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# Stochastic Dual Dynamic Programming as a modelling Tool for Power Systems with Demand Response and intermittent Renewables

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## Abstract

In modern power systems modelling notions of stochasticity and stock management have acquired importance outside of the world of hydroelectricity due the emergence of intermittent renewables, batteries and demand-side management. This paper proposes a short review and description of Stochastic Dual Dynamic Programming with the concern of applying it to such large scale power systems. An application of SDDP to North Western Europe's power 2023 power system with internal high gas prices, interconnections with neighbours having access to cheaper gas and with demand response is presented. We show that demand-response reduces market prices, outages and drives out of the market peaking gas plants while improving social welfare in the market.

**Key-words** OR in energy, Demand response, Power systems, Renewable Energy Sources, Stochastic Dual Dynamic Programming

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# 1 Introduction

In line with climate change mitigation efforts, future power systems will rely on increased shares of renewable energy sources (RES). The intermittent, random nature of most RES will bring more noise and less flexibility on the supply side of power markets. Commonly evoked paths to manage such issues include increases in storage capacities and flexibilities which may take various forms such as batteries, power-hydro storage, power-to-gas or demand response. As stocks with limited sizes, recourse to these technologies reflects intertemporal arbitrages which can only be accurately accounted for by long term representations.

Common modeling approaches for power systems operations rely on multistage optimization problem, for example with the objective of minimizing the total cost of operations until some temporal horizon. Coupling these approaches with the necessary mid- to long-term study of the dynamics of stocks involves models with numerous time stages. Moreover, with low-carbon power systems, realizations of random variables should be encompassed at each time stage. It leads to astronomically big scenario trees, so much that it compromises these long term studies.

Fortunately, similar issues arise when dealing with hydro-thermal scheduling, which is key to several water-based power systems such as that of Brazil or Norway. An optimization method, Stochastic Dual Dynamic Programming (SDDP), have thus been developed by Pereira & Pinto (1991) which treats long-term scheduling of hydro-power stations with stochastic inflows and avoids a scenario based resolution. Long limited to the hydroelectricity field, SDDP may be seen as a more general, efficient way of optimizing multistage decisions about (many) stocks affected by (many) random variables. In that sense, it adapts perfectly to whole low-carbon systems problems with different and numerous storage technologies.

Historically, SDDP was introduced by Pereira & Pinto (1991) as they sought to solve a mid- to long-term horizon hydro-thermal scheduling problem yielding optimal allocation in the hydro-dominated Brazilian power system. The method in itself is described in Section 2 and a pseudo algorithm is also given. The crucial feature of SDDP lies in its ability to compute opportunity costs with lower computational efforts than traditional stochastic dynamic programming without affecting the quality of the resulting policy. Pereira & Pinto could therefore assess the evolution of marginal costs of turbinning water of the different hydroelectric reservoirs with random inflows. Since 1991, SDDP has been widely applied to

hydro-thermal scheduling in other countries. For example, Rebennack et al. (2009) consider a hydro-thermal power system covered by an emission permits market under randomness of oil prices and water inflows. Pereira-Bonvallet et al. (2016) take up the classical field of application of SDDP, long-term hydroelectric scheduling, but add competition between agricultural and power production uses of the water in the reservoirs. Thus they seek to obtain multi-usage values of stocks along the years. Guan et al. (2018) don't extend the method but applies it to a new territory, British Columbia, and introduce supplementary constraints on the usage of water stemming from international agreements. A focus on water values is made. To conclude this overview, Diniz et al. (2020) apply SDDP to a system with numerous little reservoirs with little place for maneuvers and consider other risk measures than the expectation.

Building on the previous analysis of SDDP, a few recent studies have implemented SDDP in other contexts than hydro-thermal scheduling. Among original applications, we can cite Dowson et al. (2019) applying SDDP to dairy farms management in New Zealand. Note that the same author has co-developed in Dowson & Kapelevich (2021) a Julia package extensively used in the actual solving of the model of the present paper.

But, in a more relevant way, SDDP appears in a few studies of RES dominated power systems. Wu et al. (2016) look at the long term optimal expansion planning of interconnection capacities between productive regions of the power system. A similar work is done by Lu et al. (2020) for the Chinese system with technical modifications in the stopping rule of SDDP and a focus on RES curtailment and congestion of interconnections. The setting is rather similar than Marañón-Ledesma & Tomasgard (2019) with stages of investments framing years of hourly operations, random demand and random RES production. Even though no stock are to be found here, the authors rely on SDDP because of the lower required computational effort. They show that, under uncertainty, the resulting policy of SDDP fares better than policies determined through deterministic optimizations. Closer to the classical SDDP framework, Bodal (2016) proposes the study of a Norwegian power system marked by small hydroelectric stations and a growing share of random RES production. Papavasiliou et al. (2017) solve with SDDP the short term (24h - 15min steps) optimal dispatch in a power system with random RES production and energy storage. This model accounts for transmissions between regions of production and ramping times of generation means. As the others seek to affect a value to storage in a large-scale system and part of the production is random, recourse to SDDP seems natural. They insist on

the non-anticipativity of their optimization framework as a distinction among the literature. Finally, Stüber & Odersky (2020) implements SDDP in the energy system model *urbs* and apply it to the 2015 German system with hourly resolution, an horizon of two days, 17 regional nodes, a random wind turbinning and (hydro) storage capacities. Note that a demand-side management module focusing on load-shifting exists in *urbs* but is not used in Stüber & Odersky (2020).

Thus, SDDP appears as a flexible solving tool with reduced computational effort that can encompass numerous constraints - as long as they are linear - and state variables. Through the applications it emerges that SDDP is not only adapted to hydro-thermal scheduling but to any large-scale stock management problem under uncertainty. Its main outputs are opportunity costs of using the stock at a certain date which can assess for multi-usage value of the stock and risk aversion of the user depending on the risk measure used.

In this study, we efficiently review literature, properties and implementations of SDDP in a practical point of view seeking applications to whole low-carbon systems. Then an innovative case study using SDDP is proposed: we model a 2023 power system of North Western Europe. In 2023, this power system faces several challenges: the loss of a part of its base production with Germany suppressing its nuclear production in exchange for intermittent RES, increased carbon prices and high gas prices causing on the one hand coal power plants – and notably German lignite ones – to be competitive again, and on the other hand neighbouring countries such as the United Kingdom to export its cheaper gas through electrical interconnections. In this context we seek to evaluate the multifaceted impacts of introducing demand response. Demand response refers to an accepted deviation of a load from its ordinary pattern in response to some incentive (e.g. a direct order) or some signal (e.g. a price). Demand response is modeled as stocks thanks to an analogy with hydro-power which accounts for technical and societal constraints. Conclusions are drawn in terms of economic viability of demand response, modified recourse to traditional generation units and cost-effectiveness of the system.

The paper is structured as follows. First we present a concise review of SDDP with a focus on how it may be useful for large scale power systems modeling in Section 2. Then, we propose a simple model of a modern power system with demand response adapted to SDDP in Section 3. Finally, a short application of this model to the effect of demand

response (DR) on a copperplate North Western Europe calibrated on probable conditions of year 2023 for this system exemplifies the strengths of SDDP in dealing with long-term system level problems of decarbonized power systems.

## 2 A concise description of SDDP and one implementation

We describe here succinctly the class of algorithms coined SDDP and a possible implementation. More technical material may be found in Shapiro (2011), or for a more practical approach in the documentation Dowson & Kapelevich (2021). This section relies also on the presentation of the method *An Introduction to Stochastic Dual Dynamic Programming (SDDP)* given in 2017 by V. Leclère from CERMICS (ENPC).

### 2.1 The method

SDDP seeks to solve large scale multistage stochastic optimization problems under specific hypothesis: state and control variables are continuous (actually SDDP works also with integer variables but requires some type of convex relaxation which is costly and will be avoided here), these variables must belong to convex compact sets of finite dimensional spaces, the stage objective functions must be convex, and dynamics linear. Most of all, the noise must take values in a finite set and be a white noise i.e. noises of two distinct stages are independent. The latter hypothesis enables to fit into a dynamic programming framework. Note that, it can always be obtained at the cost of expanding the dimension of the state variable (thanks to the so-called state expansion trick, see Shapiro (2011)). Under these hypothesis, SDDP yields an optimal solution of a dynamic programming problem with piece-wise linear Bellman functions approximating the Bellman functions of the true problem. Estimating these linear forms at some stage and some state relies on the computation of the dual variable associated with a slightly modified subproblem determining the true Bellman function at this stage (see below).

