Another rational solutions to the Burgers' equation

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Abstract

From particular polynomials, we construct rational solutions to the Burgers' equation as a quotient of a polynomial of degree $n-1$ in x and $n-1-\left[\frac{n}{2}\right]$ in t, by a polynomial of degree n in x and $\left[\frac{n}{2}\right]$ in t, $[n]$ being the greater integer less or equal to n. We call these solutions, solutions of order n.
We construct explicitly these solutions for orders 1 until 20.

Key Words : Burgers equation.

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1 Introduction

We consider the Burgers' equation which can be written as

$$u_t + u_{xx} + uu_x = 0 \tag{1}$$

where the subscripts x and t denote partial derivatives.
In 1915, Bateman [1] introduced this equation (1). This equation appears in different contexts in physics as in gas dynamics [2], acoustics [3], heat conduction [4], in soil water [5], in hydrodynamics turbulence [6, 7, 8], in shock waves [9],...
The first solutions has been constructed by Bateman [1] in 1915. Other types of methods have been used to solve this equation. We can quote the exp-function method [13], the tanh-coth method [14], the groups actions on coset bundles [15], the Cole-Hopf method [16, 17, 18], the homotopy perturbation method [19],...

We can quote some recent results in connection with this study as [10], [11], [12].
Rational solutions to the Burgers' equation are constructed in this paper. We give solutions as a quotient of a polynomial of degree $n-1$ in x and $n-1-\left[\frac{n}{2}\right]$ in t by a polynomial of degree n in x and $\left[\frac{n}{2}\right]$ in t, $[p]$ being the greater integer less or equal to p.

We explicitly build these solutions for orders between 1 and 20.

2 Rational solutions to the Burger's equation

We consider the following polynomials defined by

$$p_n(x,t) = \sum_{k=0}^{n} \frac{x^k}{k!} \frac{(-t)^{\frac{n-k}{2}}}{\frac{n-k}{2}!} \left(1 - \left(n-k-2\left[\frac{n-k}{2}\right]\right)\right), \text{ for } n \geq 0, \quad (2)$$
$$p_n(x,t) = 0 \text{ for } n < 0.$$

With the choice of these polynomials, we have the following statement

Theorem 2.1 *The function v_n defined by*

$$v_n(x,t) = -2\frac{p_{n-1}(x,t)}{p_n(x,t)}, \quad (3)$$

where p_n are defined by previous relations (2), is a solution to the Burgers' equation (1)

$$u_t + u_{xx} + uu_x = 0.$$

Remark 2.1 *In the following, we will call the solution v_n, the solution of order n of the Burgers' equation (1).*

Remark 2.2 *More explicitly, the previous polynomials can be written as*

$$p_{2k}(x,t) = \sum_{l=0}^{n} \frac{x^{2l}}{(2l)!} \frac{(-t)^{k-l}}{k-l!}, \text{ for } k \geq 0,$$
$$p_{2k+1}(x,t) = \sum_{l=0}^{n} \frac{x^{2l+1}}{(2l+1)!} \frac{(-t)^{k-l}}{k-l!}, \text{ for } k \geq 0,$$
$$p_n(x,t) = 0 \text{ for } n < 0.$$

3 Explicit first order solutions

All these rational solutions are singular. At each order, we see the appearence of curves of singularities. The patterns of singularities are lines or horseshoe type depending on the order of the solution.

3.1 First order solutions

Proposition 3.1 *The function v defined by*

$$v_1(x,t) = \frac{2}{x} \tag{4}$$

is a solution to the Burgers' equation (1).

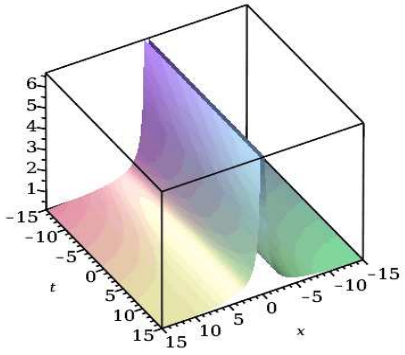

Figure 1. Solution of order 1 to (1).

3.2 Solutions of order two

Proposition 3.2 *The function v_2 defined by*

$$v_2(x,t) = \frac{-4x}{-x^2 + 2\,t}, \tag{5}$$

is a solution to the Burgers' equation (1).

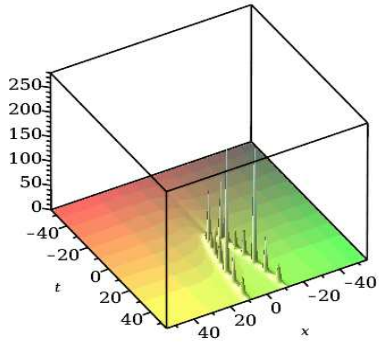

Figure 2. Solution of order 2 to (1).

3.3 Solutions of order three

Proposition 3.3 *The function v_3 defined by*

$$v_3(x,t) = 6\,\frac{-x^2+2\,t}{x\,(-x^2+6\,t)}, \tag{6}$$

is a solution to the Burgers' equation (1).

