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Forecasting crude oil volatility with exogenous predictors: As good as it GETS?

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Declaration of interest : none.

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Supplementary material to *Forecasting crude oil
volatility with exogenous predictors: As good as it
GETS ?*

February 2022

1 Harvey et al.'s (1997) test of equal predictive accuracy

Table 1: Harvey et al.'s (1997) test of equal predictive accuracy

	Squared return				Intraday range			
	MSE		QLIKE		MSE		QLIKE	
	MDM	p-val	MDM	p-val	MDM	p-val	MDM	p-val
<i>Benchmarks</i>								
GARCH(1,1)	-1,259	0,896	0,410	0,341	0,551	0,291	-0,258	0,602
GJR-GARCH(1,1,1)	-1,459	0,928	-0,063	0,525	0,187	0,426	-0,702	0,759
EGARCH(1,1)	-1,589	0,944	-0,109	0,544	-0,219	0,587	0,166	0,434
log-GARCH(1,1)	5,893	0,000***	6,829	0,000***	5,759	0,000***	2,261	0,012**
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	6,828	0,000***	5,351	0,000***	5,638	0,000***	0,465	0,321
AVOL	6,906	0,000***	5,338	0,000***	5,571	0,000***	0,404	0,343
TB3M	7,543	0,000***	5,466	0,000***	6,460	0,000***	0,718	0,236
LTGBY	5,451	0,000***	4,905	0,000***	4,523	0,000***	0,193	0,423
Basis	5,548	0,000***	5,324	0,000***	4,747	0,000***	-0,011	0,505
S&P 500	5,010	0,000***	4,871	0,000***	4,030	0,000***	-0,234	0,593
MSCI EM	6,516	0,000***	4,391	0,000***	4,432	0,000***	-0,554	0,710
GSCI	6,082	0,000***	4,002	0,000***	3,613	0,000***	-1,485	0,931
BCOM	6,285	0,000***	4,316	0,000***	3,632	0,000***	-0,614	0,730
USD-AUD	5,609	0,000***	4,768	0,000***	3,596	0,000***	-0,050	0,520
USD-CAD	2,741	0,003***	4,272	0,000***	1,969	0,025**	-0,611	0,729
USD-CLP	6,316	0,000***	4,810	0,000***	4,339	0,000***	0,047	0,481
USD-NZD	6,091	0,000***	5,037	0,000***	4,804	0,000***	0,338	0,368
USD-ZAR	5,965	0,000***	5,080	0,000***	4,416	0,000***	0,113	0,455
VIX	7,389	0,000***	5,330	0,000***	5,922	0,000***	0,179	0,429
OVX	-1,665	0,952	-1,566	0,941	-1,499	0,933	-3,780	1,000
BDI	7,178	0,000***	5,455	0,000***	6,029	0,000***	0,564	0,286
EPU	6,974	0,000***	5,492	0,000***	6,180	0,000***	0,694	0,244
<i>GARCH-MIDAS</i>								
Prod	-1,320	0,906	0,724	0,234	1,144	0,126	1,408	0,080*
Kilian	-1,378	0,916	0,750	0,227	0,870	0,192	1,678	0,047**
Prod vol	-1,259	0,896	0,738	0,230	1,238	0,108	1,556	0,060*
Kilian vol	-1,317	0,906	1,015	0,155	1,127	0,130	1,656	0,049**
Working T	-1,331	0,908	0,729	0,233	0,780	0,218	0,826	0,204

Notes: *, **, and *** mean that the null hypothesis of equal predictive accuracy is rejected at the 10%, 5%, and 1% level, respectively, against the alternative that the GETS model produce better forecasts.

2 h -step ahead forecasts

In Section 5.2.4 of the paper, we forecast crude oil volatility at longer horizons (5-day, 21-day, and 63-day ahead). For some models, R have build-in forecast functions where you just need to specify the forecast horizon. It is the case of the GARCH(1,1), GJR-GARCH(1,1), EGARCH(1,1) benchmarks and of the GETS model, but not of the log-GARCH(1,1), log-GARCH(1,1)-X, and GARCH-MIDAS models. In Section 2.1, we derive the expressions of our forecasts for these models for horizon $h > 1$. Log-GARCH-X and GETS forecasts for $h > 1$ require to build forecasts of the regressors. In our baseline specification, forecasts of each regressor x_k are produced from an ARIMA model selected according to the BIC from a general ARIMA(63,2,63) unrestricted model. We choose such a general framework to accommodate for the diversity of our regressors. In Section 2.2, we show how our results can deteriorate if we choose a simpler ARMA(5,5) specification. It implies that log-GARCH-X and GETS forecasts are very dependent on the accuracy of the regressors' forecasts, and that performance might improve provided that more attention is paid to the specific data generating processes of the regressors.

2.1 Forecast expressions

2.1.1 Log-GARCH(1,1) model

The log-GARCH(1,1) model is:

$$\ln(\sigma_t^2) = \omega + \alpha_1 \ln(\epsilon_{t-1}^2) + \beta_1 \ln(\sigma_{t-1}^2)$$

We want to forecast $\ln(\sigma_{t+h}^2)$ where h is the forecast horizon in days. For $h = 1$, all variables needed to compute $E[\ln(\sigma_{t+1}^2)|\mathcal{F}_t] = E_t[\ln(\sigma_{t+1}^2)]$ are available at time t :

$$E_t[\ln(\sigma_{t+1}^2)] = \omega + \alpha_1 \ln(\epsilon_t^2) + \beta_1 \ln(\sigma_t^2)$$

We demonstrate by recurrence that, for $h \geq 2$, $E_t[\ln(\sigma_{t+h}^2)] = (\omega + \alpha_1 E(\ln(z_{t+1}^2))) \frac{1 - (\alpha_1 + \beta_1)^{h-1}}{1 - (\alpha_1 + \beta_1)} +$

$$(\alpha_1 + \beta_1)^{h-1} E_t[\ln(\sigma_{t+1}^2)].^1$$

For $h = 2$:

$$\begin{aligned} E_t[\ln(\sigma_{t+2}^2)] &= \omega + \alpha_1 E_t[\ln(\epsilon_{t+1}^2)] + \beta_1 E_t[\ln(\sigma_{t+1}^2)] \\ &= \omega + \alpha_1 E_t[\ln(\sigma_{t+1}^2 z_{t+1}^2)] + \beta_1 E_t[\ln(\sigma_{t+1}^2)] \\ &= \omega + \alpha_1 E[\ln(z_{t+1}^2)] + (\alpha_1 + \beta_1) E_t[\ln(\sigma_{t+1}^2)] \end{aligned}$$

If it is true for h , the proposition is also true for $h + 1$:

$$\begin{aligned} E_t[\ln(\sigma_{t+h+1}^2)] &= \omega + \alpha_1 E_t[\ln(\epsilon_{t+h}^2)] + \beta_1 E_t[\ln(\sigma_{t+h}^2)] \\ &= \omega + \alpha_1 E[\ln(z_{t+h}^2)] + (\alpha_1 + \beta_1) E_t[\ln(\sigma_{t+h}^2)] \\ &= \omega + \alpha_1 E[\ln(z_{t+1}^2)] + (\alpha_1 + \beta_1) \left[(\omega + \alpha_1 E(\ln z_{t+1}^2)) \frac{1 - (\alpha_1 + \beta_1)^{h-1}}{1 - (\alpha_1 + \beta_1)} + \right. \\ &\quad \left. (\alpha_1 + \beta_1)^{h-1} E_t[\ln(\sigma_{t+1}^2)] \right] \\ &= (\omega + \alpha_1 E(\ln z_{t+1}^2)) \left[(\alpha_1 + \beta_1) \frac{1 - (\alpha_1 + \beta_1)^{h-1}}{1 - (\alpha_1 + \beta_1)} + 1 \right] + (\alpha_1 + \beta_1)^h E_t[\ln(\sigma_{t+1}^2)] \\ &= (\omega + \alpha_1 E(\ln z_{t+1}^2)) \frac{1 - (\alpha_1 + \beta_1)^h}{1 - (\alpha_1 + \beta_1)} + (\alpha_1 + \beta_1)^h E_t[\ln(\sigma_{t+1}^2)] \end{aligned}$$

2.1.2 Single-predictor log-GARCH(1,1)-X models

The log-GARCH(1,1)-X model is:

$$\ln(\sigma_t^2) = \omega + \alpha_1 \ln(\epsilon_{t-1}^2) + \beta_1 \ln(\sigma_{t-1}^2) + \eta x_{t-1}$$

We want to forecast $\ln(\sigma_{t+h}^2)$ where h is the forecast horizon in days. For $h = 1$, all variables needed to compute $E[\ln(\sigma_{t+1}^2) | \mathcal{F}_t] = E_t[\ln(\sigma_{t+1}^2)]$ are available at time t :

$$E_t[\ln(\sigma_{t+1}^2)] = \omega + \alpha_1 \ln(\epsilon_t^2) + \beta_1 \ln(\sigma_t^2) + \eta x_t$$

¹The **lgarch** package provides an estimation of $E(\ln(z_t^2))$ based on the work of Sucarrat et al. (2016).

We demonstrate by recurrence that, for $h \geq 2$, $E_t[\ln(\sigma_{t+h}^2)] = (\omega + \alpha_1 E(\ln(z_{t+1}^2))) \frac{1 - (\alpha_1 + \beta_1)^{h-1}}{1 - (\alpha_1 + \beta_1)} + (\alpha_1 + \beta_1)^{h-1} E_t[\ln(\sigma_{t+1}^2)] + \eta \sum_{k=1}^{h-1} E_t[x_{t+k}](\alpha_1 + \beta_1)^{h-1-k}$.²

For $h = 2$:

$$\begin{aligned} E_t[\ln(\sigma_{t+2}^2)] &= \omega + \alpha_1 E_t[\ln(\epsilon_{t+1}^2)] + \beta_1 E_t[\ln(\sigma_{t+1}^2)] + \eta E_t[x_{t+1}] \\ &= \omega + \alpha_1 E[\ln(z_{t+1}^2)] + (\alpha_1 + \beta_1) E_t[\ln(\sigma_{t+1}^2)] + \eta E_t[x_{t+1}] \end{aligned}$$

If it is true for h , the proposition is also true for $h + 1$:

$$\begin{aligned} E_t[\ln(\sigma_{t+h+1}^2)] &= \omega + \alpha_1 E_t[\ln(\epsilon_{t+h}^2)] + \beta_1 E_t[\ln(\sigma_{t+h}^2)] + \eta E_t[x_{t+h}] \\ &= \omega + \alpha_1 E[\ln(z_{t+h}^2)] + (\alpha_1 + \beta_1) E_t[\ln(\sigma_{t+h}^2)] + \eta E_t[x_{t+h}] \\ &= \omega + \alpha_1 E[\ln(z_{t+1}^2)] + (\alpha_1 + \beta_1) \left[(\omega + \alpha_1 E(\ln z_{t+1}^2)) \frac{1 - (\alpha_1 + \beta_1)^{h-1}}{1 - (\alpha_1 + \beta_1)} + \right. \\ &\quad \left. (\alpha_1 + \beta_1)^{h-1} E_t[\ln(\sigma_{t+1}^2)] + \eta \sum_{k=1}^{h-1} E_t[x_{t+k}](\alpha_1 + \beta_1)^{h-1-k} \right] + \eta E_t[x_{t+h}] \\ &= (\omega + \alpha_1 E(\ln z_{t+1}^2)) \frac{1 - (\alpha_1 + \beta_1)^h}{1 - (\alpha_1 + \beta_1)} + (\alpha_1 + \beta_1)^h E_t[\ln(\sigma_{t+1}^2)] + \\ &\quad \eta \sum_{k=1}^{h-1} E_t[x_{t+k}](\alpha_1 + \beta_1)^{h-k} + \eta E_t[x_{t+h}] \\ &= (\omega + \alpha_1 E(\ln z_{t+1}^2)) \frac{1 - (\alpha_1 + \beta_1)^h}{1 - (\alpha_1 + \beta_1)} + (\alpha_1 + \beta_1)^h E_t[\ln(\sigma_{t+1}^2)] + \\ &\quad \eta \sum_{k=1}^h E_t[x_{t+k}](\alpha_1 + \beta_1)^{h-k} \end{aligned}$$

²For simplicity, forecasts of x_t (i.e., $E_t[x_{t+k}]$, for $k = 1, \dots, h$) are obtained using an ARMA(p, q) model selected according to the BIC with $\max p = \max q = 5$.