Let's consider an archetypal problem solvable with SDDP, with finite horizon  $T$  yielding some decision rule  $\pi$  :

$$\begin{aligned} & \min_{\pi} \mathbb{E} \left[ \sum_{t=0}^{T-1} C_t(X_t, U_t, \mathbf{W}_t) + K(X_T) \right] \\ \text{s. t. } & X_{t+1} = f_t(X_t, U_t, W_t) \\ & U_t = \pi_t(X_t, W_t) \end{aligned} \tag{2.1}$$

where  $C_t$  is an instantaneous convex cost,  $K$  a convex final cost,  $X_t \in \mathbb{X}$  a state variable at stage  $t$ ,  $f_t$  the linear state transition dynamic from stage  $t$  to  $t + 1$ ,  $\pi_t$  the decision rule at stage  $t$ ,  $U_t \in \mathbb{U}$  the decision(s) taken at  $t$  and  $\mathbf{W}_t$  the random variable representing some noise existing between stage  $t$  and stage  $t + 1$  (excluded) with values in the finite set  $\mathbb{W}_t$ . Note that, if  $\mathbb{W}_t$  isn't finite, it may be possible to reduce it to a finite set by methods such as sample average approximation. In general, it seems reasonable to enumerate  $\mathbb{W}_t$  – which is done in SDDP – but not the whole scenario tree  $\mathbb{W}_1 \times \dots \times \mathbb{W}_T$  – which is not done in SDDP but would if one wanted to discretize the whole state space. The process  $(\mathbf{W}_t)_{t \in \{1, \dots, T\}}$  is supposed to be a white noise so that there is stage-wise independence. Sets  $\mathbb{X}$  et  $\mathbb{U}$  are compact. Problem 3.25 presented in Section 3.2 is an instance of such problem.

With these hypothesis, Problem 2.1 may be formulated in a dynamic programming framework by introducing a Bellman function  $V_t$  at each time step. This approach enables to decompose the resolution of the complete problem into that of a backward in time series of subproblems associated with each stage and the system state at this stage. The Bellman function, also called the cost-to-go, represents the expected future cost when starting from a given state. It is defined for  $x \in \mathbb{X}$ ,  $w \in \mathbb{W}$  by

$$V_T(x) = K(x) \tag{2.2}$$

$$\hat{V}_t(x, w) = \min_{u_t \in \mathbb{U}} C_t(x, u_t, w) + V_{t+1} \circ f_t(x, u_t, w) \text{ if } t < T \tag{2.3}$$

$$V_t(x) = \mathbb{E} \left[ \hat{V}_t(x, \mathbf{W}_t) \right] \text{ if } t < T. \tag{2.4}$$

The goal of SDDP is to approximate by inferior values the functions  $V_t$  with a supremum of affine functions relying heavily on the convexity of the  $V_t$ . These affine functions for stage  $t$  are obtained from the subproblem defining  $V_t$  in the classical dynamic programming framework where  $V_{t+1}$  is replaced by its approximation  $\tilde{V}_{t+1}$  computed at the previous step (we begin with  $t = T$  and final cost  $K$  is supposedly known). For a realization  $w$  of  $\mathbf{W}_t$  and a given state  $x_t$  at stage  $t$ , this subproblem writes

$$\begin{aligned} \hat{\beta}_t(w) = & \min_{x, u} C_t(x, u, w) + \tilde{V}_{t+1} \circ f_t(x, u, w) \\ \text{s. t. } & x = x_t [\hat{\lambda}_t(w)] \end{aligned} \tag{2.5}$$

The dual variable  $\hat{\lambda}_t(w)$  is associated to the state constraint of the previous subproblem. By definition of  $\hat{\lambda}_t(w)$ , we have the following inequalities for all realizations  $w$  and all states

$x$

$$\hat{\beta}_t(w) + \langle \hat{\lambda}_t(w), x - x_t \rangle \leq \hat{\beta}_t(w) \leq \hat{V}_t(x, w) \quad (2.6)$$

since  $\tilde{V}_{t+1} \leq \hat{V}_{t+1}(\cdot, w)$ . An approximation by inferior values of function  $V_t$  in the neighborhood of  $x_t$  is then given by the affine function, also called cut,

$$\beta_t + \langle \lambda_t, \cdot - x_t \rangle := \mathbb{E} \left[ \hat{\beta}_t(w) \right] + \langle \mathbb{E} \left[ \hat{\lambda}_t(w) \right], \cdot - x_t \rangle. \quad (2.7)$$

Here, only the backward phase of SDDP yielding new cuts for approximating the true value functions has been described. This phase relies notably on the drawing of relevant trajectories  $(x_t)_{t \leq T}$  for the state variables. Indeed, the cuts are only good approximations of the value functions near these points. The choice of relevant state for computation is also encountered in traditional stochastic dynamic programming and the selected approach is to grid all the state space which may be extraordinarily costly in high dimensional systems. In SDDP, computations are made only at "relevant" states i.e. those taken by the system when it follows an optimal path according to some drawn trajectory of noises. Actually, to compute this optimal path, knowledge of true value functions is required. So, these value functions are replaced by their approximates built in a previous backward phase. This describes the forward phase of SDDP. Backward and forward phases depend on each other and mutually strengthen their results. Thus SDDP alternates between forward and backward phases numerous times, until some stopping condition is met.

Under hypothesis taken here, convergence of the approximate after  $k$  iterations of forward and backward phases  $\tilde{V}_t^{(k)}$  to  $V_t$  is guaranteed when  $k$  goes to infinity (see Pereira & Pinto (1991) or Shapiro (2011)). Thus the resulting policy also converges towards the true optimal policy as the algorithm iterates. At each step ( $k$ ) (composed of a forward and a backward phase), an exact inferior bound of the value of the true problem is given by  $\underline{v}^{(k)} = \tilde{V}_0^{(k)}(x_0)$  (note that the initial state  $x_0$  is supposed to be known and set) and an exact upper bound by  $\bar{V}^{(k)} = \mathbb{E} \left[ \sum_{t=0}^{T-1} C_t(X_t^{(k)}, U_t^{(k)}, \mathbf{W}_t) + K(X_T^{(k)}) \right]$ . The latter bound may not exactly be computed and must be estimated by a Monte-Carlo method: several samples of realizations of  $\mathbf{W}$  are drawn which yields as much trajectories for  $X$  and  $U$ , then the mean on samples of the total cost  $\bar{v}^{(k)}$  is computed. A confidence interval for the value of the upper bound,  $[\bar{v}_{l,\alpha}^{(k)}, \bar{v}_{h,\alpha}^{(k)}]$ , may be derived. Noting  $v^*$  the true optimal value, we have



the following inequalities after step  $(k)$  with some confidence level  $1 - \alpha$ ,

$$\underline{v}^{(k)} \leq v^* \leq \bar{V}^{(k)} \in [\bar{v}_{l,\alpha}^{(k)}, \bar{v}_{h,\alpha}^{(k)}]. \quad (2.8)$$

Several stopping rules exist and are primarily based on inequalities 2.8. As we mentioned, the inequality gets tighter as  $k$  grows so a simple stopping rule is to set a number of iterations for the algorithm but this gives no guarantee on the quality of the approximation. The original criterion proposed by Pereira & Pinto (1991) uses more the inequalities. Setting a priori some confidence level  $1 - \alpha$ , the algorithm should stop after step  $(k)$  if  $\underline{v}^{(k)} \geq \bar{v}_{l,\alpha}^{(k)}$ . However, as pointed out by Shapiro (2011), this condition may be met very early if the confidence interval around the upper bound is very large i.e. the estimation of the upper bound is of poor quality, and no optimality guarantee may be obtained. He proposes then the following stopping rule, which will be used in the application of Section 3 and in a companion paper on the impact of demand response on future, more renewable-based power systems. It states that algorithm should stop after iteration  $(k)$  if  $|1 - \bar{v}_{h,\alpha}^{(k)}/\underline{v}^{(k)}| \leq \epsilon$  with a confidence level  $1 - \alpha$  and some precision  $\epsilon > 0$  set a priori. This stopping rule guarantees that the policy yielded by SDDP is  $\epsilon$ -optimal for the true problem with probability  $1 - \alpha/2$ . Note that these results suppose that the distribution of the upper bound is normal.

