

Validation a priori d'une mission sous-marine

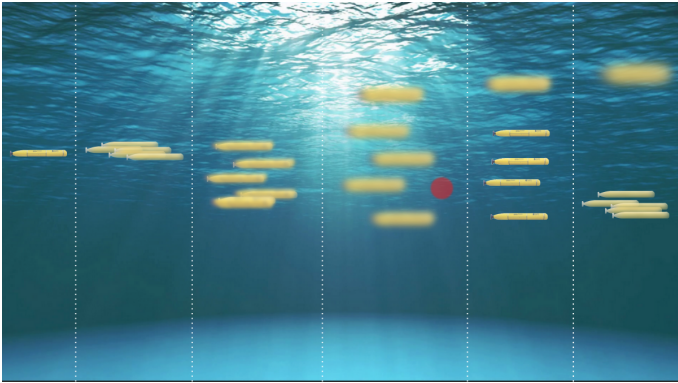
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7 octobre 2023, lac de Guerlédan
<https://drones-cap-2023.sciencesconf.org/>





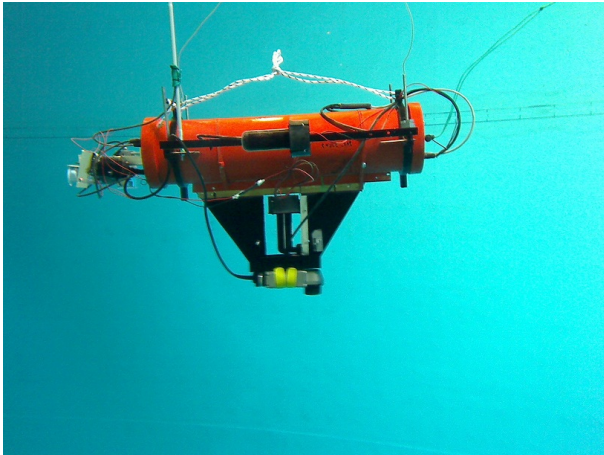
<https://youtu.be/hNqIShmMQjA>



1. Robots

A robot is a mechanical system equipped with

- actuators
- sensors
- an intelligence
- a memory.



Saucisse (ENSTA Bretagne)

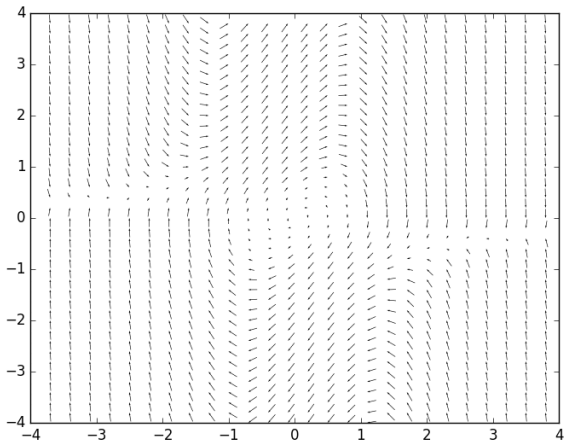
Reachability

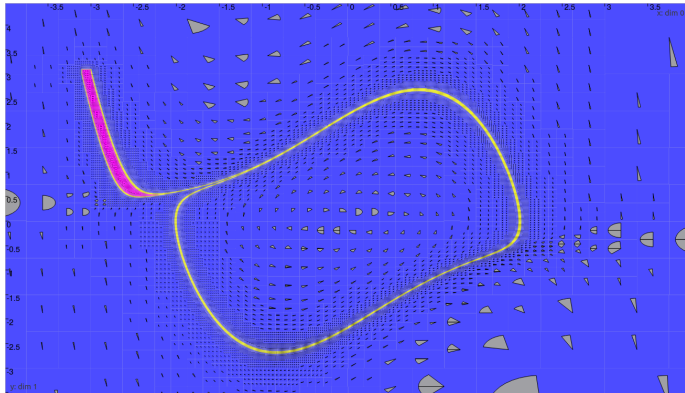
A dynamical system [Newton 1690]

