Frequency Synthesis of Interconnected Homogeneous LTI Systems

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***Abstract*— The synthesis problem of the interconnection of homogeneous Linear Time-Invariant (LTI) systems such that the frequency-response satisfies magnitude constraints is investigated. To this end, the usual synthesis approach of traditional filters, viewed as the interconnection of integrators $\frac{1}{s}$, is revisited based on the Linear Fractional Transformation (LFT) representation and the $\{x, y, z\}$-dissipative characterization. This approach, based on convex optimization, consists of two steps: the magnitude synthesis and the spectral factorization steps. When the systems are modeled by a lossless dissipative transfer function $T(s)$, it is demonstrated that each step can be extended. However, a factorization error appears when considering a general dissipative $T(s)$, preventing the direct extension of the two-step approach. It is then revealed how to overcome this issue by coupling both steps, generalizing thereby the usual synthesis approach. Finally, the interest of this work is illustrated through two applications: the design of LC-bandpass filters and the weighted $\mathcal{H}_\infty$-control of interconnected homogeneous systems.**

***Index Terms*— Frequency Design; Spectral Factorization; Interconnected systems; LFT; Dissipative systems; LMIs**

I. Introduction

The increase in complexity of modern systems along with ever-more ambitious design specifications have led to challenging engineering problems. A common approach to solving them consists in interconnecting simple local systems in a way that fulfills a global objective. When local systems are represented by the same model, the global system is denoted as the interconnection of homogeneous systems, also called homogeneous multi-agent or large-scale system. Over the last two decades, this approach has been successful in addressing diverse System and Control problems such as distributed or decentralized control [1]–[4], stability analysis of systems with generalized frequency variables [5], [6] and of time-varying systems [7], or interconnection matrix redesign [8].

Frequency filtering is one of the historical topics of interest of System Theory, especially appearing in Signal Processing, Control Theory and Electronics [9]–[11]. The synthesis problem of traditional frequency filters is well-established and can be efficiently solved based on convex optimization [9], [10]. The resolution is usually split in two steps. First, a magnitude function satisfying the frequency requirements is synthesized. Second, a stable transfer function is factorized from the magnitude function. This step, known as spectral factorization, is related to many other applications such as time series prediction [12], stochastic [13] or passive system realizations [14].

This paper focuses on the problem of synthesizing the interconnection of homogeneous Linear Time-Invariant (LTI) systems $T(s)$ to satisfy frequency filtering requirements, which plays a key role in modern applications such as the frequency design of passive electronic filters [15] or the weighted $\mathcal{H}_\infty$-control of interconnected homogeneous systems [4]. Despite efficient methods in the traditional case, the direct application of the usual filter synthesis approach to this problem leads to the presence of structural constraints on the global system representation, such as algebraic constraints on the matrices of a state-space representation or on the coefficients of a transfer matrix. Unfortunately, these structural constraints generally lead to NP-hard optimization problems [16].

The main motivation behind this work is to overcome this issue by developing a method specifically adapted to the interconnection of homogeneous LTI systems. The originality is to revisit the usual synthesis approach by viewing traditional frequency filters as the interconnection of systems with transfer function $\frac{1}{s}$, and extend it to systems with more general transfer function $T(s)$. To this end, it is proposed to take advantage from the Linear Fractional Transformation (LFT) [17], to represent the interconnection of homogeneous LTI systems, and from a frequency-domain quadratic constraint named $\{x, y, z\}$-dissipative characterization [18], [19], derived from QSR-dissipativeness [20]. The underlying idea will be to view $\frac{1}{s}$ as a lossless $\{0, 1, 0\}$-dissipative transfer function to extend the approach to systems modeled by an $\{x, y, z\}$-dissipative transfer function $T(s)$.

The main contributions of this paper are three-folds. First, the usual synthesis approach is extended step-by-step for *lossless* dissipative $T(s)$, i.e. when the quadratic constraint is defined by an equality. Based on this particular characterization and the application of adapted versions of the so-called KYP lemma [21], [22], the magnitude synthesis step is formulated as a feasibility problem under linear matrix inequality (LMI) constraints [23]. In addition, the spectral factorization step is extended by adapting the usual technique, based on the resolution of an Algebraic Riccati Equation (ARE) [24], [25]. Second, the presence of a factorization error that prevents the direct extension of the two-step approach is highlighted in

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the case of a general dissipative $T(s)$. Third, it is proposed to solve this problem based on the original idea of coupling the magnitude synthesis and spectral factorization steps, thus leading to a unifying generalization of the usual approach. Moreover, the interest of this work is illustrated on two applications: the synthesis of LC-bandpass filters and of the weights for $\mathcal{H}_\infty$-control. In particular, the second application completes the picture of previous works [4], [8], laying the foundations for the weighted $\mathcal{H}_\infty$-control of interconnected homogeneous systems.

The present paper completes and extends the preliminary work [26]. In particular, an exclusive synthesis approach is developed for lossless dissipative $T(s)$, especially leading to the extension of the spectral factorization technique. Moreover, an explicit proof of the stability of the synthesized interconnected systems is now included. The underlying technical material also provides a means for computing a stable interconnection. Furthermore, the overall conservatism of the results is reduced. Using standard loop-transformation, the interconnection is expressed as an LFT in $\frac{1}{s}$ for the satisfaction of the frequency constraints in the magnitude synthesis step, allowing the application of a version of the KYP lemma that provides necessary-and-sufficient conditions. Another important improvement is the development of the stability result for the spectral factorization, that eliminates the positive-definite constraint on the minimal-solution of the ARE.

This paper is organized as follows. In Section II, preliminaries about the LFT representation and the $\{x, y, z\}$-dissipative characterization are presented. In Section III the interconnection synthesis problem of homogeneous systems satisfying magnitude constraints is formulated and discussed. Then, in Section IV, the usual two-step approach is extended for homogeneous systems modeled by a lossless dissipative transfer function $T(s)$. In Section V, a generalized synthesis approach is proposed for the case of lossy dissipative $T(s)$. Finally, both approaches are illustrated on two applications in Section VI, while Section VII provides concluding remarks.

Notation: $\mathbb{R}^{n\times m}$ is the set of real matrices of size $n \times m$, and $\mathbb{S}_n$ (resp. $\mathbb{S}_n^+$) the symmetric (positive definite) matrices of $\mathbb{R}^{n\times n}$. The matrices I_n and $0_{n\times m}$ are the identity of $\mathbb{R}^{n\times n}$ and the zero of $\mathbb{R}^{n\times m}$. $\mathbb{C}$ is the set of complex numbers and $\mathbb{C}^+$ those with non-negative real part. The subscripts are omitted when obvious from the context. X^T and X^* stand for transpose and transpose conjugate of X while $M > (\geq)\ 0$ denotes positive (semi-) definiteness, $\Lambda(M)$ the set of eigenvalues of the matrix M and denote $(M_i)_{i=1}^{n_I} := (M_1,\ M_2,\ \ldots,\ M_{n_I})$. The symbol $\otimes$ is the Kronecker product and $\star$ the Redheffer product [17]. Bold characters denote either explicit decision variables in a problem or optimization variables in an optimization problem. For brevity, as the $\{x, y, z\}$-dissipative KYP lemma and ARE will play a key role, the additional notations are used:

$$\mathcal{LM}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, Y, X\right) = \left[\begin{array}{cc|c} A & B & \\ I & 0 & \\ \hline C & D & \end{array}\right]^T \left[\begin{array}{cc|c} Y & 0 & \\ 0 & X & \\ \hline & & \end{array}\right] \left[\begin{array}{cc|c} A & B & \\ I & 0 & \\ \hline C & D & \end{array}\right]$$

$$\mathcal{R}_{xyz}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, P, X\right) = Q_{xyz} - S_{xyz} R_{xyz}^{-1} S_{xyz}^T$$

$$\text{where} \quad Q_{xyz} := \begin{bmatrix} A \\ I \end{bmatrix}^T \begin{bmatrix} -zP & -yP \\ -yP & -xP \end{bmatrix} \begin{bmatrix} A \\ I \end{bmatrix} + C^T X C$$
$$S_{xyz} := zA^T(-P)B + y(-P)B + C^T X D$$
$$R_{xyz} := zB^T(-P)B + D^T X D$$

When $y = 1$ and $x = z = 0$, the associated equation $\mathcal{R}_{010}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, P, X\right) = 0$ is the continuous-time ARE.

II. Preliminaries

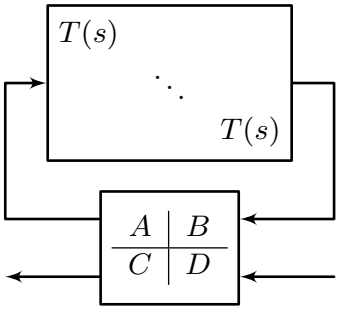

Fig. 1. Interconnection of homogeneous systems $\boldsymbol{T(s)}$.

Consider the interconnection of n homogeneous LTI systems $T(s)$ (Fig. 1) with the Laplace-domain representation:

$$\begin{cases} p(s) = (T(s) \cdot I_n)\, q(s) \\ q(s) = Ap(s) + Bw(s) \\ z(s) = Cp(s) + Dw(s) \end{cases} \tag{1}$$

where $T(s)$ is a proper transfer function and A, B, C, D real matrices. Representation (1) is a particular case of the linear fractional representation (LFR) [17]. Provided that $(I - AT(s))^{-1}$ exists, the transfer $H(T(s))$ from $w(s)$ to $z(s)$ is computed by the linear fractional transformation (LFT):

$$(T(s) \cdot I_n) \star \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = D + CT(s)\,(I - AT(s))^{-1}\,B \tag{2}$$

For the sake of brevity, (2) is denoted as an LFT representation of $H(T(s))$. The LFR (1) and the LFT (2) generalize the usual relationship between a transfer function and its state-space realization, obtained when $T(s) = \frac{1}{s}$. Moreover, for single-input-single-output $H(T(s))$, (2) is a proper, n^{th}-order rational function of $T(s)$ and any proper, n^{th}-order rational function of $T(s)$ may be represented as in (2). The LFT framework enables to easily compute standard algebraic operations, such as the product of LFTs or their inversion. Furthermore, given a state-space representation of $T(s)$

$$T(s) = \left(\frac{1}{s} \cdot I_m\right) \star \left[\begin{array}{c|c} A_T & B_T \\ \hline C_T & D_T \end{array}\right] \tag{3}$$

an equivalent LFT representation of $H(T(s))$ is provided by:

$$(T(s) \cdot I_n) \star \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left(\frac{1}{s} \cdot I_{n_s}\right) \star \left[\begin{array}{c|c} A_s & B_s \\ \hline C_s & D_s \end{array}\right]$$

where $n_s = nm$ and

$$\left[\begin{array}{c|c} A_s & B_s \\ \hline C_s & D_s \end{array}\right] = \left[\begin{array}{c|c} I_n \otimes A_T & I_n \otimes B_T \\ \hline I_n \otimes C_T & I_n \otimes D_T \end{array}\right] \star \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \tag{4}$$

In this paper, the systems will be characterized based on an input-output relation named $\{x, y, z\}$-dissipativeness, also known as (Q, S, R) dissipativeness [20].