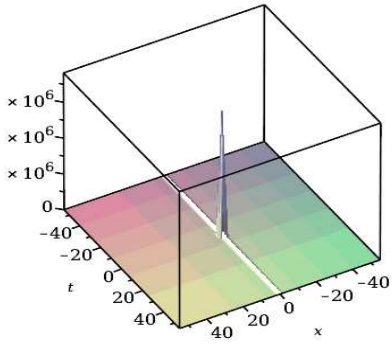

Figure 3. Solution of order 3 to (1).

3.4 Solutions of order four

Proposition 3.4 *The function v_4 defined by*

$$v_4(x,t) = -8\,\frac{x\,\left(-x^2+6\,t\right)}{x^4-12\,x^2t+12\,t^2}, \tag{7}$$

is a solution to the Burgers' equation (1).

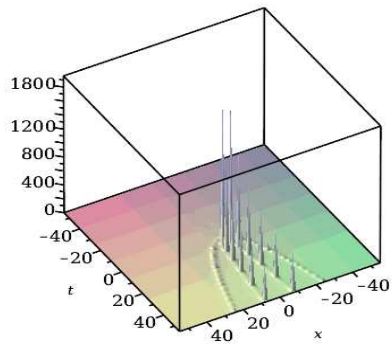

Figure 4. Solution of order 4 to (1).

3.5 Solutions of order five

Proposition 3.5 *The function v_5 defined by*

$$v_5(x,t) = 10\,\frac{x^4 - 12\,x^2t + 12\,t^2}{x\,(x^4 - 20\,x^2t + 60\,t^2)}, \tag{8}$$

is a solution to the Burgers' equation (1).

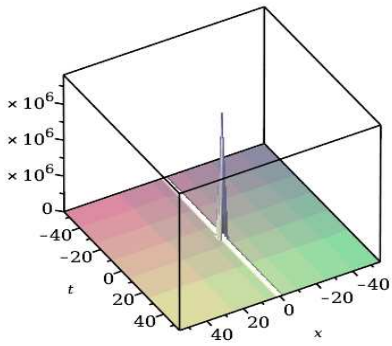

Figure 4. Solution of order 5 to (1).

3.6 Solutions of order six

Proposition 3.6 *The function v_6 defined by*

$$v_6(x,t) = -12\,\frac{x\left(x^4 - 20\,x^2t + 60\,t^2\right)}{-x^6 + 30\,x^4t - 180\,x^2t^2 + 120\,t^3}, \tag{9}$$

is a solution to the Burgers' equation (1).

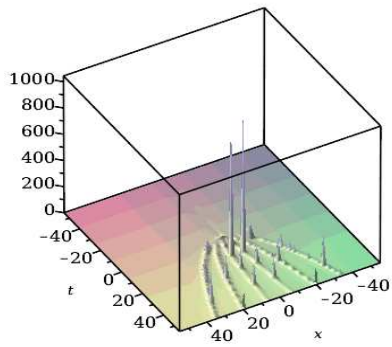

Figure 6. Solution of order 6 to (1).

3.7 Solutions of order seven

Proposition 3.7 *The function v_7 defined by*

$$v_7(x,t) = 14\,\frac{-x^6 + 30\,x^4t - 180\,x^2t^2 + 120\,t^3}{x\,(-x^6 + 42\,x^4t - 420\,x^2t^2 + 840\,t^3)}, \tag{10}$$

is a solution to the Burgers' equation (1).

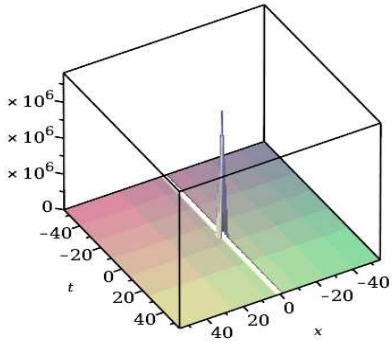

Figure 7. Solution of order 7 to (1).

3.8 Solutions of order eight

Proposition 3.8 *The function v_8 defined by*

$$v_8(x,t) = -16\,\frac{x\left(-x^6 + 42\,x^4t - 420\,x^2t^2 + 840\,t^3\right)}{x^8 - 56\,x^6t + 840\,x^4t^2 - 3360\,x^2t^3 + 1680\,t^4}, \tag{11}$$

is a solution to the Burgers' equation (1).

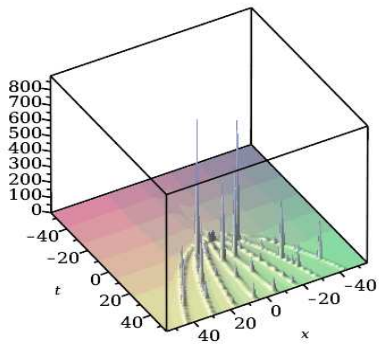

Figure 8. Solution of order 8 to (1).

3.9 Solutions of order nine

Proposition 3.9 *The function v_9 defined by*

$$v_9(x,t) = 18\,\frac{x^8 - 56\,x^6t + 840\,x^4t^2 - 3360\,x^2t^3 + 1680\,t^4}{x\left(x^8 - 72\,x^6t + 1512\,x^4t^2 - 10080\,x^2t^3 + 15120\,t^4\right)}, \tag{12}$$

is a solution to the Burgers' equation (1).