## 2.2 Employed implementation

The previously described SDDP may be implemented in several ways. For example, during a forward phase, more than one trajectory of states may be drawn and cuts computed for each of these trajectories in the backward phase. This short section presents the pseudocode for one implementation of SDDP, which is used in the Julia package by Dowson & Kapelevich (2021) on which the application of Section 3 relies. We however slightly modified it in order to use Shapiro's stopping rule described above.

A crucial advantage of SDDP lies in its treatment of the state space and of noise. As the state space is not entirely discretized but only precisely described on regions the system actually explores, the number of points where a subproblem is solved and noise has to be drawn is reduced. If the noise has dimension  $N$  at each stage and it costs  $O(A)$  to solve a subproblem of the type of 2.5 (one stage convex deterministic optimization), the complexity of a forward phase is  $O(NTA)$  as one trajectory of  $T$  noises of size  $N$  is drawn.

Moreover, if the noise at some stage  $t$  takes value in a space of cardinal  $K_t$  (i.e.  $|\mathbb{W}_t| = K_t$ ), the backward stage consists of, for each stage  $t$ , the resolution of  $K_t$  subproblems and the drawing of  $K_t$  noises for the stage which yields a total complexity of the backward phase of  $O\left((N + A) \sum_{t=1}^T K_t\right)$ . It is to be compared with a scenario-based approach in which all of the  $\prod_{t=1}^T K_t$  trajectories are considered. To compute the stopping rule,  $MT$  draws of noise are necessary leading to a complexity of  $O(MTN)$ . Thus, on the whole, the complexity of SDDP is significantly lower than that of scenario based approaches and enables to consider large scale problems with long-term horizons, high dimensional noises and an important number of scenarios as the complexity grows with  $\sum_{t=1}^T K_t$  and not  $\prod_{t=1}^T K_t$ . In this sense, SDDP is well-suited to problems related with the long term analysis of systems subject to many uncertainties and stock management such as power systems with large shares of intermittent renewables and hydro-power. Stock management affected by noises is indeed the primary goal of SDDP (see Pereira & Pinto (1991)). In practice, it enables to derive naturally water values or opportunity costs for these stocks. The opportunity cost for some stock described by the  $i$ th coordinate of the state variable is indeed given at a certain state  $x_t$  by the  $i$ th coefficient of the gradient of the value function  $V_t$  at this state which may be approximated by the gradient of  $\tilde{V}_t^{(k)}$  at the same point. When yielding  $\tilde{V}_t^{(k)}$ , SDDP returns in fact the affine function i.e. a tuple of their slope  $\lambda_t^{(k+1)}$  and constant term  $\beta_t^{(k+1)}$  that reaches at state  $x_t$  the supremum among all cuts computed for  $V_t$ . Thus, the gradient of  $\tilde{V}_t^{(k)}$  at  $x_t$  is simply  $\lambda_t^{(k+1)}$ , a natural return of SDDP, and the water value its  $i$ th coefficient. This also highlights how SDDP is particularly adapted for stock management problems.

Finally, SDDP avoids the complete enumeration of all possible scenarios for the noise which alleviates a bounding computational constraint in scenario-based method – moving from complexity growing as  $\prod_{t=1}^T K_t$  to  $\sum_{t=1}^T K_t$  with our notations. Note that, however, it doesn't prevent the curse of dimensionality arising with high dimensional state variables (which increases when the state space has been expanded in order to fit to the white noise assumption). In our notations, this source of complexity is hidden behind  $O(A)$  and affects the resolution of all subproblems. In a sense, SDDP handles better stochasticity than its counterpart but is still bounded by curses inherent to a dynamic programming framework.

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**Algorithm 1** SDDP - DOASA Implementation

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**Initialization.**

Provide an initial state  $x_0$ , a precision parameter  $\epsilon$ , a confidence level  $1 - \alpha$  and a number  $M$  of sample for MC estimations

**for**  $t = 0 \dots T - 1$  **do**

Give  $\tilde{V}_t^{(0)}$  a convex approximate of  $V_t$  so that  $\tilde{V}_t^{(0)} \leq V_t$ .  
 $\tilde{V}_T^{(0)} = K$ .

**Iterations.**

**while** Shapiro's stopping rule isn't verified at the end of step  $(k - 1)$  **do**

**Forward phase.**

Draw a trajectory of noise  $(w_0, \dots, w_{T-1}) \in \mathbb{W}^T$ .

**for**  $t = 0 \dots T - 1$  **do**

Compute  $u_t^{(k)} \in \operatorname{argmin}_{u \in \mathcal{U}} \left( C_t(x_t^{(k)}, u, w_t) + \tilde{V}_{t+1}^{(k)} \circ f_t(x_t^{(k)}, u, w_t) \right)$ .

Define a point  $x_{t+1}^{(k)}$  such that  $x_{t+1}^{(k)} = f_t(x_t^{(k)}, u_t^{(k)}, w_t)$ .

**Backward phase.**

**for**  $t = T - 1, \dots, 0$  **do**

Compute for all  $w$   $\hat{\beta}_t^{(k+1)}(w)$  and  $\hat{\lambda}_t^{(k+1)}(w)$  as in 2.5. (Hence the noise must take value in a finite set)

Compute  $\beta_t^{(k+1)} = \mathbb{E} \left[ \hat{\beta}_t^{(k+1)}(\mathbf{W}_t) \right]$  and  $\lambda_t^{(k+1)} = \mathbb{E} \left[ \hat{\lambda}_t^{(k+1)}(\mathbf{W}_t) \right]$ .

Add the new cut :  $\tilde{V}_t^{(k+1)} = \max \left( \tilde{V}_t^{(k)}, \beta_t^{(k+1)} + \langle \lambda_t^{(k+1)}, \cdot - x_t^{(k)} \rangle \right)$ .

**Stopping rule.**

Draw  $M$  trajectories of noise  $(w_t^m)_{t \leq T, m \leq M}$ .

Compute the  $M$  optimal decisions and states  $(U_t^{(k),m}, X_t^{(k),m})_{t \leq T, m \leq M}$  associated with the problem where  $V_t$  is replaced by  $\tilde{V}_t^{(k)}$  and noises  $(w_t^m)_{t \leq T, m \leq M}$ .

Compute the upper bound MC estimate

$$\bar{v}_M^{(k)} = \frac{1}{M} \sum_{m=1}^M \sum_{t=0}^{T-1} C_t(X_t^{(k),m}, U_t^{(k),m}, w_t^m) + K(X_T^{(k),m})$$

Compute the standard deviation of the sample

$$\bar{S}_M^{(k)} = \sqrt{\frac{1}{M-1} \sum_{m=1}^M \left[ \sum_{t=0}^{T-1} C_t(X_t^{(k),m}, U_t^{(k),m}, w_t^m) + K(X_T^{(k),m}) - \bar{v}_M^{(k)} \right]^2}$$

Compute the upper bound of the confidence interval of level  $1 - \alpha$  for the upper bound of the problem

$$\bar{v}_{h,\alpha,M}^{(k)} = \bar{v}_M^{(k)} + t_\alpha \frac{\bar{S}_M^{(k)}}{\sqrt{M}}$$

Test the condition

$$\left| 1 - \frac{\bar{v}_{h,\alpha,M}^{(k)}}{\tilde{V}_0^{(k)}(x_0)} \right| \leq \epsilon$$

**End of step**  $(k)$ .

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### 3 SDDP-friendly models of demand response and power systems

This section proposes an example of a contemporary interesting problem for power systems for which SDDP is particularly adapted. We seek to evaluate the operational impact of demand response on power systems in a 2023 North Western Europe (NWE) – Belgium, France, Germany, Luxembourg and Netherlands– marked by increasing CO<sub>2</sub> prices and very high prices of gas. First, we precise what the term demand response encompasses and how it is modeled here avoiding integer variables. Then, a multistage stochastic optimization problem is stated for the hourly dispatch of generation and storage means in a copperplate NWE during a year facing random demand and random productions of intermittent renewable energy sources (RES). Finally, aggregated impacts of demand response of the system are evaluated and discussed. If the reader is interested in a more detailed calibration and policy analysis of demand response in future power systems in a SDDP framework, we refer to a companion paper (Ancel & Massol (2022)).

