



HAL
open science

Interaction quenches in nonzero temperature fermionic condensates

Hadrien Kurkjian, Victor Colussi, Paul Dyke, Chris Vale, Silvia Musolino

► **To cite this version:**

Hadrien Kurkjian, Victor Colussi, Paul Dyke, Chris Vale, Silvia Musolino. Interaction quenches in nonzero temperature fermionic condensates. *Physical Review A*, 2024, 109 (4), pp.L041302. 10.1103/PhysRevA.109.L041302 . hal-04288654

HAL Id: hal-04288654

<https://hal.science/hal-04288654v1>

Submitted on 16 Nov 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Interaction quenches in nonzero temperature fermionic condensates

H. Kurkjian,¹ V. E. Colussi,^{2,3} P. Dyke,⁴ C. Vale,⁴ and S. Musolino⁵

¹*Laboratoire de Physique Théorique, Université de Toulouse, CNRS, UPS, 31400, Toulouse, France*

²*Inflation, Inc., 3030 Sterling Circle, Boulder, CO 80301, USA*

³*Pitaevskii BEC Center, CNR-INO and Dipartimento di Fisica, Università di Trento, 38123 Trento, Italy*

⁴*Optical Sciences Centre, ARC Centre of Excellence in Future Low-Energy Electronics Technologies, Swinburne University of Technology, Melbourne 3122, Australia*

⁵*Université Côte d'Azur, CNRS, Institut de Physique de Nice, 06200 Nice, France*

We revisit the study of amplitude oscillations in a pair condensate of fermions after an interaction quench, and generalize it to nonzero temperature. For small variations of the order parameter, we show that the energy transfer during the quench determines both the asymptotic pseudo-equilibrated value of the order parameter and the magnitude of the oscillations, after multiplication by, respectively, the static response of the order parameter and spectral weight of the pair-breaking threshold. Since the energy transferred to the condensed pairs decreases with temperature as the superfluid contact, the oscillations eventually disappear at the critical temperature. For deeper quenches, we generalize the regimes of persistent oscillations and monotonic decay to nonzero temperatures, and explain how they become more abrupt and are more easily entered at high temperatures when the ratio of the initial to final gap either diverges, when quenching towards the normal phase, or tends to zero, when quenching towards the superfluid phase. Our results are directly relevant for existing and future experiments on the non-equilibrium evolution of Fermi superfluids near the phase transition.

Introduction.—Fermionic condensates, unlike most of their bosonic counterparts, are made of composite objects, known as Cooper pairs. This internal structure implies more degrees of freedom beyond the usual sound waves found in bosonic systems [?]. At the individual level, single Cooper pairs can break into two unpaired fermions, which leads to a gapped spectrum of fermionic quasiparticles [?]. At the many-body level, whole wavepackets of quasiparticles can be excited for example by tuning the interparticle interaction strength [?]. This causes the amplitude of the order parameter to oscillate in a characteristic way [?], with a frequency and damping determined by the spectral distribution of the wavepacket.

In contrast with the typical picture of amplitude or Higgs modes relying on a single complex bosonic field [?] in a Mexican hat potential, amplitude oscillations in a fermionic condensate are an intrinsically many-body effect, emerging only from the superposition of individual quasiparticle vibrations [? ? ? ?]. Still, for spatially-dependent and weak perturbations of the interaction strength, the evolution of the excited quasiparticle wavepacket can be summarized by a single pole of the order-parameter response function, such that the oscillations can be interpreted as a damped collective mode [? ?].

The case of homogeneous (zero-momentum) perturbations is more subtle: one can no longer identify a pole in the order-parameter response function, such that the collective mode disappears. There remains however a non-Lorentzian singularity in the spectral function, right at the threshold energy for breaking Cooper pairs. In the time-domain, this converts into the famous power-law decaying oscillations of the order parameter [?]. The density of quasiparticle states available around the

pair-breaking threshold changes depending on whether the gapped fermionic spectrum has its minimum at zero or nonzero momentum, corresponding respectively to the Bose-Einstein Condensate (BEC) or Bardeen-Cooper-Schrieffer (BCS) regimes. The lower density of states in the BEC regime makes the damping exponent increase to $3/2$, compared to $1/2$ in the BCS regime [?].

