

# Supplementary material: Interaction quenches in nonzero temperature fermionic condensates

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## LINEARISATION OF THE HFB EQUATIONS

We give here additional details on our analytical solution of the BCS equations (1)–(2) in the regime of weak perturbations, leading to Eq. (6). We linearize the evolution around the initial equilibrium state, rather than around a virtual final equilibrium state as in Ref. [7]. At nonzero temperature, where  $\Delta(t)$  never reaches  $\Delta_f$ , this is far more intuitive.

We introduce the fluctuations  $\delta c_{\mathbf{k}} = c_{\mathbf{k}} - c_{\mathbf{k},i}$ ,  $\delta n_{\mathbf{k}} = n_{\mathbf{k}} - n_{\mathbf{k},i}$ , and  $\delta\Delta = \Delta - \Delta_i$  and linearize the BCS system (1)–(2):

$$i\partial_t \delta c_{\mathbf{k}} = 2\xi_{\mathbf{k}} \delta c_{\mathbf{k}} - 2\Delta_i \delta n_{\mathbf{k}} + \frac{\xi_{\mathbf{k}}}{\epsilon_{\mathbf{k}}} F(\epsilon_{\mathbf{k}}) \delta\Delta \quad (S1)$$

$$i\partial_t \delta n_{\mathbf{k}} = -\Delta_i (\delta c_{\mathbf{k}} - \delta c_{\mathbf{k}}^*) - \frac{\Delta_i}{2\epsilon_{\mathbf{k}}} F(\epsilon_{\mathbf{k}}) (\delta\Delta - \delta\Delta^*) \quad (S2)$$

Although the quench scenario corresponds to  $\delta n_{\mathbf{k}} = \delta c_{\mathbf{k}} = 0$  at  $t = 0^-$ , we make so far no assumption on the initial state. In this more general case, the fluctuation of  $\Delta$  has a time-dependent part caused by the  $\delta c$ 's, and a constant part  $\delta\Delta_0 = \frac{g_i - g_f}{V} \sum_{\mathbf{k}} c_{\mathbf{k},i}$  caused by the quench on  $g$ :

$$\delta\Delta(t) = \frac{g_i}{V} \sum_{\mathbf{k}} \delta c_{\mathbf{k}}(t) + \delta\Delta_0 \quad (S3)$$

In the spirit of Ref. [? ], we now move to the quasiparticle basis:

$$\alpha_{\mathbf{k}}^+ = \frac{\xi_{\mathbf{k}}}{\epsilon_{\mathbf{k}}} (\delta c_{\mathbf{k}} + \delta c_{\mathbf{k}}^*) - \frac{2\Delta_i}{\epsilon_{\mathbf{k}}} \delta n_{\mathbf{k}} \quad (S4)$$

$$\alpha_{\mathbf{k}}^- = \delta c_{\mathbf{k}} - \delta c_{\mathbf{k}}^* \quad (S4)$$

$$m_{\mathbf{k}} = \frac{2\xi_{\mathbf{k}}}{\epsilon_{\mathbf{k}}} \delta n_{\mathbf{k}} + \frac{\Delta_i}{\epsilon_{\mathbf{k}}} (\delta c_{\mathbf{k}} + \delta c_{\mathbf{k}}^*) \quad (S5)$$

so as to diagonalise the individual parts of Eqs. (??)–(??):

$$i\partial_t \alpha_{\mathbf{k}}^+ = 2\epsilon_{\mathbf{k}} \alpha_{\mathbf{k}}^- + F(\epsilon_{\mathbf{k}}) (\delta\Delta - \delta\Delta^*) \quad (S6)$$

$$i\partial_t \alpha_{\mathbf{k}}^- = 2\epsilon_{\mathbf{k}} \alpha_{\mathbf{k}}^+ + \frac{\xi_{\mathbf{k}}}{\epsilon_{\mathbf{k}}} F(\epsilon_{\mathbf{k}}) (\delta\Delta + \delta\Delta^*) \quad (S7)$$

$$i\partial_t m_{\mathbf{k}} = 0 \quad (S8)$$

In the quasiparticle basis, the fluctuations of  $\Delta$  take the form:

$$\delta\Delta = \delta\Delta_0 + \frac{g_i}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \alpha_{\mathbf{k}}^- + \frac{\xi_{\mathbf{k}}}{\epsilon_{\mathbf{k}}} \alpha_{\mathbf{k}}^+ + \frac{\Delta_i}{\epsilon_{\mathbf{k}}} m_{\mathbf{k}} \right] \quad (S9)$$