#### 3.1 Demand Response in a SDDP framework: the hydro-power analogy

Demand response (DR) encompasses all electricity consumption that can voluntarily be displaced in time or curtailed. All demand response deposit presents a form of inertia: thermal inertia when the cooling of a stock or the heating of a house is curtailed, mechanical inertia when cement mills are momentarily stopped or behavioural inertia when electric vehicles are kept parked at night so that they can be charged. Flexible loads can be found in a large diversity of electricity usage, from the biggest industrial consumers to the simplest residential water boiler. As we seek to analyze the impact of all of these flexible loads on the whole electric system, we need a common representation of demand response that should both reflect the technical features of the modeled flexibility and be compatible with the description of traditional means of electrical production.

Following an important body of work we model demand response as a negative generation technology (see Verrier (2018) or Marañón-Ledesma & Tomasgard (2019)). DR is supposedly aggregated by a new actor, the DR aggregator, and all deposits of DR are seen as controllable means by the aggregator within technical and societal limits. This approach is particularly adapted to models of electricity markets where the supply/demand balance

constraint is written explicitly, as it will be the case in the present study. In such framework, it is indeed obviously similar to subtract a term on the demand side of the power balance equation and to add a new power of the same amount on the production side. We keep this idea as it enables a unified representation of demand response and ordinary generation means. We model demand response in a similar way as Verrier (2018), each demand response technology  $dr$  being represented by an installed capacity of negative generation  $C^{dr}$ , a maximum duration of event  $\Delta^{dr}$ , a time availability  $(A_t^{dr})_{t \leq T}$  and a maximum number of activation per year  $N^{dr}$ . All of these parameters can be easily deduced from the load profiles of the consumers. The construction of load profiles and calibration of the demand response model is not described here for sake of clarity but in a companion paper (Ancel & Massol (2022)).

Demand response may be divided into two classes depending on the destiny of the flexible load: load-shedding when the demand is purely erased and load-shifting when it is only pre- or postponed and still has to be balanced at one point.

Load-shedding is to be found mainly in the industrial sector with its centralized big consumers. It represents a pure loss for the consumer (e.g. steel that wasn't produced during the shedding event) and thus has a very high activation cost. Therefore load-shedding is a peak mean of negative generation and recourse to it can only be limited in time and infrequent. These limitations lead naturally to an analogy with conventional hydro-power stations. A limited quantity of energy can indeed be removed from the demand along the year during shedding events. The shedded load ( $turb_t^{dr}$ ) can't exceed the flexible part of the power demanded by the consumer in a normal situation as a dam can't inject more power in the electric system than its installed turbinning capacity. Thus we represent load-shedding technologies as a reservoir of a size given by the product between the available capacity, the duration of an event and the maximum number of events per year. The level of this reservoir represents the quantity of energy that can still be shedded during the remaining part of the year. It can only decrease as water (i.e. energy) is turbinned (i.e. removed from demand or negatively generated). The turbinning is limited by the available capacity.

Load-shifting appears more frequently among temperature related usage (cooling, heating, air conditioning) and in the tertiary or residential sector. As underlined by H. C. Gils (2014) or Müller & Möst (2018), it represents a massive potential of flexible loads which remains currently largely untapped. With limited duration of events, load-shifting implies

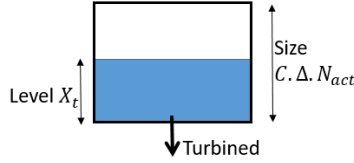


Figure 3.1: Model of a load-shedding technology.

less discomfort than load-shedding and is therefore at a quite low activation cost and may be activated a greater number of time. The exact same analogy with conventional dam wouldn't be satisfactory as it can't memorize the postponed load that has to be balanced. Load-shifting behaves indeed more similarly to hydro-power stations equipped with pumping capacity. These stations are situated between two reservoirs of water (upstream and downstream), inject power ( $turb_t^{dr}$ ) in the system by turbining water from the upstream reservoir to the downstream one and stock power out of the system by pumping water ( $pump_t^{dr}$ ) from downstream to upstream. Similarly in load-shifting, a limited quantity of energy (water) can be postponed (turbined) thus generating negatively this energy from a system perspective. It is afterward balanced (pumped) thus increasing demand at that time. Therefore load-shifting may be modeled according to this hydro-power analogy. First, we consider an upstream reservoir of size equal to the available capacity of shifting times the duration of a shifting event. Upon activation of the load-shifting technology, some water is turbined out of this reservoir, the installed turbining capacity being the available shiftable load. Contrarily to hydro-power stations, the turbine efficiency is 1 meaning that the power of water going through the turbine is perfectly transmitted to the electric system. This water ends up in a downstream reservoir of same size as the upstream one reflecting that only a limited level of consumption can be postponed at the same time. Water may then be pumped back into the upstream reservoir as the shifted load is balanced. The pump has the same properties as the turbine except for the sense of flows. In order to easily keep track of the number of activations we consider a third, contractual and disconnected from the system reservoir of size the size of the upstream reservoir times the maximal number of activations. It acts simply as a counter: water is turbined from it at the same time and of the same amount than from the upstream reservoir. Once the contractual reservoir is emptied, the technology becomes unavailable for the remaining part of the year.

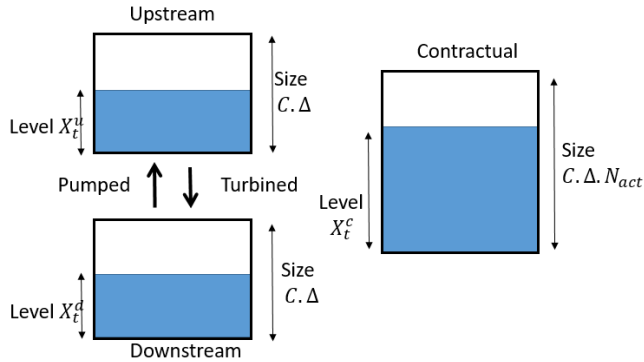


Figure 3.2: Model of a load-shifting technology.

The model of demand response technologies presented here treats them as hydro-power stations. This means that the equations describing them are all linear and that the negative generation from demand response is naturally integrated in the production side of the power balance equation as a linear term. Inclusion in the design of reservoirs of integer parameters such as the maximum number of activations or the memory of a postponed load to balance in the near future is a crucial feature of our model. It prevents from the recourse to integer variable in the optimization problem that will be written in the next section while keeping the constraints and objective linear. Moreover, this model represents explicitly demand response deposits as stocks of flexibility affected by decisions depending on random variables – which calls for a recourse to SDDP. Calibration of this model of DR on the 2023 NWE is given in Appendix A.2.

### 3.2 A multistage stochastic optimization for an energy-only competitive power market

Here we formulate the optimization problem of a system operator seeking to minimize the total cost of operation during a year facing random demand and RES production under load balance and technical constraints. This problem is a multi-stage stochastic optimization problem with 364 stages (days) denoted by index  $t$ . On each stage, the subproblem is a perfect foresight multi-step optimization problem with steps corresponding to hours and denoted by  $h$ . An extensive description of the model is given hereafter.

**Demand.** Residual demand encompasses aggregated and *non-flexible* electricity demand and the production of solar panels, wind turbines and run-of-water hydroelectric means. It is a random variable denoted by  $D$ , a realization of which at week  $t$  and hour  $h$  being written  $D_h^{\omega_t}$ . The exponent  $\omega_t$  corresponds to the draw of a residual demand scenario at stage  $t$ . The construction of these scenarios relies on public historical data for NWE<sup>1</sup>.

