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Higgs oscillations in a unitary Fermi superfluid

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Symmetry-breaking phase transitions are central to our understanding of states of matter. When a continuous symmetry is spontaneously broken, new excitations appear that are tied to fluctuations of the order parameter. In superconductors and fermionic superfluids, the phase and amplitude can fluctuate independently, giving rise to two distinct collective branches. However amplitude fluctuations are difficult to both generate and measure, as they do not couple to directly the density of fermions and have only been observed indirectly to date. Here, we excite amplitude oscillations in an atomic Fermi gas with resonant interactions by an interaction quench. Exploiting the sensitivity of Bragg spectroscopy to the amplitude of the order parameter, we measure the time-resolved response of the atom cloud, directly revealing amplitude oscillations at twice the frequency of the gap. The magnitude of the oscillatory response shows a strong temperature dependence, and the oscillations appear to decay faster than predicted by time-dependent BCS theory applied to our experimental setup.

The ability of interacting particles to act collectively underpins many of the remarkable properties of quantum matter. From superfluidity and superconductivity to magnetism and elementary particles, order parameters and their fluctuations govern a wide variety of collective quantum phenomena [1]. Phase transitions characterized by a complex bosonic order parameter are generally accompanied by the emergence of two distinct collective excitations, one corresponding to a (gapless) fluctuation of the phase and the other to a (gapped) fluctuation of the amplitude. The phase or Goldstone mode manifests as a sound wave in neutral systems [2], but is pushed up to the plasma frequency in charged systems [3, 4]. The amplitude mode has featured prominently in particle [5] and condensed matter physics [6], and is commonly known as the Higgs mode. Higgs excitations have been studied in a variety of materials including charge density wave [7–9], BCS [10] and cuprate [11, 12] superconductors as well as superfluid ^3He [13] and antiferromagnetic materials [14]. Ultracold atomic gases may also support stable Higgs modes in certain situations including Bose gases near the superfluid-Mott insulator transition [15, 16], spinor Bose-Einstein condensates (BECs) [17], atoms in optical cavities [18–20], Fermi gases [21], and supersolid phases in dipolar gases [22].

In fermionic condensates, amplitude oscillations arise through excitation of the pairing field – an intrinsically many-body property, giving rise to rich phenomenology [21, 23–25]. BCS superfluids can support a stable collective excitation branch within the pair-breaking continuum, which persists even at strong coupling [26]. In the zero-momentum limit, the spectral weight of this branch vanishes, yet amplitude oscillations still occur due

to the presence of a singularity in the amplitude response function at the threshold energy for pair-breaking excitations [25]. Within a mean-field approximation, the frequency of these “Higgs” oscillations is set at twice the gap in the fermionic excitation spectrum, and the oscillations decay according to a power-law with an exponent that varies with the interactions [23, 24, 27–29]. In the non linear excitation regime, other asymptotic behaviours become possible, including persistent oscillations [30]. In neutral Fermi gases, radio frequency (rf) studies have identified a spectroscopic peak at the pair-breaking continuum [21, 31], and both modulated [32] and quenched [33, 34] interactions have been used to study pair condensation dynamics, but no observation of the characteristic oscillations have been reported.

Here, we directly observe amplitude oscillations in an ultracold atomic Fermi condensate with resonant interactions. We excite the oscillations by a uniform (zero-momentum) quench of the interactions using a magnetic Feshbach resonance. We probe the ensuing out-of-equilibrium dynamics using a high-momentum Bragg pulse, tuned to the recoil energy of superfluid pairs, and hence very sensitive to variations of the order parameter. Our real-time experiment allows us to characterize the frequency, magnitude and decay of the oscillations. Comparing to predictions from time-dependent BCS theory, our experiment confirms the oscillation frequency at twice the value of the gap (2Δ) and shows qualitative agreement on the temperature dependence of the oscillation magnitude, with a reduction as the number of condensed pairs decreases near the critical temperature T_c . The observed oscillations decay faster than predicted by BCS theory, even when experimental effects such as in-

