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# Macroscopic dynamic capillary pressure for two-phase flow in porous media

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## 1. Introduction

A closed predictive model for the macroscopic pressure difference, often referred to as the *macroscopic capillary pressure*, for two-phase flow in homogeneous porous media is of major importance to form a complete set of governing macroscale equations. This has been a long-lasting subject of research during the past 40 years [1, 3], and the majority of the existing models remain empirical at some level of their derivation. A new closed macroscopic dynamic capillary pressure equation is reported in this work on the basis of the existence of a representative (periodic) unit cell able to locally describe microscale momentum transport in the system. This is carried out with an adjoint method together with a Green's formulation, requiring no other simplifying assumption.

## 2. Upscaled dynamic capillary pressure

The general framework of the analysis is the incompressible, creeping flow of two Newtonian fluids  $\alpha$  and  $\beta$  saturating a homogeneous porous medium whose rigid and spatially homogeneous solid skeleton is the  $\sigma$ -phase.

### 2.1. Pore-scale and adjoint problems

The pore-scale flow problem can be written in a periodic unit cell representative of the domain as follows ( $\alpha = \beta, \gamma$ )

$$\nabla \cdot \mathbf{v}_\alpha = 0, \quad \text{in } \mathcal{V}_\alpha, \quad (1a)$$

$$\mathbf{0} = \nabla \cdot \mathbf{T}_{\tilde{p}_\alpha} - \nabla \mathcal{P}_\alpha, \quad \text{in } \mathcal{V}_\alpha, \quad (1b)$$

$$\mathbf{v}_\beta = \mathbf{v}_\gamma, \quad \text{at } \mathcal{A}_{\beta\gamma}, \quad (1c)$$

$$\mathbf{n}_{\beta\gamma} \cdot \mathbf{T}_{\tilde{p}_\beta} = \mathbf{n}_{\beta\gamma} \cdot \mathbf{T}_{\tilde{p}_\gamma} + \mathbf{n}_{\beta\gamma} (\langle p_\beta \rangle^\beta - \langle p_\gamma \rangle^\gamma)_{\mathbf{r}_{\beta\gamma}} + 2H\gamma \mathbf{n}_{\beta\gamma} + \nabla_s \gamma, \quad \text{at } \mathcal{A}_{\beta\gamma}, \quad (1d)$$

$$\mathbf{v}_\alpha = \mathbf{0}, \quad \text{at } \mathcal{A}_{\alpha\sigma}, \quad (1e)$$

$$\mathbf{v}_\alpha|_{\mathcal{S}_{\alpha i}^-} = \mathbf{v}_\alpha|_{\mathcal{S}_{\alpha i}^+}, \quad i = 1, \dots, N, \quad (1f)$$

$$(\mathbf{n}_\alpha \cdot \mathbf{T}_{\tilde{p}_\alpha})_{\mathcal{S}_{\alpha i}^-} = -(\mathbf{n}_\alpha \cdot \mathbf{T}_{\tilde{p}_\alpha})_{\mathcal{S}_{\alpha i}^+}, \quad i = 1, \dots, N, \quad (1g)$$

$$\langle \tilde{p}_\alpha \rangle^\alpha = 0. \quad (1h)$$

Here,  $\mathbf{v}_\alpha$  is the velocity in the  $\alpha$ -phase, in which the pressure,  $p_\alpha$ , is decomposed as  $p_\alpha = \langle p_\alpha \rangle^\alpha + \tilde{p}_\alpha$ ,  $\mathbf{T}_{\tilde{p}_\alpha} = -\tilde{p}_\alpha \mathbf{I} + \mu_\alpha (\nabla \mathbf{v}_\alpha + \nabla \mathbf{v}_\alpha^T)$ ,  $\nabla \mathcal{P}_\alpha = \nabla \langle p_\alpha \rangle^\alpha - \rho_\alpha \mathbf{b}_\alpha$ ,  $H$  is the double mean curvature of the fluid-fluid interface,  $\mathcal{A}_{\beta\gamma}$ , which unit normal directed from the  $\beta$  phase to the  $\gamma$  phase is  $\mathbf{n}_{\beta\gamma}$ ,  $\mu_\alpha$  and  $\gamma$  are the dynamic viscosity and interfacial tension and  $\nabla_s \gamma$  stands for possible Marangoni effects;  $\mathcal{S}_{\alpha i}^\pm$  are the entrances/exits of the  $\alpha$ -phase at the  $i$ th edge of the periodic unit cell.