**Thermal and nuclear means.** Nuclear and thermal productions are aggregated respectively into one representative nuclear power plant, four representative thermal power plants, a peak gas mean (gas turbine), a base gas mean (CCGT plant), a lignite plant as it's still important in Germany and a traditional coal plant. The latter two plants are still competitive due to high gas prices even though the carbon price tends to move them out of the market. They generate a power  $g_{t,h}^{tech}$  at time  $(t, h)$ . Their capacities sums up to the installed capacities of the modeled means in the underlying real system. They are denoted by  $C^{tech}$ . Ramping time is neglected. This means that the modeled system is more responsive than the real one. This would a priori tend to reduce the recourse to demand response technologies but we will see in our application that this, even in this pessimistic context, a need for demand flexibility still exists. Thermal means are considered always available at full capacity whereas the installed nuclear capacity is weighted by an availability factor  $A_{t,h}^{nk}$ . This is inspired by the French system where a lot of maintenance on nuclear power plants takes place during the summer rendering up to 40% of the installed capacity unavailable. Note that, in 2023, all German nuclear plants will have been decommissioned. Finally, generation is made at constant unitary costs  $V^{tech}$ . Thermal means are subject to a 80€/tCO<sub>2</sub> carbon price. Their unitary costs takes it into account along with the average carbon content for this mean, their average efficiency<sup>2</sup> and their average fuel price<sup>3</sup>. Installed capacities and marginal costs of each technology used in the following simulations are displayed in Appendix A.1. Note that, the merit order of the thermal technologies is profoundly modified when going from "low" gas prices pre-dating the current surge to average 2022 gas prices.

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<sup>1</sup>See ENTSO-E Transparency Platform <https://transparency.entsoe.eu/load-domain/r2/totalLoadR2/show>

<sup>2</sup>From DIW (202X) Phasing out of coal in the German energy sector

<sup>3</sup>From argusmedia, average of ARA prices in 2022 T2 for coal and gas. For lignite see <https://www.argusmedia.com/en/news/2195015-modern-german-lignite-plant-margins-halve-from-2024>.



**Interconnections.** North Western Europe exchanges electricity with its neighbours. In a context of high gas prices, some of these neighbours, such as the UK (Figure 3.3 provides a glimpse of the change of power exchange habits between France and the UK) and to a lesser extent Spain, have access to cheaper gas, already saturate their internal use of gas and may use power interconnections with NWE to export this cheaper gas transformed into power and benefit from this price difference. Scandinavian countries produce also cheap electricity thanks to massive hydroelectric capacities. They are also quite well interconnected with NWE. These regions  $r$  lead to decision variables of import  $m_{t,h}^r$  and export  $m_{t,h}^r$ . Exchanges are supposed to be made at set prices  $V^r$  which reflect the 2022/2023 situation where the UK and Spain tend to export gas-based power at a cheaper price than that of internal CCGT or GT production and Scandinavia relies on hydro-power surplus. After these regions, Austria, Poland and Switzerland form the next three most connected countries with NWE. However, exchanges are often one way and no particularity of generation mix or gas prices would justify a modification of the traditional pattern of exchanges between NWE and these countries. So exchanges with them are not subject to decisions and only represented in the balance constraint by a net imports term  $netM^{AT+PL+CH}$ .

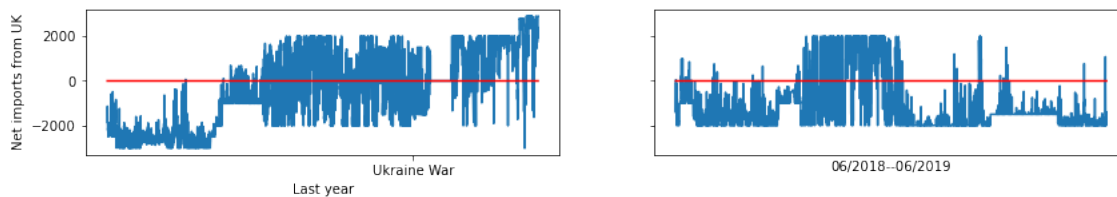


Figure 3.3: French net power imports from the UK during the last year marked by increased gas prices in NWE above all since the beginning of the Ukraine war and from June 2017 to June 2018, a more representative year. France was structurally an exporter of power during spring and since the high gas prices period has become an importer.

**Hydroelectric production.** Three main types of hydroelectric means exist: run-of-water which is not controllable and included in the residual demand in our model, conventional hydroelectricity which can turbine water from some reservoir in a controlled way and hydroelectricity with pumping capacity which can additionally store energy by pumping water. A representative station is modeled for each of the last two types and indexed  $hy$  for conventional and  $hp$  for hydroelectricity with pumps. In a similar way as thermal and nuclear means, these plants draw powers  $turb_{t,h}^{hy}$  and  $turb_{t,h}^{hp}$  in the system with an

efficiency factor  $E$  from two reservoirs with installed capacities  $C^{hy}$  and  $C^{hp}$ . Their level are respectively denoted  $X_{t,h}^{hy}$  and  $X_{t,h}^{hp}$ . For the pumping station, a power  $pump_{t,h}^{hp}$  may be consumed within the same technical limits. Generation is done at unitary constant costs  $V^{hy}$  and  $V^{hp}$ , pumping is free.

A traditional focus point of hydroelectric means modeling is to account for the random filling of water reservoirs by the meteorological water cycle. It requires the construction of representative inflow scenarios, generally obtained from historical data at station level and the addition of supplementary state variables i.e. a significant increase of computational effort. In our copperplate model, we could not assess for the diversity of climates faced by the real hydro-power stations since we only model a representative dam. Moreover, our objective is to model the operations of demand response deposits and to give them a value, and not a perfect hydroelectric scheduling. Therefore, the filling of reservoir is not modeled explicitly and our representative stations are given a fixed and finite reservoir size ( $S^{hy}$  and  $S^{hp}$ ). We choose to fix these sizes to the energy production of each mean (conventional or PHS) in 2016-2019. As they are the first two means to be called, this equates to the reproduction of the last year total consumption. The fact that water in hydro-power station should not know a period of scarcity means that the marginal cost of turbinning water through these stations does not include an opportunity cost and renders our model rather optimistic about hydroelectricity production. But, this eventual flaw will be present in our analysis of systems with and without demand response, thus not impacting the comparison.

**Demand response.** Demand response is modeled according to the hydroelectric analogy developed in the previous section and is associated with a unitary generation cost  $V^{dr}$  that compensates the end-user discomfort.

We may now write the problem of the operation of the previously described system until a finite horizon  $T$  with hourly decisions. It is a multi-stage stochastic optimization problem. Stages  $t$  correspond to beginnings of the days where hazard is realized. Then, an optimization in perfect foresight is done at the intra-day level.

**State variables** State variables of the system are the levels  $X$  of hydroelectric and demand response reservoirs.

**Decision variables.** At each time step  $h$ , a decision is made regarding thermal production ( $g_{t,h}^{CCGT}, g_{t,h}^{GT}, g_{t,h}^{Coal}$  and  $g_{t,h}^{LG}$ ), nuclear production ( $g_{t,h}^{nk}$ ), hydroelectric production ( $turb_{t,h}^{hy}, turb_{t,h}^{hp}$ ) and negative generation by demand response ( $turb_{t,h}^{dr}$ ). Decisions on energy storage in hydro-power stations with pumping capacity ( $pump_{t,h}^{hp}$ ) and in load-shifting technologies ( $pump_{t,h}^{dr}$ ) are also made. Then, imports ( $m_{t,h}^r$ ) and exports ( $x_{t,h}^r$ ) from the UK, Scandinavia and Spain (regions indexed by  $r$ ) are determined. Finally, in order to make the system always feasible - that is ensure supply/demand balance at all time - we introduce slack variables  $slack_{t,h}^+$  and  $slack_{t,h}^-$  corresponding to emergency curtailment or activation of means. This is done at the highest possible cost in the market i.e. its price cap  $PC$ .