Definition 1 ($\{x, y, z\}$-dissipative system)**.**
Let x, y, z be real scalars such that $xz - y^2 < 0$. Denote $p_{\mathbb{C}^+}$ (resp. $p_{j\mathbb{R}}$) the set of poles of $T(s)$ in $\mathbb{C}^+$ (resp. $j\mathbb{R}$). Then, $T(s)$ is said to be $\{x, y, z\}$-dissipative if

$$\forall s \in \mathbb{C}^+ \setminus p_{\mathbb{C}^+}, \qquad \begin{bmatrix} T(s) \\ 1 \end{bmatrix}^* \begin{bmatrix} x & y \\ y & z \end{bmatrix} \begin{bmatrix} T(s) \\ 1 \end{bmatrix} \geq 0 \tag{5}$$

In particular, the system is said to be lossless $\{x, y, z\}$-dissipative if it is $\{x, y, z\}$-dissipative and

$$\forall j\omega \in j\mathbb{R} \setminus p_{j\mathbb{R}}, \quad \begin{bmatrix} T(j\omega) \\ 1 \end{bmatrix}^* \begin{bmatrix} x & y \\ y & z \end{bmatrix} \begin{bmatrix} T(j\omega) \\ 1 \end{bmatrix} = 0$$

By extension, the system is said to be lossy if it is $\{x, y, z\}$-dissipative but not lossless.

Remark 1. Dissipativeness is usually defined in the time-domain. Nonetheless, it comes from [27, Theorem 27, p. 62] that Definition 1 is equivalent to (Q, S, R)-dissipativeness, under minimality assumption of the state-space representation, as single-input-single-output systems $T(s)$ are considered.

Given a transfer function $T(s)$, an $\{x, y, z\}$ triplet such that (5) holds can be found based on a graphical test on $T(j\omega)$. This test consists in verifying in the complex plane that $T(j\omega)$ belongs to a geometrical shape parametrized by $\{x, y, z\}$ and jointly satisfies a condition in the style of the Nyquist criterion. Consider the following constraint

$$\forall j\omega \in j\mathbb{R} \setminus p_{j\mathbb{R}}, \quad \begin{bmatrix} T(j\omega) \\ 1 \end{bmatrix}^* \begin{bmatrix} x & y \\ y & z \end{bmatrix} \begin{bmatrix} T(j\omega) \\ 1 \end{bmatrix} \geq 0 \tag{6}$$

If $x < 0$, (6) leads $T(j\omega)$ to be inside the disk of center $c = -\frac{y}{x}$ and radius $r = \sqrt{\frac{y^2 - xz}{x^2}}$ in the complex plane. In addition, if $T(s)$ is stable ($T(s)$ has no poles in $\mathbb{C}^+$) then (5) also holds. If $x = 0$, the conditions are that $T(j\omega)$ belongs to the half plane defined by $\Re_e(s) \geq -\frac{z}{2y}$ (resp. $\leq$) when $y > 0$ (resp. $y < 0$), $T(s)$ has no poles in $\Re_e(s) > 0$ and at most simple poles on $\Re_e(s) = 0$, and the residues of its imaginary axis poles are positive (resp. negative). If $x > 0$, the graph of $T(j\omega)$ should be outside the disk of center c and radius r, and counterclockwise encircles this disk as many times as the number of poles of $T(s)$ in $\mathbb{C}^+$. These conditions are in fact necessary and sufficient (see [27, Chap. 8, Section 3]).

Remark 2. The encompassing disk of minimal radius may be computed based on LMI optimization [15, Chap. 2].

In this paper, the stability of the interconnection $H(T(s))$ of the $\{x, y, z\}$-dissipative systems $T(s)$ defined by the LFT (2) will be understood in the sense of the internal stability. This property is equivalent to the stability of the transfer matrix $T(s)\left(I - AT(s)\right)^{-1}$, which in turn can be verified by satisfying an LMI, as stated in Lemma 1.

Lemma 1 (adapted from [20])**.**
Let $T(s)$ be an $\{x, y, z\}$-dissipative and $H(T(s))$ be defined by the LFT (2). If the following LMI constraint holds

$$\exists \mathbf{P} \in \mathbb{S}_n^+, \qquad \begin{bmatrix} A \\ I \end{bmatrix}^T \Phi_{xyz}\left(\boldsymbol{P}\right) \begin{bmatrix} A \\ I \end{bmatrix} > 0 \tag{7}$$

where

$$\Phi_{xyz}\left(\boldsymbol{P}\right) := \begin{bmatrix} -z\boldsymbol{P} & -y\boldsymbol{P} \\ -y\boldsymbol{P} & -x\boldsymbol{P} \end{bmatrix} \tag{8}$$

then $H(T(s))$ is stable.

Remark 3. Lemma 1 provides a simple LMI test to check stability of the interconnection of homogeneous systems. Furthermore, it also provides an implicit means to compute a stable interconnection matrix A. Indeed, if one computes A such that its eigenvalues λ_i satisfy $\begin{bmatrix} \lambda_i \\ 1 \end{bmatrix}^* \begin{bmatrix} -z & -y \\ -y & -x \end{bmatrix} \begin{bmatrix} \lambda_i \\ 1 \end{bmatrix} > 0$, then (7) will be satisfied. This may be achieved based on pole placement [17], [28]. See also [15, Chap. 2] for more details.

Finally, it should be noted that $\frac{1}{s}$ is lossless $\{0, 1, 0\}$-dissipative. It will be at the heart of this paper to revisit the usual synthesis approach based on this characterization, and extend it from $T(s) = \frac{1}{s}$ to any $\{x, y, z\}$-dissipative $T(s)$. To this end, the KYP lemma [21], [22], which is a fundamental result in Control and System theory allowing to test a frequency-domain quadratic constraint by solving an LMI feasibility problem, will have a key role. Lemma 2 provides an $\{x, y, z\}$-dissipative version, adapted to the interconnection of homogeneous LTI systems.

Lemma 2 ($\{x, y, z\}$-dissipative KYP Lemma)**.**
Let $T(s)$ be an $\{x, y, z\}$-dissipative transfer function. Let $H(T(s))$ be defined by the LFT (2) and be stable. Define $\Phi_{xyz}\left(\boldsymbol{P}\right)$ as in (8). Then $(i) \Rightarrow (ii)$.

(i) There exists $\boldsymbol{P} \in \mathbb{P}_n$ such that

$$\mathcal{LM}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \Phi_{xyz}\left(\boldsymbol{P}\right), X\right) \geq 0 \tag{9}$$

where $\mathbb{P}_n = \mathbb{S}_n$ if $T(s)$ is lossless $\{x, y, z\}$-dissipative, $\mathbb{P}_n = \mathbb{S}_n^+$ if $T(s)$ is lossy $\{x, y, z\}$-dissipative.

(ii) The following condition holds

$$\forall \omega \in \mathbb{R}, \quad H(T(j\omega))^* X H(T(j\omega)) \geq 0$$

Similar lemma holds by substituting the sign $\geq$ by the sign $>$.

Proof. Define $G(T(j\omega)) := (I - AT(j\omega))^{-1}B$. Right- and left- multiplying (9) by $\begin{bmatrix} T(j\omega)G(T(j\omega)) \\ I \end{bmatrix}$ and its transpose-conjugate, and using the identity:

$$\begin{bmatrix} A & B \\ I & 0 \end{bmatrix} \begin{bmatrix} T(j\omega)G(T(j\omega)) \\ I \end{bmatrix} = \begin{bmatrix} I \\ T(j\omega) \cdot I \end{bmatrix} G(T(j\omega))$$

it comes that:

$$-E_{\boldsymbol{P}}(T(j\omega)) + H(T(j\omega))^* X H(T(j\omega)) \geq 0$$

where $E_{\boldsymbol{P}}(T(j\omega))$ is given by

$$\begin{aligned} &E_{\boldsymbol{P}}(T(j\omega)) = \\ &G(T(j\omega))^* \begin{bmatrix} T(j\omega) \cdot I \\ I \end{bmatrix}^* \begin{bmatrix} x\boldsymbol{P} & y\boldsymbol{P} \\ y\boldsymbol{P} & z\boldsymbol{P} \end{bmatrix} \begin{bmatrix} T(j\omega) \cdot I \\ I \end{bmatrix} G(T(j\omega)) \\ &= G(T(j\omega))^* \begin{bmatrix} T(j\omega) \\ 1 \end{bmatrix}^* \begin{bmatrix} x & y \\ y & z \end{bmatrix} \begin{bmatrix} T(j\omega) \\ 1 \end{bmatrix} \boldsymbol{P} G(T(j\omega)) \end{aligned}$$

Then $\forall j\omega \in j\mathbb{R} \setminus p_{j\mathbb{R}}$, $E_{\boldsymbol{P}}(T(j\omega)) \geq 0$ by (5). Moreover, as $G(T(s))$ is stable by stability of $H(T(s))$, it comes by analytic continuation that $E_{\boldsymbol{P}}(T(j\omega)) \geq 0$ also holds for all $j\omega \in p_{j\mathbb{R}}$. Thus condition (ii) is satisfied for all $\omega \in \mathbb{R}$. □

III. Problem Setting

This paper focuses on the synthesis of the interconnection of n homogeneous LTI systems for frequency filtering. Each system is modeled by the same transfer function $T(s)$, which is $\{x, y, z\}$-dissipative. The interconnection is represented by an LFT in $T(s)$. The magnitude of the interconnected systems should satisfy a continuous spectral mask, that consists of constant upper and lower bounds defined on frequency intervals.

A. Problem formulation

Problem 1.
GIVEN n homogeneous systems with transfer function $T(s)$,
GIVEN an $\{x, y, z\}$-dissipative characterization of $T(s)$,
GIVEN n_U upper bounds U_u associated with n_U intervals Ω_u^U,
GIVEN n_L lower bounds L_l associated with n_L intervals Ω_l^L,
FIND if there exists a stable interconnection $\boldsymbol{W}(T(s))$ of n homogeneous systems $T(s)$, which satisfies the spectral mask:

$$\forall u \in (1, \dots, n_U), \quad \forall \omega \in \Omega_u^U, \quad |\boldsymbol{W}(T(j\omega))|^2 \leq U_u^2 \tag{10}$$

$$\forall l \in (1, \dots, n_L), \quad \forall \omega \in \Omega_l^L, \quad |\boldsymbol{W}(T(j\omega))|^2 \geq L_l^2 \tag{11}$$

IF so, COMPUTE it.

Three main issues appear in directly tackling Problem 1:

1) The decision variable $\boldsymbol{W}(T(s))$ does not belong to a finite dimensional space. In fact, $\boldsymbol{W}(T(s))$ is a rational function in $T(s)$, and the vector space of rational functions is not finite dimensional.
2) Lower bound constraint (11) is non-convex in $\boldsymbol{W}(T(j\omega))$. Indeed, (11) may be, for instance, geometrically interpreted as enforcing $\boldsymbol{W}(T(j\omega))$ to belong to the complement of a disk in the complex plane.
3) The spectral mask (10)-(11) involves the continuous frequency-variable ω, which leads to semi-infinite constraints which cannot be directly verified in finite time.