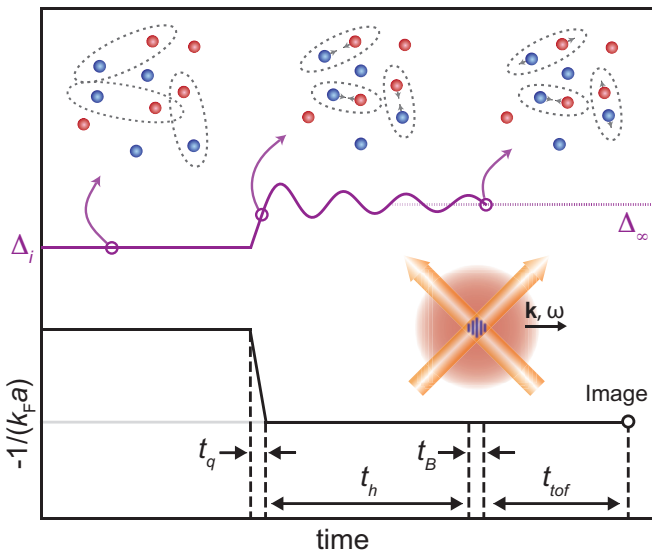


FIG. 1. (Color online) Excitation and detection of amplitude oscillations in a paired Fermi superfluid. Pairs of fermions initially at equilibrium (dashed ellipses, top left) are excited by a sudden variation of the interatomic interactions. This projects the pairs into a superposition of the more tightly bound ground state and the continuum of excited states, beginning at energy ϵ_k . The pairing field thus begins oscillating, triggering Higgs oscillations of the order parameter (purple curve). The continuum edge at $2\Delta \equiv 2\min(\epsilon_k)$ sets the frequency of the oscillations, which attenuate over time due to the spreading of the excited state wave functions and k -dependent frequencies, eventually stabilizing at Δ_∞ . At nonzero temperatures, the superfluid pairs are surrounded by a thermal cloud of unpaired atoms (isolated blue and red dots), reducing the spectral weight of the Higgs mode. To measure the oscillations, we quench the interactions from slightly on the BCS side of a broad Feshbach resonance, to unitarity in a time $t_q = 50 \mu\text{s}$ and hold the cloud for a time t_h before probing the Bragg response from the central region. The optical confinement is then switched off and the atoms allowed to expand for a time t_{tof} before being imaged to determine the momentum imparted by the Bragg pulse [35].

homogeneous broadening are taken into account.

Our experimental protocol is depicted in Fig. 1 [25]. An ultracold gas of fermionic ^6Li atoms is prepared in a balanced mixture of two spin states, initially at thermal equilibrium. Elastic collisions between atoms in these states can be tuned by an external magnetic field through a broad Feshbach resonance [35]. Interactions are characterized by the dimensionless parameter $1/(k_F a)$ where $k_F = (3\pi^2 n)^{1/3}$ is the Fermi wave vector, n is the atomic density and a is the s -wave scattering length. The cloud is initially prepared below T_c , slightly to the BCS side of the Feshbach resonance ($1/(k_F a_i) \approx -0.18 \pm 0.02$). The magnetic field is then ramped to unitarity (where $a \rightarrow \infty$) in a time $t_q = 50 \mu\text{s}$, too fast for the system to follow adiabatically, creating a superposition of the more strongly paired ground state and the continuum of excited states. As this superposition evolves, the pairing

field oscillates at a frequency set by the energy difference between the ground and excited states, leading to Higgs oscillations of the order parameter.

According to Refs.[23, 24, 30] a power-law damping of the oscillations occurs, due to the spread in energy of the lowest lying excited states. In the BCS (weak-coupling) limit, the lowest energy excitations occur at the Fermi surface, $p \approx \hbar k_F$, where the 3D density of excited states is large, and this small spread in energy leads to oscillations decaying slowly, as $t^{-1/2}$ [23]. In the opposite limit of tightly bound molecules, the dispersion minimum occurs at $p = 0$, where the density-of-state vanishes, as for free particles. The evolution of the excited wave function is thus similar to a 3D ballistic expansion and the overlap with the molecular ground state decays as $t^{-3/2}$ [24].