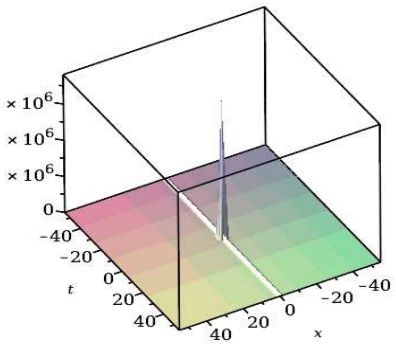

Figure 9. Solution of order 9 to (1).

3.10 Solutions of order ten

Proposition 3.10 *The function v_{10} defined by*

$$v_{10}(x,t) = 20\,\frac{x\left(x^8 - 72\,x^6t + 1512\,x^4t^2 - 10080\,x^2t^3 + 15120\,t^4\right)}{-x^{10} + 90\,x^8t - 2520\,x^6t^2 + 25200\,x^4t^3 - 75600\,x^2t^4 + 30240\,t^5}, \tag{13}$$

is a solution to the Burgers' equation (1).

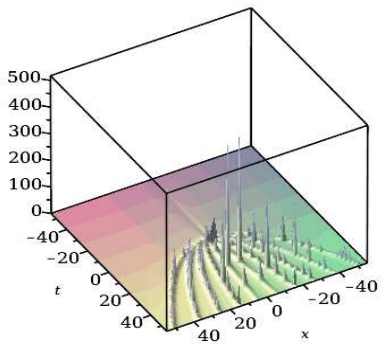

Figure 10. Solution of order 10 to (1).

3.11 Solutions of order eleven

Proposition 3.11 *The function* v_{11} *defined by*

$$v_{11}(x,t) = 22\,\frac{-x^{10}+90\,x^8t-2520\,x^6t^2+25200\,x^4t^3-75600\,x^2t^4+30240\,t^5}{x\,(-x^{10}+110\,x^8t-3960\,x^6t^2+55440\,x^4t^3-277200\,x^2t^4+332640\,t^5)}, \quad (14)$$

is a solution to the Burgers' equation (1).

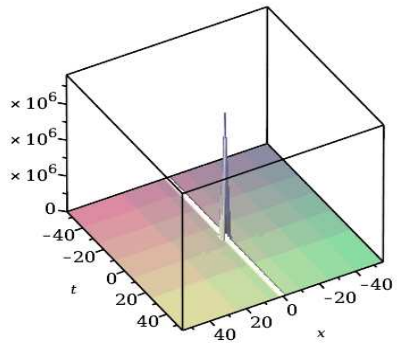

Figure 11. Solution of order 11 to (1).

3.12 Solutions of order twelve

Proposition 3.12 *The function* v_{12} *defined by* $v_{12}(x,t) = \dfrac{n(x,t)}{d(x,t)}$,

$n(x,t) = -24x(-x^{10}+110\,x^8t-3960\,x^6t^2+55440\,x^4t^3-277200\,x^2t^4+332640\,t^5)$,

$d(x,t) = x^{12}-132\,tx^{10}+5940\,t^2x^8-110880\,t^3x^6+831600\,t^4x^4-1995840\,t^5x^2+665280\,t^6$

is a solution to the Burgers' equation (1).

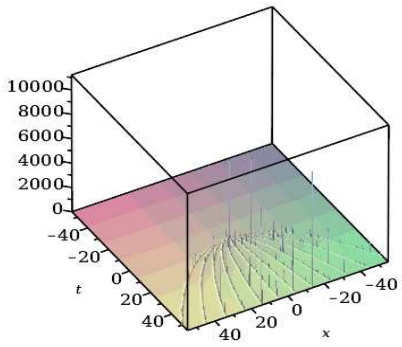

Figure 12. Solution of order 12 to (1).

3.13 Solutions of order thirteen

Proposition 3.13 *The function* v_{13} *defined by* $v_{13}(x,t) = \dfrac{n(x,t)}{d(x,t)}$,

$n(x,t) = 26(x^{12} - 132\,tx^{10} + 5940\,t^2x^8 - 110880\,t^3x^6 + 831600\,t^4x^4 - 1995840\,t^5x^2 + 665280\,t^6)$,

$d(x,t) = x(x^{12} - 156\,tx^{10} + 8580\,t^2x^8 - 205920\,t^3x^6 + 2162160\,t^4x^4 - 8648640\,t^5x^2 + 8648640\,t^6)$

is a solution to the Burgers' equation (1).

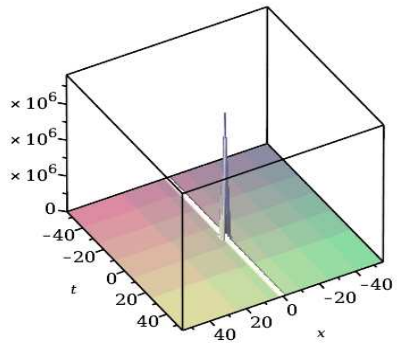

Figure 13. Solution of order 13 to (1).