2.1.3 GARCH-MIDAS models

The GARCH-MIDAS model with an asymmetry term is given by:

$$\begin{aligned}\epsilon_{t,m} &= \sqrt{\tau_m g_{t,m}} z_t \quad z_t \sim i.i.d. N(0, 1) \\ g_{t,m} &= (1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma \mathbb{1}_{\{\epsilon_{t-1,m} < 0\}}) \frac{\epsilon_{t-1,m}^2}{\tau_m} + \beta g_{t-1,m} \\ \tau_m &= \exp(\theta_0 + \theta \sum_{k=1}^K \phi_k x_{m-k})\end{aligned}$$

We want to forecast $\sigma_{t+h,m'}^2 = \tau_{m'} g_{t+h,m'}$ where h is the horizon of the forecast in days, and m' is the month of day $t + h$. $\sigma_{t+h,m'}^2$ has two components: (i) $g_{t+h,m'}$ is the short-term component, and (ii) $\tau_{m'}$ is the long-term component. The forecast in t of $\sigma_{t+h,m'}^2$ is then given by $E[\tau_{m'} | \mathcal{F}_t] E[g_{t+h,m'} | \mathcal{F}_t]$.

(i) Conditional on whether the days t and $t + h$ are in the same month we use either $E[\tau_{m'} | \mathcal{F}_t] = \tau_m$ or $E[\tau_{m'} | \mathcal{F}_t] = \tau_{m+1}$. For simplification purposes, because we do not explicitly model the dynamics of the underlying variable x_t , we follow Conrad and Kleen (2020) and use τ_{m+1} for forecast of $\tau_{m'}$ beyond one month (i.e., for $h = 63$ days). It means that we hold the long-term component constant after month $m + 1$.

(ii) For $h = 1$, all variables needed to compute $E[g_{t+1,m} | \mathcal{F}_t] = E_t[g_{t+1,m}]$ are available at time t :

$$\begin{aligned}E_t[g_{t+1,m}] &= E_t \left[(1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma \mathbb{1}_{\{\epsilon_{t,m} < 0\}}) \frac{\epsilon_{t,m}^2}{\tau_m} + \beta g_{t,m} \right] \\ &= (1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma \mathbb{1}_{\{\epsilon_{t,m} < 0\}}) g_{t,m} z_t^2 + \beta g_{t,m}\end{aligned}$$

We demonstrate by recurrence that, for $h \geq 2$, $E_t[g_{t+h,m}] = 1 + (\alpha + \gamma/2 + \beta)^{h-1} (E_t[g_{t+1,m}] - 1)$.

For $h = 2$:

$$\begin{aligned}
E_t[g_{t+2,m}] &= E_t \left[(1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma \mathbb{1}_{\{\epsilon_{t+1,m} < 0\}}) \frac{\epsilon_{t+1,m}^2}{\tau_m} + \beta g_{t+1,m} \right] \\
&= E_t \left[(1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma \mathbb{1}_{\{\epsilon_{t+1,m} < 0\}}) g_{t+1,m} z_{t+1}^2 + \beta g_{t+1,m} \right] \\
&= (1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma/2) E_t[g_{t+1,m}] E[z_{t+1}^2] + \beta E_t[g_{t+1,m}] \\
&= (1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma/2 + \beta) E_t[g_{t+1,m}] \\
&= 1 + (\alpha + \gamma/2 + \beta)(E_t[g_{t+1,m}] - 1)
\end{aligned}$$

If it is true for h , the proposition is also true for $h + 1$:

$$\begin{aligned}
E_t[g_{t+h+1,m}] &= E_t \left[(1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma \mathbb{1}_{\{\epsilon_{t+h,m} < 0\}}) \frac{\epsilon_{t+h,m}^2}{\tau_m} + \beta g_{t+h,m} \right] \\
&= (1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma/2 + \beta) E_t[g_{t+h,m}] \\
&= (1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma/2 + \beta) (1 + (\alpha + \gamma/2 + \beta)^{h-1} (E_t[g_{t+1,m}] - 1)) \\
&= 1 + (\alpha + \gamma/2 + \beta)^h (E_t[g_{t+1,m}] - 1)
\end{aligned}$$

2.2 Results with alternative X forecasts

In this section, we show our results where X forecasts are generated from a simpler ARMA(5,5) framework for $h = \{5, 21, 63\}$.

Table 2: 5-day ahead out-of-sample results with X forecasts generated from an ARMA(5,5) framework

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	60,565***	7	1,402***	3	43,675***	4	0,386***	18
GJR-GARCH(1,1)	58,778***	2	1,388***	1	42,395***	3	0,389***	19
EGARCH(1,1)	58,207***	1	1,388***	2	41,797***	2	0,412***	20
log-GARCH(1,1)	363,221**	28	1,610	28	295,203***	28	0,430**	21
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	115,712	21	1,539	23	73,263**	20	0,373***	12
AVOL	119,110	22	1,542	26	76,165**	22	0,376***	14
TB3M	111,164	19	1,541	24	72,648***	19	0,382***	17
LTGBY	126,971**	25	1,527	19	82,807***	25	0,368***	10
Basis	130,243**	26	1,535	22	85,463***	27	0,367***	9
S&P 500	134,898*	27	1,525	17	81,216***	24	0,355***	5
MSCI EM	96,285**	16	1,509*	13	53,140***	13	0,346***	3
GSCI	102,671**	18	1,493**	9	55,340***	15	0,333***	1
BCOM	91,832**	14	1,508*	12	51,958***	11	0,351***	4
USD-AUD	87,092**	12	1,520*	14	49,658***	10	0,361***	6
USD-CAD	75,820**	11	1,497*	10	40,289***	1	0,344***	2
USD-CLP	91,581**	13	1,524	15	53,492***	14	0,362***	8
USD-NZD	96,333**	17	1,531**	20	57,190***	17	0,370***	11
USD-ZAR	93,506**	15	1,525	18	52,466***	12	0,361***	7
VIX	121,810	23	1,560	27	76,942***	23	0,379***	16
OVX	66,483**	10	1,524**	16	59,172***	18	0,610**	28
BDI	115,122	20	1,541	25	73,924***	21	0,376***	15
EPU	125,263*	24	1,532	21	83,243***	26	0,374***	13
<i>GARCH-MIDAS</i>								
Prod	60,022***	3	1,410***	4	46,022***	7	0,437***	22
Kilian	60,048***	4	1,417***	5	44,796***	6	0,446**	24
Prod vol	61,052***	8	1,422***	6	46,689***	9	0,450**	25
Kilian vol	60,297***	6	1,430***	8	46,380***	8	0,455**	26
Working T	60,212***	5	1,424***	7	44,733***	5	0,444***	23
<i>AR-X-log-GARCH-X</i>								
GETS	65,404***	9	1,502***	11	56,617***	16	0,563***	27

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 3: 5-day simple forecast combinations and dynamic model averaging with X forecasts generated from an ARMA(5,5) framework

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	85,727	1,503	48,847	0,345
Trimmed mean	95,688	1,516	55,718	0,354
DMSPE	80,156	1,493	45,407	0,341
DMA	61,471	1,402	35,559	0,307
<i>GARCH-MIDAS models</i>				
Mean	60,023	1,412	45,421	0,435
Trimmed mean	60,126	1,414	45,650	0,439
DMSPE	60,495	1,425	46,065	0,447
DMA	61,544	1,409	41,219	1,422
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	80,035	1,462	46,175	0,327
Trimmed mean	77,577	1,462	44,638	0,327
DMSPE	69,320	1,431	40,017	0,321
DMA	61,472	1,401	35,610	0,307
<i>AR-X-log-GARCH-X</i>				
GETS	65,404	1,502	56,617	0,563

Notes: Bold characters signal models with the lowest information criteria.

Table 4: 21-day ahead out-of-sample results with X forecasts generated from an ARMA(5,5) framework

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	66,683***	9	1,476***	2	54,583***	9	0,477***	19
GJR-GARCH(1,1,1)	64,897***	2	1,474***	1	53,258***	5	0,498**	20
EGARCH(1,1)	64,217***	1	1,480***	3	53,625***	7	0,532**	22
log-GARCH(1,1)	231,823**	28	1,616	28	206,973***	28	0,475**	18
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	76,639**	20	1,517***	17	59,584***	20	0,416***	9
AVOL	82,668*	25	1,531***	25	64,034***	25	0,420***	13
TB3M	76,915**	21	1,522***	22	60,540***	23	0,426***	15
LTGBY	77,022**	22	1,512***	15	60,739***	24	0,418***	11
Basis	89,336**	26	1,527***	23	70,335***	26	0,412***	8
S&P 500	78,773**	24	1,504***	12	58,634***	19	0,396***	6
MSCI EM	72,827**	15	1,499***	10	53,467***	6	0,393***	3
GSCI	75,293**	18	1,497***	8	52,919***	3	0,376***	1
BCOM	71,710**	13	1,504***	13	53,098***	4	0,395***	5
USD-AUD	71,846**	14	1,512***	16	54,951***	10	0,418***	10
USD-CAD	67,156**	10	1,488***	4	49,823***	1	0,391***	2
USD-CLP	75,265**	17	1,519***	20	56,443***	16	0,411***	7
USD-NZD	73,179***	16	1,518***	18	56,431***	15	0,424***	14
USD-ZAR	71,432**	12	1,499***	11	52,335***	2	0,394***	4
VIX	96,713**	27	1,594*	27	73,119***	27	0,445***	17
OVX	66,032***	8	1,499***	9	57,540***	17	0,556*	23
BDI	77,469**	23	1,520***	21	60,507***	22	0,420***	12
EPU	75,306**	19	1,491***	5	60,264***	21	0,428***	16
<i>GARCH-MIDAS</i>								
Prod	65,164***	4	1,518***	19	55,585***	13	0,571*	26
Kilian	65,826***	7	1,493***	6	55,394***	12	0,559	24
Prod vol	67,512***	11	1,544***	26	57,705***	18	0,612	28
Kilian vol	65,157***	3	1,509***	14	56,127***	14	0,580	27
Working T	65,383***	6	1,529***	24	54,185***	8	0,568*	25
<i>AR-X-log-GARCH-X</i>								
GETS	65,369***	5	1,494***	7	55,246***	11	0,526**	21

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 5: 21-day simple forecast combinations and dynamic model averaging with X forecasts generated from an ARMA(5,5) framework

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	70,763	1,496	53,503	0,395
Trimmed mean	72,915	1,504	55,355	0,402
DMSPE	70,295	1,495	53,563	0,398
DMA	64,875	1,442	44,632	0,283
<i>GARCH-MIDAS models</i>				
Mean	65,166	1,505	55,157	0,557
Trimmed mean	65,253	1,516	55,496	0,573
DMSPE	65,014	1,507	55,433	0,559
DMA	64,206	1,440	54,012	1,568
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	70,415	1,479	54,488	0,398
Trimmed mean	69,141	1,478	53,422	0,399
DMSPE	68,430	1,472	53,241	0,399
DMA	64,400	1,444	45,018	0,465
<i>AR-X-log-GARCH-X</i>				
GETS	65,369	1,494	55,246	0,526

Notes: Bold characters signal models with the lowest information criteria.

Table 6: 63-day ahead out-of-sample results with X forecasts generated from an ARMA(5,5) framework

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	71,389***	24	1,520***	3	60,806***	14	0,523***	7
GJR-GARCH(1,1,1)	69,490***	16	1,548***	18	61,306***	17	0,608*	20
EGARCH(1,1)	68,224***	3	1,543***	15	61,545***	19	0,636	22
log-GARCH(1,1)	113,384***	28	1,576***	22	94,499***	28	0,501***	2
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	68,860***	8	1,531***	10	60,185***	7	0,533***	10
AVOL	70,755***	21	1,547***	17	60,386***	9	0,525***	8
TB3M	69,082***	13	1,538***	12	60,567***	10	0,544***	12
LTGBY	69,058***	12	1,538***	13	60,636***	11	0,544***	13
Basis	70,609***	19	1,525***	7	59,663***	5	0,495***	1
S&P 500	69,699***	17	1,551***	19	60,926***	15	0,553***	16
MSCI EM	68,583***	5	1,524***	5	59,536***	3	0,517***	6
GSCI	69,391***	15	1,528***	9	59,352***	2	0,505***	4
BCOM	68,781***	7	1,527***	8	59,622***	4	0,516***	5
USD-AUD	68,874***	9	1,540***	14	60,643***	12	0,548***	14
USD-CAD	69,152***	14	1,551***	20	61,446***	18	0,570***	17
USD-CLP	68,625***	6	1,521***	4	58,989***	1	0,503***	3
USD-NZD	69,050***	11	1,544***	16	60,728***	13	0,550***	15
USD-ZAR	68,339***	4	1,525***	6	59,689***	6	0,526***	9
VIX	90,538	27	1,674*	27	74,786**	27	0,578***	18
OVX	67,550***	2	1,515***	2	62,245***	20	0,611*	21
BDI	68,966***	10	1,533***	11	60,245***	8	0,535***	11
EPU	70,030***	18	1,647**	25	64,366***	23	0,779	27
<i>GARCH-MIDAS</i>								
Prod	71,383***	23	1,657*	26	64,583***	25	0,775	26
Kilian	71,192***	22	1,561***	21	63,131***	21	0,686	23
Prod vol	72,805***	26	1,697*	28	65,914***	26	0,817**	28
Kilian vol	70,728***	20	1,619**	23	64,496***	24	0,751	25
Working T	71,782***	25	1,643*	24	63,265***	22	0,724	24
<i>AR-X-log-GARCH-X</i>								
GETS	66,694***	1	1,489	1	61,164***	16	0,581***	19

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 7: 63-day simple forecast combinations and dynamic model averaging with X forecasts generated from an ARMA(5,5) framework

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	68,577	1,528	59,869	0,526
Trimmed mean	68,709	1,530	60,001	0,530
DMSPE	68,487	1,526	59,923	0,527
DMA	59,927	1,445	43,344	0,547
<i>GARCH-MIDAS models</i>				
Mean	70,771	1,611	63,471	0,711
Trimmed mean	70,748	1,625	63,700	0,735
DMSPE	70,492	1,611	63,444	0,713
DMA	60,235	1,427	45,605	1,680
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	69,116	1,523	60,124	0,527
Trimmed mean	68,848	1,527	60,272	0,538
DMSPE	68,750	1,518	60,022	0,527
DMA	59,909	1,433	42,974	0,691
<i>AR-X-log-GARCH-X</i>				
GETS	66,694	1,489	61,164	0,581

Notes: Bold characters signal models with the lowest information criteria.