This remarkable collective effect has recently been the center of much experimental attention, both with ultracold fermionic atoms [? ?], superconductors [?] and cavity QED simulators [? ?]. The observations in those experiments have revealed some important limits in our theoretical understanding of the oscillations. Previous studies [? ?] have been restricted to zero temperature, whereas experimentally the oscillations have been recorded from low temperature to the vicinity of the phase transition. Additionally, important observables [?], such as the oscillation amplitude, or the asymptotic limit of the order parameter, have not yet been fully understood.

Here, we show that oscillations of the order parameter for small interaction quenches in the regime of linear response have the same form at zero and nonzero temperature: the power-law damping retains the same exponent, and the oscillation frequency 2Δ simply decreases with temperature as the gap Δ . However, the presence of thermally excited quasiparticles before the quench limits the variation of the order parameter, which, in contrast to the zero-temperature case, no longer tends at long time to its value expected following an adiabatic change of the interaction strength. We interpret the magnitude of the oscillations as the product of the spectral weight of the pair-breaking threshold with the energy change during the quench, itself related to the change in the scattering length through the contact.

We also argue that nonlinear effects increase near the critical temperature since the ratio of the initial to final equilibrium gap Δ_i/Δ_f either diverges or tends to zero when the depth of the interaction quench is kept fixed. The regime of power-law damped oscillations is thus hidden by the nonlinear regimes of persistent oscillations (regime III of Ref. [?]) or overdamped evolution (regime I), and the evolution in those two regimes becomes more abrupt compared to low temperatures.

Model.—We consider a balanced two-component Fermi gas trapped in a three-dimensional volume V at temperature $T = 1/\beta$ (we use $\hbar = k_B = 1$ throughout this Letter), with contact interactions between \uparrow and \downarrow components. The density ρ of the gas fixes the Fermi wave number $k_F = (3\pi^2\rho)^{1/3}$, and the bare coupling constant g is renormalized [?] to yield the appropriate s -wave scattering length a . In the mean-field approximation, the homogeneous system evolves according to the time-dependent BCS equations [?]

$$i\partial_t c_{\mathbf{k}} = (k^2/m)c_{\mathbf{k}} + \Delta(1 - 2n_{\mathbf{k}}), \quad (1)$$

$$i\partial_t n_{\mathbf{k}} = \Delta c_{\mathbf{k}}^* - \Delta^* c_{\mathbf{k}}, \quad (2)$$

where m is the atomic mass, $n_{\mathbf{k}} = \langle \hat{a}_{\mathbf{k}\uparrow}^\dagger \hat{a}_{\mathbf{k}\uparrow} \rangle = \langle \hat{a}_{\mathbf{k}\downarrow}^\dagger \hat{a}_{\mathbf{k}\downarrow} \rangle$ is the momentum distribution, $c_{\mathbf{k}} = \langle \hat{a}_{-\mathbf{k}\downarrow} \hat{a}_{\mathbf{k}\uparrow} \rangle$ the pairing wavefunction, and $\Delta = g \int d^3k c_{\mathbf{k}} / (2\pi)^3$ the order parameter.

Before the quench, the gas is at equilibrium at temperature T_i , chemical potential μ_i and scattering length a_i . This corresponds to the static solution the BCS equations, that is, the usual BCS thermal state with $n_{\mathbf{k},i} = [1 - (1 - 2F_{\mathbf{k}})\xi_{\mathbf{k}}/\epsilon_{\mathbf{k}}]/2$ and $c_{\mathbf{k},i} = -(1 - 2F_{\mathbf{k}})\Delta_i/2\epsilon_{\mathbf{k}}$, in terms of the free-fermion and BCS dispersion relations, $\xi_{\mathbf{k}} = k^2/2m - \mu_i$ and $\epsilon_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_i^2}$, and Fermi-Dirac distribution $F_{\mathbf{k}} = 1/(1 + e^{\epsilon_{\mathbf{k}}/T_i})$. The abrupt variation of a from a_i to a_f leaves the microscopic variables unchanged ($n_{\mathbf{k}}(t = 0^+) = n_{\mathbf{k},i}$ and similarly for $c_{\mathbf{k}}$) but affects the coupling constant through

$$\frac{1}{g_f} - \frac{1}{g_i} = \left(\frac{1}{a_f} - \frac{1}{a_i} \right) \frac{m}{4\pi}. \quad (3)$$

obtained via the Lippmann-Schwinger equation [? ?].

