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# Asymptotic multiscale approach for building dispersion curves: the generic case

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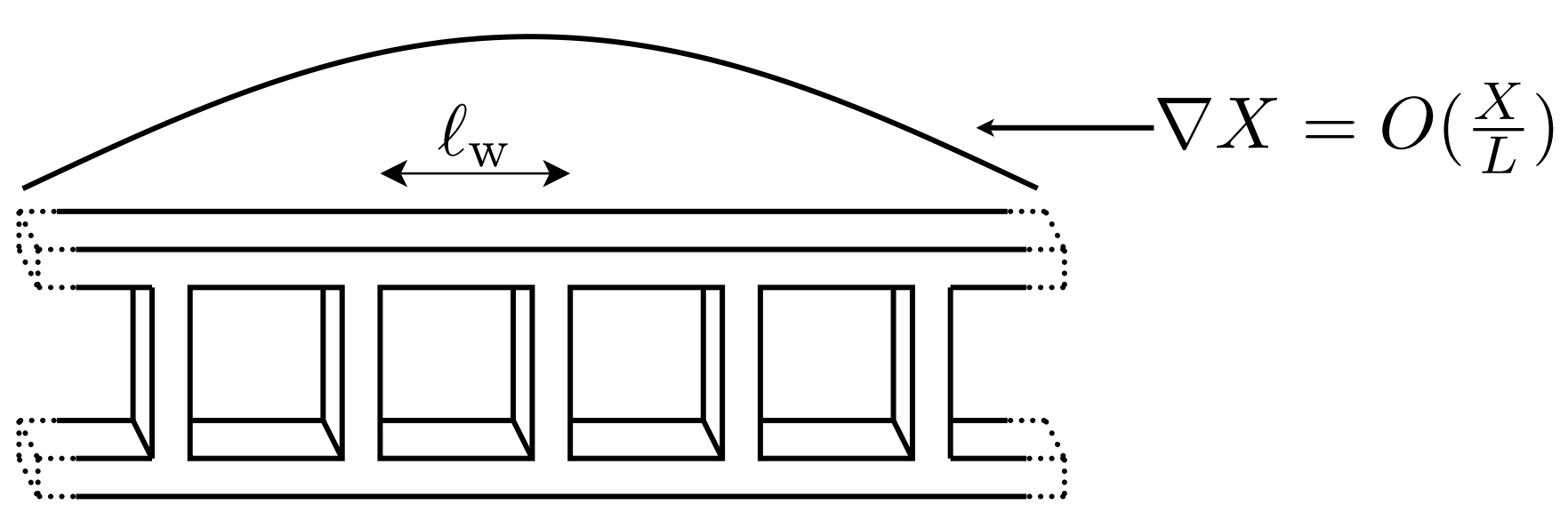


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## Motivation

Global description of transversal macroscopic dynamics of reticulated beams for wavelengths  $L$  on the order of the cell size  $\ell_w$ ?

Classical  
homogenization :

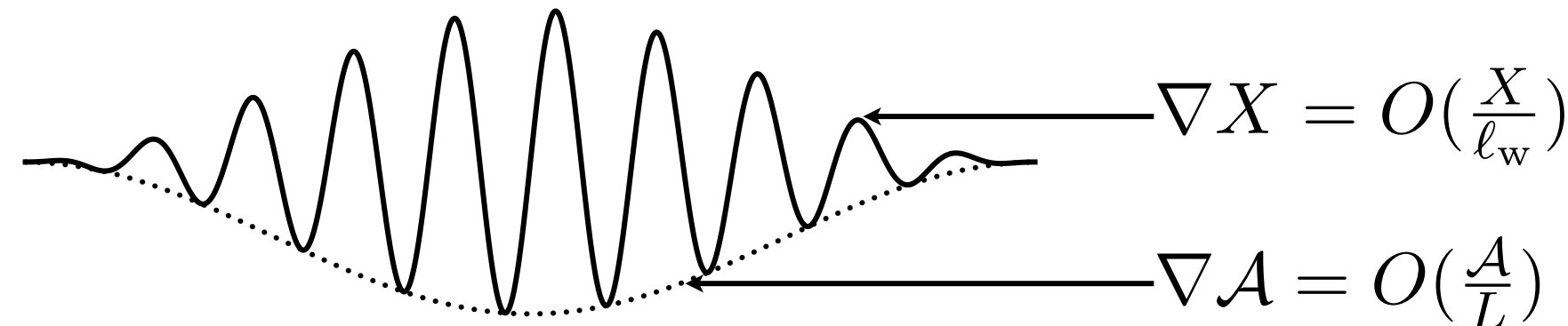


$L \gg \ell_w$

Quasi-static local

Scale separation :  $X$

At higher frequencies :