3 Alternative rolling window size

In this section, we check whether our results are robust to an alternative window size. Specifically, we consider an alternative specification where we add 500 observations to the in-sample (IS) period. It results in a rolling window scheme of 2065 observations and 542 point forecasts. The IS period then corresponds to a little bit more than 8 years and roughly 80% of observations. The first IS window starts on July 2, 2009 and ends on September 12, 2017. Therefore, in this specification, I produce out-of-sample crude oil volatility forecasts from September 13, 2017 to November 12, 2019. Tables 8 to 15 summarize our results for forecast 1-day, 5-day, 21-day, and 63-day horizons.

It should first be noted that the large differences in the loss functions between the two window-size specifications are entirely due to the period considered and not to some asymp-

totic benefits of estimating the models with more observations. Indeed, we computed the MSPE and QLIKE from our baseline forecasts on the last 542 observations and found basically the same results as in this robustness test. Beside, we overall find results to be consistent with our baseline specification. There are still a few notable differences. At the 1-day horizon, the GETS does not perform as well as in the baseline specification comparatively to other models, while, on the opposite, at the 5-day horizon, it does considerably better and ranks 1st with squared return as our proxy of true volatility with both loss functions. We also note that, at the 1-day horizon, GARCH-MIDAS are comparatively more accurate than in the baseline specification. This result translates in good accuracy of forecast combinations and DMA applied to the pool of GARCH-MIDAS models. Finally, although DMA still performs very well globally, it does not reach the same dominance as in the baseline specification.

Table 8: 1-day ahead out-of-sample results with an alternative rolling window size

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	46,421***	6	1,514***	4	26,606**	9	0,378***	16
GJR-GARCH(1,1)	46,461***	7	1,509***	2	25,148***	1	0,368***	6
EGARCH(1,1)	46,468***	8	1,516***	5	25,570***	3	0,395***	22
log-GARCH(1,1)	66,102	27	1,669	28	34,920*	27	0,384***	21
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	60,141	18	1,661	23	30,958*	22	0,378***	14
AVOL	58,695	13	1,654	17	30,111**	16	0,374***	10
TB3M	60,117	17	1,658	19	30,815*	19	0,375***	11
LTGBY	65,610	26	1,660	21	34,047	26	0,377***	12
Basis	61,548	24	1,663	25	30,943**	21	0,368***	8
S&P 500	77,702	28	1,649	15	37,577**	28	0,358***	3
MSCI EM	56,775	11	1,646	13	29,080**	11	0,369***	9
GSCI	65,128	25	1,628	11	29,978**	14	0,340***	1
BCOM	60,039	16	1,642	12	29,665**	12	0,364***	4
USD-AUD	60,224	19	1,660	20	30,616*	18	0,379***	18
USD-CAD	57,594	12	1,647	14	28,193**	10	0,367***	5
USD-CLP	60,472	20	1,666	27	30,915	20	0,383***	19
USD-NZD	60,656	22	1,664	26	31,411**	25	0,384***	20
USD-ZAR	58,963	15	1,656	18	30,332**	17	0,379***	17
VIX	61,231	23	1,651	16	30,993**	23	0,368***	7
OVX	46,137***	2	1,497***	1	25,212***	2	0,349***	2
BDI	60,655	21	1,660	22	31,235*	24	0,377***	13
EPU	58,798	14	1,662	24	29,994**	15	0,378***	15
<i>GARCH-MIDAS</i>								
Prod	46,010***	1	1,513***	3	26,086**	7	0,411***	26
Kilian	46,347***	5	1,527***	9	25,922**	4	0,415***	27
Prod vol	47,706***	10	1,523**	7	26,149***	8	0,398***	23
Kilian vol	46,332***	4	1,520***	6	26,034**	6	0,404***	24
Working T	46,255***	3	1,524**	8	26,034**	5	0,411***	25
<i>AR-X-log-GARCH-X</i>								
GETS	47,245***	9	1,545**	10	29,819**	13	0,449*	28

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 9: 1-day simple forecast combinations and dynamic model averaging with an alternative rolling window size

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	57,413	1,634	28,144	0,355
Trimmed mean	58,845	1,647	29,346	0,365
DMSPE	57,086	1,626	27,814	0,349
DMA	50,794	1,537	27,439	0,310
<i>GARCH-MIDAS models</i>				
Mean	46,345	1,519	25,860	0,402
Trimmed mean	46,219	1,519	25,906	0,406
DMSPE	46,468	1,523	25,835	0,403
DMA	46,975	1,540	24,123	1,521
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	52,840	1,582	25,653	0,327
Trimmed mean	52,696	1,585	25,957	0,329
DMSPE	51,816	1,560	25,196	0,318
DMA	50,758	1,541	27,335	0,310
<i>AR-X-log-GARCH-X</i>				
GETS	47,245	1,545	29,819	0,449

Notes: Bold characters signal models with the lowest information criteria.

Table 10: 5-day ahead out-of-sample results with an alternative rolling window size

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	47,435***	8	1,522***	6	27,491***	4	0,398***	19
GJR-GARCH(1,1,1)	46,836***	5	1,506***	2	26,274***	1	0,400***	20
EGARCH(1,1)	46,754***	4	1,512***	4	26,727***	2	0,434***	22
log-GARCH(1,1)	65,304	27	1,655	28	35,506*	27	0,390***	18
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	58,088	18	1,638	22	30,528***	17	0,377***	12
AVOL	57,322	15	1,635	18	30,091***	14	0,375***	9
TB3M	57,950	17	1,633	17	30,423***	16	0,375***	8
LTGBY	61,817	25	1,639	26	32,893*	26	0,380***	13
Basis	59,746	23	1,636	19	31,204***	22	0,373***	7
S&P 500	72,550	28	1,638	21	37,980***	28	0,365***	2
MSCI EM	55,970	11	1,626	14	29,402***	12	0,372***	6
GSCI	61,882	26	1,609**	11	31,488***	25	0,351***	1
BCOM	57,802	16	1,619*	12	30,695***	18	0,371***	4
USD-AUD	58,283	19	1,638	24	30,884**	20	0,383***	15
USD-CAD	56,067	12	1,624	13	29,091***	10	0,372***	5
USD-CLP	58,483	21	1,639	25	30,972*	21	0,383***	16
USD-NZD	58,621	22	1,642	27	31,238***	23	0,386***	17
USD-ZAR	57,179	14	1,631	15	30,356***	15	0,381***	14
VIX	60,413	24	1,638	23	31,379***	24	0,371***	3
OVX	46,740***	3	1,507***	3	26,958***	3	0,415***	21
BDI	58,432	20	1,637	20	30,848***	19	0,377***	11
EPU	56,274	13	1,633	16	29,481***	13	0,375***	10
<i>GARCH-MIDAS</i>								
Prod	46,591***	2	1,516***	5	27,534***	5	0,456*	26
Kilian	47,235***	6	1,534**	9	27,548***	6	0,464*	27
Prod vol	50,445***	10	1,532***	8	28,893***	9	0,443*	23
Kilian vol	47,306***	7	1,531**	7	27,716***	7	0,448*	24
Working T	47,541***	9	1,556**	10	28,070***	8	0,485	28
<i>AR-X-log-GARCH-X</i>								
GETS	45,943***	1	1,505***	1	29,345***	11	0,454*	25

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 11: 5-day simple forecast combinations and dynamic model averaging with an alternative rolling window size

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	55,968	1,612	28,772	0,359
Trimmed mean	57,192	1,626	29,703	0,369
DMSPE	55,591	1,604	28,937	0,356
DMA	49,687	1,600	23,357	0,307
<i>GARCH-MIDAS models</i>				
Mean	47,431	1,530	27,560	0,451
Trimmed mean	47,081	1,527	27,544	0,453
DMSPE	47,563	1,547	27,855	0,469
DMA	49,228	1,587	25,624	1,511
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	52,466	1,570	27,007	0,340
Trimmed mean	52,333	1,573	27,113	0,342
DMSPE	51,787	1,555	27,129	0,340
DMA	49,532	1,587	23,436	0,307
<i>AR-X-log-GARCH-X</i>				
GETS	45,943	1,505	29,345	0,454

Notes: Bold characters signal models with the lowest information criteria.

Table 12: 21-day ahead out-of-sample results with an alternative rolling window size

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	50,600***	8	1,604***	2	31,330***	5	0,478***	19
GJR-GARCH(1,1,1)	50,464***	6	1,606***	3	30,596***	1	0,496**	20
EGARCH(1,1)	50,121***	2	1,616***	5	31,202***	4	0,529*	22
log-GARCH(1,1)	62,757	28	1,688*	28	37,871*	28	0,443**	18
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	54,460*	17	1,642***	18	32,116***	15	0,415***	10
AVOL	54,882*	20	1,645***	22	32,077***	14	0,410***	6
TB3M	53,716**	11	1,640***	16	31,784***	13	0,421***	15
LTGBY	54,846**	19	1,636***	12	32,239***	17	0,415***	9
Basis	57,238*	23	1,650***	23	33,270***	24	0,408***	5
S&P 500	58,721**	26	1,634***	10	34,509***	26	0,396***	1
MSCI EM	53,871**	13	1,632***	9	31,423***	7	0,404***	4
GSCI	57,734*	25	1,639***	15	33,003***	23	0,398***	2
BCOM	55,233*	21	1,642***	19	32,545***	21	0,416***	12
USD-AUD	54,534*	18	1,643***	21	32,347***	20	0,421***	14
USD-CAD	53,828	12	1,635***	11	31,768***	12	0,413***	7
USD-CLP	55,944*	22	1,650***	24	32,839***	22	0,419***	13
USD-NZD	54,367**	15	1,643***	20	32,291***	19	0,421***	16
USD-ZAR	54,032*	14	1,639***	14	32,247***	18	0,422***	17
VIX	58,794*	27	1,656***	25	33,679***	25	0,401***	3
OVX	50,891***	9	1,600***	1	30,642***	2	0,515*	21
BDI	54,433*	16	1,641***	17	32,173***	16	0,415***	11
EPU	52,546***	10	1,626***	7	31,343***	6	0,414***	8
<i>GARCH-MIDAS</i>								
Prod	49,138***	1	1,617***	6	30,895***	3	0,558*	26
Kilian	50,301***	5	1,638***	13	31,563***	9	0,573	27
Prod vol	57,378***	24	1,656***	26	36,051***	27	0,554	23
Kilian vol	50,299***	4	1,632***	8	31,635***	10	0,554	24
Working T	50,265***	3	1,662***	27	31,758***	11	0,593	28
<i>AR-X-log-GARCH-X</i>								
GETS	50,586***	7	1,615***	4	31,520***	8	0,557	25

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 13: 21-day simple forecast combinations and dynamic model averaging with an alternative rolling window size

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	53,859	1,626	31,406	0,402
Trimmed mean	54,373	1,636	31,782	0,408
DMSPE	53,546	1,619	31,278	0,399
DMA	50,157	1,604	31,133	0,307
<i>GARCH-MIDAS models</i>				
Mean	50,543	1,637	31,447	0,553
Trimmed mean	49,965	1,630	31,333	0,560
DMSPE	50,666	1,649	31,901	0,562
DMA	49,498	1,596	28,456	1,550
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	52,476	1,605	30,765	0,401
Trimmed mean	52,330	1,606	30,731	0,401
DMSPE	52,061	1,595	30,723	0,402
DMA	50,236	1,597	31,432	0,331
<i>AR-X-log-GARCH-X</i>				
GETS	50,586	1,615	31,520	0,557

Notes: Bold characters signal models with the lowest information criteria.