The initial kink in the order parameter then follows from the gap equation

$$\Delta(t = 0^+) - \Delta_i = \frac{g_f - g_i}{g_i} \Delta_i. \quad (4)$$

This kink corresponds to an energy variation that is proportional to the extensive contact $C \equiv d(E/V)/d(1/a)$ [? ? ? ? ?]:

$$\epsilon \equiv \frac{E_f - E_i}{V} = -\frac{C}{4\pi m} \left(\frac{1}{a_f} - \frac{1}{a_i} \right). \quad (5)$$

While Eq. (??) is valid in geneneral, the BCS approximation of the contact is $C_{\text{BCS}} = m^2\Delta^2$. This expression vanishes at the critical temperature as BCS theory

approximates the normal phase by an ideal gas, and restricts the contact to the contribution of the condensed pairs. For the general description of the gas, this is a rather crude approximation in particular near the critical temperature, but for the amplitude oscillations studied in this Letter, the superfluid contact is precisely the important quantity¹.

Linear response.—Shallow quenches are generally characterized by a small injected energy per particle although this rule is brought into question later in this Letter. In this weakly-excited regime, one can linearize the BCS system Eqs. (??)–(??) around the initial equilibrium state and solve using the Laplace transformation [? ?]. With the initial condition (??), the phase of the order parameter is not excited, and only its modulus evolves as:

$$\Delta(t) = \Delta_\infty - \epsilon \int_{\omega_{\text{th}}}^{+\infty} \frac{2d\omega \cos \omega t}{\pi \omega} \text{Im} f(\omega + i0^+). \quad (6)$$

This expression is composed of an asymptotic value Δ_∞ reached when $t \rightarrow +\infty$, and a time-dependent, oscillatory part, written as the frequency integral of the order-parameter modulus-modulus response function

$$f(z) = -\frac{M_{11}(z)}{\Delta_i(M_{11}(z)M_{22}(z) - M_{12}^2(z))}. \quad (7)$$

The linear response matrix M_{ij} appearing here is given by integrals over the internal degrees of freedom of the Cooper pairs $M_{11} = (z^2 - 4\Delta_i^2)M_{22}/z^2 = z^2 \int \frac{d^3k}{(2\pi)^3} \frac{1-2F(\epsilon_{\mathbf{k}})}{2\epsilon_{\mathbf{k}}(z^2-4\epsilon_{\mathbf{k}}^2)}$ and $M_{12} = M_{21} = \int \frac{d^3k}{(2\pi)^3} \frac{z\xi_{\mathbf{k}}[1-2F(\epsilon_{\mathbf{k}})]}{\epsilon_{\mathbf{k}}(z^2-4\epsilon_{\mathbf{k}}^2)}$.

Quite intuitively, the final shift in Δ , obtained when the oscillations have decayed, is the product of the transferred energy and static modulus response $f(\omega = 0)$:

$$\Delta_\infty = \Delta_i + f(0)\epsilon. \quad (8)$$

We identify here an important effect of temperature on the post-quench dynamics. When $T_i = 0$ the asymptotic gap Δ_∞ matches the equilibrium gap Δ_f that would be reached after an adiabatic change of the scattering length from a_i to a_f . This is due to the static modulus response saturating the injected energy $f(0) = d\Delta/d\epsilon$. This is no longer true for $T_i > 0$, and instead

$$|\Delta_\infty - \Delta_i| < |\Delta_f - \Delta_i|. \quad (9)$$

¹ While the inverse coupling constants $1/g_i$ and $1/g_f$ diverge linearly with a momentum cutoff, their difference does not according to Eq. (??). Thus the injected energy (Eq. (??)) remains finite and nonzero, while the discontinuity in Δ (Eq. (??)) vanishes. This is a consequence of the formally divergent interaction energy $E_{\text{int}} = \Delta^2/g$ according to BCS theory.