**Objective.** At each  $t$  (beginning of the day), an immediate cost  $IC_t$  is calculated by summing the costs of production over the day,

$$\begin{aligned}
IC_t = \sum_h \left[ & V^{CCGT} g_{t,h}^{CCGT} + V^{GT} g_{t,h}^{GT} + V^{Coal} g_{t,h}^{Coal} + V^{LG} g_{t,h}^{LG} + V^{nk} g_{t,h}^{nk} \quad (3.1) \\
& + E(V^{hy} turb_{t,h}^{hy} + V^{hp} turb_{t,h}^{hp}) + \sum_{dr} V^{dr} turb_{t,h}^{dr} \\
& + PC(slack_{t,h}^+ + slack_{t,h}^-) + \sum_{region} V^{inter,r} (m_{t,h}^r + x_{t,h}^r) \quad \Big]
\end{aligned}$$

The objective function of the problem is formed by this stage cost and by the expected cost of the next stages i.e. it is equal to the Bellman value function associated with stage  $t$ .

**Constraints.** Constraints deal with the supply/demand balance in the electricity market, with state transitions and bound control and state variables.

Supply/demand balance at stage  $t$ , hour  $h$  for the realization  $\omega_t$  of the random residual demand:

$$\begin{aligned}
D_{t,h}^{\omega_t} = & g_{t,h}^{CCGT} + g_{t,h}^{GT} + g_{t,h}^{Coal} + g_{t,h}^{LG} + g_{t,h}^{nk} + E turb_{t,h}^{hy} + E(turb_{t,h}^{hp} - pump_{t,h}^{hp}) \quad (3.2) \\
& + \sum_{dr} (turb_{t,h}^{dr} - pump_{t,h}^{dr}) + \sum_{region} (m_{t,h}^r - x_{t,h}^r) + netM^{AT+PL+CH} + slack^- - slack^+
\end{aligned}$$

The dual variable of this constraint yields a (shadow) market price at time  $(t, h)$  that is capped at  $PC$ , the activation cost of variables  $slack^+$  and  $slack^-$ .

Constraints on thermal production for all technologies  $tech$ :

$$0 \leq g_{t,h}^{tech} \leq C^{th} \quad (3.3)$$

Constraints on interconnections for region  $r$ :

$$0 \leq m_{t,h}^r \leq C^{inter,r} \quad (3.4)$$

$$0 \leq x_{t,h}^r \leq C^{inter,r} \quad (3.5)$$

Constraints on nuclear production:

$$0 \leq g_{t,h}^{nk} \leq C^{nk} A_{t,h}^{nk} \quad (3.6)$$

Constraints on conventional hydroelectric production:

$$0 \leq turb_{t,h}^{hy} \leq C^{hy}/E \quad (3.7)$$

$$0 \leq X_{t,h}^{hy} \leq S^{hy} \quad (3.8)$$

$$X_{t,h}^{hy} = X_{t,h-1}^{hy} - turb_{t,h}^{hy} \quad (3.9)$$

Constraints on hydroelectric with pumping capacity production:

$$0 \leq turb_{t,h}^{hp} \leq C^{hp}/E \quad (3.10)$$

$$0 \leq pump_{t,h}^{hp} \leq C^{hp}/E \quad (3.11)$$

$$0 \leq X_{t,h}^{hp} \leq S^{hp} \quad (3.12)$$

$$X_{t,h}^{hp} = X_{t,h-1}^{hp} - turb_{t,h}^{hp} + pump_{t,h}^{hp} \quad (3.13)$$

Constraints on the load-shedding technology  $dr$ :

$$0 \leq turb_{t,h}^{dr} \leq C^{dr} A_{t,h}^{dr} \quad (3.14)$$

$$0 \leq X_{t,h}^{dr} \leq C^{dr} \Delta^{dr} N^{dr} \quad (3.15)$$

$$X_{t,h}^{dr} = X_{t,h-1}^{dr} - turb_{t,h}^{dr} \quad (3.16)$$

Constraints on the load-shifting technology  $dr$ :

$$0 \leq turb_{t,h}^{dr} \leq C^{dr} A_{t,h}^{dr,turb} \quad (3.17)$$

$$0 \leq pump_{t,h}^{dr} \leq C^{dr} A_{t,h}^{dr,pump} \quad (3.18)$$

$$0 \leq X_{t,h}^{dr,up} \leq C^{dr} \Delta^{dr} \quad (3.19)$$

$$X_{t,h}^{dr,up} = X_{t,h-1}^{dr,up} - turb_{t,h}^{dr} + pump_{t,h}^{dr} \quad (3.20)$$

$$0 \leq X_{t,h}^{dr,do} \leq C^{dr} \Delta^{dr} \quad (3.21)$$

$$X_{t,h}^{dr,do} = X_{t,h-1}^{dr,do} + turb_{t,h}^{dr} - pump_{t,h}^{dr} \quad (3.22)$$

$$0 \leq X_{t,h}^{dr,cont} \leq C^{dr} \Delta^{dr} N^{dr} \quad (3.23)$$

$$X_{t,h}^{dr,cont} = X_{t,h-1}^{dr,cont} - turb_{t,h}^{dr}. \quad (3.24)$$

**Problem.** Let's denote  $u_t$  the vector of decision variables at stage  $t$ ,  $X_t$  the state variables one and  $F$  the transition function. The admissible space for  $u_t$ ,  $\mathcal{U}_t(X_t, D_t)$ , is given by equation 3.2 to 3.24. We may then explicitly write the operator problem as a multi-stage stochastic optimization problem

$$\min_{u_1 \in \mathcal{U}_1(X_1, D_1)} \left( IC_1 + \mathbb{E} \left[ \min_{\substack{u_2 \in \mathcal{U}_2(X_2, D_2) \\ X_2 = F(X_1, u_1)}} \left( IC_2 + \dots + \mathbb{E} \left[ \min_{\substack{u_T \in \mathcal{U}_T(X_T, D_T) \\ X_T = F(X_{T-1}, u_{T-1})}} IC_T \right] \right) \right] \right) \quad (3.25)$$

## 4 Application: Impacts and Value of Demand Response in North Western Europe

With previous parameters calibrated with the values of Appendices A.1 and A.2, Problem 3.25 is solved with SDDP for four scenarios of demand response installed capacity and gas prices: full integration i.e. 100% of the capacities presented in Table A.2 are available and a baseline without DR crossed with mean prices of gas respectively pre and post dating their 2022 surge. They are denoted in the following respectively by DR+, DRLP, NoDR and NoDRLP. A gradation of DR integration is also monitored by looking at scenarios (with high gas prices) of DR levels corresponding to 10%, 50% and 75% of the potentials of Table A.2, denoted respectively 01\_DR, 05\_DR and 075\_DR. In order to measure DR impacts, we may be interested in several aggregated indicators. Once the optimal policy is obtained, 1000 years are simulated in order to measure its efficiency and derive the aggregated in-

dicators. Resulting indicators are shown in Figure 4.1 for different levels of DR with high gas prices and in Figure 4.2 for a comparison between scenarios DR+, DRLP, NoDR and NoDRLP.

First, the mean (over the simulations) amount of emissions of the system during the year reflects the use of thermal means. As DR should competes with peak means at lower costs, it should move a part of these means out of the market. We thus expect a reduction of emissions. In Figure 4.1, it appears that the distribution of emissions remains almost unchanged with DR, with even a slight growing trend with the integration of more DR. This might be explained by the fact that lignite, the dirtiest generation mean, has the lowest marginal cost among thermal means so that it may benefit from DR decreasing market prices and driving more often out of the market the other emitter means. In that sense, we conclude that DR, due to its limited capacity, may only push out of the market the last called means so that DR reduces emissions of the power system only if these peak units are also the dirtiest. This analysis is confirmed by a robustness check with unrealistically high DR potentials which triple that of scenario DR+ where emissions are decreased because that massive load-shifting enables the system to avoid not only recourse to gas turbines but also to coal plants which are the second most expensive production mean.

In terms of security of supply, we may account for the mean number of hours where the market price is equal to the market price cap. This situation only happens in our model when slack variables are positive i.e. when an outage occurs. As a flexibility provider, DR – and particularly load-shifting – should enable the system to avoid these times of tension. This prediction is confirmed by Figure 4.1 where a direct trend linking DR integration and total time at market price cap appears.