Table 14: 63-day ahead out-of-sample results with an alternative rolling window size

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	55,271**	20	1,636***	19	36,059	18	0,509	19
GJR-GARCH(1,1,1)	55,505**	21	1,674**	21	37,205	21	0,587	21
EGARCH(1,1)	54,439***	14	1,647***	20	36,671	20	0,578	20
log-GARCH(1,1)	57,050**	25	1,628***	17	36,046***	17	0,444***	10
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	53,545***	6	1,598***	9	34,280***	8	0,448***	14
AVOL	53,879***	12	1,601***	12	33,424***	1	0,417***	1
TB3M	54,121***	13	1,621***	16	35,357***	16	0,496*	18
LTGBY	53,547***	7	1,595***	8	34,303***	9	0,447***	13
Basis	54,656***	16	1,609***	14	34,385***	11	0,434***	5
S&P 500	54,688***	17	1,610***	15	34,775***	14	0,444***	9
MSCI EM	53,410***	5	1,589***	1	33,542***	2	0,423***	2
GSCI	54,579***	15	1,607***	13	34,687***	13	0,445***	11
BCOM	53,609***	10	1,595***	7	34,214***	7	0,443***	7
USD-AUD	53,394***	4	1,593***	4	34,135***	5	0,443***	8
USD-CAD	53,590***	9	1,599***	11	34,470***	12	0,454***	16
USD-CLP	53,613***	11	1,593***	5	33,673***	3	0,426***	4
USD-NZD	53,376***	3	1,594***	6	34,182***	6	0,446**	12
USD-ZAR	53,371***	2	1,593***	3	34,003***	4	0,440***	6
VIX	58,549**	28	1,635***	18	36,414***	19	0,425***	3
OVX	58,506*	27	1,767*	26	41,810	28	0,787	26
BDI	53,550***	8	1,599***	10	34,368***	10	0,451***	15
EPU	53,022***	1	1,591***	2	35,255***	15	0,494*	17
<i>GARCH-MIDAS</i>								
Prod	55,213***	19	1,738*	23	37,873	22	0,709	23
Kilian	55,939**	23	1,755*	25	39,438	25	0,769	25
Prod vol	55,781**	22	1,745*	24	38,629	24	0,722	24
Kilian vol	55,111***	18	1,709*	22	37,997	23	0,673	22
Working T	56,413**	24	1,823	28	39,858	26	0,809	27
<i>AR-X-log-GARCH-X</i>								
GETS	57,621**	26	1,805*	27	41,010	27	0,841	28

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 15: 63-day simple forecast combinations and dynamic model averaging with an alternative rolling window size

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	53,687	1,596	34,374	0,446
Trimmed mean	53,670	1,597	34,237	0,443
DMSPE	53,451	1,588	34,259	0,443
DMA	52,696	2,074	34,665	0,727
<i>GARCH-MIDAS models</i>				
Mean	55,570	1,739	38,638	0,713
Trimmed mean	55,623	1,744	38,791	0,726
DMSPE	55,526	1,737	38,635	0,714
DMA	52,988	1,610	31,298	1,555
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	53,989	1,605	35,075	0,472
Trimmed mean	53,855	1,603	34,932	0,468
DMSPE	53,774	1,596	35,017	0,470
DMA	52,698	1,612	34,252	0,690
<i>AR-X-log-GARCH-X</i>				
GETS	57,621	1,805	41,010	0,841

Notes: Bold characters signal models with the lowest information criteria.

4 Alternative crude oil series

In this section, we display our results for an alternative oil series, brent crude oil.

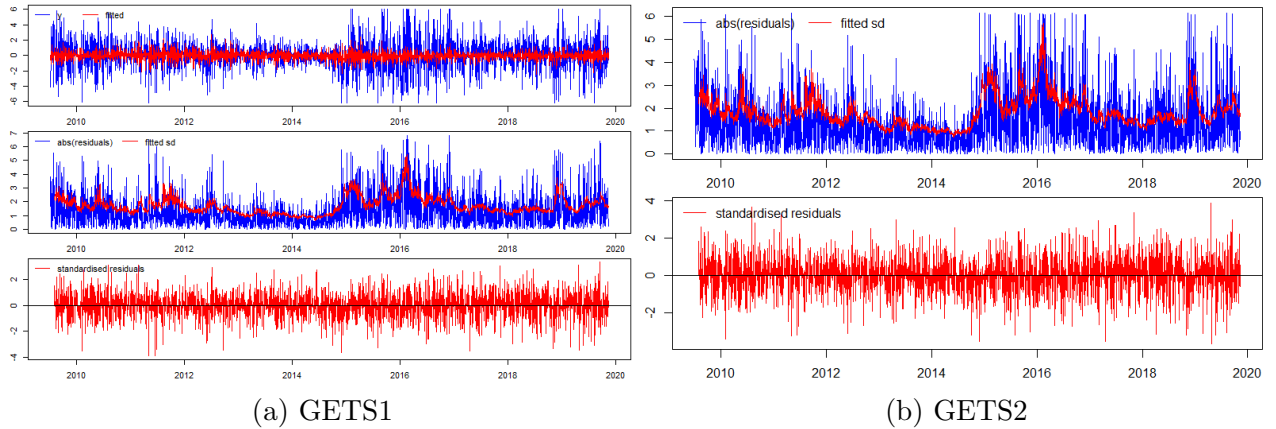


Figure 1: Representation of fitted GETS models for crude oil volatility (Brent)

Table 16: In-sample estimation results (Brent)

	GETS1		GETS2	
	Coef	t-stat	Coef	t-stat
<i>Mean equation</i>				
Mean constant	-0.157	-2.131**		
y_{t-1}	0.155	6.069***		
Basis	0.006	14.756***		
EPU	0.001	2.407**		
<i>Volatility equation</i>				
Volatility constant	-0.654	13.247***	-0.595	10.639***
$\ln(\text{EqWMA}_{20})$	0.396	4.444***	0.386	4.130***
USD-AUD	0.233	3.606***	0.253	3.772***
OVX	0.038	5.533***	0.038	5.346***
<i>Information criteria</i>				
Log-likelihood	-4960.91		-5101.41	
BIC	3.837		3.932	
<i>Diagnostic tests</i>				
LB AR-X (lag)	0.10 (2)		0.08 (1)	
LB ARCH-X (lag)	0.71 (1)		0.93 (1)	

Notes: *, **, and *** denote statistical significance at the 10%, 5%, and 1% significance level, respectively.

Table 17: Out-of-sample results (Brent)

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	50,959***	8	1,415***	2	40,371***	3	0,341***	2
GJR-GARCH(1,1)	49,454***	2	1,408***	1	38,781***	2	0,330***	1
EGARCH(1,1)	49,382***	1	1,436***	8	37,888***	1	0,372***	8
log-GARCH(1,1)	113,451	27	1,609	28	81,272*	27	0,455*	27
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	106,557	20	1,591	21	72,132**	21	0,438**	20
AVOL	112,860	26	1,604	27	73,848*	22	0,444**	24
TB3M	108,538	21	1,585	19	78,716**	26	0,439**	21
LTGBY	112,798	25	1,587	20	74,351*	24	0,434***	17
Basis	110,229	23	1,598	24	75,477**	25	0,442*	23
S&P 500	111,381	24	1,560	15	65,340**	18	0,405***	11
MSCI EM	90,889	15	1,556	14	52,790***	14	0,407***	13
GSCI	90,203	14	1,538	11	52,246***	13	0,383***	9
BCOM	83,362	12	1,545	12	50,942***	12	0,398***	10
USD-AUD	85,108	13	1,573	16	56,491**	15	0,430***	16
USD-CAD	78,878	11	1,556	13	48,812**	10	0,409**	14
USD-CLP	93,361	16	1,585	18	63,178**	17	0,437*	19
USD-NZD	97,305	18	1,598	25	66,572***	19	0,450***	26
USD-ZAR	93,933	17	1,576	17	59,429**	16	0,425***	15
VIX	109,143	22	1,597	23	74,061*	23	0,440**	22
OVX	53,941	10	1,458***	9	42,610***	8	0,406***	12
BDI	102,769	19	1,591	22	67,814**	20	0,434***	18
EPU	120,759	28	1,603	26	84,640**	28	0,445**	25
<i>GARCH-MIDAS</i>								
Prod	50,340***	4	1,424***	4	41,972***	5	0,359***	3
Kilian	50,477***	5	1,429***	7	41,290***	4	0,360***	4
Prod vol	50,648***	7	1,424***	5	42,430	7	0,364***	6
Kilian vol	50,323***	3	1,420***	3	42,111***	6	0,361***	5
Working T	50,624***	6	1,424***	6	42,826***	9	0,366***	7
<i>AR-X-log-GARCH-X</i>								
GETS	52,414***	9	1,499***	10	49,307***	11	0,581***	28

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

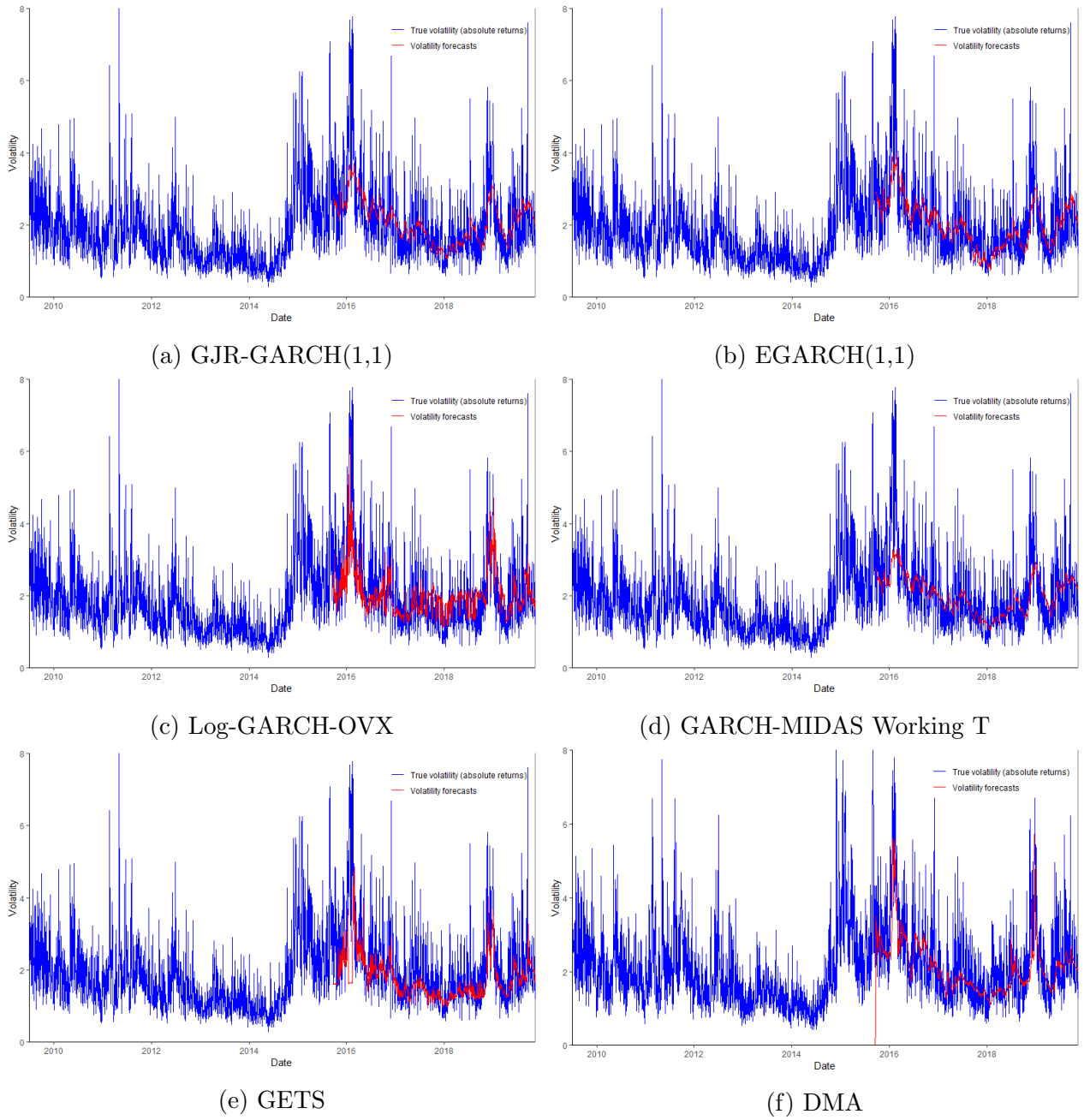
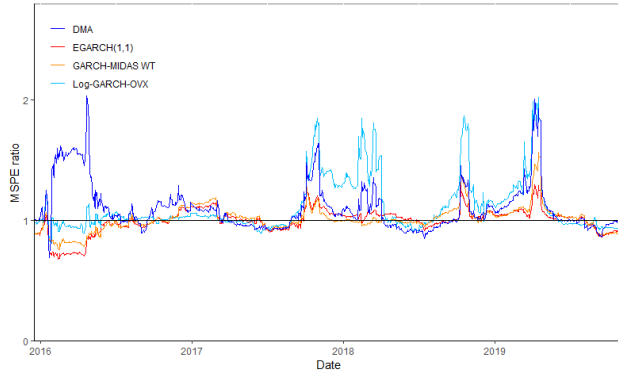