These issues can be solved for traditional analog and digital filters, i.e. with $T(s) = \frac{1}{s}$ and $T(s) = e^{\tau_s s}$. Issue 1) can be addressed based on the coprime factorization [17, Chap. 5] $\boldsymbol{W}(s) = \boldsymbol{W_N}(s)\boldsymbol{W_D^{-1}}(s)$, where $\boldsymbol{W_N}(s)$ and $\boldsymbol{W_D}(s)$ are proper rational functions with fixed denominator $d(s)$ of degree n. The resulting synthesis problem is then equivalently reformulated in terms of $\boldsymbol{W_N}(s)$ and $\boldsymbol{W_D}(s)$ which belong to finite dimensional spaces [10], [22]. In order to overcome both issues 2) and 3), the usual synthesis approach is based on two steps [9], [10], [29], [30]. First, a change of decision variables is made: the magnitudes $\boldsymbol{M_N}(j\omega) = |\boldsymbol{W_N}(j\omega)|^2$ and $\boldsymbol{M_D}(j\omega) = |\boldsymbol{W_D}(j\omega)|^2$ are considered instead of $\boldsymbol{W_N}(s)$ and $\boldsymbol{W_D}(s)$. The spectral mask constraints (10)-(11) thus become linear in the new variables $\boldsymbol{M_N}(j\omega)$ and $\boldsymbol{M_D}(j\omega)$. The resulting semi-infinite convex optimization problem is then equivalently reformulated as a finite-dimensional LMI feasibility problem using the frequency-interval KYP lemma [22]. The second step, also known as the spectral factorization technique, consists in computing a stable transfer function $\boldsymbol{W_N}(s)$ and a stable and inversely stable $\boldsymbol{W_D}(s)$, such that the modulus of the frequency responses corresponds to the previously synthesized magnitudes $|\boldsymbol{W_N}(j\omega)|^2 = M_N(j\omega)$ and $|\boldsymbol{W_D}(j\omega)|^2 = M_D(j\omega)$.

A natural strategy to address Problem 1 would be to express $\boldsymbol{W}(T(s))$ as an LFT in $\frac{1}{s}$ and to directly apply the usual synthesis approach. Unfortunately, this would result in structural constraints similar to (4), which usually leads to NP-hard optimization problems [16]. The use of T(s) overcomes this issue by encapsulating these structural constraints, but requires generalizing the synthesis approach, which is the focus of this paper.

The remainder of this section deals with the finite-dimensional parametrization to $\boldsymbol{W}(T(s))$. Then, Section IV extends the two-step approach for lossless dissipative $T(s)$, and Section V deals with the case of lossy dissipative $T(s)$.

B. Finite-dimensional parametrization

Issue 1) is tackled using an equivalent finite-dimensional parametrization of rational functions in $T(s)$. Similarly to the traditional case, $\mathbf{W}(T(s))$ is factorized as follows:

$$\mathbf{W}(T(s)) := \mathbf{W_N}(T(s))\mathbf{W_D^{-1}}(T(s)) \tag{12}$$

where $\mathbf{W_N}$ and $\mathbf{W_D}$ belong to the vector space of proper, real rational functions having d as denominator, and d is a fixed polynomial of degree n chosen such that $1/d(T(s))$ is stable. This space is finite dimensional and so admits a finite basis. Denote $\mathcal{B}$ the column concatenation of the $n+1$ elements of a given basis and define an LFT representation by:

$$\mathcal{B}(T(s)) := (T(s) \cdot I_n) \star \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \tag{13}$$

Without loss of generality, it is assumed that d is chosen such that A satisfies (7) (see for instance Remark 3). Then, $\mathbf{W_N}(T(s))$ and $\mathbf{W_D}(T(s))$ are decomposed such as:

$$\mathbf{W_N}(T(s)) = \mathbf{\Gamma_N^T}\mathcal{B}(T(s)) \qquad \mathbf{W_D}(T(s)) = \mathbf{\Gamma_D^T}\mathcal{B}(T(s))$$

where $\mathbf{\Gamma_N}, \mathbf{\Gamma_D} \in \mathbb{R}^{n+1}$. The LFT representations of $\mathbf{W_N}(T(s))$ and $\mathbf{W_D}(T(s))$ are then given by:

$$\mathbf{W_N}(T(s)) = (T(s) \cdot I_n) \star \left[\begin{array}{c|c} A & B \\ \hline \mathbf{C_N} & \mathbf{D_N} \end{array}\right] \tag{14}$$

$$\mathbf{W_D}(T(s)) = (T(s) \cdot I_n) \star \left[\begin{array}{c|c} A & B \\ \hline \mathbf{C_D} & \mathbf{D_D} \end{array}\right] \tag{15}$$

with $\mathbf{C_N} = \mathbf{\Gamma_N^T}C$, $\mathbf{D_N} = \mathbf{\Gamma_N^T}D$, $\mathbf{C_D} = \mathbf{\Gamma_D^T}C$, $\mathbf{D_D} = \mathbf{\Gamma_D^T}D$.

Therefore, Problem 1 can be equivalently formulated as finding if there exist real matrices $\boldsymbol{C_N}, \boldsymbol{C_D} \in \mathbb{R}^{1\times n}$ and $\boldsymbol{D_N}, \boldsymbol{D_D} \in \mathbb{R}$ such that the interconnection $\boldsymbol{W}(T(s))$, defined by (12)-(15), is stable and satisfies (10)-(11).

C. Reverse parametrization

Reversely, an LFT representation of $\mathbf{W}(T(s))$

$$\boldsymbol{W}(T(s)) := (T(s) \cdot I_n) \star \left[\begin{array}{c|c} \boldsymbol{A_W} & \boldsymbol{B_W} \\ \hline \boldsymbol{C_W} & \boldsymbol{D_W} \end{array}\right] \tag{16}$$

is obtained from (12), (14), (15) as follows [17, Chap. 10]:

$$\left[\begin{array}{c|c} \mathbf{A_W} & \mathbf{B_W} \\ \hline \mathbf{C_W} & \mathbf{D_W} \end{array}\right] = \left[\begin{array}{c|c} A - B\mathbf{D_D^{-1}}\mathbf{C_D} & B\mathbf{D_D^{-1}} \\ \hline \mathbf{C_N} - \mathbf{D_N}\mathbf{D_D^{-1}}\mathbf{C_D} & \mathbf{D_N}\mathbf{D_D^{-1}} \end{array}\right] \tag{17}$$

IV. Lossless-dissipative LFT filter synthesis

In this section, the magnitude synthesis and the spectral factorization steps are independently extended from traditional analog filters, viewed as LFTs in $\frac{1}{s}$, to systems represented by an LFT in $T(s)$, where $T(s)$ is a lossless dissipative transfer function. By bringing together all the extended steps, a procedure is obtained for the synthesis of the interconnection of the n homogeneous systems $T(s)$, based on LMI optimization.

A. Convex formulation via magnitude synthesis

In order to address issue 2), a change of variable is achieved, leading to a convex formulation. Instead of directly synthesizing the transfer functions $\mathbf{W_N}(T(s))$ and $\mathbf{W_D}(T(s))$, the magnitude functions $\mathbf{M_N}(T(j\omega)) := |\mathbf{W_N}(T(j\omega))|^2$ and $\mathbf{M_D}(T(j\omega)) := |\mathbf{W_D}(T(j\omega))|^2$ are taken as decision variables. The associated synthesis problem is then convex, as constraints (10)-(11) become linear in the new decision variables. By defining $\boldsymbol{X_N}, \boldsymbol{X_D} \in \mathbb{S}_{n+1}$ such that $\forall \omega \in \mathbb{R}$,

$$\mathcal{B}(T(j\omega))^* \boldsymbol{X_N} \mathcal{B}(T(j\omega)) = \mathbf{M_N}(T(j\omega)) \tag{18}$$
$$\mathcal{B}(T(j\omega))^* \boldsymbol{X_D} \mathcal{B}(T(j\omega)) = \mathbf{M_D}(T(j\omega)) \tag{19}$$

a lower bound l for instance is now formulated such as:

$$\forall \omega \in \Omega_l^L, \quad \mathcal{B}(T(j\omega))^* \left(\boldsymbol{X_N} - L_l^2 \boldsymbol{X_D}\right) \mathcal{B}(T(j\omega)) \geq 0$$

Moreover, this formulation also enables to obtain a finite-dimensional optimization problem. Indeed, by applying the KYP lemma [21], the frequency-interval generalization [22], [31] or the $\{x, y, z\}$-dissipative version (Lemma 2), a feasibility problem under LMI constraints, independent of the frequency variable ω, is obtained as presented in Theorem 1.

Theorem 1.
Assume $T(s)$ is lossless $\{x, y, z\}$-dissipative. Consider the stable $\mathcal{B}(T(s))$ defined in (13) and denote A_s, B_s, C_s, D_s the matrices of the equivalent LFT representation in $\frac{1}{s}$:

$$\mathcal{B}(T(s)) = \left(\frac{1}{s} \cdot I_{n_s}\right) \star \left[\begin{array}{c|c} A_s & B_s \\ \hline C_s & D_s \end{array}\right] \tag{20}$$

Then $(i) \Rightarrow (ii)$.

(i) $\exists \boldsymbol{X_N}, \boldsymbol{X_D} \in \mathbb{S}_{n+1}$, $\exists \boldsymbol{P_N}, \boldsymbol{P_D} \in \mathbb{S}_n$, $\exists (\boldsymbol{D_u} \in \mathbb{S}_{n_s}^+)_{u=1}^{n_U}$, $\exists (\boldsymbol{P_u} \in \mathbb{S}_{n_s})_{u=1}^{n_U}$, $\exists (\boldsymbol{D_l} \in \mathbb{S}_{n_s}^+)_{l=1}^{n_L}$, $\exists (\boldsymbol{P_l} \in \mathbb{S}_{n_s})_{l=1}^{n_L}$,

$$\mathcal{LM}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \Phi_{xyz}(\boldsymbol{P_N}), \boldsymbol{X_N}\right) \geq 0$$

$$\mathcal{LM}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \Phi_{xyz}(\boldsymbol{P_D}), \boldsymbol{X_D}\right) > 0$$

$$\mathcal{LM}\left(\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix}, \Pi_{\Omega_u^U}(\boldsymbol{D_u}, \boldsymbol{P_u}), U_u^2 \boldsymbol{X_D} - \boldsymbol{X_N}\right) \geq 0 \tag{21}$$

$$\mathcal{LM}\left(\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix}, \Pi_{\Omega_l^L}(\boldsymbol{D_l}, \boldsymbol{P_l}), \boldsymbol{X_N} - L_l^2 \boldsymbol{X_D}\right) \geq 0 \tag{22}$$

where $\Phi_{xyz}(\boldsymbol{P})$ is defined in (8),

$$\Pi_\Omega(\boldsymbol{D}, \boldsymbol{P}) := \begin{bmatrix} -z_\Omega \boldsymbol{D} & -y_\Omega^* \boldsymbol{D} - \boldsymbol{P} \\ -y_\Omega \boldsymbol{D} - \boldsymbol{P} & -x_\Omega \boldsymbol{D} \end{bmatrix}$$

and x_Ω, y_Ω, z_Ω depend on Ω and are defined as follows:

$$\left.\begin{array}{ll} \begin{bmatrix} x_\Omega & y_\Omega \\ y_\Omega^* & z_\Omega \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \text{for } \Omega = \mathbb{R} \\ \begin{bmatrix} x_\Omega & y_\Omega \\ y_\Omega^* & z_\Omega \end{bmatrix} = \begin{bmatrix} \omega_0^2 & 0 \\ 0 & -1 \end{bmatrix} & \text{for } \Omega = [0, \omega_0] \\ \begin{bmatrix} x_\Omega & y_\Omega \\ y_\Omega^* & z_\Omega \end{bmatrix} = \begin{bmatrix} -\omega_1\omega_2 & -j\frac{\omega_1+\omega_2}{2} \\ j\frac{\omega_1+\omega_2}{2} & -1 \end{bmatrix} & \text{for } \Omega = [\omega_1, \omega_2] \\ \begin{bmatrix} x_\Omega & y_\Omega \\ y_\Omega^* & z_\Omega \end{bmatrix} = \begin{bmatrix} -\omega_0^2 & 0 \\ 0 & 1 \end{bmatrix} & \text{for } \Omega = [\omega_0, +\infty) \end{array}\right\}$$