$L' \approx \ell_w$

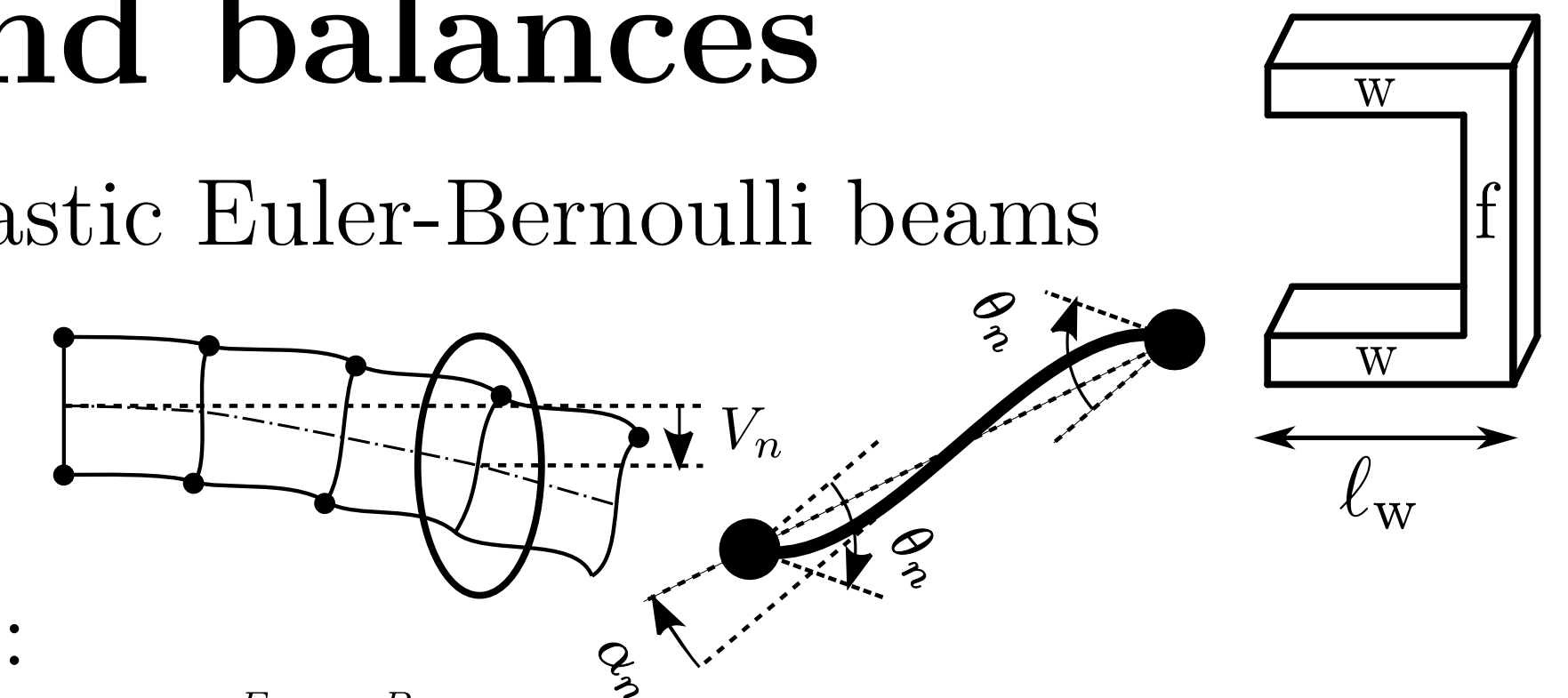
Dynamic local

Scale separation : amplitude  $\mathcal{A}$  of the periodic modes (ref [1], [2], [3], [4])