We model this dynamics using time-dependent BCS theory [36]. The initial state of the gas is treated in first approximation as a homogeneous BCS state at nonzero temperature, containing both superfluid pairs and unpaired thermal atoms with a Fermi-Dirac distribution $n_F(\epsilon_{\mathbf{k},i}) = 1/(1 + \exp(\epsilon_{\mathbf{k},i}/k_B T))$, where $\epsilon_{\mathbf{k},i} = \sqrt{(\hbar^2 k^2/2m - \mu_i)^2 + \Delta_i^2}$ is the initial spectrum, Δ_i and μ_i the initial gap and chemical potential, respectively. Following the quench, the initial momentum distribution of the atoms $n_{\mathbf{k}}(t=0) = n_{\mathbf{k},i}$ and pair correlation function $c_{\mathbf{k}}(t=0) = c_{\mathbf{k},i}$ are out-of-equilibrium and evolve according to the time-dependent BCS equations:

$$i\hbar\partial_t n_{\mathbf{k}} = \Delta c_{\mathbf{k}}^* - c_{\mathbf{k}} \Delta^*, \quad (1)$$

$$i\hbar\partial_t c_{\mathbf{k}} = (\hbar^2 k^2/m)c_{\mathbf{k}} + (1 - 2n_{\mathbf{k}})\Delta \quad (2)$$

where a non-linearity is caused by the gap equation $\Delta(t) = g_0 \int d^3 k c_{\mathbf{k}}/(2\pi)^3$ with g_0 the coupling constant of the short-range interactions.

For temperatures well below T_c , our quench is shallow ($|\Delta_i - \Delta(t)| \ll \Delta_i$), and the cloud remains close to equilibrium. In this limit, the dynamical system (1)–(2) can be treated within linear response and the time-evolution of Δ expressed as a Fourier transform of the amplitude-amplitude response function $\chi_{|\Delta||\Delta|}$ [35]:

$$\Delta(t) - \Delta_\infty \propto \int_{2\Delta/\hbar}^{+\infty} \frac{\cos \omega t}{\omega} \chi''_{|\Delta||\Delta|}(\omega) d\omega, \quad (3)$$

where the asymptotic value $\Delta_\infty = \Delta(t \rightarrow +\infty)$ is not necessarily the equilibrium state in this integrable theory. This frequency integral covers the superposition of all excited states with energy $2\epsilon_k$, giving rise to the collective response of $\Delta(t)$. The gapped BCS spectrum sets the lower bound $2\Delta/\hbar$, and the behavior near this pair-breaking threshold governs the long-time behavior of $\Delta(t)$. In the BCS regime ($\mu_i > 0$, which includes unitarity), the amplitude response has a square-root singularity at the continuum edge, $\chi''_{|\Delta||\Delta|} \propto_{\omega \rightarrow 2\Delta/\hbar} 1/\sqrt{\omega - 2\Delta/\hbar}$ leading to power-law damped oscillations of the form

$$\frac{\Delta(t) - \Delta_\infty}{\Delta_i - \Delta_\infty} \underset{t \gg \tau_{\text{TF}}}{=} A_{\text{th}} \frac{\cos(2\Delta t/\hbar + \pi/4)}{(2\Delta t/\hbar)^{\gamma_{\text{th}}}}. \quad (4)$$

We find that the amplitude A_{th} decreases with temperature, whereas the damping exponent $\gamma_{\text{th}} = 1/2$ stays constant. For larger quenches triggering nonlinear dynamics, the oscillatory form (4) can remain valid but the oscillation frequency ω_{H} deviates from $2\Delta/\hbar$ [30, 35].

We probe these dynamics using Bragg spectroscopy. Our experiments use atom clouds confined in an oblate harmonic potential, formed by a combination of optical and magnetic fields [35], leading to a non-uniform density distribution. As a consequence the pairing gap $\Delta(\mathbf{r})$, set by the local Fermi energy, $E_{\text{F}}(\mathbf{r}) = \hbar^2(3\pi^2n(\mathbf{r})^{2/3})/(2m)$, varies with position \mathbf{r} across the cloud. To overcome this, we probe only a small, near-homogeneous volume of the cloud using two-photon Bragg scattering. At the end of the hold time t_h , we send in two tightly-focused Bragg lasers (Fig. 1), that intersect in the centre of the trapped cloud, where the density distribution is most uniform [37, 38]. We define the average density in the Bragg volume $\bar{n} = \int \Omega_{\text{Br}}(\mathbf{r})n(\mathbf{r})d^3\mathbf{r} / \int \Omega_{\text{Br}}(\mathbf{r})d^3\mathbf{r}$, where $\Omega_{\text{Br}}(\mathbf{r})$ is the spatially dependent two-photon Rabi frequency. In the experiments presented here, we find $\bar{n} = (0.955 \pm 0.018)n_0$, where n_0 is the peak density in the trap centre, to be independent of temperature within our experimental resolution [35]. The remaining small inhomogeneities can be accounted for in our theoretical description within the local density approximation [35]. They cause an additional damping of the oscillations, as regions oscillating at different frequencies gradually de-phase.