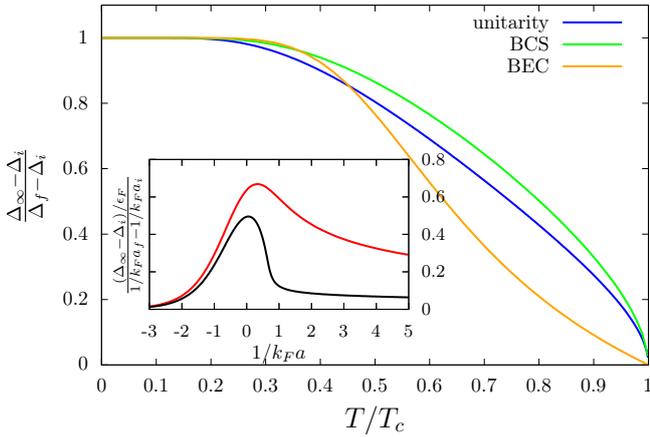


FIG. 1: (Main panel) The asymptotic change of the order parameter $\Delta_\infty - \Delta_i$ measured relative to the change $\Delta_f - \Delta_i$ under an adiabatic evolution, as a function of temperature at unitarity, in the BCS and BEC limits. At $T = 0$, $\Delta_\infty = \Delta_f$ despite the non-adiabatic nature of the quench. (Inset) The change $\Delta_\infty - \Delta_i$ (in units of ϵ_F) relative to the change in the scattering length $1/k_{Fa_f} - 1/k_{Fa_i}$ throughout the BEC-BCS crossover at $T = 0$ (red curve) and near T_c ($T/T_c = 0.99$, black curve). In both cases, a maximum is reached near unitarity. Note that the change remains non zero (in units of ϵ_F) in the limit $T \rightarrow T_c$, which means the linear approximation breaks down (if the quench depth $1/k_{Fa_f} - 1/k_{Fa_i}$ is kept independent of temperature).

In other words, the order parameter remains closer to its initial value than it would under an adiabatic transformation, as a part of the injected energy is absorbed by the thermally excited quasiparticles. Both at zero and nonzero initial temperature, the state reached asymptotically is not an equilibrium state and, in particular, does not have a well-defined temperature. To describe equilibration, the integrable BCS system should be replaced by an ergodic model.

Eq. (??) provides a criterion for the validity of the linear regime. For the deviation of the order parameter to remain small, it is necessary and sufficient that $|\Delta_\infty - \Delta_i| \ll \Delta_i$. At low temperatures, Δ_i is comparable to the Fermi energy ϵ_F , so this condition simply translates into $|a_f - a_i| \ll a_i$, which is not a particularly demanding constraint, especially near unitarity ($1/|a| = 0$). Near T_c however, $(\Delta_\infty - \Delta_i)/(1/k_{Fa_f} - 1/k_{Fa_i})$ is comparable to ϵ_F (as shown by the black curve in Fig. ??) and hence much larger than Δ_i . This leads to a stricter condition $|a_f - a_i| \ll a_i \Delta_i / \epsilon_F$ for the validity of the linear approximation. For a quench depth $a_f - a_i$ fixed independently of temperature, which corresponds to the experimental scenario studied in [?], this condition will *always* be violated when T_i is sufficiently close to T_c .

We now turn to the time-evolution described by Eq. (??). The continuity of $\Delta(t)$ at $t = 0$ is guaranteed by the sum-rule of the modulus-modulus response function:

$\int_{-\infty+i0^+}^{+\infty+i0^+} dz f(z) / 2i\pi z = 0$. Then, at long times, the nature of the oscillations of $\Delta(t)$ depends on the behavior of f in the vicinity of the pair-breaking threshold ω_{th} . In the BCS regime ($\mu_i > 0$) and, irrespective of the temperature, the response function has a squareroot divergence near $\omega_{th} = 2\Delta_i$. Conversely, in the BEC regime ($\mu_i < 0$) at all temperatures, the response function is cancelled as a squareroot near the dimer-breaking threshold:

$$\text{Im}f(\omega + i0^+) \underset{\omega \rightarrow \omega_{th}}{\sim} \begin{cases} f_{th} \sqrt{\frac{\omega_{th}}{\omega - \omega_{th}}} & \text{when } \mu_i > 0 \\ f_{th} \sqrt{\frac{\omega - \omega_{th}}{\omega_{th}}} & \text{when } \mu_i < 0 \end{cases} . \quad (10)$$