Figure 2: Volatility forecasts of selected models (Brent)

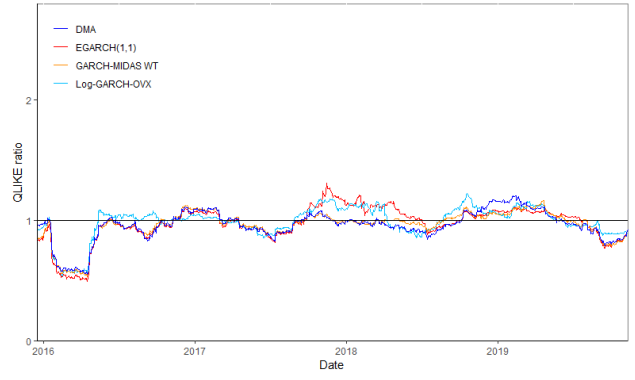
Table 18: Simple forecast combinations and dynamic model averaging (Brent)

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	84,104	1,553	52,334	0,403
Trimmed mean	89,722	1,568	56,731	0,416
DMSPE	76,604	1,535	48,429	0,389
DMA	53,509	1,430	34,606	0,306
<i>GARCH-MIDAS models</i>				
Mean	50,368	1,422	42,011	0,359
Trimmed mean	50,436	1,423	42,065	0,360
DMSPE	50,376	1,423	42,231	0,364
DMA	53,020	1,444	38,060	1,452
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	69,690	1,498	43,421	0,360
Trimmed mean	69,435	1,500	43,655	0,362
DMSPE	59,397	1,456	38,303	0,330
DMA	53,061	1,430	34,498	0,304
<i>AR-X-log-GARCH-X</i>				
GETS	52,414	1,499	49,307	0,581

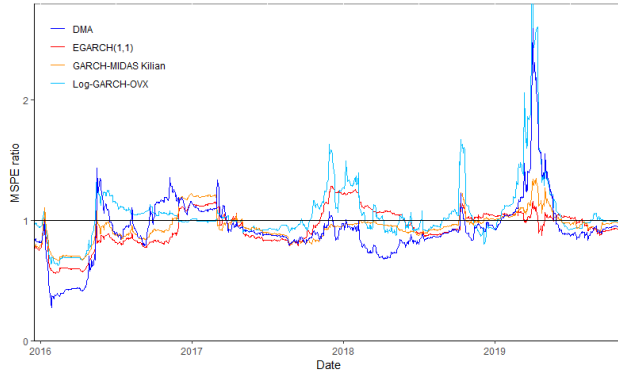
Notes: Bold characters signal models with the lowest information criteria.



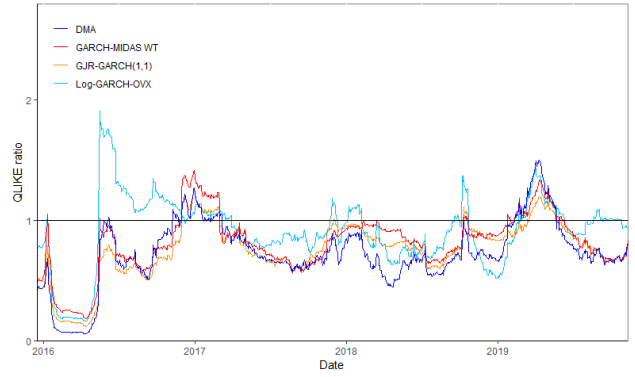
(a) Benchmark: squared return - LF: MSPE



(b) Benchmark: squared return - LF: QLIKE



(c) Benchmark: intraday range - LF: MSPE



(d) Benchmark: intraday range - LF: QLIKE

Figure 3: Rolling window loss function ratio relative to the GETS (Brent)

Table 19: 5-day ahead out-of-sample results (Brent)

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	51,838***	9	1,426***	1	42,875***	5	0,367***	2
GJR-GARCH(1,1)	50,495***	2	1,426***	2	41,026***	3	0,362***	1
EGARCH(1,1)	50,473***	1	1,462***	9	40,680***	2	0,411***	14
log-GARCH(1,1)	106,426*	28	1,604	27	81,493*	28	0,459*	27
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	96,089*	21	1,581	22	70,450*	21	0,437***	21
AVOL	105,267	26	1,599	26	76,198*	24	0,448***	24
TB3M	97,702*	22	1,581	23	76,441*	26	0,444***	23
LTGBY	98,124*	23	1,579	20	70,491*	22	0,435***	19
Basis	99,194*	24	1,587	24	73,027*	23	0,441***	22
S&P 500	92,566*	20	1,551	13	59,688**	17	0,400***	10
MSCI EM	83,050*	14	1,552	14	52,717***	13	0,405***	12
GSCI	86,675*	17	1,541	10	53,507***	14	0,390***	5
BCOM	77,906*	12	1,546	12	51,269***	11	0,403***	11
USD-AUD	80,413*	13	1,574	17	56,777**	15	0,433***	18
USD-CAD	74,727	11	1,554	15	47,680*	10	0,408***	13
USD-CLP	86,219	16	1,577	18	62,575*	18	0,435**	20
USD-NZD	91,742*	18	1,597	25	66,770***	20	0,452***	25
USD-ZAR	83,309	15	1,569	16	56,851**	16	0,422***	15
VIX	103,154	25	1,606	28	76,280*	25	0,455**	26
OVX	51,618***	8	1,440***	5	37,335***	1	0,368***	3
BDI	91,785	19	1,580	21	66,174*	19	0,433***	17
EPU	105,705*	27	1,577	19	78,410**	27	0,431***	16
<i>GARCH-MIDAS</i>								
Prod	50,825***	3	1,436***	3	43,237***	6	0,383***	4
Kilian	51,156***	5	1,450***	8	42,669***	4	0,391***	6
Prod vol	51,516***	7	1,443***	6	44,077***	8	0,397***	8
Kilian vol	50,926***	4	1,447***	7	43,714***	7	0,393***	7
Working T	51,183***	6	1,439***	4	44,383***	9	0,398***	9
<i>AR-X-log-GARCH-X</i>								
GETS	54,858***	10	1,543**	11	52,191***	12	0,633***	28

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

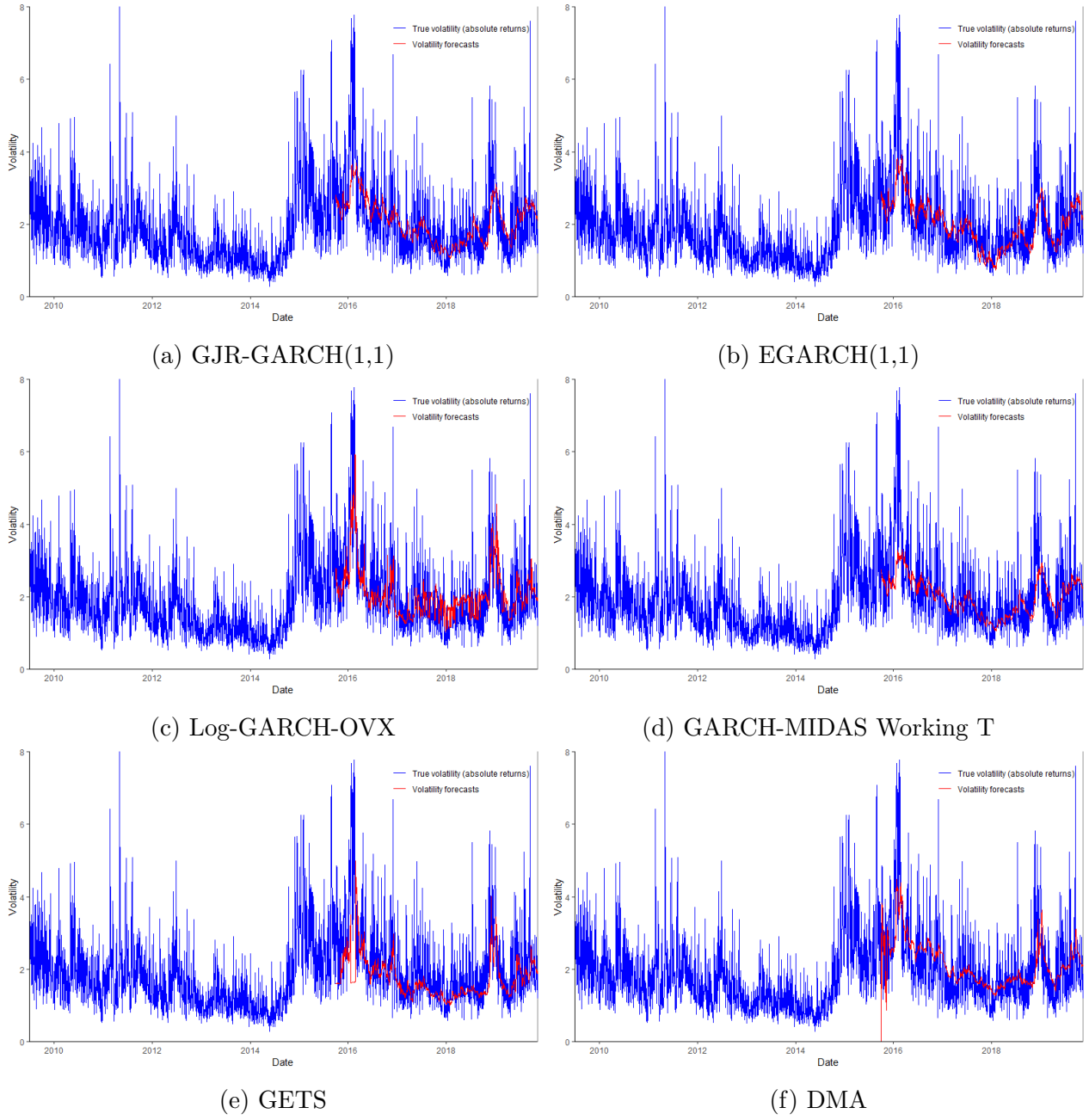
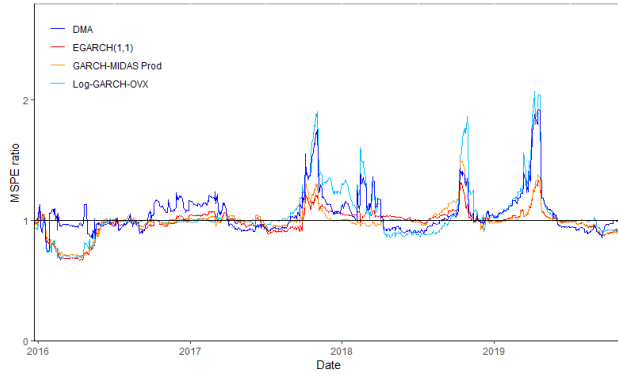


Figure 4: 5-day volatility forecasts of selected models (Brent)

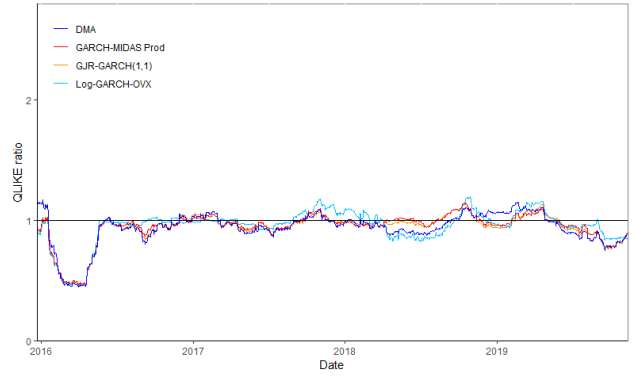
Table 20: 5-day ahead simple forecast combinations and dynamic model averaging

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	78,168	1,549	52,917	0,404
Trimmed mean	81,691	1,561	56,154	0,415
DMSPE	73,873	1,534	49,936	0,394
DMA	51,855	1,436	48,896	0,336
<i>GARCH-MIDAS models</i>				
Mean	50,968	1,439	43,463	0,389
Trimmed mean	50,946	1,439	43,491	0,389
DMSPE	50,797	1,436	43,346	0,389
DMA	55,000	1,457	39,525	1,447
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	66,123	1,498	45,260	0,366
Trimmed mean	65,888	1,499	45,427	0,369
DMSPE	59,654	1,466	41,073	0,346
DMA	51,879	1,436	44,704	0,608
<i>AR-X-log-GARCH-X</i>				
GETS	54,858	1,543	52,191	0,633

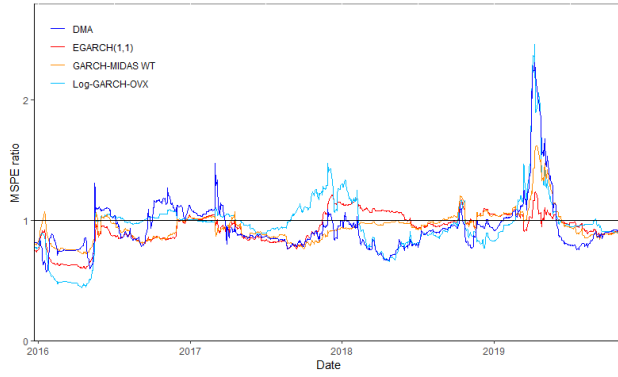
Notes: Bold characters signal models with the lowest information criteria.