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}).$$

Example. Van der Pol oscillator

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -(x_1^2 - 1)x_2 - x_1 \end{cases}$$
$$\mathbf{x}(0) \in \mathbb{X}_0$$





Vehicle

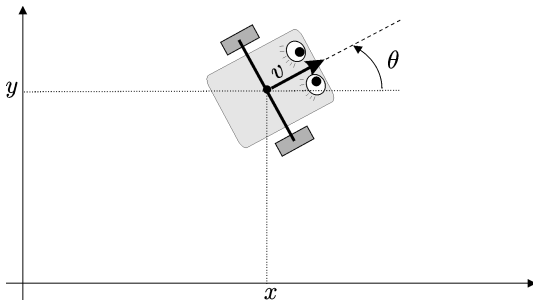
A **vehicle** is a dynamical system with actuators

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}).$$

Example. The Dubin's car (1957).

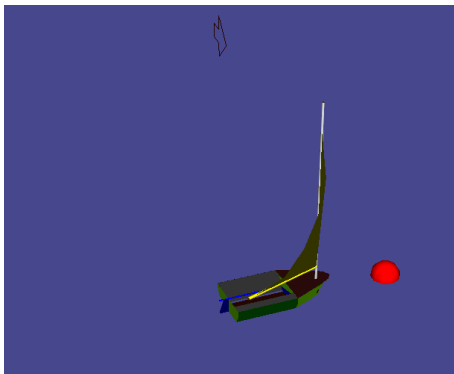
$$\begin{cases} \dot{x} &= \cos \theta \\ \dot{y} &= \sin \theta \\ \dot{\theta} &= u \end{cases}$$

with $u \in [-1, 1]$.



Simulation (formule d'Euler, 1770)

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \quad \sim \quad \mathbf{x}(t_{k+1}) = \mathbf{x}(t_k) + \delta \cdot \mathbf{f}(\mathbf{x}(t_k), \mathbf{u}(t_k))$$



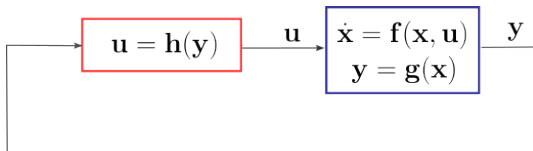
Intelligence

A robot is a vehicle with sensors, actuators and an intelligence:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \quad (\text{evolution})$$

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) \quad (\text{observation})$$

$$\mathbf{u} = \mathbf{h}(\mathbf{y}). \quad (\text{control})$$

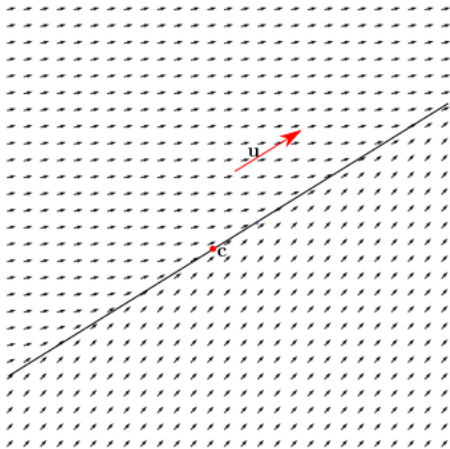


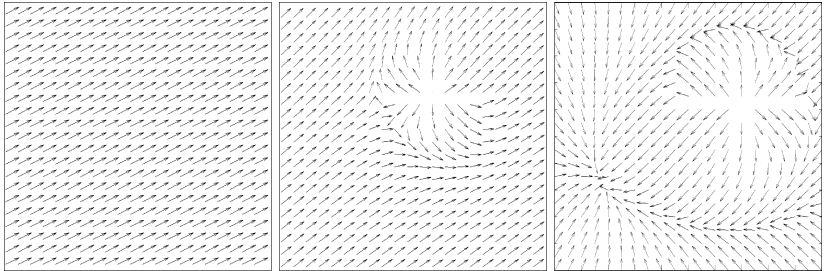
We have

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{h}(\mathbf{g}(\mathbf{x}))) = \boldsymbol{\psi}(\mathbf{x})$$