To solve the time-dependent system, we introduce the Laplace transform of the variables:

$$A_{\mathbf{k}}^{\pm}(\omega) = \int_{0^-}^{+\infty} e^{i\omega t} \alpha_{\mathbf{k}}^{\pm}(t) dt \quad (S10)$$

and similarly for  $M_{\mathbf{k}}(\omega)$  and  $\delta_{\pm}(\omega)$  the transform of  $m_{\mathbf{k}}(t)$  and  $\delta\Delta(t) \pm \delta\Delta^*(t)$  respectively. This allows us to express the microscopic variables in terms of the fluctuations of  $\Delta$ ,

$$A_{\mathbf{k}}^+(\omega) = \frac{i\omega \alpha_{\mathbf{k}}^+(0^-) + 2i\epsilon_{\mathbf{k}} \alpha_{\mathbf{k}}^-(0^-)}{\omega^2 - 4\epsilon_{\mathbf{k}}^2} + \frac{2\xi_{\mathbf{k}} F(\epsilon_{\mathbf{k}})}{\omega^2 - 4\epsilon_{\mathbf{k}}^2} \delta_+(\omega) + \frac{\omega F(\epsilon_{\mathbf{k}})}{\omega^2 - 4\epsilon_{\mathbf{k}}^2} \delta_-(\omega) \quad (S11)$$

$$A_{\mathbf{k}}^-(\omega) = \frac{i\omega \alpha_{\mathbf{k}}^-(0^-) + 2i\epsilon_{\mathbf{k}} \alpha_{\mathbf{k}}^+(0^-)}{\omega^2 - 4\epsilon_{\mathbf{k}}^2} + \frac{2\epsilon_{\mathbf{k}} F(\epsilon_{\mathbf{k}})}{\omega^2 - 4\epsilon_{\mathbf{k}}^2} \delta_-(\omega) + \frac{\omega \xi_{\mathbf{k}} F(\epsilon_{\mathbf{k}})}{\epsilon_{\mathbf{k}} (\omega^2 - 4\epsilon_{\mathbf{k}}^2)} \delta_+(\omega) \quad (S12)$$

$$M_{\mathbf{k}}(\omega) = \frac{im_{\mathbf{k}}(0^-)}{\omega} \quad (S13)$$

and finally to eliminate them using the resummation

Eq. (??), yielding a closed system of equations on  $\delta_{\pm}$ :

$$M \begin{pmatrix} \delta - \delta^* \\ \delta + \delta^* \end{pmatrix} = -i \begin{pmatrix} S_- \\ S_+ \end{pmatrix} \quad (S14)$$

The fluctuation matrix  $M$  is introduced below equation (7) of the main text, and the sums encoding the initial conditions on  $\Delta$  are given by

$$S_+ = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[ \frac{\omega \xi_{\mathbf{k}} \alpha_{\mathbf{k}}^+(0^-) + 2\xi_{\mathbf{k}} \epsilon_{\mathbf{k}} \alpha_{\mathbf{k}}^-(0^-)}{\epsilon_{\mathbf{k}}(\omega^2 - 4\epsilon_{\mathbf{k}}^2)} + \frac{\Delta_i m_{\mathbf{k}}(0^-)}{\epsilon_{\mathbf{k}} \omega} \right] + \frac{\delta\Delta_0 + \delta\Delta_0^*}{g_i \omega} \quad (\text{S15})$$

$$S_- = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\omega \alpha_{\mathbf{k}}^-(0^-) + 2\epsilon_{\mathbf{k}} \alpha_{\mathbf{k}}^+(0^-)}{\omega^2 - 4\epsilon_{\mathbf{k}}^2} + \frac{\delta\Delta_0 - \delta\Delta_0^*}{g_i \omega} \quad (\text{S16})$$

Inverting Eq. (??) and switching back to the time domain [using the inverse Laplace transformation  $f(t) = -\frac{1}{2\pi} \int_{+\infty+i\eta}^{-\infty+i\eta} dz e^{-izt} F(z)$ ], yields, for the time-evolution of  $\Delta$ :

$$\begin{pmatrix} \delta\Delta(t) - \delta\Delta^*(t) \\ \delta\Delta(t) + \delta\Delta^*(t) \end{pmatrix} = - \int_{+\infty+i\eta}^{-\infty+i\eta} \frac{dz}{2i\pi} e^{-izt} M^{-1} \begin{pmatrix} S_- \\ S_+ \end{pmatrix} \quad (\text{S17})$$

We now input the initial condition corresponding to the interaction quench, that is (as explained above Eq. (3) of the main text)  $\alpha_{\mathbf{k}}^{\pm} = m_{\mathbf{k}} = 0$ , and  $\delta\Delta_0 = \epsilon g_i / \Delta_i$ , which converts into  $S_- = 0$  and  $S_+ = 2\epsilon / \omega \Delta_i$ . Finally, we

derive Eq. (6) of the main text by closing the integration contour in Eq. (??) around the branch cuts of  $M$  (see Fig. 1 in Ref. [7]), and by remarking that the modulus-modulus response function (Eq. (7)) is related to  $M$  by  $f(z) = -(M^{-1})_{22} / \Delta_i$ .