In the same idea, as load-shifting is cheaper than many of the other generation capacity, these deposits should be called whenever its possible and their opportunity costs are not too high. This should reduce the market prices in traditionally sensitive periods for the system such as the evening peaks in winter. That’s why our third indicator is the mean (over the simulations) average (over the days) 7 pm winter weekdays market price. In the same way as the previous indicator, integration of more demand response in the system seems to reduce general tension of the system and thus market prices at these times. However, after 50% of the DR capacities of Table A.2 are introduced, no further significant benefits in terms of market prices seems created by more DR.

Finally, the system interest in DR may be quantified by the difference in total cost of operating the system with and without DR. This difference is indeed equal to the social welfare variation implied by the introduction of DR as this difference equates to the difference in producer surplus while consumer surplus is constant since DR deposits encompass for all flexible demand and count as negative production means so that the remaining demand is perfectly inelastic. The distribution of welfare variation with the introduction of DR lies entirely in positive numbers in all scenarios except 01\_DR in which negative values may be reached without this event having a probability greater than 0.5 (see Figure 4.1). A trend seems to exist where more DR implies an increased welfare. However, similarly to the previous indicator, this effect stagnates for the highest levels of DR integration as if the need for short term flexibility of the system was saturated. Note that demand response and particularly load-shifting may only provide short-term flexibility as shifted load have to be balanced whereas PHS or flexibility provided by interconnections provide more mid-term flexibility – battery storage is still not relevant for 2023 but would provide short-, mid- and long-term flexibility. The conclusion remains that, from the 2023 NWE power system point of view, demand response yield positive impacts in terms of operational costs, market prices and outages duration.

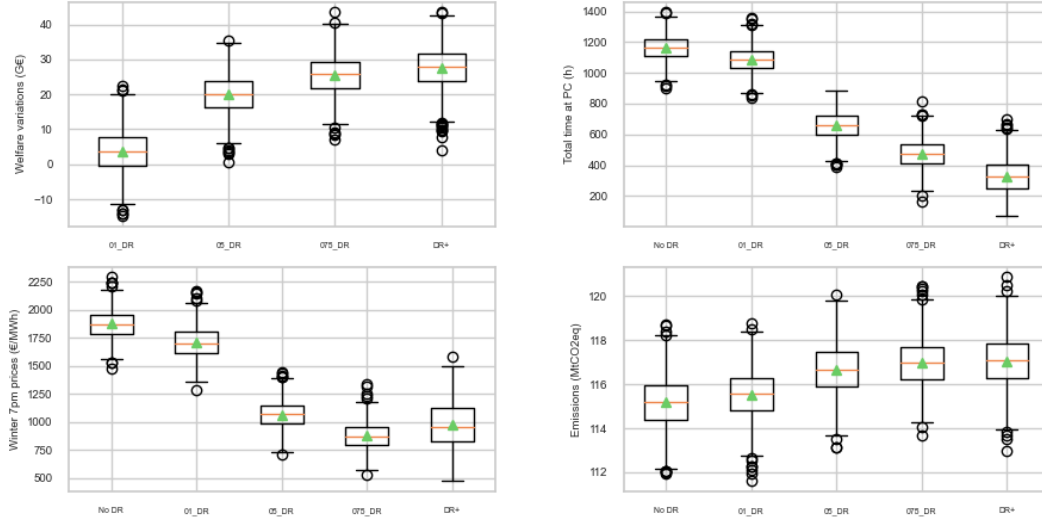


Figure 4.1: Distributions of the four aggregated indicators of the system status with different integration of DR and high gas prices.

To contrast these positive impacts of demand response on the tensed system of 2023, we look at scenarios NoDRLP and DRLP reflecting more ordinary gas prices during the last decade, and therefore a more traditional merit order of the production means. There is already a difference between the no DR baselines depending on the gas prices: with cheaper gas, imports from the UK are reduced of around a third and halved from Spain. More importantly, translating the change in merit order, emissions are around 15% lower with low gas prices as lignite and coal are less called. This change doesn't affect the security of supply nor the volatility of market prices (but, obviously, mean prices are decreased). These results underlines how the phasing out from coal policies, especially coupled with a phase out of nuclear as in Germany with the *Energiewende*, were not conceived for gas prices so high they modify the merit order of generation technologies and hence lack robustness.

Introduction of DR benefits slightly more to the low gas prices system as displayed in Figure 4.2. With this low gas price baseline, the effect of DR on increasing the security of supply and reducing and stabilizing market prices is also a bit higher, but of similar order than with higher gas prices. The most interesting difference in impact resides in the variation of emissions of the power system when DR is introduced. With high gas prices, emissions tend to increase when DR is integrated at the maximum of its envisioned potential (we saw above that they decrease if much more DR is introduced but that's less realistic). On the opposite, with lower gas prices, emissions decrease. This result might be explained in a rather simple way: the carbon intensity of each generation mean is constant while the merit order changes with the considered increase in gas prices. With high gas prices, as displayed in Table A.1, gas power plants have higher marginal costs than their coal counterparts, while the situation is reversed in the low gas prices scenario. But DR, and here more precisely load-shifting, suppresses calls to the most costly generation plants. Thus, with low gas prices, DR pushes out of the market the dirty coal power plants while relatively cleaner gas plants are affected with high gas prices. Depending on the fuel price context, the introduction of demand response in the NWE power system, up to a realistic limit, may be environmentally detrimental. A simple way out would be to link the carbon price and the fuel prices so that it remains always more attractive to use cleaner fuels.

Finally, as mentioned earlier, opportunity costs are natural outputs of SDDP. We illustrate this fact in our framework by displaying the evolution of the opportunity cost of the cement-related demand response in scenario DR+ in Figure 4.3. It has to be compared to market prices in order to explain the decisions of usage of this deposit. In Figure 4.3, it



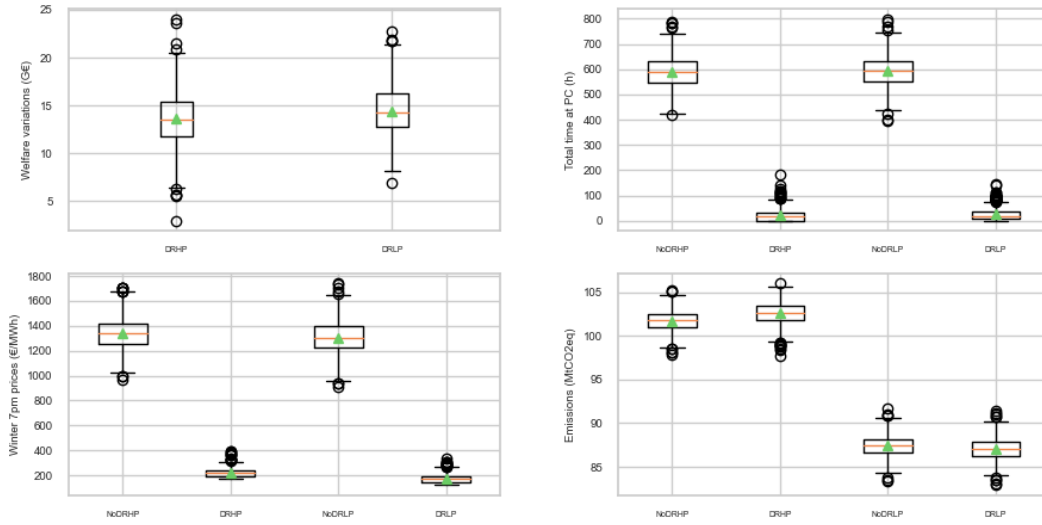


Figure 4.2: Distributions of the four indicators of the system status with different gas prices.

appears that there exists an extended period before spring when market prices are too low for the use of the cement deposit to generate infra-marginal rent. During this period, almost no turbinage from this deposit is done reflecting how opportunity costs guide decisions in this model. Water value becomes null at the end of the year because we didn't give a value to remaining stocks of flexibility at the end of the year in our model since the number of possible activations of DR events reboots at the beginning of the year – whereas the water in a hydro-power station doesn't which explains positive water values at the end of the year in this case. Similar results and curves may be obtained, in this SDDP framework, for all considered reservoirs, DR and water alike.