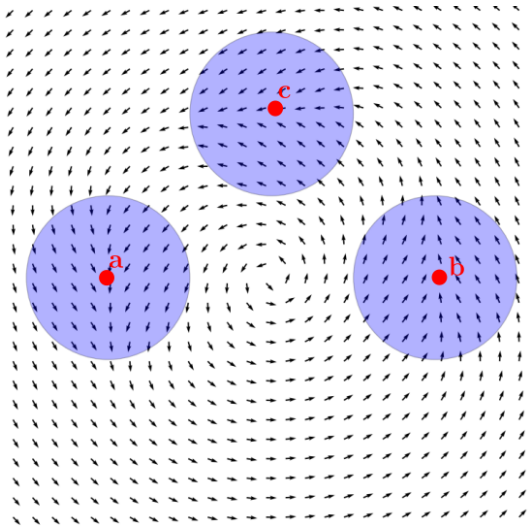
Thus an intelligent vehicle is a dynamical system.

Control





Artificial potential fields



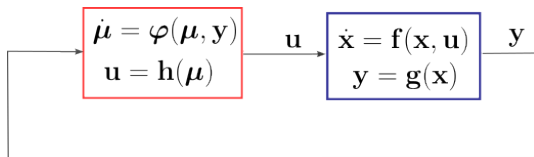
A robot has memory

A robot is a intelligent vehicle with memory

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) && \text{(ontic evolution)} \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t)) && \text{(observation)} \\ \mu_{k+1} &= \varphi(\mu_k, \mathbf{y}(t_k)) && \text{(epistemic evolution)} \\ \mathbf{u}(t_k) &= \mathbf{h}(\mu_k) && \text{(control)}\end{aligned}$$

With an analog controller

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) && \text{(ontic evolution)} \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t)) && \text{(observation)} \\ \dot{\boldsymbol{\mu}}(t) &= \boldsymbol{\varphi}(\boldsymbol{\mu}(t), \mathbf{y}(t)) && \text{(epistemic evolution)} \\ \mathbf{u}(t) &= \mathbf{h}(\boldsymbol{\mu}(t)). && \text{(control)}\end{aligned}$$



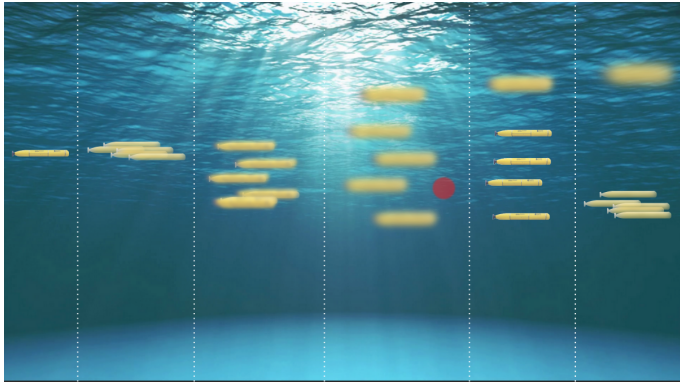
$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{h}(\boldsymbol{\mu}(t))) && \text{(ontic evolution)} \\ \dot{\boldsymbol{\mu}}(t) &= \boldsymbol{\varphi}(\boldsymbol{\mu}(t), \mathbf{g}(\mathbf{x}(t))) && \text{(epistemic evolution)}\end{aligned}$$