To resonantly excite pairs with zero center-of-mass momentum, we set the frequency difference between the two lasers to half of the atomic recoil ($\hbar\omega_r/2 = \hbar^2k^2/(4m)$) [38, 39]. Bragg scattered pairs begin moving with a velocity $\hbar\mathbf{k}/(2m)$ where $\mathbf{k} = \mathbf{k}_a - \mathbf{k}_b$ is the difference of the wave-vectors of the two Bragg lasers. We use $k \simeq 4k_{\text{F}}$ to ensure that $\hbar\omega$ is large compared to E_{F} , and the Bragg pulse duration ($t_B = 50 \mu\text{s}$) provides good spectral resolution, while remaining 3 to 4 times shorter than the typical oscillation period ($\tau_{\text{H}} = 2\pi/\omega_{\text{H}}$) so the oscillations remain visible. We estimate that the observed oscillation magnitude is reduced by less than 15% due to this time-averaging [35].

The resulting center of mass displacement $S = \Delta X_{\text{CoM}}$ following time-of-flight expansion, is proportional to the momentum transferred to the atoms by the Bragg lasers [35], hence to the imaginary part of density-density response function $\chi''_{nn}(\omega_r/2, k = 4k_{\text{F}})$ [37]. At large k , χ''_{nn} has a sharp peak at the continuum threshold which coincides approximately with the pair recoil frequency [40, 41]. Both the height and energy of this peak are sensitive to variations in Δ . When $t_B \ll \tau_{\text{H}}$, the Higgs oscillations are approximately stationary during the Bragg pulse and the time-dependent Bragg response can be written as

$$\chi''_{nn}(\omega, k, t) \approx \chi''_{nn}(\omega, k; \Delta_i) + \frac{d\chi''_{nn}}{d\Delta}(\Delta(t) - \Delta_i), \quad (5)$$

Our Bragg frequency $\omega = \omega_r/2$ sits just on the high energy slope of the threshold peak [35], where χ''_{nn} is very

sensitive to variations of Δ . Experimentally, we observe that the Bragg response at $\omega = \omega_r/2$ shows a strong dependence on the condensate fraction, reflecting the temperature dependence of the spectral weight of this threshold peak [35].

Armed with this capability, we use local Bragg scattering as a sensitive, temporally resolved probe for oscillations of the order parameter. Fig. 2 shows examples of the measured Bragg response, as a function of hold time t_h , in units of the local Fermi time $\tau_{\text{F}} = \hbar/E_{\text{F}}$, for a range of temperatures¹ [35]. A damped oscillation is clear in the Bragg response of the colder clouds, giving a direct signature of the Higgs oscillations. The magnitude of the oscillations decreases for warmer clouds, until non-oscillatory behavior is observed for $T \gtrsim 0.15T_{\text{F}}$. Also shown are fits of the data to a function of the form $S(t) = A_{\text{ex}} \cos(\omega_{\text{H}}t + \phi)/t^\gamma + S_\infty$ where A_{ex} , ω_{H} , ϕ , γ and S_∞ are fit parameters that characterize the oscillations.

To compare our experimental measurements to theory, we obtain the asymptotic Bragg response S_∞ ($t \rightarrow \infty$), and the separately measure the responses S_i and S_f at thermal equilibrium with the initial and final scattering lengths. From these we construct the ratio $(S(t) - S_\infty)/(S_f - S_i)$, which we directly compare to the theoretical equivalent $(\Delta(t) - \Delta_\infty)/(\Delta_f - \Delta_i)$. The advantage of comparing these quantities is that they do not depend on the experimental sensitivity $d\chi''_{nn}/d\Delta$ or the offset in the experimental data due to the normal phase response $\chi_{nn}(T > T_c)$, which is not captured in BCS theory. Note the experimental and theoretical temperatures are scaled by the respective critical temperatures of the initial clouds $T_{c,i}$. In Fig. 2(b) we see good agreement in the dynamics at short times and lower temperatures, however at later times, the experimental signal decays faster than theoretically predicted. This is emphasized in Fig. 3(b) which shows the root-mean-square of $(S(t) - S_\infty)/(S_i - S_f)$ in the time interval $3 \leq t/\tau_{\text{H}} \leq 20$ where experiment and theory differ by roughly a factor of two.