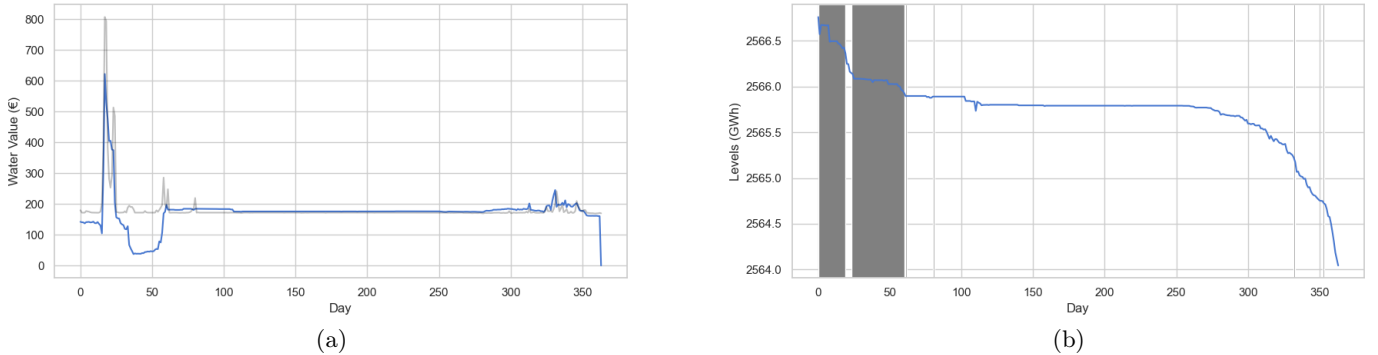


Figure 4.3: (a) Daily water values for the cement mill DR deposit in blue and market prices in grey. (b) Daily level of the contractual reservoir of this deposit. Grey areas correspond to times when market prices are lower than the sum of the activation cost and the opportunity cost (water value) of this deposit.

## 5 Conclusion

In this paper, a powerful method for solving multistage stochastic (convex-linear) optimization problems, SDDP, has been presented with the concern of applying it to modern or future power systems with more intermittent renewable energy sources, storage capacities and demand-side management. SDDP requires a particular framework, notably regarding random variables: they must have values in a finite space and be stage-wise independent. Its main features are, on the one hand, an ability to handle large sets of accessible values for random variables at each stage because the complexity of the method grows only with the sum on all stages of the size of these sets, while traditional scenario-based approaches have growing complexity with the product of these sizes. On the other hand, as SDDP has been historically designed for (water) stock management problems, opportunity costs of using the considered stocks at a some stage given the state of the system – or to be more precise, an approximation of these costs – are natural outputs of the method. These advantages help dramatically the analysis of optimal behaviours of stock-managing agents (hydro-power producers, but also demand response aggregators or battery managers) in power systems under multiple stochastic inputs such as demand, water inflows, renewable production or even unattended needs for maintenance.

Then, a short application of SDDP is proposed. We focus on a simple model of the

power system of North Western Europe under current particular circumstances of high gas prices and competitive use of power interconnections by neighbouring countries such as the UK. In this simple, copperplate model we show the benefits of introducing demand response for alleviating tension in this power system. Even with relatively small available capacity, demand response enables to shave demand peaks and partially absorb renewable overproduction at lower costs than increasing interconnections, PHS capacities or installing batteries. The power system has thus access to more flexibility which improves social welfare (as defined in Section 4). Developing demand response, and more generally a demand-side approach of power markets, proves to be an efficient low-cost mid term way out of the current power crisis in NWE. It drives indeed out of the market the most marginally costly generation technologies which are currently gas-related ones thus reducing both electricity wholesale prices and dependency toward imported gas. However, it may be in our model at the benefit of coal power plants, which is verified currently with recent decisions postponing the phasing out of coal in France or Germany.

Finally, SDDP appears as a powerful modelling tool for modern power systems involving more stocks on both sides of the market. With more complex models such as *Empire* or *urbs* starting to account more precisely for demand-side management in a bottom-up approach (see Marañón-Ledesma & Tomasgard (2019) or Stüber & Odersky (2020)), SDDP is deemed to be included in such frameworks. It may also be used as in this paper, in simpler models, in order to provide not necessarily trustworthy values but valuable insights which may guide short to mid term responses to a particular crisis.

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# A Calibration Data

## A.1 Power system

Technology	Belgium	France	Germany & Luxembourg	Netherlands	Total	Carbon content (tCO <sub>2</sub> /MWh)	Cost (LP-HP) (€/MWh)
Lignite	0	0	6240	0	6240	0.428	95.1
Coal	0	1817	7908.6	4006	13731.6	0.342	165.1
CCGT	3542.23	11506	15618.7	9442.89	40109.8	0.202	91.7 – 171.7
GT	250.236	1061.2	1103.36	667.08	3081.88	0.202	128.3 – 240.4
Nuclear	5943	61370	0	486	67799	0	23
Hydro Conv.	0	7188	1408	0	8596	0	7.53
PHS	1308	4656	9280	0	15244	0	9.54
Interco. UK	750	2014	0	1016	3780	0	95 – 150
Interco. Spain	0	3000	0	0	3000	0	96 – 160
Interco. Scand.	0	0	3209	1400	4609	0	15
Run-of-water	177	10955	3743	38	14913	0	0
Wind	4883	13610	63584	11060	93137	0	0
PV	4788	10213	56567	16074	85617	0	0

Table A.1: Installed capacity by country (MW), carbon content (tCO<sub>2</sub>/MWh) and Variable cost (€/MWh) in the modeled NWE power system. For thermal means, we suppose a carbon price set at 80€/tCO<sub>2</sub> in line with the 2023 EU-ETS projections. Their efficiency is taken into account in installed capacity numbers.

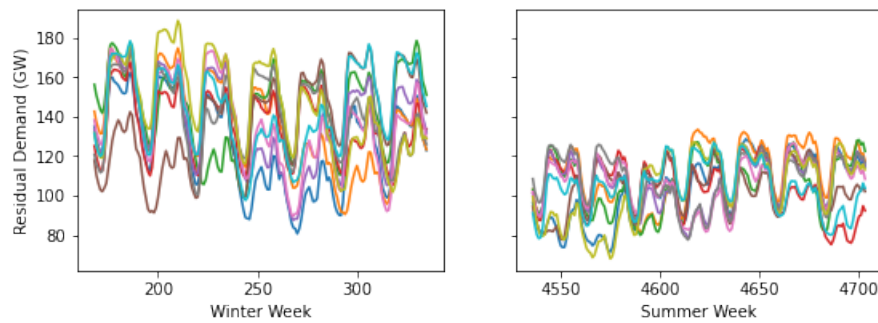


Figure A.1: Ten scenarios of residual demand in NWE for a winter week and a summer week. Structural cyclical of demand is affected by intermittent RES production – particularly in Germany due to higher RES integration.

## **A.2 Demand response potentials**

Method and Data are based on the works of Verrier (2018) and H. C. Gils (2014); H. Gils (2015). For greater details, please refer to Ancel & Massol (2022).



Technology	Installed capacity (MW)	Mean available capacity (MW)	Activation cost (€/MWh)	Event duration (h)	Number of activations	Temporal profile
Steel - electric furnace	2222	2222	411	4	50	
Aluminium - electric furnace	447	447	164	4	50	
Chlorine electrolysis	1366	1366	96	4	50	
Cement mills	1083	391	10	3	260	
Paper and pulp	2062	1595	10	3	260	
Industrial cooling	3166	1083	16	1	364	
Industrial cross-tech ventilation	693	302	16	1	364	
Tertiary cooling	39753	7962	11	1	364	
Tertiary heating	47042	15974	11	4	364	
Residential cooling	30460	24873	11	1	364	
Residential heating	7052	3004	11	1	364	

Table A.2: Characteristics of modeled demand response means. Availability profiles are given on the most relevant time scale for each technology. For temperature dependent technologies (e.g. the last four) intra-day variations also exist. Red : Available load reduction. Black : Available load increase.