3.14 Solutions of order fourteen

Proposition 3.14 *The function* v_{14} *defined by* $v_{14}(x,t) = \dfrac{n(x,t)}{d(x,t)}$,

$n(x,t) = -28x(x^{12} - 156\,tx^{10} + 8580\,t^2x^8 - 205920\,t^3x^6 + 2162160\,t^4x^4 - 8648640\,t^5x^2 + 8648640\,t^6)$,

$d(x,t) = -x^{14} + 182\,tx^{12} - 12012\,t^2x^{10} + 360360\,t^3x^8 - 5045040\,t^4x^6 + 30270240\,t^5x^4 - 60540480\,t^6x^2 + 17297280\,t^7$

is a solution to the Burgers' equation (1).

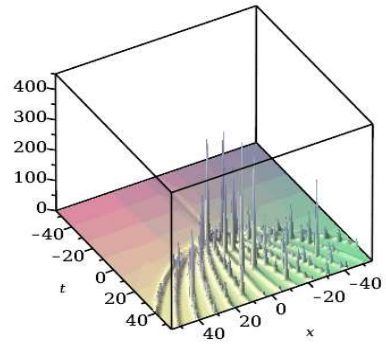

Figure 14. Solution of order 14 to (1).

3.15 Solutions of order fifthteen

Proposition 3.15 *The function* v_{15} *defined by* $v_{15}(x,t) = \dfrac{n(x,t)}{d(x,t)}$,

$n(x,t) = 30(-x^{14}+182\,tx^{12}-12012\,t^2x^{10}+360360\,t^3x^8-5045040\,t^4x^6+30270240\,t^5x^4-60540480\,t^6x^2+17297280\,t^7)$,

$d(x,t) = x(-x^{14}+210\,tx^{12}-16380\,t^2x^{10}+600600\,t^3x^8-10810800\,t^4x^6+90810720\,t^5x^4-302702400\,t^6x^2+259459200\,t^7)$

is a solution to the Burgers' equation (1).

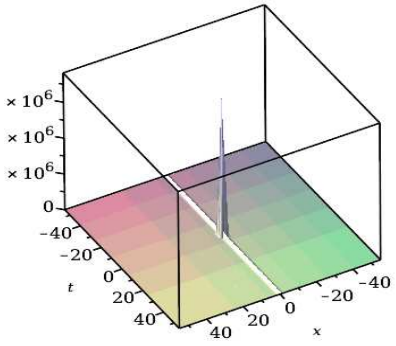

Figure 15. Solution of order 15 to (1).

3.16 Solutions of order sixteen

Proposition 3.16 *The function* v_{16} *defined by* $v_{16}(x,t) = \dfrac{n(x,t)}{d(x,t)}$,

$n(x,t) = -32x(-x^{14} + 210\,tx^{12} - 16380\,t^2x^{10} + 600600\,t^3x^8 - 10810800\,t^4x^6 + 90810720\,t^5x^4 - 302702400\,t^6x^2 + 259459200\,t^7)$,

$d(x,t) = x^{16} - 240\,tx^{14} + 21840\,t^2x^{12} - 960960\,t^3x^{10} + 21621600\,t^4x^8 - 242161920\,t^5x^6 + 1210809600\,t^6x^4 - 2075673600\,t^7x^2 + 518918400\,t^8$

is a solution to the Burgers' equation (1).

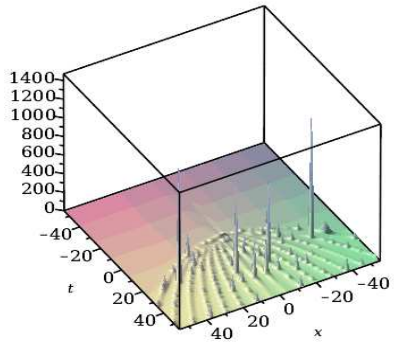

Figure 16. Solution of order 15 to (1).

3.17 Solutions of order seventeen

Proposition 3.17 *The function* v_{17} *defined by* $v_{17}(x,t) = \dfrac{n(x,t)}{d(x,t)}$,

$n(x,t) = 34(x^{16} - 240\,tx^{14} + 21840\,t^2x^{12} - 960960\,t^3x^{10} + 21621600\,t^4x^8 - 242161920\,t^5x^6 + 1210809600\,t^6x^4 - 2075673600\,t^7x^2 + 518918400\,t^8)$,

$d(x,t) = x(x^{16} - 272\,tx^{14} + 28560\,t^2x^{12} - 1485120\,t^3x^{10} + 40840800\,t^4x^8 - 588107520\,t^5x^6 + 4116752640\,t^6x^4 - 11762150400\,t^7x^2 + 8821612800\,t^8)$

is a solution to the Burgers' equation (1).

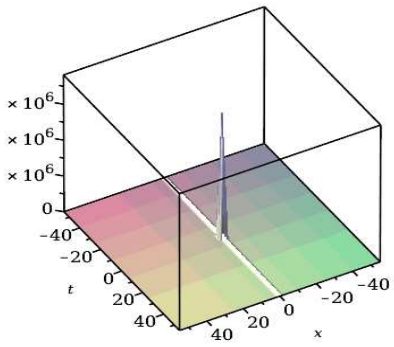

Figure 17. Solution of order 17 to (1).