From the fits to the experimental data we extract the oscillation frequency ω_{H} and damping exponent γ . Fig. 3(a) shows $\hbar\omega_{\text{H}}/2E_{\text{F}}$ versus temperature, along with a selection of previous measurements and calculations of the pairing gap Δ . Theoretically, we expect $\hbar\omega_{\text{H}}$ to provide a lower bound on 2Δ , and to approach this value at low temperatures when our quench is in the shallow regime. Our measurements lie mostly in the range $0.4 \lesssim \hbar\omega_{\text{H}}/2E_{\text{F}} \lesssim 0.5$, in good agreement with previous studies [31, 37, 43], as well as advanced calculations based on many-body T-matrix methods [44, 45] and quantum Monte-Carlo techniques [46, 47]. Although Δ is expected

¹ Note that the temperature of the cloud was measured after the quench at unitarity. This will therefore include some heating that occurs due to the non-adiabatic experimental quench, which is not accounted for by BCS theory.

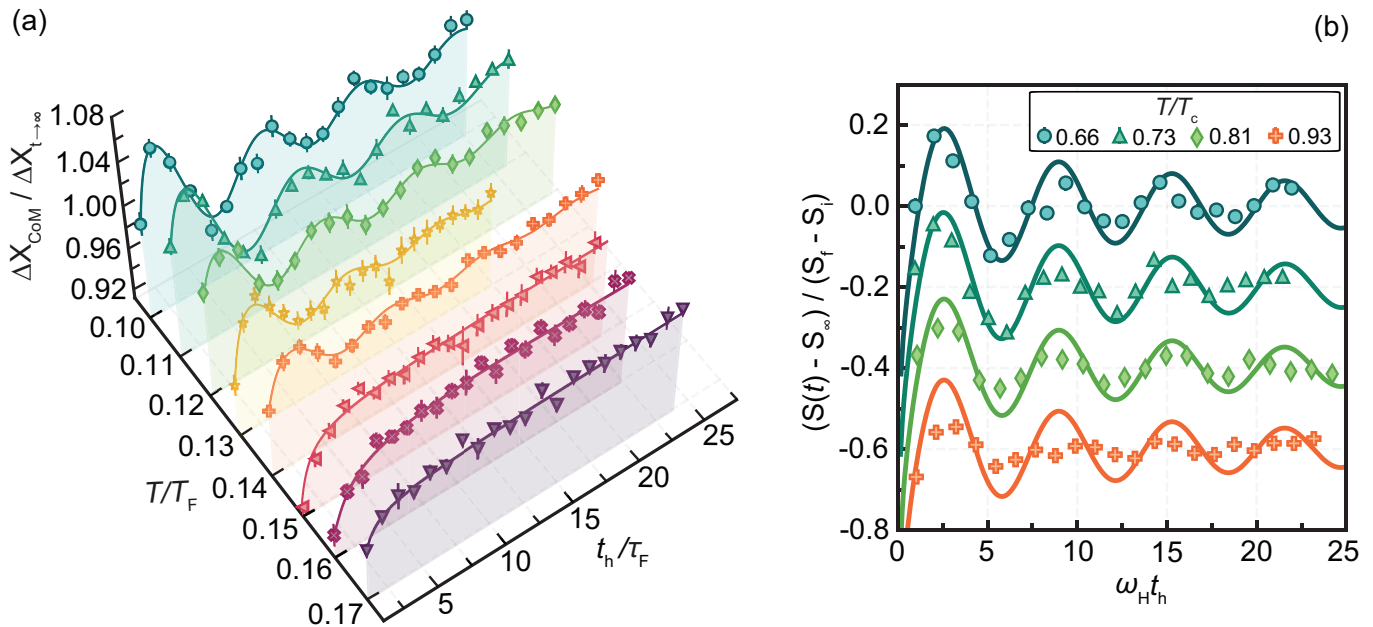


FIG. 2. (Color online) (a) Bragg response (centre of mass displacement S), relative to the asymptotic response S_∞ ($t \rightarrow \infty$), as function of hold time after the quench for a selection of (final) equilibrium cloud temperatures. Points are the experimental measurements and solid lines are fits to the data of a power-law damped sinusoidal function (see text). (b) Comparison with time-dependent BCS theory including experimental effects [35]. The experimental points are shown as a function of $\omega_H t_h$ and T/T_c using the fitted value of $\hbar\omega_H/\epsilon_F$ and the estimated value $T_{c,i}/T_F \simeq 0.15$ [42] at $1/k_F a = -0.18$. The Bragg signal $S(t) - S_\infty$ is scaled to its variation $S_f - S_i$ under an adiabatic sweep of the scattering length, which we measured independently, and the theoretical curves are offset by the delay acquired during the ramp time [35]

to vanish with a critical exponent of $\nu \simeq 0.62$ at T_c [48], we do not observe a noticeable reduction of ω_H in the temperature range we probe.