For the derivation of the upscaled model, an adjoint problem is now considered that writes

$$\nabla \cdot \mathbf{f}_\beta = \frac{1}{V_\beta}, \quad \text{in } \mathcal{V}_\beta, \quad \nabla \cdot \mathbf{f}_\gamma = -\frac{1}{V_\gamma}, \quad \text{in } \mathcal{V}_\gamma, \quad (2a)$$

$$\mathbf{0} = \nabla \cdot \mathbf{T}_{f_\alpha}, \quad \text{in } \mathcal{V}_\alpha, \quad (2b)$$

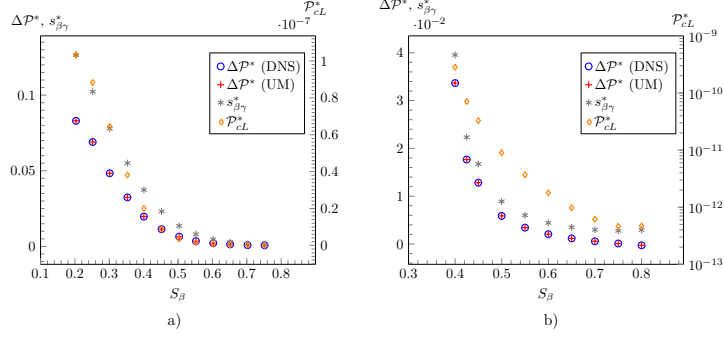


Figure 1:  $\Delta\mathcal{P}^*$ , obtained from direct numerical simulation (DNS) and predicted from the upscaled equation (3) (UE), along with the interfacial term contribution,  $s_{\beta\gamma}^*$ , and classical Laplace capillary pressure,  $\mathcal{P}_{cL}^*$  (see text), versus the wetting phase saturation,  $S_\beta$ . a)  $\mu^* = 0.1$ ,  $Ca = 10$ ; b)  $\mu^* = 10$ ,  $Ca = 0.1$ .  $\varepsilon = 0.8$ .

$$\mathbf{f}_\beta = \mathbf{f}_\gamma, \quad \mathbf{n}_{\beta\gamma} \cdot \mathbf{T}_{f_\beta} = \mathbf{n}_{\beta\gamma} \cdot \mathbf{T}_{f_\gamma}, \quad \text{at } \mathcal{A}_{\beta\gamma}, \quad (2c)$$

$$\mathbf{f}_\alpha = \mathbf{0}, \quad \text{at } \mathcal{A}_{\alpha\sigma}, \quad (2d)$$

$$\mathbf{f}_\alpha|_{\mathcal{S}_{\alpha i}^-} = \mathbf{f}_\alpha|_{\mathcal{S}_{\alpha i}^+}, \quad (\mathbf{n}_\alpha \cdot \mathbf{T}_{f_\alpha})_{\mathcal{S}_{\alpha i}^-} = -(\mathbf{n}_\alpha \cdot \mathbf{T}_{f_\alpha})_{\mathcal{S}_{\alpha i}^+}, \quad i = 1, \dots, N, \quad (2e)$$

$$f_\alpha = 0, \quad \text{at } \mathbf{r}_\alpha^0. \quad (2f)$$

## 2.2. Macroscopic capillary pressure

Following the procedure detailed in [2], a Green's formula can be used to combine the physical flow problem and the adjoint (closure) problem that leads to the following macroscopic equation for the dynamic capillary pressure