3.18 Solutions of order eighteen

Proposition 3.18 *The function* v_{18} *defined by* $v_{18}(x,t) = \dfrac{n(x,t)}{d(x,t)}$,

$n(x,t) = -36x(x^{16} - 272\,tx^{14} + 28560\,t^2x^{12} - 1485120\,t^3x^{10} + 40840800\,t^4x^8 - 588107520\,t^5x^6 + 4116752640\,t^6x^4 - 11762150400\,t^7x^2 + 8821612800\,t^8)$,

$d(x,t) = -x^{18} + 306\,tx^{16} - 36720\,t^2x^{14} + 2227680\,t^3x^{12} - 73513440\,t^4x^{10} + 1323241920\,t^5x^8 - 12350257920\,t^6x^6 + 52929676800\,t^7x^4 - 79394515200\,t^8x^2 + 17643225600\,t^9$

is a solution to the Burgers' equation (1).

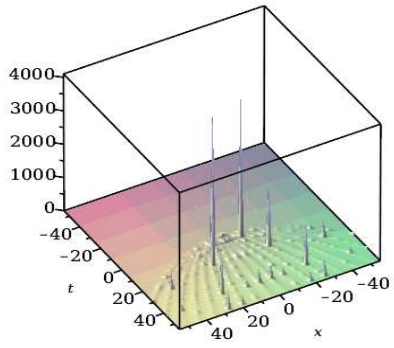

Figure 18. Solution of order 18 to (1).

3.19 Solutions of order nineteen

Proposition 3.19 *The function* v_{19} *defined by* $v_{19}(x,t) = \dfrac{n(x,t)}{d(x,t)}$,

$n(x,t) = 38(-x^{18} + 306\,tx^{16} - 36720\,t^2x^{14} + 2227680\,t^3x^{12} - 73513440\,t^4x^{10} + 1323241920\,t^5x^8 - 12350257920\,t^6x^6 + 52929676800\,t^7x^4 - 79394515200\,t^8x^2 + 17643225600\,t^9)$,

$d(x,t) = x(-x^{18} + 342\,tx^{16} - 46512\,t^2x^{14} + 3255840\,t^3x^{12} - 126977760\,t^4x^{10} + 2793510720\,t^5x^8 - 33522128640\,t^6x^6 + 201132771840\,t^7x^4 - 502831929600\,t^8x^2 + 335221286400\,t^9)$

is a solution to the Burgers' equation (1).

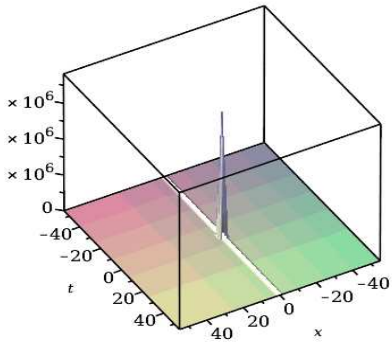

Figure 19. Solution of order 19 to (1).

3.20 Solutions of order twenty

Proposition 3.20 *The function* v_{20} *defined by* $v_{20}(x,t) = \frac{n(x,t)}{d(x,t)}$,

$n(x,t) = -40x(-x^{18}+342\,tx^{16}-46512\,t^2x^{14}+3255840\,t^3x^{12}-126977760\,t^4x^{10}+2793510720\,t^5x^8-33522128640\,t^6x^6+201132771840\,t^7x^4-502831929600\,t^8x^2+335221286400\,t^9)$,

$d(x,t) = x^{20}-380\,tx^{18}+58140\,t^2x^{16}-4651200\,t^3x^{14}+211629600\,t^4x^{12}-5587021440\,t^5x^{10}+83805321600\,t^6x^8-670442572800\,t^7x^6+2514159648000\,t^8x^4-3352212864000\,t^9x^2+670442572800\,t^{10}$

is a solution to the Burgers' equation (1).

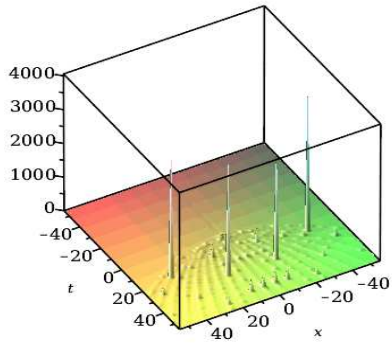

Figure 20. Solution of order 20 to (1).

4 Conclusion

We have given an expression of rational solutions to the Burgers' equation involving particular polynomials.
In particular, we have constructed explicit solutions to the Burgers' equation for the orders $n = 1$ until $n = 20$.
All these solutions are singular. We can classify them by the pattern of their singulatities.
The singularities of these solutions depend on the orders of the solutions. When we consider odd order solutions we have always the line $x = 0$ of singularities. In the case of even order solutions $n = 2p$, the singularities form horseshoe patterns with p branches.
It will be interesting to construct solutions of this equation depending on some real parameters.

