



**HAL**  
open science

## Doppler Gyroscopes: Frequency vs Phase Estimation

John Howell, Merav Kahn, Einav Grynszpan, Ziv Roi Cohen, Stefania Residori,  
Umberto Bortolozzo

► **To cite this version:**

John Howell, Merav Kahn, Einav Grynszpan, Ziv Roi Cohen, Stefania Residori, et al.. Doppler Gyroscopes: Frequency vs Phase Estimation. *Physical Review Letters*, 2022, 129 (11), pp.113901. <10.1103/PhysRevLett.129.113901>. <hal-04286614>

**HAL Id: hal-04286614**

**<https://hal.science/hal-04286614v1>**

Submitted on 15 Nov 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

# Doppler Gyroscopes: Frequency vs Phase Estimation

John C. Howell,<sup>1,2</sup> Merav Kahn,<sup>1</sup> Einav Grynszpan,<sup>1</sup> Ziv Roi Cohen,<sup>1</sup> Stefania Residori,<sup>3</sup> and Umberto Bortolozzo<sup>3</sup>

<sup>1</sup>*Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem, Israel, 91904*

<sup>2</sup>*Institute for Quantum Studies, Chapman University, 1 University Drive, Orange, CA 92866, USA*

<sup>3</sup>*HOASYS SAS, 1047 route des Dolines, Business Pôle, 06560 Valbonne, France*

We consider the fundamental roles of frequency versus phase in parameter estimation, specifically in the Sagnac effect. We describe a novel, ultra-sensitive gyroscope based on the extremely steep frequency-dependent gain of a liquid crystal light valve. We provide compelling experimental evidence that the Doppler shift is fundamental in the Sagnac effect giving clarity to a long-debated question. We experimentally show orders of magnitude improvement in sensitivity relative to the standard quantum limit of a gyroscope based on phase estimation.

The burgeoning field of quantum metrology seeks to find “quantum advantages” over existing classical measurement schemes [1–12]. Owing to its importance in gyroscopes [2] and gravitational wave detection [1, 13] as well as its fundamental nature in all branches of interferometry, phase estimation beyond the standard quantum limit has been the prototypical example [1–3, 5, 7, 12]. Pragmatically, due to loss, quantum phase estimation techniques have, so far, only offered a few percent improvement over the standard quantum limit in the few-photon regime [10] or a few dB improvement in the high power regime [9, 11]. However, what if phase estimation for a class of experiments is suboptimal? Depending on the measurement apparatus, phase estimation may have different fundamental limits than frequency estimation [14]. Here we demonstrate that by using an ultra-steep, frequency-dependent gain measurement rather than performing phase estimation in a passive gyroscope, we can achieve orders of magnitude improvement below the phase-estimation standard quantum limit of a single-loop Sagnac interferometer of the same size. Further, we provide important insights into a long-debated question about the role of Doppler shifts in the Sagnac effect.

Gyroscopes are powerful tools in tests of fundamental physics, guidance systems, inertial navigation, accelerometry, geodesy, seismology and geophysics to name a few. Significant advances in micromechanical [15], atomic [16–18], chip-based systems [19] and ring laser gyros [20–26] have been achieved. It may come as a surprise, then, that there are still open questions about the fundamental underpinnings of gyroscopes. For the sake of clarity, we consider an optical gyroscope to be a system in which a light source and a detector, in the same reference frame, rotating at a constant angular speed, can measure the rotation rate of the system. However, instead of measuring differential phase shifts, as is typically done, we measure differential frequency shifts.

The estimation of the rotation speed is closely related to a long-debated question of the role of Doppler shifts in the Sagnac effect [27]. In a standard Sagnac interferometer, the light is split using a beamsplitter into two counter-propagating beams that return to the same

beamsplitter. Special relativity predicts an in-plane, closed-path relative phase acquired by the two beams in an optical Sagnac, given by

$$\Delta\phi = \frac{8\pi\Omega A}{\lambda c} \quad (1)$$

where  $\Omega$  is the angular frequency,  $A$  is the gyroscope area,  $\lambda$  is the wavelength of light and  $c$  is the speed of light. Hence, once the phase is known, the angular rotation frequency can be determined. The shot noise limited rotation sensitivity can then be found by substituting the standard quantum limit for phase, namely

$$\Delta\phi = \frac{1}{2\sqrt{N}}, \quad (2)$$

where  $N$  is the number of photons.