$$\Delta\mathcal{P} = \langle p_\gamma \rangle^\gamma|_{\mathbf{x}_\gamma} - \langle p_\beta \rangle^\beta|_{\mathbf{x}_\beta} = -\psi_\beta \cdot \nabla \langle p_\beta \rangle^\beta + \rho_\beta \mathbf{b}_\beta \cdot \boldsymbol{\varphi}_\beta - \psi_\gamma \cdot \nabla \langle p_\gamma \rangle^\gamma + \rho_\gamma \mathbf{b}_\gamma \cdot \boldsymbol{\varphi}_\gamma + s_{\beta\gamma}, \quad (3)$$

where  $\boldsymbol{\varphi}_\alpha = \int_{\mathcal{V}_\alpha} \mathbf{f}_\alpha dV$ ,  $\psi_\alpha = \boldsymbol{\varphi}_\alpha - \int_{\mathcal{A}_{\beta\gamma}} \mathbf{n}_{\alpha\kappa} \cdot \mathbf{f}_\alpha \mathbf{z}_\alpha dA$ , and  $s_{\beta\gamma} = \int_{\mathcal{A}_{\beta\gamma}} (2H\gamma \mathbf{n}_{\beta\gamma} + \nabla_s \gamma) \cdot \mathbf{f}_\beta dA$ . Here,  $\mathbf{z}_\alpha$  denotes the position of a point located at  $\mathcal{A}_{\beta\gamma}$  relative to the  $\alpha$ -phase barycenter in the unit cell.

## 3. Results

This predictive equation is tested in the simple case of two-phase flow within a square pattern of parallel cylinders of circular cross section (see figure 1 in [2]) with flow along a principal axis of the structure orthogonal to the cylinders axes. The wetting fluid flows under the form of sheets aligned with the mean flow and attached to the cylinders, whereas the non-wetting phase flows in the core, between the wetting fluid sheets. The test consists in comparing the results on the dimensionless macroscopic dynamic capillary pressure  $\Delta\mathcal{P}^*$  obtained from the solution of the pore-scale flow problem (DNS), on the one hand, and from the upscaled equation (3) (UE), on the other hand. Results are reported in figure 1 versus the wetting fluid saturation  $S_\beta$  for a porosity  $\varepsilon = 0.8$ , two viscosity ratios  $\mu^* = \mu_\beta/\mu_\gamma$  and two values of the capillary number  $Ca = h\ell^2/\gamma$ ,  $h$  being the pressure gradient magnitude in both phases and  $\ell$  the periodic unit cell size. No volume force and no Marangoni effect are considered. The classical Laplace pressure  $\mathcal{P}_{cL}^* = 2\langle H^* \rangle_{\beta\gamma}/Ca$ , where  $\langle H^* \rangle_{\beta\gamma} = \frac{1}{A_{\beta\gamma}^*} \int_{\mathcal{A}_{\beta\gamma}} H^* dA^*$  and  $A_{\beta\gamma}^*$  is the dimensionless measure of  $\mathcal{A}_{\beta\gamma}$ , is also reported in these figures. These results show the excellent agreement between DNS and the upscaled equation, and highlight the fact that the classical Laplace relationship cannot represent a satisfactory estimate of  $\Delta\mathcal{P}^*$ . The influence of the pressure gradient in each phase (dynamic effects) is also pinpointed.

## References

- [1] HASSANIZADEH, S. M. & GRAY, W. G. 1990 Mechanics and thermodynamics of multiphase flow in porous media including interphase boundaries. *Advances in Water Resources* **13** (4), 169–186.
- [2] LASSEUX, D. & VALDÉS-PARADA, F. J. 2023 Upscaled dynamic capillary pressure for two-phase flow in porous media. *Journal of Fluid Mechanics* **959**, R2.
- [3] MARLE, C.-M. 1981 From the pore scale to the macroscopic scale: equations governing multiphase fluid flow through porous media. *Flow and transport in porous media. Proc. Euromech 143, Delft, 1981* pp. 57–61.