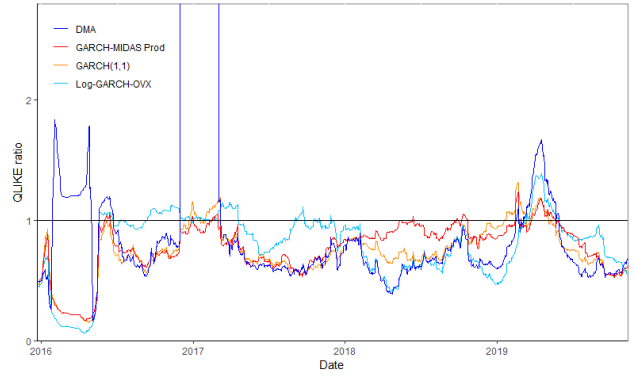
(a) Benchmark: squared return - LF: MSPE



(b) Benchmark: squared return - LF: QLIKE



(c) Benchmark: intraday range - LF: MSPE



(d) Benchmark: intraday range - LF: QLIKE

Figure 5: 5-day rolling window loss function ratio relative to the GETS (Brent)

Table 21: 21-day ahead out-of-sample results (Brent)

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	53,847**	8	1,474***	6	49,151***	8	0,464***	12
GJR-GARCH(1,1)	51,913***	3	1,465***	1	46,972***	2	0,462***	10
EGARCH(1,1)	51,999***	4	1,517***	9	47,590***	4	0,559***	27
log-GARCH(1,1)	91,604*	25	1,603	27	78,074*	26	0,490***	24
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	75,542*	18	1,566*	17	63,946*	19	0,461***	8
AVOL	89,083*	24	1,593	25	74,837*	24	0,478***	19
TB3M	82,167*	23	1,573*	20	70,785*	23	0,496***	25
LTGBY	74,420*	16	1,560*	16	63,413*	18	0,461***	9
Basis	77,552*	20	1,567*	18	65,675*	21	0,460***	7
S&P 500	68,257*	13	1,519***	10	55,608***	14	0,420***	1
MSCI EM	68,020*	12	1,541**	12	53,857***	12	0,428***	3
GSCI	131,718*	28	1,587*	23	106,607*	28	0,469***	16
BCOM	100,493*	26	1,582	22	77,994*	25	0,463***	11
USD-AUD	71,868*	14	1,569*	19	59,939*	15	0,464***	13
USD-CAD	76,983*	19	1,546	13	52,418*	11	0,433***	4
USD-CLP	79,122*	22	1,580	21	64,899*	20	0,468***	14
USD-NZD	78,079*	21	1,588**	24	65,717***	22	0,481***	20
USD-ZAR	67,247*	11	1,548**	14	51,758***	10	0,433***	5
VIX	107,993*	27	1,652	28	92,492*	27	0,529***	26
OVX	58,510**	10	1,472***	3	51,197***	9	0,470***	17
BDI	72,409*	15	1,558**	15	60,875**	16	0,452***	6
EPU	74,738*	17	1,520***	11	63,113***	17	0,426***	2
<i>GARCH-MIDAS</i>								
Prod	51,745***	1	1,467***	2	47,431***	3	0,468***	15
Kilian	52,277***	6	1,472***	5	46,916***	1	0,470***	18
Prod vol	53,196**	7	1,480***	8	49,090***	7	0,488***	23
Kilian vol	51,790***	2	1,475***	7	47,761***	5	0,483***	21
Working T	52,153***	5	1,472***	4	48,381***	6	0,484***	22
<i>AR-X-log-GARCH-X</i>								
GETS	57,357**	9	1,600**	26	54,823***	13	0,693	28

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

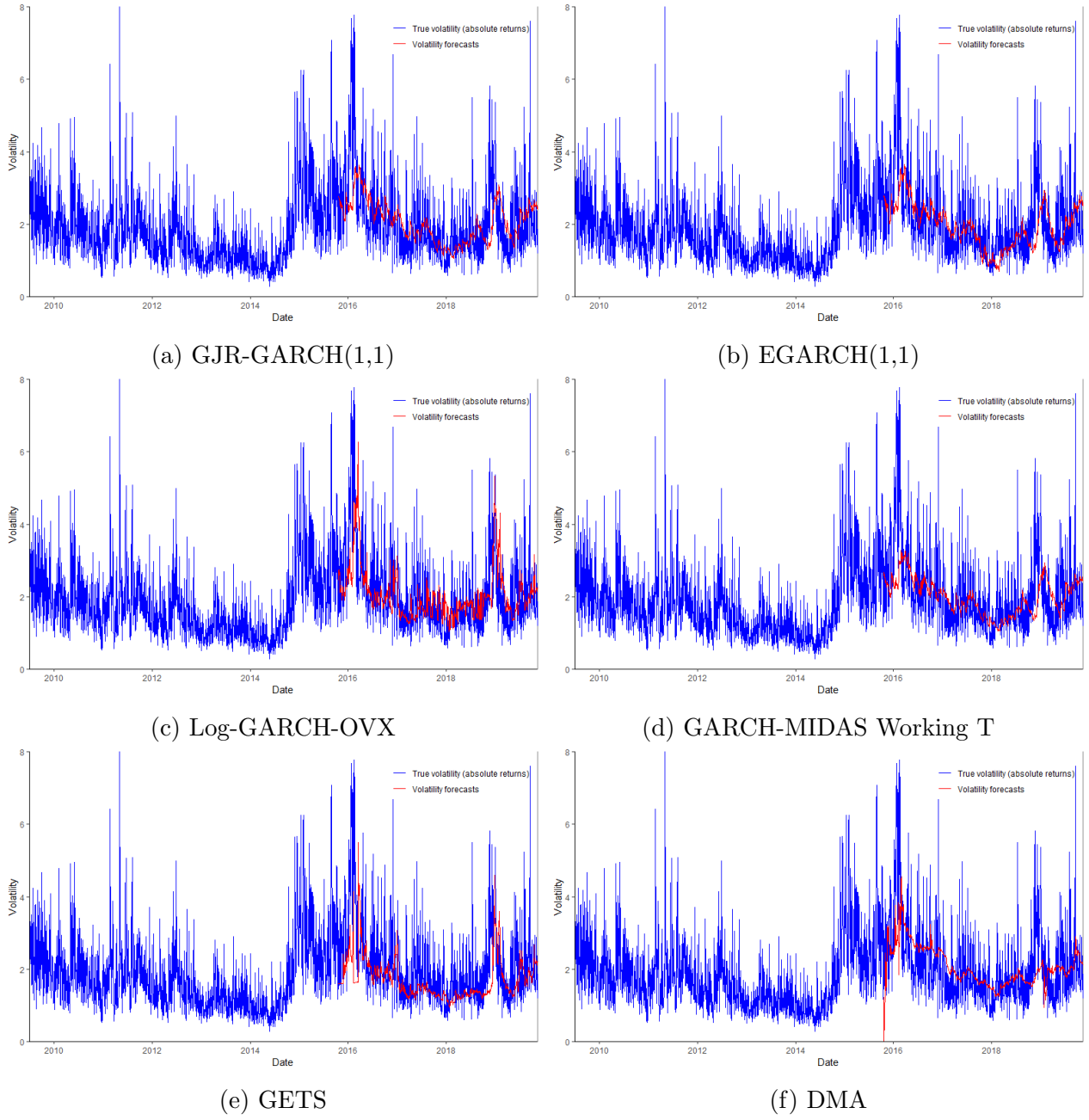
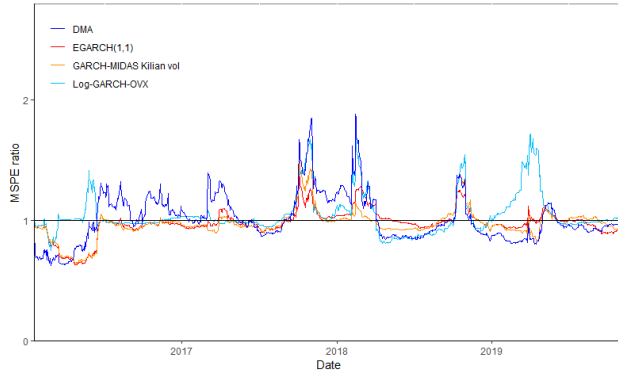


Figure 6: 21-day volatility forecasts of selected models (Brent)

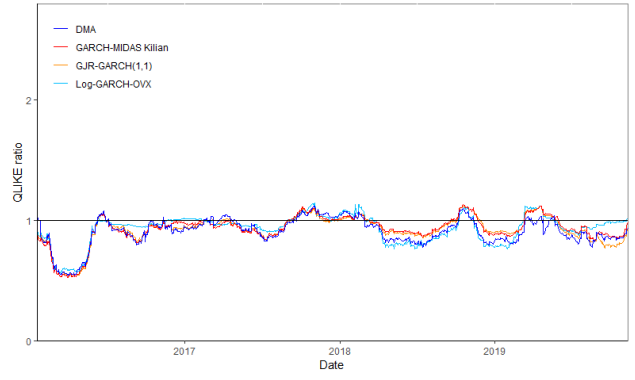
Table 22: 21-day ahead simple forecast combinations and dynamic model averaging

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	67,466	1,537	54,638	0,430
Trimmed mean	68,759	1,545	55,976	0,437
DMSPE	66,716	1,530	55,018	0,429
DMA	51,878	1,448	41,322	0,336
<i>GARCH-MIDAS models</i>				
Mean	51,918	1,468	47,602	0,472
Trimmed mean	51,886	1,468	47,709	0,474
DMSPE	52,171	1,476	47,820	0,480
DMA	52,577	1,454	50,800	1,504
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	61,892	1,503	50,476	0,410
Trimmed mean	60,989	1,503	50,684	0,413
DMSPE	59,121	1,485	49,393	0,404
DMA	52,008	1,447	42,834	0,434
<i>AR-X-log-GARCH-X</i>				
GETS	57,357	1,600	54,823	0,693

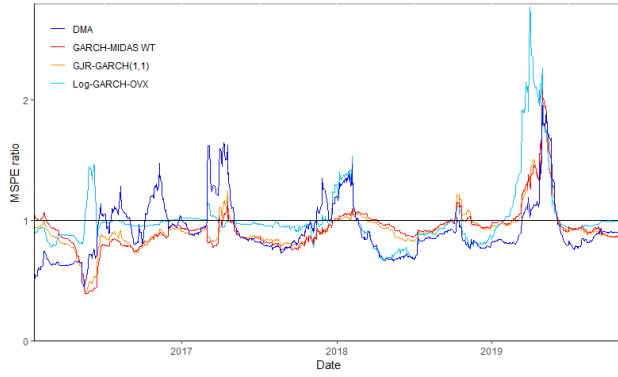
Notes: Bold characters signal models with the lowest information criteria.