The global state is $\mathbf{z} = (\mathbf{x}, \mu)$. We have

$$\dot{\mathbf{z}} = \psi(\mathbf{z})$$

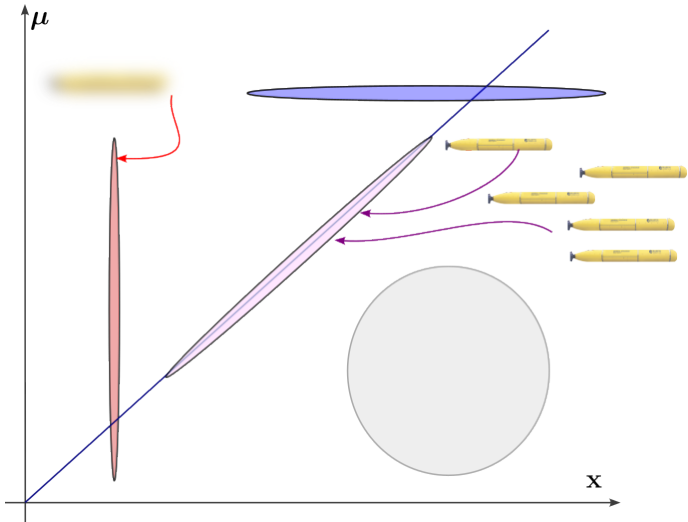
An intelligent vehicle with memory is thus a dynamical system.

2. Uncertainties



State of the robot : $\mathbf{z} = (\mathbf{x}, \mu)$, with

- \mathbf{x} : the ontic state
- μ : the epistemic state

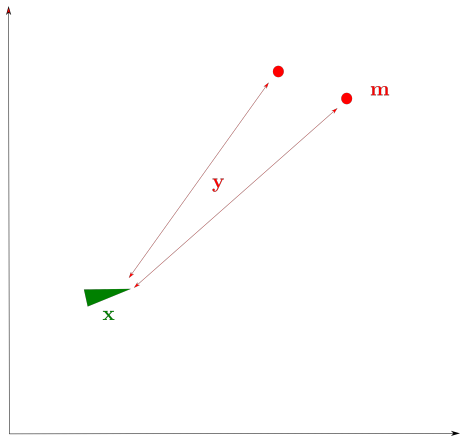


Perception : We measure \mathbf{x}
Communication : we measure μ

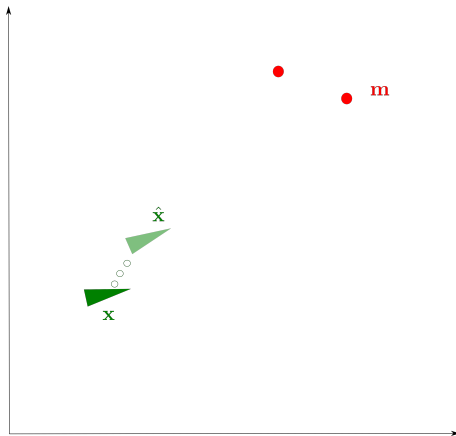
Swarm



<https://youtu.be/xlgp9P0SY1Y>
t=1:40



$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} &= \mathbf{g}(\mathbf{x})\end{aligned}$$



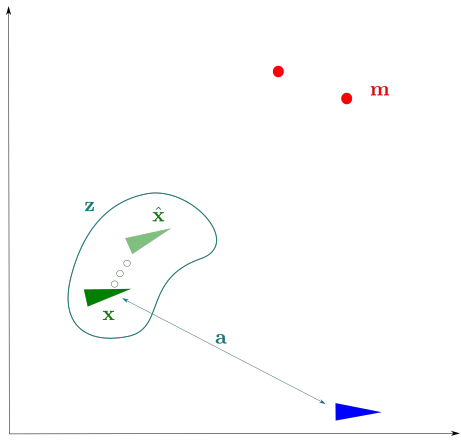
If we set $\mathbf{z} = (\mathbf{x}, \hat{\mathbf{x}})$, we get