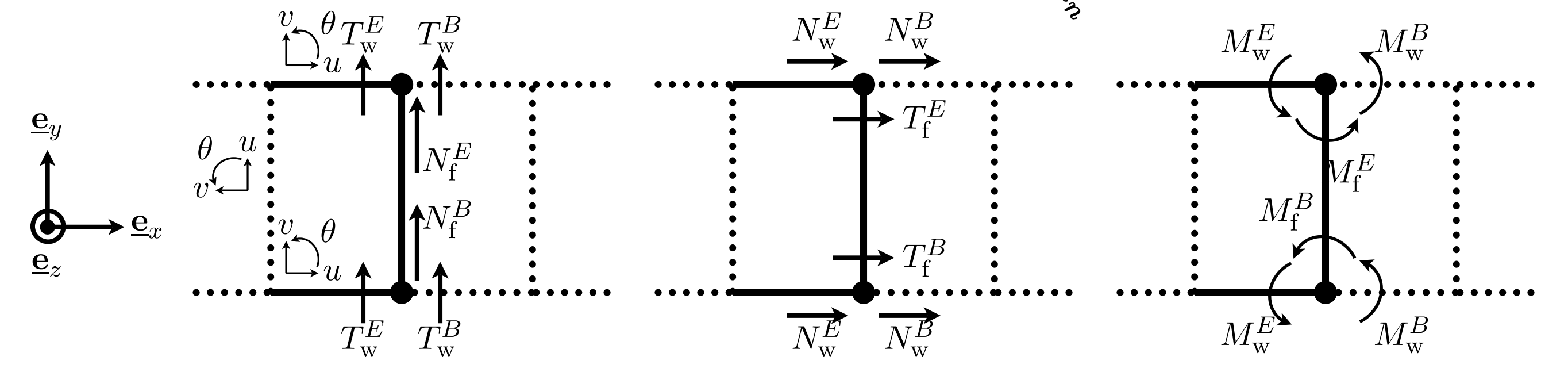
## Kinematics and balances

● Period  $\Omega$  : 3 linear elastic Euler-Bernoulli beams

● Transversal kinematics



● Balances at each level :



$$\text{Transversal balance : } \underline{\underline{H}}^{EB} \cdot \underline{\underline{v}}_{n-1} + \underline{\underline{H}}_T \cdot \underline{\underline{v}}_n + {}^T \underline{\underline{H}}^{EB} \cdot \underline{\underline{v}}_{n+1} = \underline{\underline{0}} \quad (1)$$

## Dispersion equations

Introducing  $\underline{\underline{v}}_n = (\frac{1}{2}\alpha_n \ell_f, V_n, \theta_n) = \underline{\underline{v}}_0 e^{-ik_\omega n \ell_w}$  ;  $k_\omega \in [0, \frac{\pi}{\ell_w}[$

$$\text{into (1) yields : } \underline{\underline{D}}_T \cdot \underline{\underline{v}}_0 = \begin{bmatrix} \tilde{a}_T & 0 & b_T \\ 0 & \tilde{c}_T & \tilde{e}_T \\ b_T & -\tilde{e}_T & \tilde{d}_T \end{bmatrix} \cdot \underline{\underline{v}}_0 = \underline{\underline{0}}$$

$$\text{Dispersion of the transversal waves : } \mathcal{E}_T(\omega, k_\omega) = 0 = \det(\underline{\underline{D}}_T)$$

## Bloch modes

● Consider  $\Omega_p = \bigcup_p \Omega$  of length  $\ell_p = p \ell_w$

● Eigenmodes  $\{\underline{\underline{\Phi}}^J, \underline{\underline{\Psi}}^J\}$  of  $\Omega_p$  with periodic boundary conditions

● Periodic wavenumbers (Bloch numbers) :  $k_{\omega_J} = \frac{2\pi q}{\ell p} \in [0, \frac{\pi}{\ell}[$

● Eigenfrequencies of are solutions of :  $\mathcal{E}_T(\omega_J, k_{\omega_J}) = 0$

## Asymptotic approach (see [1])

● Scale ratio  $\varepsilon_p = \ell_p \kappa_\omega / 2\pi \ll 1$  where  $\kappa_\omega$  modulation number