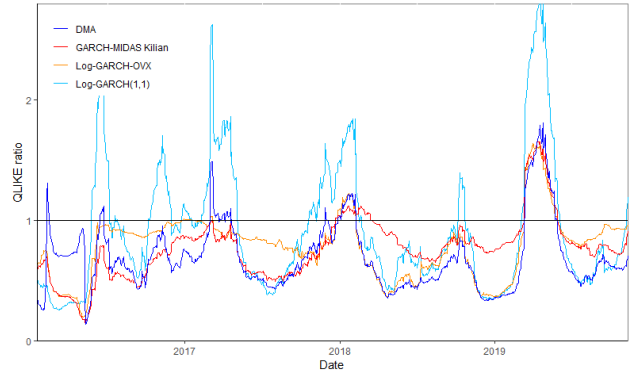
(a) Benchmark: squared return - LF: MSPE



(b) Benchmark: squared return - LF: QLIKE



(c) Benchmark: intraday range - LF: MSPE



(d) Benchmark: intraday range - LF: QLIKE

Figure 7: 21-day rolling window loss function ratio relative to the GETS (Brent)

Table 23: 63-day ahead out-of-sample results (Brent)

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	56,714***	8	1,524***	5	53,350***	5	0,510***	9
GJR-GARCH(1,1)	55,109***	3	1,556***	14	52,858***	1	0,575***	15
EGARCH(1,1)	55,027***	1	1,705***	23	53,872***	6	0,750***	24
log-GARCH(1,1)	68,085***	22	1,561***	17	63,051***	20	0,505***	7
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	58,278***	15	1,525***	6	55,102***	15	0,496***	5
AVOL	69,928***	23	1,569***	18	65,123***	23	0,522***	10
TB3M	64,674***	20	3,247***	28	61,257***	19	1,841***	28
LTGBY	57,430***	12	1,527***	7	54,612***	11	0,505***	6
Basis	58,792***	16	1,521***	4	55,368***	16	0,489***	4
S&P 500	56,996***	9	1,534***	8	55,081***	13	0,534***	12
MSCI EM	57,496***	13	1,512***	2	53,276***	4	0,471***	1
GSCI	1458,841	28	2,188***	27	1427,701	28	1,394***	27
BCOM	657,772*	27	1,934***	26	635,227*	27	0,995***	26
USD-AUD	61,219***	18	1,634***	22	54,750***	12	0,581***	16
USD-CAD	199,809***	25	1,601***	20	187,568***	25	0,651***	22
USD-CLP	90,861***	24	1,543***	11	86,959***	24	0,532***	11
USD-NZD	61,003***	17	1,544***	12	57,218***	18	0,509***	8
USD-ZAR	57,592***	14	1,507***	1	52,860***	2	0,474***	2
VIX	212,241***	26	1,738***	24	207,796***	26	0,681***	23
OVX	67,397***	21	1,560***	16	64,624***	22	0,586***	17
BDI	57,175***	10	1,515***	3	54,084***	8	0,487***	3
EPU	57,233***	11	1,605***	21	54,424***	9	0,611***	20
<i>GARCH-MIDAS</i>								
Prod	55,474***	5	1,557***	15	54,487***	10	0,598***	19
Kilian	55,450***	4	1,535***	9	52,904***	3	0,546***	13
Prod vol	56,181***	7	1,548***	13	55,087***	14	0,589***	18
Kilian vol	55,590***	6	1,584***	19	55,407***	17	0,650***	21
Working T	55,096***	2	1,542***	10	53,950***	7	0,571***	14
<i>AR-X-log-GARCH-X</i>								
GETS	61,941***	19	1,754***	25	64,036***	21	0,936	25

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

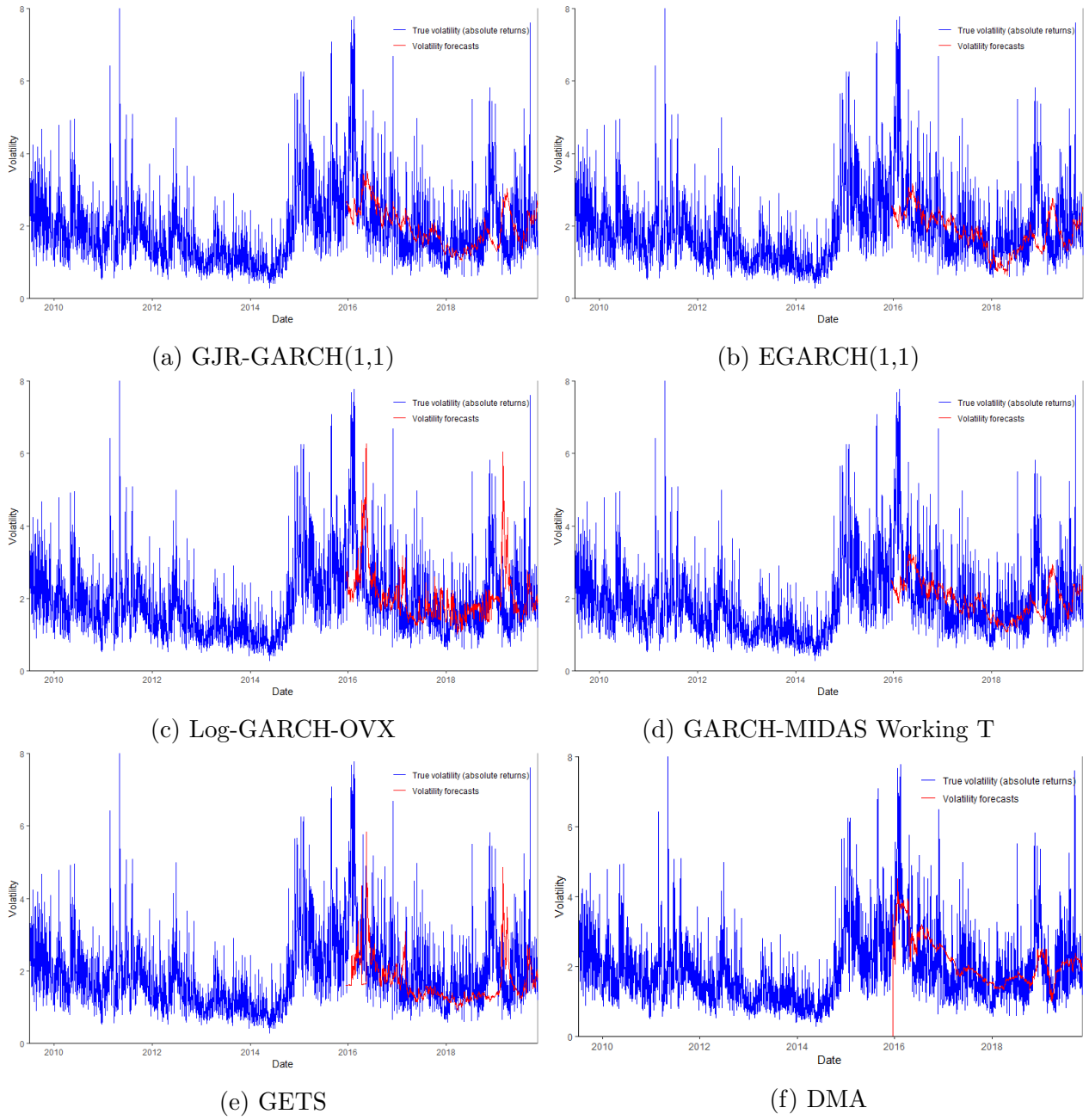
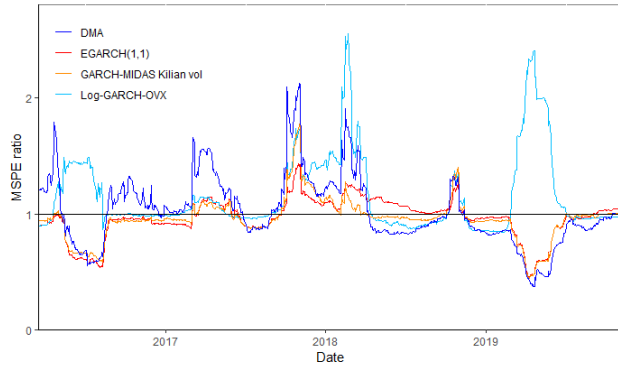


Figure 8: 63-day volatility forecasts of selected models (Brent)

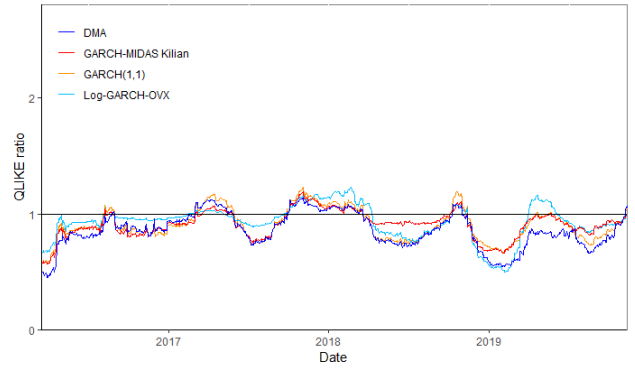
Table 24: 63-day ahead simple forecast combinations and dynamic model averaging

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	56,187	1,511	52,459	0,470
Trimmed mean	56,302	1,511	52,791	0,474
DMSPE	57,091	1,514	53,423	0,478
DMA	51,240	1,432	37,837	0,369
<i>GARCH-MIDAS models</i>				
Mean	55,316	1,546	54,124	0,580
Trimmed mean	55,402	1,547	54,301	0,585
DMSPE	55,486	1,545	54,157	0,579
DMA	51,481	1,448	42,780	1,517
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	61,661	1,507	56,496	0,461
Trimmed mean	55,806	1,499	52,632	0,472
DMSPE	56,157	1,496	52,052	0,460
DMA	51,289	1,435	37,961	0,370
<i>AR-X-log-GARCH-X</i>				
GETS	61,941	1,754	64,036	0,936

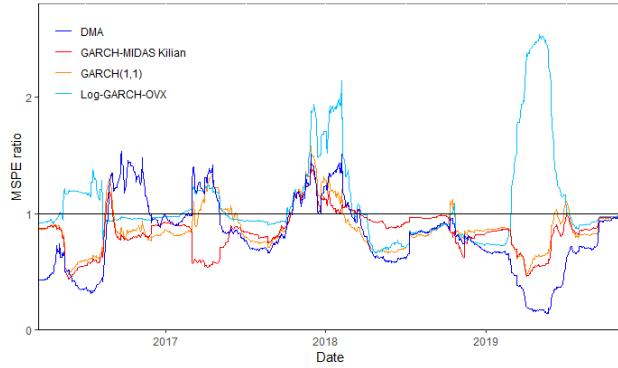
Notes: Bold characters signal models with the lowest information criteria.



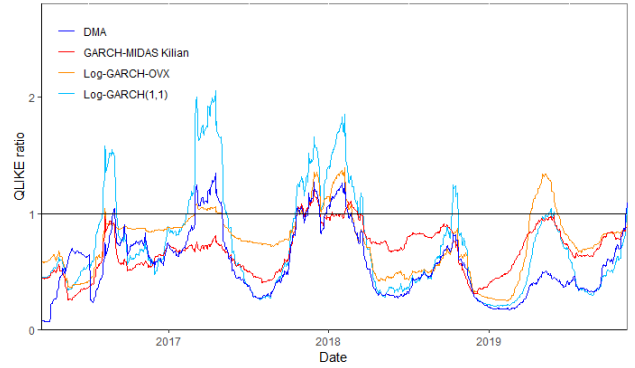
(a) Benchmark: squared return - LF: MSPE



(b) Benchmark: squared return - LF: QLIKE



(c) Benchmark: intraday range - LF: MSPE



(d) Benchmark: intraday range - LF: QLIKE

Figure 9: 63-day rolling window loss function ratio relative to the GETS (Brent)

Table 25: 1-day ahead out-of-sample results with an alternative rolling window size (Brent)

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	36,683**	8	1,402***	1	22,699***	4	0,344***	2
GJR-GARCH(1,1)	36,225**	2	1,402***	2	22,277***	1	0,340***	1
EGARCH(1,1)	36,306**	4	1,404***	3	22,719***	5	0,348***	3
log-GARCH(1,1)	53,938	16	1,582	24	35,208	15	0,461	24
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	54,645	18	1,578	20	35,407	17	0,456	20
AVOL	55,438	24	1,580	22	36,065	21	0,457	21
TB3M	51,329	14	1,561	15	32,585	13	0,441	14
LTGBY	64,001	28	1,589	27	46,312	28	0,471	27
Basis	54,773	21	1,581	23	35,650	19	0,458	22
S&P 500	60,680	27	1,556	13	41,402	27	0,433	12
MSCI EM	53,216	15	1,564	16	36,224	22	0,447	16
GSCI	54,586	17	1,548	11	35,986	20	0,424*	11
BCOM	50,474	12	1,551	12	33,124	14	0,435	13
USD-AUD	54,812	22	1,584	25	36,895	23	0,467	25
USD-CAD	50,223	11	1,560	14	31,970	11	0,441	15
USD-CLP	58,278	25	1,585	26	40,081	25	0,468	26
USD-NZD	59,382	26	1,597	28	40,343	26	0,478	28
USD-ZAR	55,128	23	1,576	17	37,087	24	0,461	23
VIX	54,741	20	1,576	18	35,489	18	0,453	17
OVX	38,739**	10	1,436***	10	23,803***	10	0,364***	7
BDI	54,652	19	1,580	21	35,241	16	0,455	18
EPU	50,783	13	1,578	19	32,440	12	0,456	19
<i>GARCH-MIDAS</i>								
Prod	36,256**	3	1,417**	5	22,579***	2	0,359**	5
Kilian	36,602**	7	1,427	9	22,879***	7	0,370*	9
Prod vol	36,743**	9	1,421**	8	22,978***	9	0,356**	4
Kilian vol	36,591**	6	1,418**	6	22,904***	8	0,363*	6
Working T	36,499**	5	1,420**	7	22,833***	6	0,364*	8
<i>AR-X-log-GARCH-X</i>								
GETS	35,464***	1	1,411***	4	22,618***	3	0,405*	10