After the frequency integration, these behaviours near ω_{th} translate into power-law attenuated oscillations of $\Delta(t)$:

$$\frac{\Delta(t) - \Delta_\infty}{\Delta_i - \Delta_\infty} \underset{t \rightarrow +\infty}{\sim} \begin{cases} \frac{f_{th}}{f(0)} \sqrt{\frac{4}{\pi\omega_{th}t}} \cos(\omega_{th}t + \frac{\pi}{4}), & \mu_i > 0 \\ \frac{f_{th}}{f(0)} \frac{1}{\sqrt{\pi\omega_{th}^3 t^3}} \cos(\omega_{th}t + \frac{3\pi}{4}), & \mu_i < 0 \end{cases} \quad (11)$$

The spectral weight f_{th} which characterizes the asymptotic behaviors at the threshold is shown in Fig. ?? as a function of the interaction regime. Comparing the zero-temperature case (solid curves) to the vicinity of T_c (dashed curves), we observe a suppression of the relative weight $f_{th}/f(0)$ on the BEC side but an increase on the BCS side. While this increase a priori favors the observability of the power-law damped oscillations, we note that $f_{th}/f(0)$ characterizes the amplitude of the signal only when scaled to the asymptotic change in Δ , see Eq. (??). Scaled to the adiabatic variation $\Delta_f - \Delta_i$, the amplitude will vanish as $\Delta_\infty - \Delta_i$ as shown by Fig. ??.

Quenches in the nonlinear regime. —The fact that the nonlinearity increases with temperature (as long as the quench depth $|a_i - a_f|$ is fixed) suggests extending our study to the nonlinear regime. We do this numerically by simulating Eqs. (??)–(??) on a fine momentum grid.

We recall the zero-temperature quench diagram of Ref. [?] (see Fig. 5 therein) that identified three qualitatively distinct regimes in the (Δ_i, Δ_f) plane. In regime I, there are no oscillations as $\Delta(t)$ is overdamped; this regime includes in particular the limit $\Delta_i \gg \Delta_f$. Regime II is the regime of power-law damped oscillations, which contains the linear regime on the diagonal $\Delta_i \simeq \Delta_f$. Finally, a regime III of undamped oscillations was identified around the limit $\Delta_f \gg \Delta_i$.

We show now how regimes I and III generalize to nonzero temperatures and tend to hide regime II when the initial state approaches the critical point ($T_i \rightarrow T_{c,i}$) and the quench depth is fixed. For an initial state in the regime $|T - T_c| \ll T_c$, that is, $\Delta_i \ll \epsilon_F$, quenches in the direction of the superfluid phase end up in $\Delta_f \approx \epsilon_F \gg \Delta_i$, and therefore in regime III of persistent oscillations. Conversely, quenches towards the normal phase yield $\Delta_i \gg \Delta_f = 0$, and thus fall into the overdamped regime I.

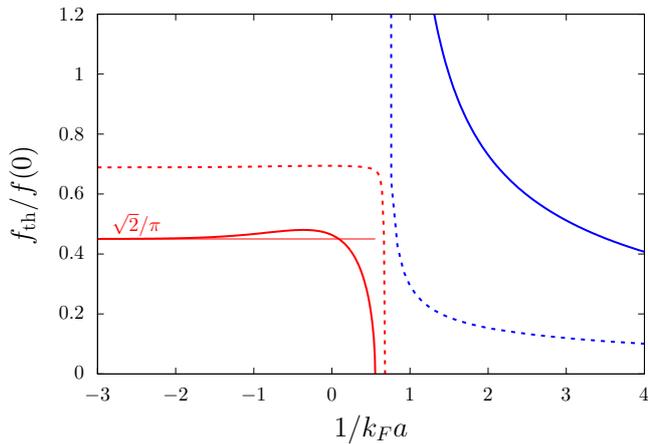


FIG. 2: The spectral weight of the pair-breaking edge f_{th} (relative to the static response $f(0)$) as a function of the interaction strength at $T = 0$ (solid curves) and $T = 0.999T_c$ (dashed curves). In red the BCS regime where the edge exhibits a squareroot divergence (upper line of Eq. (??)), in blue the BEC regime where instead this edge is a squareroot cancellation (lower line of Eq. (??)). Note that the boundary between those two regimes depends weakly on temperature.

