Springer, Berlin, Heidelberg* (2009).
- [11] Taghirad, Hamid D. and Bedoustani, Yousef B. “An Analytic-Iterative Redundancy Resolution Scheme for Cable-Driven Redundant Parallel Manipulators.” *IEEE Transactions on Robotics* (2011)DOI [10.1109/TRO.2011.2163433](https://doi.org/10.1109/TRO.2011.2163433).
- [12] Gosselin, C. and Grenier, M. “On the determination of the force distribution in overconstrained cable-driven parallel mechanisms.” *Meccanica* (2011)DOI <https://doi.org/10.1007/s11012-010-9369-x>.
- [13] Di Paola, Vincenzo, Goldsztejn, Alexandre, Zoppi, Matteo and Caro, Stéphane. “Analytic Center Based Tension Distribution for Cable-Driven Platforms (CDPs).” *Working paper or preprint* (2023)URL <https://hal.science/hal-04023175>.
- [14] Castillo, Enrique, Mínguez, Roberto and Castillo, Carmen. “Sensitivity analysis in optimization and reliability problems.” *Reliability Engineering and System Safety* (2008)DOI <https://doi.org/10.1016/j.res.2008.03.010>.



- [15] Patel, Sarosh and Sobh, Tarek. "Manipulator performance measures-a comprehensive literature survey." *Journal of Intelligent & Robotic Systems* (2015).
- [16] Yoshikawa, Tsuneo. "Analysys and control of robot manipulators with redundancy." *Proc. of the 1st Int. Symp. Robotics Research* (1984).
- [17] Salisbury, J Kenneth and Craig, John J. "Articulated hands: Force control and kinematic issues." *The International journal of Robotics research* (1982).
- [18] Olds, Kevin C. "Global indices for kinematic and force transmission performance in parallel robots." *IEEE Transactions on Robotics* (2015).
- [19] Cardou, Philippe, Bouchard, Samuel and Gosselin, Clément. "Kinematic-sensitivity indices for dimensionally non-homogeneous jacobian matrices." *IEEE Transactions on Robotics* (2010).
- [20] Aref, Mohammad, Gholami, Pooneh and Taghirad, Hamid. "Dynamic and sensitivity analysis of KNTU CDRPM: A cable driven redundant parallel manipulator." *IEEE/ASME International Conference on Mechatronics and Embedded Systems and Applications* (2008)DOI [10.1109/MESA.2008.4735650](https://doi.org/10.1109/MESA.2008.4735650).
- [21] Mattioni, V., Ida, E. and Carricato, M. "Force-distribution sensitivity to cable-tension errors in overconstrained cable-driven parallel robots." *Mechanism and Machine Theory* (2022)DOI <https://doi.org/10.1016/j.mechmachtheory.2022.104940>.
- [22] Bouchard, S., Gosselin, C. and Moore, B. "On the ability of a cable-driven robot to generate a prescribed set of wrenches." *Journal of Mechanisms and Robotics* (2010)DOI [10.1115/1.4000558](https://doi.org/10.1115/1.4000558).
- [23] Pott, Andreas. "An improved force distribution algorithm for over-constrained cable-driven parallel robots." *6th International Workshop on Computational Kinematics (CK)* (2013).
- [24] Ruiz, Ana Lucia Cruz, Caro, Stéphane, Cardou, Philippe and Guay, Francois. "ARACHNIS: Analysis of Robots Actuated by Cables with Handy and Neat Interface Software." *2nd International Conference on Cable-Driven Parallel Robots* (2015).
- [25] Boyd, Stephen and Vandenberghe, Lieven. "Convex optimization." *Cambridge university press* (2004).
- [26] Mottola, Giovanni, Gosselin, Clement and Carricato, Marco. "Effect of actuation errors on a purely-translational spatial cable-driven parallel robot." *IEEE 9th Annual International Conference on CYBER Technology in Automation, Control, and Intelligent Systems (CYBER)* (2019).
- [27] Bernstein, Dennis S. "Matrix mathematics." *Princeton university press* (2009).