References

[1] H. Bateman, Some recent researches on the motion of fluids, Monthly Weather Review, V. 43, 163-170, 1915

[2] XJ.M. Burgers, A mathematical model illustrating the theory of turbulence, Adv. In Appl. Mech., V. 1, 171-199, 1948

[3] N. Sugimoto, Burgers equation with a fractional derivative: hereditary effects on nolinear acoustic waves, Jour. Of Fluid Mech., V. 225, 631-633, 1991

[4] G.W. Bluman, J.D. Cole, The general similarity solution of the heat equation, Jour. Of Math. And Mech., V. 18, 1025-1042, 1969

[5] N. Su, P.C. Jim, W.V. Keith, E.C. Murray, R. Mao, Some recent researches on the motion of fluids, Ausr. Jour. Soil Res., V. 42, 42-, 2004

[6] J.M. Burgers, Mathematical examples illustating the relations occuring in the theory of turbulent fluid motion, Trans. Roy. Neth. Acad. Sci. Amsterdam, V. 17, 1-53, 1939

[7] J.M. Burgers, The non-linear diffusion equation: asymptotic solutions and statistical problems, Springer, 1974

[8] J.D. Cole, On a quasi-linear parabolic equation occuring in aerodynamics, Quat. Of Appl. Math., V. 9, 225-236, 1951

[9] G.W. Whitham, Linear and nonlinear waves, Pure an Appl. Math., V. 42, Wiley And Sons, 2011

[10] Q. Gao, M.Y. Zou, An analytical solution for two and three dimensional, Applied Mathematical Modelling, V. 45, 255270, 2017

[11] X.J. Yang, F. Gao, H.M. Srivastava, M.Y. Zou, Exact travelling wave solutions for the local fractional two-dimensional Burgers-type equations, Computers and Mathematics with Applications, V. 73, 203210, 2017

[12] F.L. Xia, M. Sajjad Hashemi, M. Inc, P. Ashraf, Explicit solutions of higher dimensional Burgers equations, Journal of Ocean Engineering and Science, doi.org/10.1016/j.joes.2022.04.032, 2022

[13] A. Ebaid, Exact solitary wave solution for some nonlinear evolution equations via exp-function method, Phys. Lett. A, V. 365, N. 3, 213-219, 2007

[14] A.M. Wazwaz, Multiple front solutions for the Burgers equation and the coupled Burgers equations, Appl. Math. And Comp., V. 190, N. 2, 1198-1206, 207

[15] K.B. Wolf, L. Hlavaty, S. Steinberg, Nonlinear differential equations as invariants under group action on coset bundles: Burgers and Korteweg de Vries equation families, Jour. Of Math. Ana. And Appl., V. 114, 340-359, 1986

[16] E. Hopf, The partial differential equation $u_t + uu_x = u_{xx}$, Comm. On Pure And Appl. Math., V. 3, 201-230, 1950

[17] T. Ohwada, Cole-Hopf transformation as numerical tool for the Burgers equation, Appl. And Comp. Math., V. 8, N. 1, 107-113, 2009

[18] B.M. Vaganan, Cole-Hopf transformation for higher dimensional Burgers equation with variable coefficients, Stu. In Appl. Math., V. 129, N. 3, 300-308, 2012

[19] E.R. Srivastava, G.W. Awasthi, (1+ n)-Dimensional Burgers' equation and its analytical solution: A comparative study of HPM, ADM and DTM, Ain Shams Uni., V. 5, 625-629, 2014

[20] P. Gaillard, V.B. Matveev, Wronskian addition formula and its applications, Max-Planck-Institut für Mathematik, MPI 02-31, V. **161**, 2002

[21] P. Gaillard, A new family of deformations of Darboux-Pöschl-Teller potentials, Lett. Math. Phys., V. **68**, 77-90, 2004

[22] P. Gaillard, V.B. Matveev, New formulas for the eigenfunctions of the two-particle Calogero-Moser system, Lett. Math. Phys., V. **89**, 1-12, 2009

[23] P. Gaillard, V.B. Matveev, Wronskian and Casorai determinant representations for Darboux-Pöschl-Teller potentials and their difference extensions, J. Phys A : Math. Theor., V. **42**, 404409-1-16, 2009

[24] P. Dubard, P. Gaillard, C. Klein, V. B. Matveev, On multi-rogue wave solutions of the NLS equation and positon solutions of the KdV equation, Eur. Phys. J. Spe. Top., V. **185**, 247-258, 2010

[25] P. Gaillard, Families of quasi-rational solutions of the NLS equation and multi-rogue waves, J. Phys. A : Meth. Theor., V. **44**, 435204-1-15, 2011

[26] P. Gaillard, Wronskian representation of solutions of the NLS equation and higher Peregrine breathers, J. Math. Sciences : Adv. Appl., V. **13**, N. 2, 71-153, 2012

[27] P. Gaillard, Degenerate determinant representation of solution of the NLS equation, higher Peregrine breathers and multi-rogue waves, J. Math. Phys., V. **54**, 013504-1-32, 2013

[28] P. Gaillard, Wronskian representation of solutions of NLS equation and seventh order rogue waves, J. Mod. Phys., V. **4**, N. 4, 246-266, 2013

[29] P. Gaillard, V.B. Matveev, Wronskian addition formula and Darboux-Pöschl-Teller potentials, J. Math., V. **2013**, ID 645752, 1-10, 2013