An alternative theory is that the Doppler effect is the fundamental mechanism of the Sagnac effect (see references in [27]). However, Malykin elucidates two reasons against the use of the Doppler effect [27]. First, in a closed-loop system, the beamsplitter plays both the role of the emitter and the detector. He states, “...the radiation source and detector must be in motion relative to each other if the Doppler effect is to be manifest”. This implies that the net Doppler shift *outside* the closed interferometer is zero. However, we feel this is not a good argument against a Doppler shift model, since proponents argue that it is the Doppler shifts between the source and the emitter (leading to a differential phase) that matter and not what happens external to the interferometer. We go beyond this assertion by exploring symmetry-breaking designs where the detector is not the same element as the emitter allowing us to measure differential frequency shifts even at the detector.

Malykin’s second argument is that if there is a material medium, a Doppler-shift theory differs from the standard prediction by a factor of  $2n^2$  where  $n$  is the index of refraction of the material. Even in vacuum, the result still differs by a factor of 2. The vacuum result is rectified by proponents of the Doppler effect by using the length of the interferometer in the interferometer’s frame. However, Malykin points out that it is inconsistent to assume

a Doppler shift in the lab frame and the loop length in the rotating frame.

Instead of resolving the theoretical inconsistencies, we endeavor to show that Doppler shifts do exist within the system. We ask the question: if a moving mirror imparts a Doppler shift, why would such a shift vanish if the mirror is inside an interferometer? It is our opinion that measuring the phase alone leads to ambiguous interpretations. To parse out the role of the Doppler effect, we need a system that measures *only* differential frequency shifts and *not* differential phase.

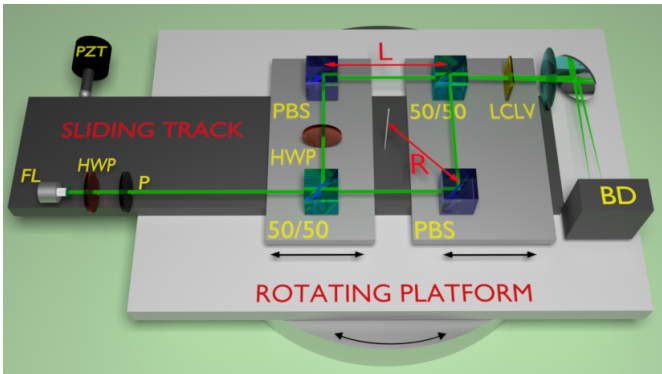


FIG. 1. Experimental setup. A laser beam is coupled into a square Mach-Zehnder interferometer on top of a rotating platform. The size and distance of the interferometer relative to the axis of rotation can be controlled by moving small tables with the beamsplitters. A liquid crystal light valve acts as the relativistic “detector” by providing spectrally-dependent gain, which is then measured by balanced detectors. All elements are on the rotating platform.

For our spectral estimation technique, we incorporate an ultra-sensitive, wave-mixing spectrometer based on an extremely steep frequency-dependent gain to measure frequency offsets. We use a liquid crystal light valve (LCLV), to measure the relative Doppler shift of two counter-propagating paths, but not the relative phase. Two-beam interference in the crystal creates a self-induced index grating from which the beams scatter. The various scattering orders from the two beam-interference inside an LCLV can lead to phase-insensitive steep spectral gain dependence [14] or extremely slow or fast group velocities [28]. It was shown [14] that the shot noise limited spectral sensitivity is given by

$$\Delta f = \frac{1}{|\chi|\sqrt{N}} \quad (3)$$

where  $|\chi|$  is the slope of the spectral gain and  $N$  is the number of measured photons. Because the slope of the gain curve can be large, we can achieve very high precision. This is the type of “slow light” advantage that was sought for almost two decades [29–32].