In Fig. ??, we illustrate the onset of regime III at high temperatures when quenching in the direction of the superfluid phase. Going from the BCS side ($a_i < 0$) to unitarity, with a quench depth sufficiently low to be in regime II at $T = 0$, as in Fig. ??(a), we notice an increase of the oscillation amplitude (scaled to Δ_∞), which precedes the appearance of persistent oscillations at temperatures close to $T_{c,i}$. The persistent oscillations also become much more abrupt than at low temperature, as illustrated by Fig. ??(b), where the quench depth is chosen to be in regime III already at $T = 0$.

In Fig. ??, we consider the opposite case of quenches towards the normal phase with $1/a_i = 0$ and a_f on the BCS side. As shown in Fig. ??(a), quenches sufficiently shallow to be in regime II at low temperatures undergo a gradual decrease of their asymptotic limit and oscillation frequency (both determined by Δ_∞) with temperature, up to a point where the order parameter tends to zero and no longer oscillates. In Fig. ??(a), this occurs at $T/T_c = 0.999$, corresponding to $\Delta_f/\Delta_i \sim 2 \times 10^{-4}$. This threshold of regime I is reached at a lower temperature for larger quench depths. When the quench is sufficiently deep to be in regime I already at $T = 0$, it remains in this regime at all temperatures, and the decay of $\Delta(t)$ becomes more abrupt as illustrated by Fig. ??(b).

Conclusion. —We have studied amplitude oscillations in a nonzero temperature fermionic condensate within time-dependent BCS theory. We showed how the magnitude of the oscillations and the asymptotic change of the order parameter $\Delta_\infty - \Delta_i$ are both proportional to the BCS contact. The oscillations thus fade out as this contact vanishes at the phase transition. While the

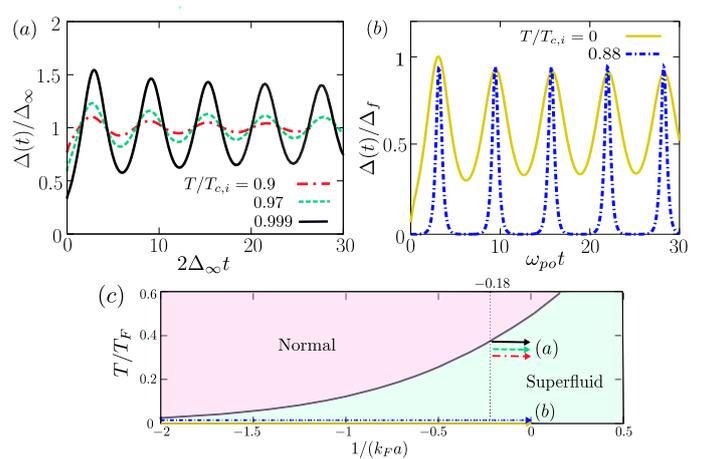


FIG. 3: (a) The onset of regime III (persistent oscillations) in a quench from $1/(k_F a_i) = -0.18$ to $1/(k_F a_f) = 0$ when raising the initial temperature. (b) Effect of temperature on the persistent oscillations for a quench from $1/(k_F a_i) = -2$ to $1/(k_F a_f) = 0$ belonging to regime III at all temperatures. Here, we find that the oscillation frequency ω_{po} is smaller than $2\Delta_f$. (c) Illustration of the quenches studied in (a) and (b) in the $(1/(k_F a), T/T_F)$ plot.