(ii) *The magnitudes $M_N(T(j\omega))$ and $M_D(T(j\omega))$ defined by (18)-(19) are such that*

$$\forall \omega \in \mathbb{R}, \qquad M_N(T(j\omega)) \geq 0$$
$$\forall \omega \in \mathbb{R}, \qquad M_D(T(j\omega)) > 0$$
$$\forall u = 1, \ldots, n_U, \quad \forall \omega \in \Omega_u^U, \qquad \frac{M_N(T(j\omega))}{M_D(T(j\omega))} \leq U_u^2$$
$$\forall l = 1, \ldots, n_L, \quad \forall \omega \in \Omega_l^L, \qquad \frac{M_N(T(j\omega))}{M_D(T(j\omega))} \geq L_l^2$$

Proof. First, applying the $\{x, y, z\}$-dissipative KYP lemma (Lemma 2) to the two first LMIs of condition (i) leads to: $\forall \omega \in \mathbb{R}$, $\mathcal{B}(T(j\omega))^* \boldsymbol{X_N} \mathcal{B}(T(j\omega)) \geq 0$ and $\mathcal{B}(T(j\omega))^* \boldsymbol{X_D} \mathcal{B}(T(j\omega)) > 0$. Then, by the frequency-interval KYP lemma [31] with $\mathcal{B}(T(s))$ given by (20), the LMIs (21) and (22) are equivalent to: $\forall \omega \in \Omega_u^U$, $\mathcal{B}(T(j\omega))^* \left(U_u^2 \boldsymbol{X_D} - \boldsymbol{X_N}\right) \mathcal{B}(T(j\omega)) \geq 0$ and $\forall \omega \in \Omega_l^L$, $\mathcal{B}(T(j\omega))^* \left(\boldsymbol{X_N} - L_l^2 \boldsymbol{X_D}\right) \mathcal{B}(T(j\omega)) \geq 0$, or equivalently $\frac{\mathcal{B}(T(j\omega))^* \boldsymbol{X_N} \mathcal{B}(T(j\omega))}{\mathcal{B}(T(j\omega))^* \boldsymbol{X_D} \mathcal{B}(T(j\omega))} \leq U_u^2$ and $\frac{\mathcal{B}(T(j\omega))^* \boldsymbol{X_N} \mathcal{B}(T(j\omega))}{\mathcal{B}(T(j\omega))^* \boldsymbol{X_D} \mathcal{B}(T(j\omega))} \geq L_l^2$. Condition (ii) is finally obtained using (18)-(19). □

Therefore, the previous change of variable enables to efficiently synthesize the squared magnitude functions $M_N(T(j\omega))$ and $M_D(T(j\omega))$ such that the spectral mask is satisfied. Nonetheless, the reverse change of variables, i.e. the spectral factorization step, needs now to be considered.

Remark 4. Constraints (21)-(22) result from the application of the frequency-interval KYP lemma [31] by representing $\mathcal{B}(T(s))$ as an LFT in $\frac{1}{s}$. Alternatively, an $\{x, y, z\}$-dissipative version is applied in [26], where $\mathcal{B}(T(s))$ is represented as an LFT in $T(s)$. This lowers the number of optimization variables but comes at the prize of a higher conservatism, that would be even more important in the lossy dissipative case.

B. LFT spectral factorization for lossless dissipative T(s)

The LFT spectral factorization step tackles the following problem: from a given squared magnitude function, say for instance $M_N(T(j\omega))$, is there a stable transfer function $\mathbf{W_N}(T(s))$ represented by an LFT in $T(s)$ such that

$$\forall \omega \in \mathbb{R}, \qquad |\mathbf{W_N}(T(j\omega))|^2 = M_N(T(j\omega))$$

And, if so, it is aimed to compute it.

A straightforward necessary condition for the existence of such $\mathbf{W_N}(T(s))$ is $\forall \omega \in \mathbb{R}$, $M_N(T(j\omega)) \geq 0$. When $T(s) = \frac{1}{s}$, it turns out that this condition is also sufficient [32], [33]. The remaining issue is then the computation of the spectral factor $\boldsymbol{W_N}(T(s))$. While it may be analytically computed,

especially for particular analog filters such as Butterworth or Chebyshev [29, Chap. 7], a more efficient computational method is generally based on the resolution of an Algebraic Riccati Equation (ARE) [17, Chap. 13]. However, a direct extension of this method to any lossless dissipative $T(s)$ would lead to solve an $\{x,y,z\}$-dissipative version of the ARE:

$$\mathcal{R}_{xyz}\left(\begin{bmatrix}A & B\\ C & D\end{bmatrix}, \boldsymbol{P_R}, X\right) = 0 \tag{23}$$

for which, to the best of the authors' knowledge, there is no resolution algorithm. Instead, it is demonstrated in Theorem 2 that an adapted extension leads to solve again a usual ARE, based on a congruency relation provided by Lemma 3.

Lemma 3.
Consider real scalars x,y,z given such that $xz-y^2<0$. Denote ϵ the number such as $\epsilon=1$ if $y\geq 0$, $\epsilon=-1$ otherwise. In addition, define $\rho := 2(y^2-xz)(\epsilon y+\sqrt{y^2-xz})$ and

$$\alpha := \frac{x}{\sqrt{\rho}} \qquad \beta := \frac{\epsilon y+\sqrt{y^2-xz}}{\sqrt{\rho}} \qquad \gamma := \frac{z}{\sqrt{\rho}}$$

Then the following matrices are congruent

$$Q(P) := \mathcal{LM}\left(\begin{bmatrix}A & B\\ C & D\end{bmatrix}, \Phi_{xyz}(P), X\right)$$
$$\widehat{Q}(P) := \mathcal{LM}\left(\begin{bmatrix}\widehat{A} & \widehat{B}\\ \widehat{C} & \widehat{D}\end{bmatrix}, \Phi_{010}(P), X\right)$$

where
$$\widehat{A} = (\epsilon\gamma A+\beta I)(\epsilon\beta A+\alpha I)^{-1}$$
$$\widehat{B} = (\epsilon\beta A+\alpha I)^{-1}B$$
$$\widehat{C} = \epsilon(\alpha\gamma-\beta^2)C(\epsilon\beta A+\alpha I)^{-1}$$
$$\widehat{D} = D-\epsilon C(\epsilon\beta A+\alpha I)^{-1}\beta B$$

Proof. See App. I. □

Remark 5. From a System perspective, the congruency relation of Lemma 3 can be interpreted in terms of an internal loop transformation, called loop-shifting [34]. As an insight, if $\alpha\neq 0$, one may verify that the matrices associated with $\widehat{Q}$ may be computed using the Redheffer star product as:

$$\left[\begin{array}{c|c}\widehat{A} & \widehat{B}\\ \hline \widehat{C} & \widehat{D}\end{array}\right] = \left[\begin{array}{c|c}\beta\alpha^{-1}I & \alpha^{-1}I\\ \hline \epsilon(\gamma-\beta^2\alpha^{-1})I & -\epsilon\beta\alpha^{-1}\end{array}\right] \star \left[\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right]$$

The LFT extension of the spectral factorization technique to lossless dissipative $T(s)$ is now stated.

Theorem 2.
Let $T(s)$ be lossless $\{x,y,z\}$-dissipative. Given $X\in\mathbb{S}_{n+1}$ and the stable $\mathcal{B}(T(s))$ defined in (13), define Q and $\widehat{Q}$ as in Lemma 3. Assume the pair $(\widehat{A},\widehat{B})$ is controllable. Moreover, assume the regularity assumption $\widehat{D}^TX\widehat{D}>0$ holds.
Then $(i)\Leftrightarrow(ii)\Rightarrow(iii)$.

(i) There exists a solution $\boldsymbol{P}\in\mathbb{S}_n$ to the LMI $Q(\boldsymbol{P})\geq 0$.
(ii) There exists a solution $\boldsymbol{P_R}\in\mathbb{S}_n$ to the ARE

$$\mathcal{R}_{010}\left(\begin{bmatrix}\widehat{A} & \widehat{B}\\ \widehat{C} & \widehat{D}\end{bmatrix}, \boldsymbol{P_R}, X\right) = 0 \tag{24}$$

(iii) There exists a couple $(\boldsymbol{C_F},\boldsymbol{D_F})\in\mathbb{R}^{1\times n}\times\mathbb{R}$ associated with $\boldsymbol{P_R}$ such that the transfer function $\boldsymbol{F}(T(s))$:

$$\boldsymbol{F}(T(s)) := (T(s)\cdot I_n)\star\left[\begin{array}{c|c}A & B\\ \hline \boldsymbol{C_F} & \boldsymbol{D_F}\end{array}\right]$$

is stable and satisfies

$$\forall\omega\in\mathbb{R},\ |\boldsymbol{F}(T(j\omega))|^2 = \mathcal{B}(T(j\omega))^*X\mathcal{B}(T(j\omega)) \tag{25}$$

Moreover, if there exists $\boldsymbol{P}\in\mathbb{S}_n$ such that $Q(\boldsymbol{P})>0$, the couple $(\boldsymbol{C_F},\boldsymbol{D_F})$ associated with the minimal solution $\boldsymbol{P_R^{min}}$ of the ARE (24) leads $\boldsymbol{F}(T(s))$ to be inversely stable.