[30] P. Gaillard, Two-parameters determinant representation of seventh order rogue waves solutions of the NLS equation, J. Theor. Appl. Phys., V. **7**, N. **45**, 1-6, 2013

[31] P. Gaillard, Deformations of third order Peregrine breather solutions of the NLS equation with four parameters, Phys. Rev. E, V. **88**, 042903-1-9, 2013

[32] P. Gaillard, Two parameters deformations of ninth Peregrine breather solution of the NLS equation and multi rogue waves, J. Math., V. **2013**, 1-111, 2013

[33] P. Gaillard, Six-parameters deformations of fourth order Peregrine breather solutions of the NLS equation, J. Math. Phys., V. **54**, 073519-1-22, 2013

[34] P. Gaillard, The fifth order Peregrine breather and its eight-parameters deformations solutions of the NLS equation, Commun. Theor. Phys., V. **61**, 365-369, 2014

[35] P. Gaillard, Ten parameters deformations of the sixth order Peregrine breather solutions of the NLS equation, Phys. Scripta, V. **89**, 015004-1-7, 2014

[36] P. Gaillard, Higher order Peregrine breathers, their deformations and multi-rogue waves, J. Of Phys. : Conf. Ser., V. **482**, 012016-1-7, 2014

[37] P. Gaillard, M. Gastineau, Eighteen parameter deformations of the Peregrine breather of order ten solutions of the NLS equation, Int. J. Mod. Phys. C, V. **26**, N. 2, 1550016-1-14, 2014

[38] P. Gaillard, Two parameters wronskian representation of solutions of nonlinear Schrödinger equation, eight Peregrine breather and multi-rogue waves, J. Math. Phys., V. **5**, 093506-1-12, 2014

[39] P. Gaillard, Hierarchy of solutions to the NLS equation and multi-rogue waves, J. Phys. : Conf. Ser., V. **574**, 012031-1-5, 2015

[40] P. Gaillard, Tenth Peregrine breather solution of the NLS, Ann. Phys., V. **355**, 293-298, 2015

[41] P. Gaillard, M. Gastineau, The Peregrine breather of order nine and its deformations with sixteen parameters solutions of the NLS equation Phys. Lett. A, V. **379**, 1309-1313, 2015

[42] P. Gaillard, Other 2N-2 parameters solutions to the NLS equation and 2N+1 highest amplitude of the modulus of the N-th order AP breather, J. Phys. A: Math. Theor., V. **48**, 145203-1-23, 2015

[43] P. Gaillard, Multi-parametric deformations of the Peregrine breather of order N solutions to the NLS equation and multi-rogue waves, Adv. Res., V. **4**, N. **5**, 346-364, 2015

[44] P. Gaillard, Higher order Peregrine breathers solutions to the NLS equation, Jour. Phys. : Conf. Ser., V. **633**, 012106-1-6, 2016

[45] P. Gaillard, M. Gastineau Patterns of deformations of Peregrine breather of order 3 and 4, solutions to the NLS equation with multi-parameters, Journal of Theoretical and Applied Physics, V. **10**,1-7, 2016

[46] P. Gaillard, M. Gastineau Twenty parameters families of solutions to the NLS equation and the eleventh Peregrine breather, Commun. Theor. Phys, V. **65**, N. **2**, 136-144, 2016

[47] P. Gaillard, Rational solutions to the KPI equation and multi rogue waves, Annals Of Physics, V. **367**, 1-5, 2016

[48] P. Gaillard, M. Gastineau Twenty two parameters deformations of the twelfth Peregrine breather solutions to the NLS equation, Adv. Res., V. **10**, 83-89, 2016

[49] P. Gaillard, Towards a classification of the quasi rational solutions to the NLS equation, Theor. And Math. Phys., V. **189**, N. **1**, 1440-1449, 2016

[50] P. Gaillard, Fredholm and Wronskian representations of solutions to the KPI equation and multi-rogue waves, Jour. of Math. Phys., V. **57**, 063505-1-13, doi: 10.1063/1.4953383, 2016

[51] P. Gaillard, M. Gastineau Families of deformations of the thirteenth Peregrine breather solutions to the NLS equation depending on twenty four parameters, Jour. Of Bas. And Appl. Res. Int., V. **21**, N. 3, 130-139, 2017

[52] P. Gaillard, From Fredholm and Wronskian representations to rational solutions to the KPI equation depending on $2N-2$ parameters, Int. Jour. of Appl. Sci. And Math., V. **4**, N. 3, 60-70, 2017

[53] P. Gaillard, Families of rational solutions of order 5 to the KPI equation depending on 8 parameters, New Hor. in Math. Phys., V. 1, N. 1, 26-31, 2017

[54] P. Gaillard, 6-th order rational solutions to the KPI Equation depending on 10 parameters, Jour. Of Bas. And Appl. Res. Int., V. **21**, N. 2, 92-98, 2017

[55] P. Gaillard, N-Order rational solutions to the Johnson equation depending on $2N-2$ parameters, Int. Jour. of Adv. Res. in Phys. Sci., V. **4**, N. 9, 19-37, 2017