● Rescale parameters : Lineic mass  $\Lambda_f = O(\varepsilon_p^2 \Lambda_w)$  ; Bending stiffness  $K_f = O(\varepsilon_p^2 K_w)$  ; Compression stiffness  $EA_f = O(\varepsilon_p EA_w)$

● Leading order kinematics :  $\underline{\underline{v}}(x) = \mathcal{A}(x)\underline{\underline{\Phi}}^J + \mathcal{B}(x)\underline{\underline{\Psi}}^J$  ( $\mathcal{A}, \mathcal{B}$ ) modulation amplitudes of the (simple or double) modes  $\{\underline{\underline{\Phi}}^J, \underline{\underline{\Psi}}^J\}$

● Dispersion relation :  $k_\omega = \frac{2\pi q}{\ell w p} + \kappa_\omega$  with  $\kappa_\omega \ll \frac{2\pi q}{\ell w p}$  introducing  $\mathcal{A}(x) = \mathcal{A}_0 e^{-i\kappa_\omega x}$  in the modulation equations

## Numerical values

$$\Lambda_w = 15.6 \text{ kg/m} = 8\Lambda_f ; K_f = 0.5 \text{ MN} = 8K_w ; EA_w = 21.9 \text{ MN} = 2EA_f$$

## Modulation equations

● Modulation based on  $\Omega$  ( $k_\omega = 0$ )

Generic beam

$$\omega_J = 0 : \frac{2EI_w EI_s}{K_f} V^{(6)} - (2EI_w + EI_s) V^{(4)} - \frac{EI_s \Lambda_w \omega^2}{K_f} V^{(2)} + \Lambda_w \omega^2 V = 0$$

$$\text{Shear stress } T = -K_w (V' - \alpha)$$

$$\mathcal{T}' = (T - M_I')' = \Lambda_w \omega^2 V$$

$$\text{Global moment } M_G = -EI_s \alpha'$$

$$M_G' = -T$$

$$\text{Intern moment } M_I = -2EI_w V^{(2)}$$

$$K_w (V' - \theta) = 0$$

$$\omega_J > 0 : \mathcal{M}_{\Omega}^J(\omega)(\omega^2 - \omega_J^2)\mathcal{A} + \mathcal{K}_{\Omega}^J(\omega)\mathcal{A}^{(2)} = 0$$

● Modulation based on  $\Omega_2$  ( $k_\omega = \frac{\pi}{\ell_w}$ ) (specific simple modes of  $\Omega_2$ )