Now, we consider the Doppler shift from each reflecting surface of a rotating interferometer, like the one shown

in Fig. 1. We recently showed [33] that the Ashworth-Davies [34] non-relativistic Doppler shift  $\Delta f_m$  of a laser of wavelength  $\lambda$  from a mirror moving with angular velocity  $\Omega$  and radius  $R$  is given by

$$\Delta f_m = \frac{2\Omega R}{\lambda} \cos(\phi) \cos(\alpha), \quad (4)$$

where, with respect to the mirror’s surface normal,  $\alpha$  is the direction of propagation of the mirror and  $\phi$  is the angle of incidence.

Consider a simple Doppler model in which the axis of rotation is at the center of a square Mach-Zehnder interferometer like the one shown in Fig. 1. The distance from the axis of rotation to each reflective element is then  $R = L/\sqrt{2}$ , where  $L$  is the length of a side of the square. In this scenario  $\alpha = 0$  for the first 50/50 beamsplitter and  $\alpha = 90^\circ$  for the two corner mirrors (we use polarizing beamsplitters (PBS) effectively as mirrors). The final beamsplitter has an angle  $\alpha = \epsilon/2$ , where  $\epsilon \ll \phi$ . The incidence angle is the same for all surfaces, namely  $\phi = 45^\circ$  for all surfaces, except the last beamsplitter which is  $\phi = 45^\circ + \epsilon/2$ . In this scenario  $\alpha = 0^\circ$  for the first beamsplitter  $\alpha = \epsilon/2$  for the final beamsplitter. Keeping only terms first order in  $\epsilon$ , the relative Doppler shifts between the two paths is given by

$$\Delta f \approx \frac{\sqrt{2}L(\cos(\phi) - \cos(\phi + \epsilon/2))}{\lambda} \quad (5)$$

yielding the differential frequency approximation

$$\Delta f \approx \frac{L\Omega\epsilon}{2\lambda}. \quad (6)$$

The final beamsplitter, with its small relative angle, breaks the symmetry in the system. It should also be noted that the frequency differential is zero when the angle between the output beams  $\epsilon = 0$  as observed for closed systems. This model predicts, using parameters from our system (see section IV in the supplementary material for further information), an estimated shot noise limited rotation sensitivity down to  $\Omega \approx 100$  pRad/s/Hz $^{-1/2}$  (approximately 5 orders of magnitude below the SQL for phase estimation of an interferometer of the same size and same number of measured photons).

For our experiment, light from a 532 nm fiber-coupled laser was launched on a rotation mount. A half-wave plate (HWP) and polarizer (P) were used to adjust the beam intensity and make the light vertically polarized (necessary for the LCLV). The 50/50 beam splitter and a polarizing beamsplitter (PBS) were placed on a small movable platform on top of a track. The other 50/50 beamsplitter and the other polarizing beamsplitter (PBS) were placed on another movable platform on the same track. The two independently movable platforms allowed us to measure the sensitivity of the system versus the position of the axis of rotation and the inter-

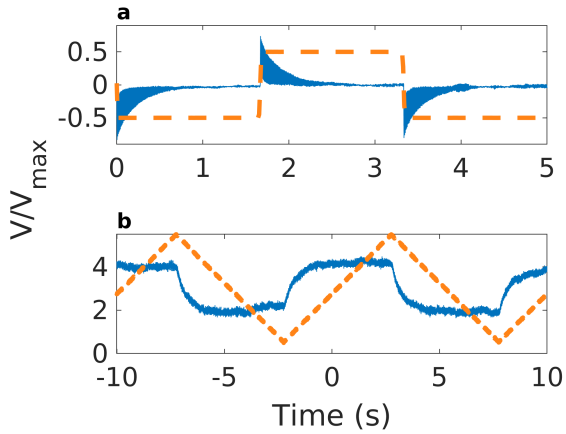


FIG. 2. LCLV response vs input signals. In **a**, an experiment was performed similar to the one in [14] in which the laser and detector are in the laboratory frame and a mirror undergoes linear translations according to a piezo electric input signal. However, the LCLV response always re-equilibrates to zero meaning the LCLV, after relaxing, has no response to constant phase. The experimental setup **b** utilizes the turntable shown in Fig. 1 unlike **a**. A triangle wave is applied to the piezo-actuator which rotates the turntable yielding the expected square wave signal.