Proof. $(i)\Leftrightarrow(ii)$
By Lemma 3, the matrices $Q(\boldsymbol{P})$ and $\widehat{Q}(\boldsymbol{P})$ are congruent. This implies that $Q(\boldsymbol{P})\geq 0\Leftrightarrow\widehat{Q}(\boldsymbol{P})\geq 0$. Then, it is known [24], [25] that, under the regularity and controllability assumptions, the equivalence $(ii)\Leftrightarrow\exists\boldsymbol{P},\ \widehat{Q}(\boldsymbol{P})\geq 0$ holds.
$\Rightarrow(iii)$
By Schur complement lemma [23], $\widehat{Q}(\boldsymbol{P_R})$ is congruent to

$$\begin{bmatrix}\mathcal{R}_{010}\left(\begin{bmatrix}\widehat{A} & \widehat{B}\\ \widehat{C} & \widehat{D}\end{bmatrix}, \boldsymbol{P_R}, X\right) & 0\\ 0 & \widehat{D}^TX\widehat{D}\end{bmatrix}$$

Then, if $\boldsymbol{P_R}$ is solution to the ARE, by successive congruency:

$$\operatorname{rank}\left(Q(\boldsymbol{P_R})\right) = \operatorname{rank}\left(\widehat{Q}(\boldsymbol{P_R})\right) = \operatorname{rank}\left(\widehat{D}^TX\widehat{D}\right) = 1$$

Thus, there exists a couple $(\boldsymbol{C_F},\boldsymbol{D_F})\in\mathbb{R}^{1\times n}\times\mathbb{R}$ such that

$$Q(\boldsymbol{P_R}) = \begin{bmatrix}\boldsymbol{C_F} & \boldsymbol{D_F}\end{bmatrix}^T\begin{bmatrix}\boldsymbol{C_F} & \boldsymbol{D_F}\end{bmatrix} \tag{26}$$

Define $G(T(j\omega)) := (I-AT(j\omega))^{-1}B$, right- and left-multiply (26) by $\begin{bmatrix}T(j\omega)G(T(j\omega))\\ I\end{bmatrix}$ and its transpose-conjugate. It comes then: $\forall\omega\in\mathbb{R}$,

$$-E_{\boldsymbol{P_R}}(T(j\omega)) + \mathcal{B}(T(j\omega))^*X\mathcal{B}(T(j\omega)) = |\boldsymbol{F}(T(j\omega))|^2 \tag{27}$$

where $E_{\boldsymbol{P_R}}(T(j\omega))$ is given by (cf proof of Lemma 2)

$$E_{\boldsymbol{P_R}}(T(j\omega)) =$$
$$G(T(j\omega))^*\begin{bmatrix}T(j\omega)I\\ I\end{bmatrix}^*\begin{bmatrix}x\boldsymbol{P_R} & y\boldsymbol{P_R}\\ y\boldsymbol{P_R} & z\boldsymbol{P_R}\end{bmatrix}\begin{bmatrix}T(j\omega)I\\ I\end{bmatrix}G(T(j\omega))$$
$$= G(T(j\omega))^*\begin{bmatrix}T(j\omega)\\ 1\end{bmatrix}^*\begin{bmatrix}x & y\\ y & z\end{bmatrix}\begin{bmatrix}T(j\omega)\\ 1\end{bmatrix}\boldsymbol{P_R}G(T(j\omega))$$

As $T(s)$ is lossless $\{x,y,z\}$-dissipative, $E_{\boldsymbol{P_R}}(T(j\omega))=0$ and thus (25) holds. Finally, stability of $\boldsymbol{F}(T(s))$ is implied by the stability of $\mathcal{B}(T(s))$.

Moreover, using the inversion formula of LFT, it comes:

$$\boldsymbol{F}^{-1}(T(s)) = (T(s)\cdot I_n)\star\left[\begin{array}{c|c}A-B\boldsymbol{D_F^{-1}}\boldsymbol{C_F} & B\boldsymbol{D_F^{-1}}\\ \hline \boldsymbol{D_F^{-1}}\boldsymbol{C_F} & \boldsymbol{D_F^{-1}}\end{array}\right]$$

Denoting $\boldsymbol{A^{inv}} := A-B\boldsymbol{D_F^{-1}}\boldsymbol{C_F}$, it comes by (26):

$$\boldsymbol{A^{inv}} = A-B(\boldsymbol{D_F^T}\boldsymbol{D_F})^{-1}\boldsymbol{D_F^T}\boldsymbol{C_F} = A-B\boldsymbol{R_{xyz}^{-1}}\boldsymbol{S_{xyz}^T}$$

The condition that there exists $\boldsymbol{P}\in\mathbb{S}_n$ such that $Q(\boldsymbol{P})>0$ implies [25] the existence of a minimal solution $\boldsymbol{P_R^{min}}$ and a maximal solution $\boldsymbol{P_R^{max}}$ of the ARE (24) that are such

that $\boldsymbol{\Delta} = \boldsymbol{P_R^{\max}} - \boldsymbol{P_R^{\min}} > 0$. In addition, by successive congruency, this implies that $\boldsymbol{P_R^{\min}}$ and $\boldsymbol{P_R^{\max}}$ are also solutions of the dissipative version (23) of the ARE. Denote $\boldsymbol{A_{\min}^{inv}}$, $\boldsymbol{R_{\min}}$, $\boldsymbol{S_{\min}}$ the matrices $\boldsymbol{A^{inv}}$, $\boldsymbol{R_{xyz}}$, $\boldsymbol{S_{xyz}}$ associated with $\boldsymbol{P_R^{\min}}$, and $\boldsymbol{R_{\max}}$, $\boldsymbol{S_{\max}}$ the matrices $\boldsymbol{R_{xyz}}$ and $\boldsymbol{S_{xyz}}$ associated with $\boldsymbol{P_R^{\max}}$. Now, it is aimed to prove that:

$$\begin{bmatrix} \boldsymbol{A_{\min}^{inv}} \\ I \end{bmatrix}^T \Phi_{xyz}(\boldsymbol{\Delta}) \begin{bmatrix} \boldsymbol{A_{\min}^{inv}} \\ I \end{bmatrix} > 0 \tag{28}$$

To this end, simple calculation provides the next identities:

$$\begin{bmatrix} A \\ I \end{bmatrix}^T \Phi_{xyz}(\boldsymbol{\Delta}) \begin{bmatrix} A \\ I \end{bmatrix} = \boldsymbol{S_{\max}} \boldsymbol{R_{\max}^{-1}} \boldsymbol{S_{\max}^T} - \boldsymbol{S_{\min}} \boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}^T}$$

$$\begin{aligned} &\begin{bmatrix} -B\boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}^T} \\ 0 \end{bmatrix}^T \Phi_{xyz}(\boldsymbol{\Delta}) \begin{bmatrix} A \\ I \end{bmatrix} \\ &\qquad + \begin{bmatrix} A \\ I \end{bmatrix}^T \Phi_{xyz}(\boldsymbol{\Delta}) \begin{bmatrix} -B\boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}^T} \\ 0 \end{bmatrix} \\ =& 2\boldsymbol{S_{\min}} \boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}^T} - \boldsymbol{S_{\min}} \boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\max}^T} - \boldsymbol{S_{\max}} \boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}^T} \end{aligned}$$

$$\begin{aligned} &\begin{bmatrix} -B\boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}^T} \\ 0 \end{bmatrix}^T \Phi_{xyz}(\boldsymbol{\Delta}) \begin{bmatrix} -B\boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}^T} \\ 0 \end{bmatrix} \\ =& \boldsymbol{S_{\min}} \boldsymbol{R_{\min}^{-1}} \boldsymbol{R_{\max}} \boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}^T} - \boldsymbol{S_{\min}} \boldsymbol{R_{\min}^{-1}} \boldsymbol{R_{\min}} \boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}^T} \end{aligned}$$

Using these identities, (28) is equivalent to

$$\begin{bmatrix} \boldsymbol{S_{\max}} \\ \boldsymbol{S_{\min}} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{R_{\max}^{-1}} & -\boldsymbol{R_{\min}^{-1}} \\ -\boldsymbol{R_{\min}^{-1}} & \boldsymbol{R_{\min}^{-1}} \boldsymbol{R_{\max}} \boldsymbol{R_{\min}^{-1}} \end{bmatrix} \begin{bmatrix} \boldsymbol{S_{\max}} \\ \boldsymbol{S_{\min}} \end{bmatrix} > 0$$

This inequality is also equivalent to

$$\begin{bmatrix} \boldsymbol{R_{\max}^{-1}} \boldsymbol{S_{\max}} \\ \boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{R_{\max}} & -\boldsymbol{R_{\max}} \\ -\boldsymbol{R_{\max}} & \boldsymbol{R_{\max}} \end{bmatrix} \begin{bmatrix} \boldsymbol{R_{\max}^{-1}} \boldsymbol{S_{\max}} \\ \boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}} \end{bmatrix} > 0$$

which is always true by strict positivity of $\boldsymbol{R_{\max}} > 0$ and $\boldsymbol{R_{\max}^{-1}} \boldsymbol{S_{\max}} \neq \boldsymbol{R_{\min}^{-1}} \boldsymbol{S_{\min}}$. Finally, by Lemma 1, $\boldsymbol{F}^{-1}(T(s))$ is stable. □

Remark 6. Compared to [26, Theorem 3], Theorem 2 provides a less conservative condition (i), as $\boldsymbol{P} > 0$ is not required.

Remark 7. If $Q(\mathbf{P}) > 0$ then $\widehat{Q}(\mathbf{P}) > 0$ by congruency, implying that the regularity assumption $\widehat{D}^T X \widehat{D} > 0$ holds.

C. Synthesis procedure

Given a lossless $\{x, y, z\}$-dissipative $T(s)$, the synthesis procedure consists of four steps.

1. Define a stable basis $\mathcal{B}(T(s))$ with a controllable LFT representation (13) such that (7) holds.
2. Compute $\boldsymbol{X_N}$ and $\boldsymbol{X_D}$ by solving feasibility problem (i) of Theorem 1 augmented with the regularity assumption $\widehat{D}^T \boldsymbol{X_N} \widehat{D} > 0$.
3. If there is a solution to step 2., compute $\boldsymbol{P_{N_R}}$ and $\boldsymbol{P_{D_R}^{min}}$ as a solution and the minimal-solution of the AREs:

$$\mathcal{R}_{010}\left(\begin{bmatrix} \widehat{A} & \widehat{B} \\ \widehat{C} & \widehat{D} \end{bmatrix}, \boldsymbol{P_{N_R}}, X_N \right) = 0 \tag{29}$$

$$\mathcal{R}_{010}\left(\begin{bmatrix} \widehat{A} & \widehat{B} \\ \widehat{C} & \widehat{D} \end{bmatrix}, \boldsymbol{P_{D_R}^{min}}, X_D \right) = 0 \tag{30}$$

Then, compute $\boldsymbol{C_N}$, $\boldsymbol{D_N}$, $\boldsymbol{C_D}$, $\boldsymbol{D_D}$ such as

$$\mathcal{LM}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \Phi_{xyz}(P_{N_R}), X_N \right) = \begin{bmatrix} \boldsymbol{C_N^T} \\ \boldsymbol{D_N^T} \end{bmatrix} \begin{bmatrix} \boldsymbol{C_N} & \boldsymbol{D_N} \end{bmatrix}$$

$$\mathcal{LM}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \Phi_{xyz}(P_{D_R}^{\min}), X_D \right) = \begin{bmatrix} \boldsymbol{C_D^T} \\ \boldsymbol{D_D^T} \end{bmatrix} \begin{bmatrix} \boldsymbol{C_D} & \boldsymbol{D_D} \end{bmatrix}$$

By applying Theorem 2, the transfer functions $W_N(T(s))$ and $W_D(T(s))$ defined by (14)-(15) are stable and $W_D(T(s))$ is inversely stable.