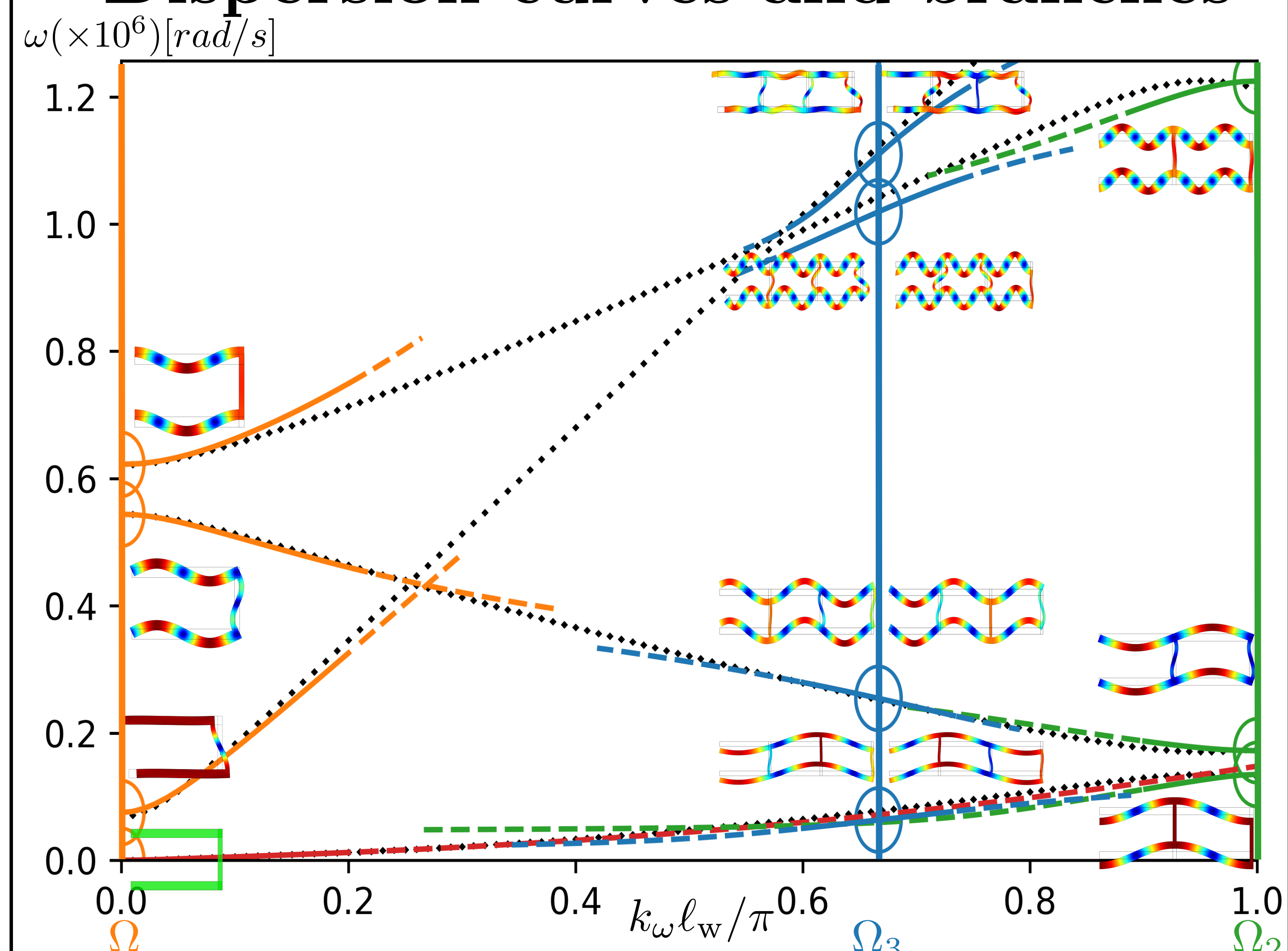
$$\mathcal{M}_{\Omega_2}^J(\omega)(\omega^2 - \omega_J^2)\mathcal{A} + \mathcal{K}_{\Omega_2}^J(\omega)\mathcal{A}^{(2)} = 0$$

● Modulation based on  $\Omega_3$  ( $k_\omega = \frac{2\pi}{3\ell_w}$ ) (specific double modes of  $\Omega_3$ )

$$\left. \begin{aligned} \mathbb{A}_{\Omega_3}^J(\omega)\mathcal{B}' + \mathbb{B}_{\Omega_3}^J(\omega)(\omega - \omega_J)\mathcal{A} &= 0 \\ \mathbb{A}_{\Omega_3}^J(\omega)\mathcal{A}' - \mathbb{B}_{\Omega_3}^J(\omega)(\omega - \omega_J)\mathcal{B} &= 0 \end{aligned} \right\} \Rightarrow \mathcal{M}_{\Omega_3}^J(\omega)(\omega - \omega_J)^2\mathcal{A} + \mathcal{K}_{\Omega_3}^J(\omega)\mathcal{A}^{(2)} = 0$$

Simple reconstruction by three parts

## Dispersion curves and branches



[1] A. Rallu, S. Hans, and C. Boutin, "Asymptotic analysis of high-frequency modulation in periodic systems. Analytical study of discrete and continuous structures," J. Mech. Phys. Solids, vol. 117, pp. 123–156, 2018

[2] E. M. Daya, B. Braikat, N. Damil, and M. Potier-Ferry, "Continuum modeling for the modulated vibration modes of large repetitive structures," 2002.

[3] R. V. Craster, J. Kaplunov, and A. V. Pichugin, "High-frequency homogenization for periodic media," Proc. R. Soc. A Math. Phys. Eng. Sci., vol. 466, no. 2120, pp. 2341–2362, Aug. 2010.

[4] C. Boutin, A. Rallu, and S. Hans, "Large scale modulation of high frequency acoustic waves in periodic porous media.," J. Acoust. Soc. Am., vol. 132, no. 6, pp. 3622–36, Dec. 2012,