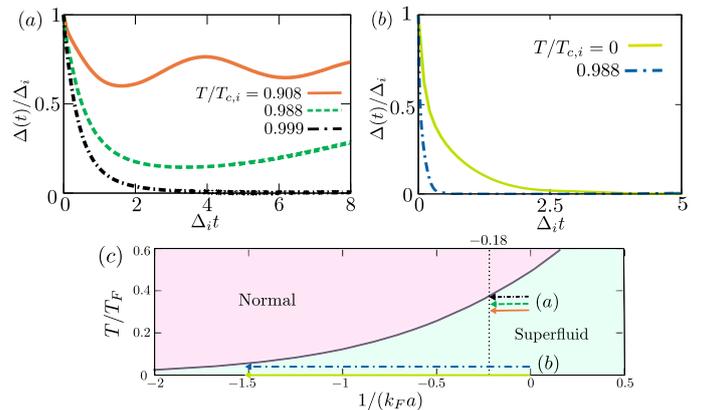


FIG. 4: (a) The onset of regime I (overdamped evolution) when raising the initial temperature of quenches from $1/(k_F a_i) = 0$ to $1/(k_F a_f) = -0.18$. (b) The decay of $\Delta(t)$ becomes more abrupt at higher temperature, as shown here for a quench from $1/(k_F a_i) = 0$ to $1/(k_F a_f) = -1.5$ belonging to regime I at all temperatures. (c) Illustration of the quenches studied in (a) and (b) in the $(1/(k_F a), T/T_F)$ plot.

oscillation frequency is predicted to vanish at T_c with Δ on the BCS side, it stays nonzero on the BEC side and coincides with the molecular binding energy $E_{\text{mol}} = 2|\mu|$. The fact that time-dependent BCS theory does not correctly describe the normal phase of the interacting gas limits our description of what happens outside the superfluid phase, in during particular dynamical phase transitions [?]. Extending BCS theory to correctly describe the nonequilibrium evolution across the phase transition would be a major achievement. One can also imagine

that amplitude oscillations of the pairing field still occur in the normal phase when a pseudogap appears in the single-particle spectral density [?].

We thank Denise Ahmed-Braun, Servaas Kokkelmans for valuable discussions. V.E.C. acknowledges financial support from the Provincia Autonoma di Trento, the Italian MIUR under the PRIN2017 project CEnTraL and the National Science Foundation under Grant No. NSF PHY-1748958. S. M. acknowledges financial support from the ANR-21-CE47-0009 Quantum-SOPHA project.