4. The transfer function $W(T(s))$, defined by (16) with (17), is stable and satisfies the spectral mask (10)-(11).

V. General LFT filter synthesis

The aim of this section is to address Problem 1 in the case of a general $\{x, y, z\}$-dissipative transfer function $T(s)$. Unfortunately, the usual approach based on two uncoupled steps, the magnitude synthesis and the spectral factorization steps, is not applicable for lossy dissipative characterizations. Indeed, while the magnitude synthesis step can be straightly extended, the spectral factorization step induces a factorization error, due to $E_{P_R}(T(j\omega)) \neq 0$ in (27), leading to $W_N(T(s))$ and $W_D(T(s))$ such that:

$$|W_N(T(j\omega))|^2 = -E_{P_{N_R}}(T(j\omega)) + \mathcal{B}(T(j\omega))^* X_N \mathcal{B}(T(j\omega))$$
$$|W_D(T(j\omega))|^2 = -E_{P_{D_R}}(T(j\omega)) + \mathcal{B}(T(j\omega))^* X_D \mathcal{B}(T(j\omega))$$

As a result, the synthesized filter $W(T(s))$ would be such that:

$$|W(T(j\omega))|^2 = \frac{|W_N(T(j\omega))|^2}{|W_D(T(j\omega))|^2} \neq \frac{\mathcal{B}(T(j\omega))^* X_N \mathcal{B}(T(j\omega))}{\mathcal{B}(T(j\omega))^* X_D \mathcal{B}(T(j\omega))}$$

and it could not be guaranteed that $W(T(s))$ would satisfy the spectral mask. Instead, it is proposed to include the factorization errors inside the magnitude synthesis. This implies a coupling between the two steps which requires to be solved simultaneously, generalizing then the usual approach.

A. Coupled magnitude synthesis and spectral factorization

In order to include the factorization errors into the magnitude synthesis, the magnitudes $|\boldsymbol{W_N}(T(j\omega))|^2$ and $|\boldsymbol{W_D}(T(j\omega))|^2$ are re-written such as:

$$|\boldsymbol{W_N}(T(j\omega))|^2 = \overline{\mathcal{B}}(T(j\omega))^* \overline{\boldsymbol{X}}_{\boldsymbol{N}} \overline{\mathcal{B}}(T(j\omega))$$
$$|\boldsymbol{W_D}(T(j\omega))|^2 = \overline{\mathcal{B}}(T(j\omega))^* \overline{\boldsymbol{X}}_{\boldsymbol{D}} \overline{\mathcal{B}}(T(j\omega))$$

with

$$\overline{\boldsymbol{X}}_{\boldsymbol{N}} := \begin{bmatrix} -x\boldsymbol{P_{N_R}} & -y\boldsymbol{P_{N_R}} & 0 \\ -y\boldsymbol{P_{N_R}} & -z\boldsymbol{P_{N_R}} & 0 \\ 0 & 0 & \boldsymbol{X_N} \end{bmatrix} \tag{31}$$

$$\overline{\boldsymbol{X}}_{\boldsymbol{D}} := \begin{bmatrix} -x\boldsymbol{P_{D_R}} & -y\boldsymbol{P_{D_R}} & 0 \\ -y\boldsymbol{P_{D_R}} & -z\boldsymbol{P_{D_R}} & 0 \\ 0 & 0 & \boldsymbol{X_D} \end{bmatrix} \tag{32}$$

$$\overline{\mathcal{B}}(T(s)) := \begin{bmatrix} T(s)G(T(s)) \\ G(T(s)) \\ \mathcal{B}(T(s)) \end{bmatrix} = (T(s) \cdot I_n) \star \left[\begin{array}{c|c} A & B \\ \hline \overline{C} & \overline{D} \end{array}\right] \tag{33}$$

where $G(T(s)) = (I - AT(s))^{-1}B$, $\overline{C} := \begin{bmatrix} I & A^T & C^T \end{bmatrix}^T$ and $\overline{D} := \begin{bmatrix} 0 & B^T & D^T \end{bmatrix}^T$.

The aim is then to synthesize $\boldsymbol{X_N}$, $\boldsymbol{X_D}$, $\boldsymbol{P_{N_R}}$ and $\boldsymbol{P_{D_R}}$ such that $\frac{\overline{\mathcal{B}}(T(j\omega))^* \overline{\boldsymbol{X}}_{\boldsymbol{N}} \overline{\mathcal{B}}(T(j\omega))}{\overline{\mathcal{B}}(T(j\omega))^* \overline{\boldsymbol{X}}_{\boldsymbol{D}} \overline{\mathcal{B}}(T(j\omega))}$ satisfies the spectral mask while $\boldsymbol{P_{N_R}}$, $\boldsymbol{P_{D_R}}$ being solutions of the ARE. Theorem 3 provides a sufficient condition to achieve this.

Theorem 3.
Consider the stable $\mathcal{B}(T(s))$ defined in (13), denote $\widehat{A}$, $\widehat{B}$, $\widehat{C}$, $\widehat{D}$ as in Lemma 3, and assume the pair $(\widehat{A}, \widehat{B})$ is controllable. Moreover, define $\overline{\mathcal{B}}(T(s))$ as (33) and $\overline{A}_s$, $\overline{B}_s$, $\overline{C}_s$, $\overline{D}_s$ as:

$$\overline{\mathcal{B}}(T(s)) = \left(\frac{1}{s} \cdot I_{n_s}\right) \star \left[\begin{array}{c|c} \overline{A}_s & \overline{B}_s \\ \hline \overline{C}_s & \overline{D}_s \end{array}\right]$$

If the optimization problem (i) is feasible, then problem (ii) admits a solution.

(i) $\exists \boldsymbol{X_N}, \boldsymbol{X_D} \in \mathbb{S}_{n+1}$, $\exists \boldsymbol{P_{N_R}}, \boldsymbol{P_{D_R}} \in \mathbb{S}_n$, $\exists \boldsymbol{P_D^{pos}} \in \mathbb{S}_n^+$, $\exists (\boldsymbol{D_u} \in \mathbb{S}_{n_s}^+)_{u=1}^{n_U}$, $\exists (\boldsymbol{P_u} \in \mathbb{S}_{n_s})_{u=1}^{n_U}$, $\exists (\boldsymbol{D_l} \in \mathbb{S}_{n_s}^+)_{l=1}^{n_L}$, $\exists (\boldsymbol{P_l} \in \mathbb{S}_{n_s})_{l=1}^{n_L}$,

$$\widehat{D}^T \boldsymbol{X_N} \widehat{D} > 0 \tag{34}$$

$$\mathcal{R}_{010}\left(\begin{bmatrix} \widehat{A} & \widehat{B} \\ \widehat{C} & \widehat{D} \end{bmatrix}, \boldsymbol{P_{N_R}}, \boldsymbol{X_N}\right) = 0 \tag{35}$$

$$\mathcal{R}_{010}\left(\begin{bmatrix} \widehat{A} & \widehat{B} \\ \widehat{C} & \widehat{D} \end{bmatrix}, \boldsymbol{P_{D_R}}, \boldsymbol{X_D}\right) = 0 \tag{36}$$

$$\mathcal{LM}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \Phi_{xyz}(\boldsymbol{P_D^{pos}}), \boldsymbol{X_D}\right) > 0 \tag{37}$$

$$\mathcal{LM}\left(\begin{bmatrix} \overline{A}_s & \overline{B}_s \\ \overline{C}_s & \overline{D}_s \end{bmatrix}, \Pi_{\Omega_u^U}(\boldsymbol{D_u}, \boldsymbol{P_u}), U_u^2 \overline{\boldsymbol{X}}_{\boldsymbol{D}} - \overline{\boldsymbol{X}}_{\boldsymbol{N}}\right) \geq 0 \tag{38}$$

$$\mathcal{LM}\left(\begin{bmatrix} \overline{A}_s & \overline{B}_s \\ \overline{C}_s & \overline{D}_s \end{bmatrix}, \Pi_{\Omega_l^L}(\boldsymbol{D_l}, \boldsymbol{P_l}), \overline{\boldsymbol{X}}_{\boldsymbol{N}} - L_l^2 \overline{\boldsymbol{X}}_{\boldsymbol{D}}\right) \geq 0 \tag{39}$$

where $\overline{\boldsymbol{X}}_{\boldsymbol{N}}$ and $\overline{\boldsymbol{X}}_{\boldsymbol{D}}$ given by (31)-(32), and $\Phi_{xyz}(\boldsymbol{P})$ and $\Pi_\Omega(\boldsymbol{D}, \boldsymbol{P})$ defined in (8) and Theorem 1.

(ii) *There exist stable transfer functions $\boldsymbol{W_N}(T(s))$ and $\boldsymbol{W_D}(T(s))$ defined by (14)-(15) with $\boldsymbol{C_N}, \boldsymbol{C_D} \in \mathbb{R}^{1\times n}$ and $\boldsymbol{D_N}, \boldsymbol{D_D} \in \mathbb{R}$, such that $\boldsymbol{W}(T(s))$, given by (12), satisfies the spectral mask (10)-(11).*
In addition, if $\boldsymbol{P_{D_R}}$ is the minimal solution $\boldsymbol{P_{D_R}^{\min}}$ of the ARE (36), then $\boldsymbol{W}(T(s))$ is stable.

Proof. First, similarly to proof of Theorem 2, (34)-(36) imply that there exist $\boldsymbol{C_N}, \boldsymbol{C_D} \in \mathbb{R}^{1\times n}$, $\boldsymbol{D_N}, \boldsymbol{D_D} \in \mathbb{R}$ such that:

$$Q(\boldsymbol{P_{N_R}}) = \begin{bmatrix} \boldsymbol{C_N} & \boldsymbol{D_N} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{C_N} & \boldsymbol{D_N} \end{bmatrix}$$
$$Q(\boldsymbol{P_{D_R}}) = \begin{bmatrix} \boldsymbol{C_D} & \boldsymbol{D_D} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{C_D} & \boldsymbol{D_D} \end{bmatrix}$$

Moreover, the transfer functions $\boldsymbol{W_N}(T(s))$ and $\boldsymbol{W_D}(T(s))$ defined by (14)-(15) are stable and satisfy: $\forall \omega \in \mathbb{R}$,

$$|\boldsymbol{W_N}(T(j\omega))|^2 = \overline{\mathcal{B}}(T(j\omega))^* \overline{\boldsymbol{X}}_{\boldsymbol{N}} \overline{\mathcal{B}}(T(j\omega))$$
$$|\boldsymbol{W_D}(T(j\omega))|^2 = \overline{\mathcal{B}}(T(j\omega))^* \overline{\boldsymbol{X}}_{\boldsymbol{D}} \overline{\mathcal{B}}(T(j\omega))$$

In addition, by the last part of the proof of Theorem 2, (37) implies that the minimal solution $\boldsymbol{P_{D_R}^{\min}}$ of (36) leads $\boldsymbol{W_D}(T(s))$ to be inversely stable and so $\boldsymbol{W}(T(s))$ is stable.