Fig. 3(c) shows the fitted damping exponents γ which all lie close to unity. While the uncertainties in γ are relatively large, our measurements are not consistent with either the BEC or BCS exponents and display no obvious temperature dependence. The average of our measured damping coefficients is $\bar{\gamma} = 0.98 \pm 0.15$. This is significantly above the theoretical prediction of $\gamma_{\text{th}} = 0.50 \pm 0.02$ [35] where we take into account the inhomogeneous density and the finite experimental time window. These effects lead to compensating shifts on the BCS prediction $\gamma_{\text{th}} = 1/2$, resulting in a correction that is small compared to the difference between BCS and BEC limits. We note that the prediction of power-law damping is based on integrable theories [30] and may be violated at long times in the (ergodic) experimental system. Indeed, fitting an exponentially decaying cosine function to the experimental data gives a statistically indistinguishable quality of fit such that we cannot rule out exponential decay or that γ is affected by other ergodic processes. In the vicinity of T_c , the local density approximation may also break down for describing delocalised pairs. Effects of the inhomogeneity of the cloud may thus become enhanced even in the nearly-uniform

region probed by our Bragg beams.

Fifty years after their prediction [23], we present the direct observation of amplitude oscillations in a weakly-excited Fermi superfluid. Using Bragg spectroscopy we probe the real-time dynamics in a unitary Fermi gas, in qualitative agreement with time-dependent BCS theory, both at low temperatures and near the phase transition. Our work opens a wide avenue of research, with possible direct extensions to the BCS and BEC regimes, different quench regimes [30] or dynamical crossings of the phase transition [34]. Our work also opens pathways to investigate ergodic evolution and the possibility of achieving pre-equilibrated states in strongly interacting quantum matter.

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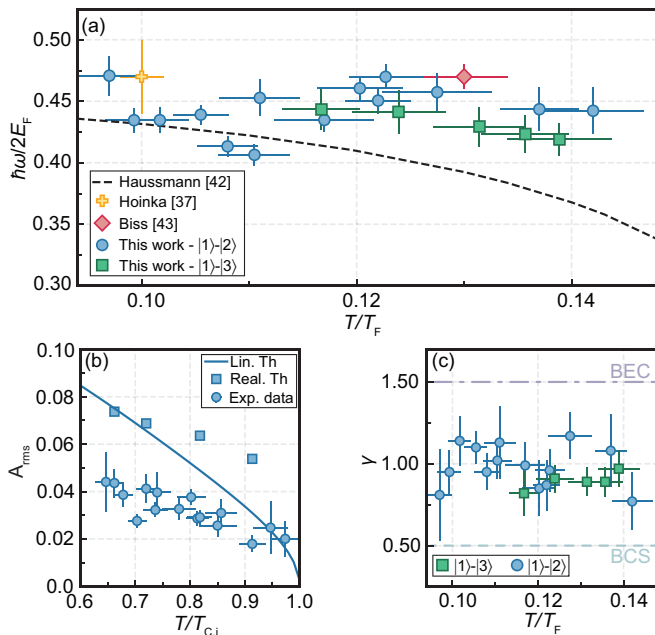


FIG. 3. (Color online) (a) Frequency of the Higgs oscillation versus the normalised temperature T/T_F along with previous measurements and a theoretical calculation (dashed line) of the gap. Blue circles and green squares represent measurements using different combinations of internal states (but for the same interaction quench) [35]. (b) Comparison of the experimental and theoretical averaged magnitudes of the oscillations $A_{\text{rms}} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} dt (S(t) - S_\infty)^2 / (S_f - S_i)^2}$ with $t_1 = 3\tau_H$ and $t_2 = 20\tau_H$. The influence of the choice of t_1 and t_2 is discussed in the Supplementary Material [35]. (c) The fitted damping exponents of the Higgs oscillation lie approximately midway between the BCS and BEC values.

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