-
- [] L. Pitaevskii and S. Stringari, *Bose–Einstein Condensation and Superfluidity* (Oxford University Press, Oxford, 2016).
 - [] R. Haussmann, M. Punk, and W. Zwerger, Phys. Rev. A **80**, 063612 (2009), URL <https://link.aps.org/doi/10.1103/PhysRevA.80.063612>.
 - [] R. G. Scott, F. Dalfovo, L. P. Pitaevskii, and S. Stringari, Phys. Rev. A **86**, 053604 (2012), URL <https://link.aps.org/doi/10.1103/PhysRevA.86.053604>.
 - [] A. Volkov and C. M. Kogan, Zh. Eksp. Teor. Fiz. **65**, 2038 (1973), [Sov. Phys. JETP, Vol. 38, No. 5, p. 1018], URL <http://www.jetp.ras.ru/cgi-bin/e/index/r/65/5/p2038?a=list>.
 - [] D. Pekker and C. Varma, Annual Review of Condensed Matter Physics **6**, 269 (2015), URL <https://doi.org/10.1146/annurev-conmatphys-031214-014350>.
 - [] A. Schmid, Physik der kondensierten Materie **8**, 129 (1968), ISSN 1431-584X, URL <https://doi.org/10.1007/BF02422735>.
 - [] V. Gurarie, Phys. Rev. Lett. **103**, 075301 (2009), URL <https://link.aps.org/doi/10.1103/PhysRevLett.103.075301>.
 - [] E. A. Yuzbashyan, M. Dzero, V. Gurarie, and M. S. Foster, Phys. Rev. A **91**, 033628 (2015), URL <https://link.aps.org/doi/10.1103/PhysRevA.91.033628>.
 - [] V. A. Andrianov and V. N. Popov, Theoretical and Mathematical Physics **28**, 829 (1976), URL <https://doi.org/10.1007/BF01029175>.
 - [] H. Kurkjian, S. N. Klimin, J. Tempere, and Y. Castin, Phys. Rev. Lett. **122**, 093403 (2019), URL <https://link.aps.org/doi/10.1103/PhysRevLett.122.093403>.
 - [] A. Behrle, T. Harrison, J. Kombe, K. Gao, M. Link, J. S. Bernier, C. Kollath, and M. Köhl, Nature Physics (2018), URL <https://doi.org/10.1038/s41567-018-0128-6>.
 - [] P. Dyke, S. Musolino, H. Kurkjian, D. J. M. Ahmed-Braun, A. Pennings, I. Herrera, S. Hoinka, S. J. J. M. F. Kokkelmans, V. E. Colussi, and C. J. Vale, *Higgs oscillations in a unitary Fermi superfluid*, arXiv:2310.03452 (2023), URL <https://arxiv.org/abs/2310.03452>.
 - [] R. Matsunaga, Y. I. Hamada, K. Makise, Y. Uzawa, H. Terai, Z. Wang, and R. Shimano, Phys. Rev. Lett. **111**, 057002 (2013), URL <https://link.aps.org/doi/10.1103/PhysRevLett.111.057002>.
 - [] R. J. Lewis-Swan, D. Barberena, J. R. K. Cline, D. J. Young, J. K. Thompson, and A. M. Rey, Phys. Rev. Lett. **126**, 173601 (2021), URL <https://link.aps.org/doi/10.1103/PhysRevLett.126.173601>.
 - [] D. J. Young, A. Chu, E. Y. Song, D. Barberena, D. Wellnitz, Z. Niu, V. M. Schäfer, R. J. Lewis-Swan, A. M. Rey, and J. K. Thompson, *Observing Dynamical Phases of a Bardeen-Cooper-Schrieffer Superconductor in a Cavity QED Simulator*, arXiv:2306.00066 (2023), URL <https://arxiv.org/abs/2306.00066>.
 - [] W. Zwerger, ed., *The BCS-BEC Crossover and the Unitary Fermi Gas* (Springer, Berlin, 2012).
 - [] J.-P. Blaizot and G. Ripka, *Quantum Theory of Finite Systems* (MIT Press, Cambridge, Massachusetts, 1985).
 - [] J. R. Taylor, *Scattering theory: the quantum theory of nonrelativistic collisions* (Courier Corporation, 2006).
 - [] Y. Castin, in *Ultra-cold Fermi Gases*, edited by M. Inguscio, W. Ketterle, and C. Salomon (Società Italiana di Fisica, Bologna, 2007).
 - [] F. Werner and Y. Castin, Phys. Rev. A **86**, 013626 (2012), URL <https://link.aps.org/doi/10.1103/PhysRevA.86.013626>.
 - [] R. Qi, Z. Shi, and H. Zhai, Phys. Rev. Lett. **126**, 240401 (2021), URL <https://link.aps.org/doi/10.1103/PhysRevLett.126.240401>.
 - [] S. Tan, Annals of Physics **323**, 2952 (2008), ISSN 0003-4916, URL <https://www.sciencedirect.com/science/article/pii/S0003491608000456>.
 - [] S. Tan, Annals of Physics **323**, 2971 (2008), ISSN 0003-4916, URL <https://www.sciencedirect.com/science/article/pii/S0003491608000432>.
 - [] S. Tan, Annals of Physics **323**, 2987 (2008), ISSN 0003-4916, URL <https://www.sciencedirect.com/science/article/pii/S0003491608000420>.
 - [] E. Braaten, D. Kang, and L. Platter, Phys. Rev. A **78**, 053606 (2008), URL <https://link.aps.org/doi/10.1103/PhysRevA.78.053606>.
 - [] Y. Castin and H. Kurkjian, Comptes Rendus. Physique **21**, 253 (2020).
 - [] P. Dyke, A. Hogan, I. Herrera, C. C. N. Kuhn, S. Hoinka, and C. J. Vale, Phys. Rev. Lett. **127**, 100405 (2021), URL <https://link.aps.org/doi/10.1103/PhysRevLett.127.100405>.
 - [] H. Hu, X.-J. Liu, P. D. Drummond, and H. Dong, Phys. Rev. Lett. **104**, 240407 (2010), URL <https://link.aps.org/doi/10.1103/PhysRevLett.104.240407>.