Finally, by the frequency-interval version of the KYP Lemma [31], (38)-(39) are equivalent to:

$$\forall \omega \in \Omega_u^U, \quad \frac{\overline{\mathcal{B}}(T(j\omega))^* \overline{\boldsymbol{X}}_{\boldsymbol{N}} \overline{\mathcal{B}}(T(j\omega))}{\overline{\mathcal{B}}(T(j\omega))^* \overline{\boldsymbol{X}}_{\boldsymbol{D}} \overline{\mathcal{B}}(T(j\omega))} \leq U_u^2$$
$$\forall \omega \in \Omega_l^L, \quad \frac{\overline{\mathcal{B}}(T(j\omega))^* \overline{\boldsymbol{X}}_{\boldsymbol{N}} \overline{\mathcal{B}}(T(j\omega))}{\overline{\mathcal{B}}(T(j\omega))^* \overline{\boldsymbol{X}}_{\boldsymbol{D}} \overline{\mathcal{B}}(T(j\omega))} \geq L_l^2$$

□

Remark 8. The purpose of (37) is to ensure that the minimal solution of (36) is stabilizing, i.e. $\boldsymbol{W}(T(s))$ is stable.

B. Heuristic resolution of coupled AREs using LMI optimization

The coupling of the magnitude synthesis and the spectral factorization steps leads to solve the feasibility problem of condition (i) of Theorem 3. While avoiding a factorization error, this requires to solve the LMI feasibility problem of the magnitude synthesis augmented with the AREs (35)-(36), leading to a non-convex formulation.

To overcome this issue, an heuristic is proposed here in order to obtain a linear minimization problem under LMI constraints. Indeed, the minimal solution of the ARE (24) can be computed [35] by minimizing the trace of $\boldsymbol{P_R}$ subject to the LMI $\widehat{Q}(\boldsymbol{P_R}) \geq 0$. Notice that the LMI $Q(\boldsymbol{P_R}) \geq 0$ may be rather used, as both constraints are equivalent by Lemma 3. Based on a similar idea, it is then proposed to substitute both AREs (35)-(36) by a linear minimization problem over two LMI constraints. This leads to solve the following problem:

$$\begin{cases} \min\limits_{\substack{\boldsymbol{X_N},\ \boldsymbol{X_D} \in \mathbb{S}_{n+1}, \\ \boldsymbol{P_{N_R}},\ \boldsymbol{P_{D_R}} \in \mathbb{S}_n,\ \boldsymbol{P_D^{pos}} \in \mathbb{S}_n^+, \\ \boldsymbol{D_u},\ \boldsymbol{D_l} \in \mathbb{S}_n^+,\ \boldsymbol{P_u},\ \boldsymbol{P_l} \in \mathbb{S}_n}} & \operatorname{trace}(\boldsymbol{P_{N_R}}) + \operatorname{trace}(\boldsymbol{P_{D_R}}) \\ \text{such that} & (34), (37) - (39), (40) - (41) \end{cases}$$

where

$$\mathcal{LM}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \Phi_{xyz}(\boldsymbol{P_{N_R}}), \boldsymbol{X_N}\right) \geq 0 \tag{40}$$

$$\mathcal{LM}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \Phi_{xyz}(\boldsymbol{P_{D_R}}), \boldsymbol{X_D}\right) \geq 0 \tag{41}$$

If this optimization problem converges to a solution and if additionally the obtained solutions $P_{N_R}^\diamond$ and $P_{D_R}^\diamond$ are respectively solution of (35) and minimal solution of (36), then Problem 1 admits a stable solution $\boldsymbol{W}(T(s))$.

Remark 9. If the following constraint is added: $\exists \boldsymbol{P_N^{pos}} \in \mathbb{S}_n^+$,

$$\mathcal{LM}\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \Phi_{xyz}(\boldsymbol{P_N^{pos}}), \boldsymbol{X_N}\right) > 0$$

and $P_{N_R}^\diamond$ is a minimal solution of (35), then the computed $\boldsymbol{W}(T(s))$ will be inversely stable.

VI. Applications

The two approaches developed in this paper are now illustrated on two applications: the design of LC-ladder bandpass-filters, involving a lossless $\{0, 1, 0\}$-dissipative $T(s)$, and the $\mathcal{H}_\infty$-weights synthesis, with a lossy $\{x, y, z\}$-dissipative $T(s)$. The computations are made on MATLAB, using the Robust Control toolbox for solving LMI optimization problems.

A. Design of LC-ladder bandpass-filters

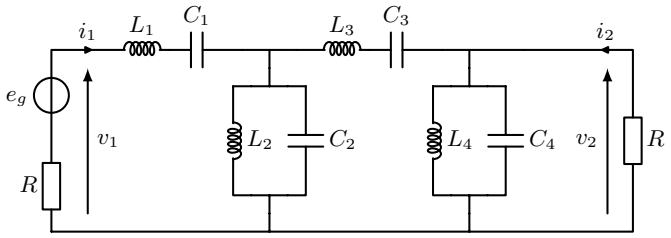

Fig. 2. A four-elements, $\boldsymbol{LC}$-ladder bandpass filter

In this example, it is aimed to design an LC-ladder bandpass-filter [36]. These filters are typically implemented as the cascade of serial and parallel LC-elements (Fig. 2). The frequency requirements are set on the scattering function s_{21} that can be defined here as the transfer from the incident signal $\alpha_1 := \frac{v_1+Ri_1}{2\sqrt{R}}$ to the reflected signal $\beta_2 := \frac{v_2-Ri_2}{2\sqrt{R}}$: $\beta_2(s) = s_{21}(s)\alpha_1(s)$. The design problem is to find if there exist positive element values $\{L_i > 0,\ C_i > 0\}_{i=1}^n$ such that the resulting s_{21} satisfies the following spectral mask:

$$\forall \omega \in [0, \omega_{U_1}], \qquad |s_{21}(j\omega)|^2 \leq U_1^2 \tag{42}$$

$$\forall \omega \in [\omega_{L_1}, \omega_{L_2}], \qquad |s_{21}(j\omega)|^2 \geq L^2 \tag{43}$$

$$\forall \omega \in [\omega_{U_2}, +\infty), \qquad |s_{21}(j\omega)|^2 \leq U_2^2 \tag{44}$$

with $\omega_{U_1} := 1.6\pi$ rad/s, $U_1|_{dB} := -20$, $\omega_{L_1} := 2\pi$ rad/s, $\omega_{L_2} := 3\pi$ rad/s, $L|_{dB} := -0.28$ and $\omega_{U_2} := 4.4\pi$ rad/s, $U_2|_{dB} := -12$, using the notation $K|_{dB} := 20\log_{10}(K)$.

In order to simplify the design problem, the following common assumption [36] is made:

$$\forall i = 1, \ldots, n, \quad \frac{1}{L_i C_i} = \omega_0^2 \qquad \text{with} \quad \omega_0 = \sqrt{\omega_{L_1}\omega_{L_2}}$$

In particular, this implies that the admittance Y_{s_j} of each serial element j and the impedance Z_{p_k} of each parallel element k are given by $Y_{s_j}(s) = \frac{1}{L_j}T(s)$ and $Z_{p_k}(s) = \frac{1}{C_k}T(s)$, where

$$T(s) := \frac{s}{s^2 + \omega_0^2}$$

For simplicity, it is assumed in the sequel that $R = 1\Omega$.

1) Formulation as a lossless-dissipative LFT filter synthesis problem:

First, notice that $T(s)$ is lossless $\{0,1,0\}$-dissipative. Then, it was demonstrated in [37] (see also [15, Chap. 4]) that a sufficient condition for a transfer function $W(T(s))$ to be the scattering function s_{21} of a bandpass LC-ladder filter is that $W(T(s))$ is stable and is such that:

$$\forall \omega \in \mathbb{R}, \qquad |W(T(j\omega))|^2 \leq 1 \tag{45}$$

$$|W(T(j\omega))|^2 = \frac{|T(j\omega)|^{2n}}{d_0 + d_2|T(j\omega)|^2 + \ldots + d_{2n}|T(j\omega)|^{2n}} \tag{46}$$

$$d_{2n} = 1 \tag{47}$$

The design problem can then be recast as the synthesis of a stable interconnection $W(T(s))$ of homogeneous lossless $\{0,1,0\}$-dissipative systems $T(s)$ such that the spectral mask (42)-(45) is satisfied and the structural constraints (46)-(47) hold. This is the lossless dissipative version of Problem 1 augmented with two structural constraints. Fortunately, these constraints can be straightforwardly included into the approach developed in Section IV, as illustrated in the sequel.

2) Adapted synthesis procedure:

The following adapted procedure of subsection IV-C is applied. First, the following stable basis $\mathcal{B}(T(s))$ is chosen:

$$\mathcal{B}(T(s)) = \frac{1}{(1+T(s))^n}\begin{bmatrix} 1 \\ T(s) \\ \vdots \\ T(s)^n \end{bmatrix} \tag{48}$$

with an LFT realization (13) given by:

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] := \left[\begin{array}{cccc|c} 0 & 1 & & & 0 \\ & \ddots & \ddots & & \vdots \\ & & 0 & 1 & 0 \\ -a_0 & \ldots & -a_{n-2} & -a_{n-1} & 1 \\ \hline -a_0 & \ldots & -a_{n-2} & -a_{n-1} & 1 \\ & & & 1 & 0 \\ & & 1 & & 0 \\ & \iddots & & & \vdots \\ 1 & & & & 0 \end{array}\right]$$

where A satisfies (7) with $a_0, \ldots, a_{n-1}$ such that

$$a_0 + a_1 \cdot T(s) + \ldots + a_{n-1} \cdot T(s)^{n-1} + T(s)^n = (1+T(s))^n$$

Second, feasibility problem (i) of Theorem 1 is solved with the additional linear constraints $\boldsymbol{X_N} = \begin{bmatrix} 0_{n\times n} & 0_{n\times 1} \\ 0_{1\times n} & 1 \end{bmatrix}$ and

$$\boldsymbol{X_D} = \begin{bmatrix} \boldsymbol{X_{D_{1,1}}} & \cdots & \boldsymbol{X_{D_{1,n}}} & \boldsymbol{X_{D_{1,n+1}}} \\ \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{X_{D_{1,n}}} & \cdots & \boldsymbol{X_{D_{n,n}}} & \boldsymbol{X_{D_{n,n+1}}} \\ \boldsymbol{X_{D_{1,n+1}}} & \cdots & \boldsymbol{X_{D_{n,n+1}}} & 1 \end{bmatrix}$$

Third, $W_N(T(s))$ and $W_D(T(s))$ are factorized such as (14)-(15) where $\begin{bmatrix} C_N & D_N \end{bmatrix} = \Gamma_N^T \begin{bmatrix} C & D \end{bmatrix}$, with $\Gamma_N = \begin{bmatrix} 0_{n\times 1} \\ 1 \end{bmatrix}$, and C_D, D_D associated with the minimum solution of the ARE (30). Finally, the LFT representation of $W(T(s))$ is obtained using (17).