$$\dot{\mathbf{z}} = \psi(\mathbf{z})$$

Assume that we can observe the motion of the robot

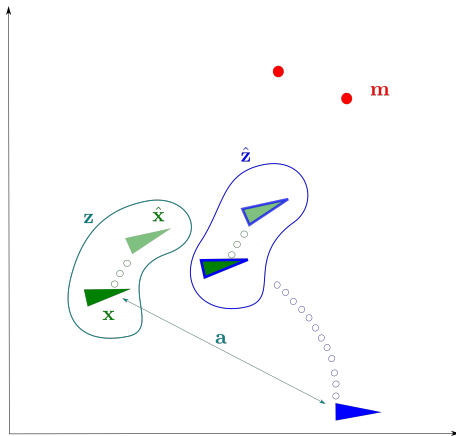
$$\dot{\mathbf{z}} = \boldsymbol{\psi}(\mathbf{z})$$

$$\mathbf{a} = \boldsymbol{\eta}(\mathbf{x})$$

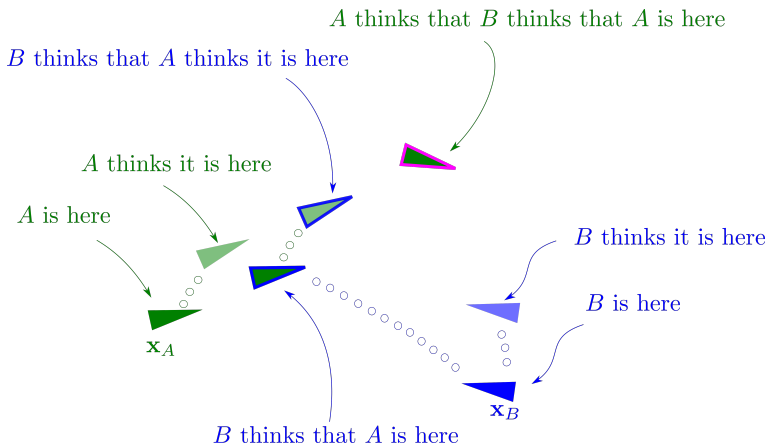


We can build an observer for \mathbf{z} :

$$\begin{aligned}\dot{\mathbf{z}} &= \boldsymbol{\psi}(\mathbf{z}) \\ \mathbf{a} &= \boldsymbol{\eta}(\mathbf{x}) \\ \dot{\hat{\mathbf{z}}} &= \hat{\boldsymbol{\psi}}(\mathbf{a}, \hat{\mathbf{z}})\end{aligned}$$

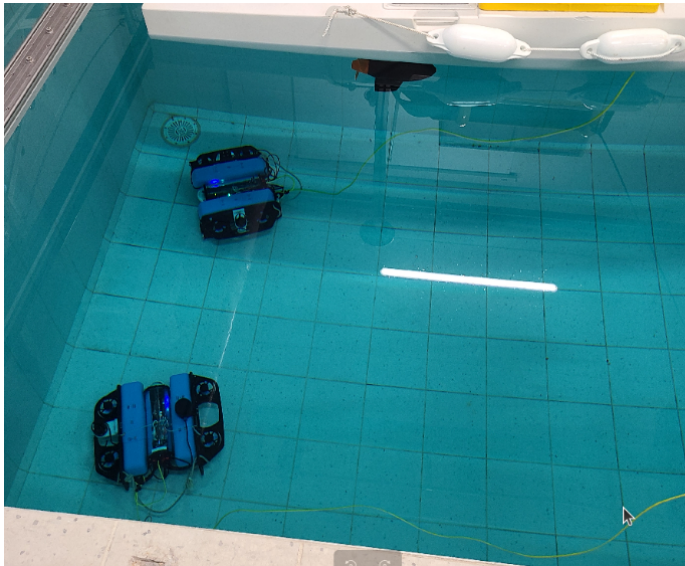


Distributed knowledge







Experiments





References

- 1 Interval analysis [7, 3, 4]
- 2 Stability with intervals : [6]
- 3 SLAM with intervals : [2]
- 4 Interval tubes [8], [1]
- 5 Reachability [5]

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