[56] P. Gaillard, Families of rational solutions to the KPI equation of order 7 depending on 12 parameters, Int. Jour. of Adv. Res. in Phys. Sci., V. **4**, N. 11, 24-30, 2017

[57] P. Gaillard, Rational solutions to the Johnson equation and rogue waves, Int. Jour. of Inn. In Sci. and Math., V. **6**, N. 1, 14-19, 2018

[58] P. Gaillard, Multiparametric families of solutions of the KPI equation, the structure of their rational representations and multi-rogue waves, Theo. And Mat. Phys., V. **196**, N. 2, 1174-1199, 2018

[59] P. Gaillard, The Johnson equation, Fredholm and wronskian representations of solutions and the case of order three, Adv. In Math. Phys., V. **2018**, 1-18, 2018

[60] P. Gaillard, Families of Solutions of Order 5 to the Johnson Equation Depending on 8 Parameters, NHIMP, V. **2**, N. 4, 53-61, 2018

[61] P. Gaillard, Multiparametric families of solutions to the Johnson equation, J. Phys. : Conf. Series, V. **1141**, 012102-1-10, 2018

[62] P. Gaillard, Rational solutions to the Boussinesq equation, Fund. Jour. Of Math. And Appl., V. , 109-112, 2019

[63] P. Gaillard, Differential relations for the solutions to the NLS equation and their different representations, Comm. In Adv. Math. Sci., V. **2**, N. 4, 1-4, 2019

[64] P. Gaillard, Multi-parametric families of solutions of order N to the Boussinesq and KP equations and the degenerate rational case, UJMA, V. **3**, N. 2, 44-52, 2020

[65] P. Gaillard, The mKdV equation and multi-parameters rational solutions, Wave Motion, V. **100**, 102667-1-9, 2021

[66] P. Gaillard, Degenerate Riemann theta functions, Fredholm and wronskian representations of the solutions to the KdV equation and the degenerate rational case, Jour. Of Geom. And Phys., V. **161**, 104059-1-12, 2021

[67] P. Gaillard, 2N parameter solutions to the Burgers equation, Jour. Of Appl. Nonlin. Dyn., V. **1**, N. **1**, 1-6, 2021

[68] P. Gaillard, Multi-Parametric rational Solutions to the KdV equation, Asi. Jour. Of Res. and Rev. In Phys., V. **3**, N. 3, 14-21, 2021

[69] P. Gaillard, Multiparametric solutions to the Gardner equation and the degenerate rational case, Jour. Of Appl. Ana. And Appl., V. **11**, N. 4, 2102-2113, 2021

[70] P. Gaillard, Other families of rational solutions to the KPI equation, Asi. Res. Jour Of Math., V. **17**, N. 6, 27-34, 2021

[71] P. Gaillard, Rational solutions to the mKdV equation associated to particular polynomials, Wave Motion, V. **107**, 102824-1-11, 2021

[72] P. Gaillard, Multiparametric Rational Solutions of Order N to the KPI Equation and the Explicit Case of Order 3, Arch. Of cur. Res. Int., V. **21**, N . 6, 58-71, 2021

[73] P. Gaillard, Rational solutions to the KPI equation from particular polynomials, Wave Motion, V. **108**, 102828-1-9, 2022

[74] P. Gaillard, Rogue Waves of the Lakshmanan Porsezian Daniel Equation Depending on Multi-parameters, As. Jour. Of Adv. Res. And Rep., V 16, N. 3, 32-40, 2022

[75] P. Gaillard, Quasi-rational and rational solutions to the defocusing NLS equation, Jour. Of Math. Sci. And Mod. Res. And Rep., V 5, N. 1, 24-34, 2022

[76] P. Gaillard, Rational solutions to the KdV equation in terms of particular polynomials, Jour. Of Nonlin. Mod. and Ana., V. 4, N. 4, 615-626, 2022

[77] P. Gaillard, Rational solutions to the Painlev II equation from particular polynomials, Int. J. Of Appl. Nonlin. Sci., V. 3, N. 3, 189-196, 2022

[78] P. Gaillard, Multi-lump solutions to the KPI equation with a zero degree of derivation, Jour. Of Theor. And Appl. Phys., V. 16, N. 4, 1-8, 2022

[79] P. Gaillard, Families of solutions to the KPI equation given by an extended Darboux transformation, Part. Diff. Equ. And Appl., V. 3, N. 80, 1-14, 2022

[80] P. Gaillard, Rogue waves of the Hirota equation in terms of quasi-rational solutions depending on multi-parameters, WSEAS Trans. On Math., DOI: 10.37394/23206.2023.22.24, V. 22, 190-203, 2023

[81] P. Gaillard, Different representations of the solutions to the cylindrical nonlinear Schrödinger equation, Dis. Nonlin. And Compl., V. 12, N. 3, 539-553, 2023

[82] P. Gaillard, Rational solutions to the KdV equation from Riemann theta functions, Jour. Of Theor. And Appl. Phys., V. 17, N. 3, 1-5, 2023

[83] P. Gaillard, On Hadamard's Coefficients for Two Dimensional Schrdinger Operators, Int. Jour. Of Theor. Phys., V. 62, N. 205, 1-16, 2023