3) Result and comparison:

By applying the previous procedure, a solution is obtained for $n = 4$ given by the LFT representation matrices $\left[\begin{array}{c|c} A_W & B_W \\ \hline C_W & D_W \end{array}\right] =$

$$\left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -83.17 & -66.727 & -26.65 & -6.082 & -83.17 \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}\right]$$

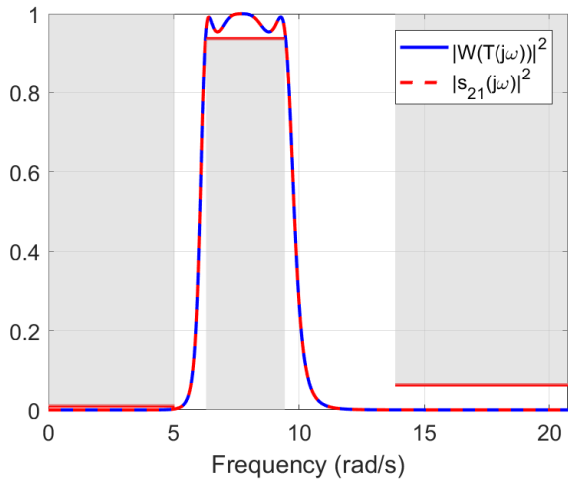

Fig. 3. Plot of the squared modulus $|\boldsymbol{W}(\boldsymbol{T}(\boldsymbol{j\omega})|^2$ of the synthesized LFT filter and of $|\boldsymbol{s_{21}}(\boldsymbol{j\omega})|^2$ of the resulting $\boldsymbol{LC}$-ladder bandpass filter.

leading to

$$W(T(s)) = \frac{b_4 T(s)^4}{1 + a_1 T(s) + a_2 T(s)^2 + a_3 T(s)^3 + a_4 T(s)^4}$$

where $a_1 = 6.082$, $a_2 = 26.65$, $a_3 = 66.73$, $a_4 = 83.17$ and $b_4 = -a_4$. It can be observed in Fig. 3 that the resulting $W(T(s))$ satisfies the spectral mask. Moreover, the element values of the LC-ladder bandpass filter are extracted (see Table I), using the procedure provided in [15, Chap. 6]. It can be verified in Fig. 3 that the computed scattering parameter s_{21} is such that $|s_{21}(j\omega)|^2 = |W(T(j\omega))|^2$.

TABLE I
ELEMENT VALUES OF THE ASSOCIATED $\boldsymbol{LC}$-LADDER BANDPASS FILTER.

Element	1	2	3	4
L_i (mH)	44.27	460.4	35.64	289.0
C_i (mF)	381.5	36.68	473.9	58.43

In order to get insight on the interest of the approach on this example, the standard Butterworth and Chebyshev filters, based on low-pass prototypes and frequency and element transformations [36], are synthesized using MATLAB. As a result, both filters lead to five-elements LC-ladder filters. Notice that, while a Chebyshev prototype filter of order 4 may be first found, an even-order Chebyshev filter can not be implemented with an LC-ladder structure as it does not fulfill the realization constraint (47). One has then to increment the order to 5 (see [15, Chap. 6]). Therefore, both filters need an additional element compared to the previously synthesized filter. This may be critical for nowadays application where frequency filters have stringent requirements (performance, power consumption, cost,...).

B. Weights synthesis for $\mathcal{H}_\infty$-control

In this example, it is aimed to synthesize a frequency LFT filter in the perspective of the weighted $\mathcal{H}_\infty$-control of interconnected homogeneous systems [4], [8]. For instance, the synthesis of a transfer function as an LFT in $T(s)$, instead of a usual state-space representation, is exploited in [4] in order to transform the global performance objective of the interconnection into a local performance constraint for each system. In particular, this enables to solve the design problem of decentralized $\mathcal{H}_\infty$-control with a dramatic complexity reduction. However, without a computational method such as proposed in the present paper, the weights are calculated by hand through a trial-and-error approach.

The illustrative problem considered here is to find $W(T(s))$ represented by (16) as an LFT in

$$T(s) = \frac{1}{(s+1)(s+2)}$$

such that $W(T(s))$ is stable and satisfies the spectral mask:

$$\begin{aligned} \forall \omega \in [0, \omega_L], \quad & |W(T(j\omega))|^2 \geq L^2 \\ \forall \omega \in [\omega_U, +\infty), \quad & |W(T(j\omega))|^2 \leq U^2 \end{aligned}$$

with $\omega_L = 1$ rad/s, $L|_{dB} = 10$, $\omega_U = 10$ rad/s, $U|_{dB} = -10$.

The approach developed in Section V is applied. First, computing the dissipative disk with minimal radius, a lossy dissipative characterization of $T(s)$ is obtained with $\{x, y, z\} = \{-1, 0.193, 0.0624\}$. Then, the optimization problem of Section V-B is solved, where $\mathcal{B}(T(s))$ is again defined as in (48). A solution is found for $n = 2$ and the resulting matrices are given by:

$$X_N = \begin{bmatrix} -0.2569 & -0.6584 & -0.2867 \\ -0.6584 & 9.9577 & -0.0184 \\ -0.2867 & -0.0184 & 9.9920 \end{bmatrix}$$

$$X_D = \begin{bmatrix} 0.0322 & -0.1308 & 0.4477 \\ -0.1308 & 1.1230 & -3.0811 \\ 0.4477 & -3.0811 & 8.9062 \end{bmatrix}$$

$$P_{N_R} = \begin{bmatrix} -9.9741 & -1.5555 \\ -1.5555 & -4.2802 \end{bmatrix} \quad P_{D_R} = \begin{bmatrix} -8.6052 & 1.4135 \\ 1.4135 & -1.1142 \end{bmatrix}$$

These solutions satisfy the dissipative AREs adapted from (23), from which P_{D_R} is the minimal solution. Finally, computing the spectral factors $W_N(T(s))$ and $W_D(T(s))$, it comes by Theorem 3, that $W(T(s)) = W_N(T(s))W_D^{-1}(T(s))$ given by

$$W(T(s)) = (T(s) \cdot I_2) \star \left[\begin{array}{cc|c} 0 & 1.00 & 0 \\ -1.7204 & 0.0386 & -3.1359 \\ \hline 0.0397 & -2.6307 & 0.3161 \end{array}\right]$$

is stable and satisfies the spectral mask, as illustrated in Fig. 4. Furthermore, it can be observed in Fig. 4 that: $\forall \omega \in \mathbb{R}$,

$$|W(T(j\omega))|^2 = \frac{\overline{\mathcal{B}}(T(j\omega))^* \overline{X}_N \overline{\mathcal{B}}(T(j\omega))}{\overline{\mathcal{B}}(T(j\omega))^* \overline{X}_D \overline{\mathcal{B}}(T(j\omega))}$$

Finally, in order to get insight on the relevance of including the spectral factorization errors, an adapted version of the usual two-step approach, with uncoupled magnitude synthesis and spectral factorization steps, is applied. The resulting magnitude $M_{wem}(T(j\omega)) = \frac{\mathcal{B}(T(j\omega))^* X_N \mathcal{B}(T(j\omega))}{\mathcal{B}(T(j\omega))^* X_D \mathcal{B}(T(j\omega))}$ and squared magnitude of the computed factor $W_{wem}(T(s))$ are plotted in Fig. 4. As expected, while $M_{wem}(T(j\omega))$ satisfies the spectral mask, the factorization errors introduce such a deviation that $W_{wem}(T(s))$ does not satisfy the spectral mask.

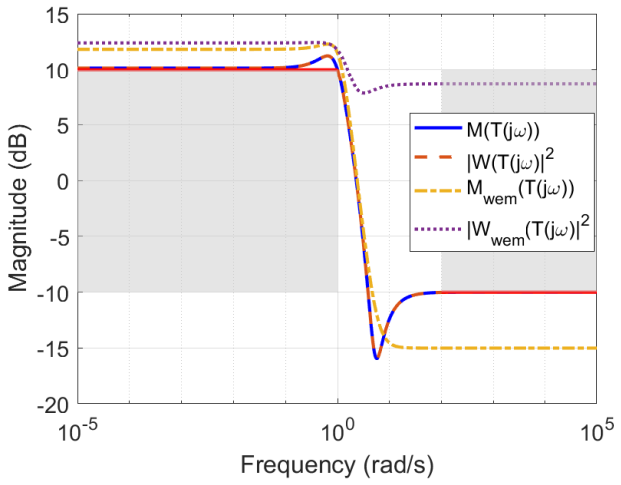

Fig. 4. Plot of the result obtained with error management $\boldsymbol{M(T(j\omega))}$, $\boldsymbol{|W(T(j\omega))|^2}$ and without error management $\boldsymbol{M_{wem}(T(j\omega))}$, $\boldsymbol{|W_{wem}(T(j\omega))|^2}$.

VII. Conclusions and Perspectives

In this paper, the filter design problem of synthesizing the interconnection of homogeneous LTI systems such that the global frequency-response satisfies magnitude constraints was investigated. Using the LFT framework and a dissipative characterization, the synthesis approach of traditional frequency filters, based on the successive magnitude synthesis and the spectral factorization steps, was revisited and extended. It was especially demonstrated that this usual two-step approach can be extended to the case where the interconnected systems are modeled by a lossless dissipative transfer function $T(s)$. Furthermore, it was revealed that the two-step cannot be extended separately for a $T(s)$ characterized by a general dissipative constraint, as a factorization error appears. Therefore, a generalized approach that coupled both steps was developed. An heuristic method was then proposed in order to get an LMI optimization problem. Last but not least, both developed approaches were applied on two appliccations. In particular, the second application highlighted the needs of taking into account the factorization error.

Further work especially includes the search of convergence conditions for the proposed heuristic, and the extension of traditional System problems related to the spectral factorization technique to interconnected homogeneous systems.

Appendix I
Proof of Lemma 3

Proof. Assume temporarily that $(\epsilon\beta A + \alpha I)^{-1}$ exists. The proof is based on congruency relations. First, it can be verified:

$$\begin{bmatrix} z(-P) & y(-P) \\ y(-P) & x(-P) \end{bmatrix} = L^T \begin{bmatrix} 0 & -P \\ -P & 0 \end{bmatrix} L$$

with $L := \frac{1}{\epsilon(\alpha\gamma-\beta^2)} \begin{bmatrix} \epsilon\gamma I & \beta I \\ \epsilon\beta I & \alpha I \end{bmatrix}$. Then, by defining

$$V := \begin{bmatrix} \epsilon(\alpha\gamma - \beta^2)(\epsilon\beta A + \alpha I)^{-1} & -\epsilon(\epsilon\beta A + \alpha I)^{-1}\beta B \\ 0 & I \end{bmatrix}$$

it comes that

$$L \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} V = \begin{bmatrix} \widehat{A} & \widehat{B} \\ I & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} C & D \end{bmatrix} V = \begin{bmatrix} \widehat{C} & \widehat{D} \end{bmatrix}$$

Therefore, the congruency $V^T Q(P) V = \widehat{Q}(P)$ is obtained.

Now, suppose that $(\epsilon\beta A + \alpha I)$ is singular, i.e. there exists a non-zero vector $v \neq 0 \in \mathbb{R}^n$ such that $\epsilon\beta A v = -\alpha v$, or equivalently that $Av = -\frac{\alpha}{\epsilon\beta} v$. This implies that $\lambda := -\frac{\alpha}{\epsilon\beta}$ is an eigenvalue of A associated with the eigenvector v. However, for any positive-definite $P \in \mathbb{S}_n^+$, this leads to:

$$v^T \begin{bmatrix} A \\ I \end{bmatrix}^T \begin{bmatrix} -zP & -yP \\ -yP & -xP \end{bmatrix} \begin{bmatrix} A \\ I \end{bmatrix} v = 0$$

thus contradicting (7), that holds by assumption on $\mathcal{B}(T(s))$, and so $(\epsilon\beta A + \alpha I)$ is non-singular. □

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