## CALCULATION OF $M$

We numerically evaluate the elements of  $M$  at nonzero temperature using the Kramers-Kronig relation

$$\frac{M_{11}}{\omega_0^2} = \frac{M_{22}}{\omega_0^2 - 4\Delta^2} = \int_{-\infty}^{+\infty} \frac{\rho_f(\omega) d\omega}{\omega_0 - \omega} \quad (\text{S18})$$

$$M_{12} = \int_{-\infty}^{+\infty} \frac{\rho_g(\omega) d\omega}{\omega_0 - \omega} \quad (\text{S19})$$

In dimensionless units ( $\check{\rho} = (2\pi)^3 \Delta \rho / k_{\Delta}^3$ ,  $\check{\omega} = \omega / 2\Delta$ ,  $\check{\beta} = \beta \Delta$ ), we have

$$\check{\rho}_f = \frac{\pi}{2\check{\omega}} \frac{\text{th}(\check{\beta}\check{\omega})}{\sqrt{\check{\omega}^2 - 1}} (k_1(\check{\omega}) + k_2(\check{\omega})) \quad (\text{S20})$$

$$\check{\rho}_g = \frac{\pi}{2} \text{th}(\check{\beta}\check{\omega}) (k_1(\check{\omega}) - k_2(\check{\omega})) \quad (\text{S21})$$

with

$$\begin{cases} k_1(\omega) = \Theta(\check{\omega} - 1) \sqrt{\check{\mu} + \sqrt{\check{\omega}^2 - 1}} \\ k_2(\omega) = \Theta(\check{\omega} - 1) \Theta(\sqrt{1 + \check{\mu}^2} - \check{\omega}) \sqrt{\check{\mu} - \sqrt{\check{\omega}^2 - 1}} \end{cases} \text{ if } \check{\mu} > 0 \quad \begin{cases} k_1(\omega) = \Theta(\check{\omega} - \sqrt{1 + \check{\mu}^2}) \sqrt{\check{\mu} + \sqrt{\check{\omega}^2 - 1}} \\ k_2(\omega) = 0 \end{cases} \text{ if } \check{\mu} < 0 \quad (\text{S22})$$

The divergence of the integrals (??)–(??) in  $\omega = \omega_0$  is easily compensated by adding/subtracting  $\rho(\omega_0)$ . Still, the evaluation can become difficult when  $\check{\omega}_0$  is close to

the angular points 1 or  $\sqrt{1 + \check{\mu}^2}$ . To overcome this, we subtract the nearly diverging behavior of the integrand. (For here on, all quantities are implicitly dimensionless).

- When  $\omega_0 \rightarrow 1^+$ , the integrand nearly diverges around  $\omega \rightarrow 1^+$ . We set  $e = (\omega - 1) / (\omega_0 - 1)$  and derive:

$$\frac{\rho_f(\omega) - \rho_f(\omega_0)}{\omega_0 - \omega} = \frac{\pi \text{th}\beta}{\sqrt{2}} \frac{\sqrt{\mu}}{(\omega_0 - 1)^{3/2}} \frac{1}{\sqrt{e} + e} + O(\omega_0 - 1)^{-1/2} \quad (\text{S23})$$

$$\frac{\rho_g(\omega) - \rho_g(\omega_0)}{\omega_0 - \omega} = O(\omega_0 - 1)^{-1/2} \quad (\text{S24})$$

- When  $\omega_0 \rightarrow \omega_3^{\pm}$ , the integrand nearly diverges around  $\omega \rightarrow \omega_3^{\mp}$ . We set  $e = (\omega - \omega_3) / (\omega_3 - \omega_0)$  and derive:

$$\frac{\rho_f}{\omega_0 - \omega} = \frac{\pi \text{th}(\beta\omega_3)}{\sqrt{2}} \frac{1}{\omega_3 \sqrt{\mu}} \frac{1}{\omega_0 - \omega_3} \frac{1}{1 + e} + O(1) \quad (\text{S25})$$

$$\frac{\rho_g}{\omega_0 - \omega} = \frac{\pi \text{th}(\beta\omega_3)}{\sqrt{2}} \frac{\sqrt{\mu}}{\omega_0 - \omega_3} \frac{1}{1 + e} + O(1) \quad (\text{S26})$$

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