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 26: 1-day simple forecast combinations and dynamic model averaging with an alternative rolling window size (Brent)

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	51,059	1,549	32,933	0,430
Trimmed mean	52,608	1,565	34,323	0,445
DMSPE	50,141	1,537	32,516	0,420
DMA	37,474	1,407	22,533	0,317
<i>GARCH-MIDAS models</i>				
Mean	36,478	1,419	22,774	0,360
Trimmed mean	36,479	1,421	22,769	0,364
DMSPE	36,531	1,419	22,948	0,366
DMA	37,330	1,431	22,720	1,417
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	44,252	1,488	27,269	0,378
Trimmed mean	44,424	1,492	27,473	0,383
DMSPE	42,404	1,461	26,146	0,358
DMA	37,534	1,412	22,647	0,323
<i>AR-X-log-GARCH-X</i>				
GETS	35,464	1,411	22,618	0,405

Notes: Bold characters signal models with the lowest information criteria.

Table 27: 5-day ahead out-of-sample results with an alternative rolling window size (Brent)

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	37,173***	5	1,414***	1	23,556***	2	0,364***	1
GJR-GARCH(1,1)	36,781***	1	1,416***	2	23,264***	1	0,367***	2
EGARCH(1,1)	37,065***	4	1,426***	4	23,896***	4	0,380***	4
log-GARCH(1,1)	53,515	20	1,584	24	35,775	18	0,464*	23
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	53,303	17	1,575	18	35,377	16	0,454*	19
AVOL	54,009	21	1,575	19	35,863	19	0,454*	18
TB3M	49,908	12	1,554	11	32,638	12	0,438**	12
LTGBY	61,295	28	1,589	26	43,907	28	0,472	27
Basis	53,315	18	1,578	22	35,484	17	0,457*	21
S&P 500	58,002	26	1,558	14	40,442	26	0,435**	11
MSCI EM	52,410	15	1,565	17	35,873	20	0,448*	16
GSCI	54,987	24	1,555	12	37,321	24	0,432**	10
BCOM	50,758	14	1,558	13	33,922	14	0,440**	13
USD-AUD	54,333	22	1,590	27	37,303	23	0,470	26
USD-CAD	50,158	13	1,563	15	33,190	13	0,446*	15
USD-CLP	56,675	25	1,585	25	39,059	25	0,465	25
USD-NZD	58,314	27	1,600	28	40,573	27	0,480**	28
USD-ZAR	53,459	19	1,577	21	36,556	21	0,461	22
VIX	54,814	23	1,581	23	36,563	22	0,457*	20
OVX	39,103***	10	1,425***	3	24,449***	9	0,373***	3
BDI	53,295	16	1,577	20	35,267	15	0,453*	17
EPU	48,347	11	1,563	16	31,247	11	0,445*	14
<i>GARCH-MIDAS</i>								
Prod	37,055***	3	1,448	5	23,760***	3	0,403**	6
Kilian	37,351***	7	1,453	8	24,053***	6	0,414**	9
Prod vol	38,002***	9	1,451*	7	24,517***	10	0,398**	5
Kilian vol	37,609***	8	1,459	9	24,248***	8	0,411**	7
Working T	37,272***	6	1,449*	6	24,046***	5	0,413**	8
<i>AR-X-log-GARCH-X</i>								
GETS	36,900***	2	1,470*	10	24,092***	7	0,464*	24

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 28: 5-day simple forecast combinations and dynamic model averaging with an alternative rolling window size (Brent)

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	50,314	1,550	33,146	0,432
Trimmed mean	51,721	1,566	34,383	0,446
DMSPE	49,974	1,539	32,875	0,426
DMA	38,230	1,431	23,732	0,335
<i>GARCH-MIDAS models</i>				
Mean	37,362	1,450	24,029	0,405
Trimmed mean	37,287	1,450	23,976	0,409
DMSPE	37,274	1,447	24,038	0,407
DMA	37,969	1,439	23,633	1,425
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	44,305	1,494	28,151	0,386
Trimmed mean	44,476	1,498	28,287	0,390
DMSPE	43,074	1,471	27,293	0,374
DMA	38,057	1,427	23,634	0,334
<i>AR-X-log-GARCH-X</i>				
GETS	36,900	1,470	24,092	0,464

Notes: Bold characters signal models with the lowest information criteria.

Table 29: 21-day ahead out-of-sample results with an alternative rolling window size (Brent)

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	40,702***	7	1,490***	2	27,243***	7	0,466***	14
GJR-GARCH(1,1)	39,816***	2	1,485***	1	26,678***	2	0,478***	19
EGARCH(1,1)	40,022***	3	1,494***	3	27,106***	3	0,489***	22
log-GARCH(1,1)	52,517	21	1,595**	25	35,495*	22	0,475***	17
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	49,985	17	1,576**	17	33,250**	19	0,461***	9
AVOL	51,107	20	1,578**	21	34,381**	20	0,463***	12
TB3M	48,218	12	1,566**	12	31,805**	12	0,457***	5
LTGBY	55,282	27	1,595*	24	37,011*	26	0,478***	20
Basis	49,868	15	1,578**	19	33,121**	17	0,461***	10
S&P 500	52,548	22	1,558***	11	34,438***	21	0,439***	2
MSCI EM	50,279	19	1,574***	15	32,775***	14	0,450***	3
GSCI	54,000	25	1,571***	13	36,275**	25	0,453***	4
BCOM	49,921	16	1,575**	16	32,814**	15	0,457***	6
USD-AUD	52,655	23	1,593**	23	35,564*	23	0,476***	18
USD-CAD	49,236	13	1,573**	14	32,622**	13	0,458***	7
USD-CLP	53,395	24	1,586*	22	36,201**	24	0,468***	15
USD-NZD	54,718	26	1,601***	27	37,295***	27	0,482***	21
USD-ZAR	49,758	14	1,578**	20	32,850**	16	0,463***	13
VIX	56,459	28	1,601**	26	38,192*	28	0,475***	16
OVX	44,246*	11	1,530***	9	27,976***	9	0,463***	11
BDI	50,026	18	1,577**	18	33,248**	18	0,460***	8
EPU	43,472*	10	1,533***	10	28,308***	11	0,429***	1
<i>GARCH-MIDAS</i>								
Prod	39,486***	1	1,503***	4	26,431***	1	0,494***	23
Kilian	40,229***	5	1,522***	6	27,220***	6	0,525**	27
Prod vol	41,266***	8	1,518***	5	27,874***	8	0,499***	24
Kilian vol	40,263***	6	1,524***	8	27,111***	5	0,519***	25
Working T	40,076***	4	1,523***	7	27,107***	4	0,520***	26
<i>AR-X-log-GARCH-X</i>								
GETS	41,333***	9	1,624**	28	28,255***	10	0,648	28

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 30: 21-day simple forecast combinations and dynamic model averaging with an alternative rolling window size (Brent)

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	48,709	1,558	31,792	0,442
Trimmed mean	49,573	1,570	32,646	0,453
DMSPE	47,847	1,544	31,277	0,433
DMA	39,158	1,489	25,161	0,335
<i>GARCH-MIDAS models</i>				
Mean	40,132	1,514	27,016	0,505
Trimmed mean	40,018	1,515	26,991	0,513
DMSPE	40,427	1,524	27,356	0,520
DMA	39,547	1,472	25,647	1,449
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	45,022	1,519	29,128	0,419
Trimmed mean	45,028	1,522	29,219	0,422
DMSPE	44,060	1,501	28,570	0,410
DMA	39,246	1,468	25,025	0,359
<i>AR-X-log-GARCH-X</i>				
GETS	41,333	1,624	28,255	0,648

Notes: Bold characters signal models with the lowest information criteria.

Table 31: 63-day ahead out-of-sample results with an alternative rolling window size (Brent)

	Squared return				Intraday range			
	MSPE	Rank	QLIKE	Rank	MSPE	Rank	QLIKE	Rank
<i>Benchmarks</i>								
GARCH(1,1)	44,052***	18	1,553***	18	31,923*	20	0,540	19
GJR-GARCH(1,1)	44,112***	20	1,594*	21	32,205*	23	0,594	21
EGARCH(1,1)	43,589***	15	1,590**	20	31,782**	18	0,592	20
log-GARCH(1,1)	44,149***	21	1,507***	16	29,952***	14	0,430***	12
<i>Single-predictor log-GARCH(1,1)-X</i>								
AOI	42,543***	3	1,488***	4	29,043***	4	0,427***	11
AVOL	42,646***	6	1,490***	5	28,872***	1	0,425***	9
TB3M	44,078***	19	1,574*	19	30,365**	15	0,500*	18
LTGBY	43,569***	14	1,493***	8	29,660***	13	0,426***	10
Basis	42,565***	5	1,488***	3	28,965***	3	0,424***	6
S&P 500	44,179***	22	1,506***	15	31,026**	17	0,450**	15
MSCI EM	43,360***	11	1,492***	7	29,507***	12	0,421***	2
GSCI	44,310***	25	1,502***	13	30,751**	16	0,437***	14
BCOM	43,064***	8	1,490***	6	29,387***	8	0,423***	5
USD-AUD	43,530***	12	1,498***	11	29,458***	9	0,424***	7
USD-CAD	43,051***	7	1,499***	12	29,490***	11	0,434***	13
USD-CLP	43,258***	10	1,495***	9	29,300***	7	0,423***	4
USD-NZD	43,224***	9	1,497***	10	29,141***	6	0,422***	3
USD-ZAR	42,506***	2	1,487***	2	28,887***	2	0,420***	1
VIX	52,478*	27	1,551**	17	38,065**	27	0,466***	16
OVX	53,553*	28	1,679	27	41,665*	28	0,705	27
BDI	42,553***	4	1,486***	1	29,095***	5	0,425***	8
EPU	42,013***	1	1,505***	14	29,459***	10	0,468***	17
<i>GARCH-MIDAS</i>								
Prod	43,944***	16	1,610*	24	32,065*	22	0,627	24
Kilian	44,187***	23	1,625*	26	32,320*	25	0,639	26
Prod vol	44,227***	24	1,595*	22	32,315*	24	0,622	23
Kilian vol	43,537***	13	1,617*	25	31,792**	19	0,628	25
Working T	44,018***	17	1,601*	23	31,948*	21	0,600	22
<i>AR-X-log-GARCH-X</i>								
GETS	47,122*	26	1,869	28	36,695	26	0,999	28

Notes: Bold characters signal models with the lowest information criteria. *, **, and *** mean that the model is in $\widehat{M}_{90\%}^*$, $\widehat{M}_{75\%}^*$, and $\widehat{M}_{60\%}^*$, respectively.

Table 32: 63-day simple forecast combinations and dynamic model averaging with an alternative rolling window size (Brent)

	Squared return		Intraday range	
	MSPE	QLIKE	MSPE	QLIKE
<i>Single-predictor log-GARCH-X models</i>				
Mean	42,850	1,491	29,343	0,428
Trimmed mean	42,891	1,493	29,272	0,427
DMSPE	42,658	1,486	29,165	0,425
DMA	40,540	1,507	27,033	0,373
<i>GARCH-MIDAS models</i>				
Mean	43,931	1,604	32,036	0,616
Trimmed mean	44,042	1,612	32,114	0,622
DMSPE	43,826	1,593	31,905	0,611
DMA	40,993	1,525	27,450	1,435
<i>Benchmarks + log-GARCH-X models + GARCH-MIDAS models</i>				
Mean	42,896	1,492	29,790	0,441
Trimmed mean	42,684	1,490	29,551	0,438
DMSPE	42,680	1,485	29,601	0,438
DMA	40,604	1,558	26,987	0,370
<i>AR-X-log-GARCH-X</i>				
GETS	47,122	1,869	36,695	0,999

Notes: Bold characters signal models with the lowest information criteria.