ferometer's size. The two polarizing beamsplitters were used in place of mirrors allowing for amplitude control of the two paths. The beams were then directed from the polarizing beamsplitters and the final 50/50 beamsplitter such that they overlapped on the LCLV, but had a relative angle of approximately 10 mRad. The beam waists at the crystal were approximately 2.5 mm with combined intensity of between 1.5 mW and 2.5 mW impinging on the crystal. The beams then pass through a lens and are sent to a balanced detector (BD) in the focal plane of the lens where the beams have separated. The half wave plate (HWP) inside the Mach-Zehnder was adjusted until the beams were intensity-balanced in the detector. The differential balanced detector signal was passed through a low-noise preamp. From the fiber launch to the photodetectors, all equipment is on a single rotating platform atop a turntable driven by a piezoactuator (PZT).

To test the properties of this system, we used a linear piezo-electric actuator to exert a transverse force on the sliding track that also acts as a lever arm. The horizontal distance from the PZT to the axis of rotation was 28 cm. The actuator has a linear response of approximately 60 nm/V. We used a range of amplitudes and frequencies for driving sinusoidal, triangular and square wave oscillations. To demonstrate that we are measuring a frequency offset and not a phase offset, we show that constant phase offsets (*not* phase gradients) do not contribute to the LCLV response. It is important to note that for demonstrating that the system is not sensitive to a constant phase offset we are *not* using a setup in Fig.

1. Rather, the experimental setup for this experiment is based on [14] in which a mirror is linearly translated with respect to the lab frame in which the detector and laser are stationary. Fig. 2**a** shows the response of the LCLV to sudden changes in phase (square wave driven piezo). The Square wave peak-to-peak oscillation corresponds approximately to 17 degrees of phase shift. It can be seen that after an initial sudden change in phase (Doppler shift) the system relaxes once again to the equilibrium position (zero) even though the phase offset remains constant until the next fluctuation. This shows that the LCLV does not respond to constant phase offsets as would be the case in standard Sagnac interferometers undergoing uniform rotation. Hence, the signal must be from frequency shifts and not from phase.

We now turn to the results for the rotating platform shown in Fig. 1 undergoing small oscillatory motions. As can be seen in Fig. 2**b**, a 100 mHz triangle wave with peak to peak angular displacement of approximately 1  $\mu$ Rad is applied to the piezo actuator. At the turning points, there is a sudden acceleration at which there is a rapid change in response. After a finite crystal relaxation time, the system settles to the new equilibrium. For a system of constant rotational speed, the system approaches a constant voltage offset proportional to the speed as expected.

Fig. 3 shows the driving signal and the system response of a 0.003 m<sup>2</sup> interferometer, from a 200 mHz sine wave (a 20 second interval is shown in the inset). The driving signal resulted in a 1.3  $\mu$ Rad/s amplitude for the angular velocity. The length of the total measurement was 1000 seconds. The associated amplitude spectral density of the signal is shown. It can be seen that over a large range of the spectrum, the noise floor is more than 2 orders of magnitude below the shot noise limited (the standard quantum limit) response of a standard passive gyroscope of the same area and approaches 3 orders of magnitude close to 1 Hz. Fig. 4 shows the Allan deviation. The bias drift of the system is calculated to be approximately 17 nRad/s. For emphasis, we note that we are still several orders of magnitude above the shot noise limit of the frequency differential measurement of the LCLV implying that many orders of magnitude improvement are still possible.

We tested the dependence of the system on several parameters: the position of the axis of rotation, the size of the interferometer, and the amplitude and frequency of the piezo actuated movement. For the experiments shown in Fig. 5, triangle waves of various amplitudes and frequencies were used. In Fig. 5**a**, the two movable platforms were translated together, while preserving the interferometer area, by a distance  $\Delta L$  to find the system sensitivity to position of the interferometer relative to the axis of rotation. It can be seen that there is a linear dependence on distance from the axis of rotation, thus differing from other passive optical gyroscopes. Fig. 5**b**

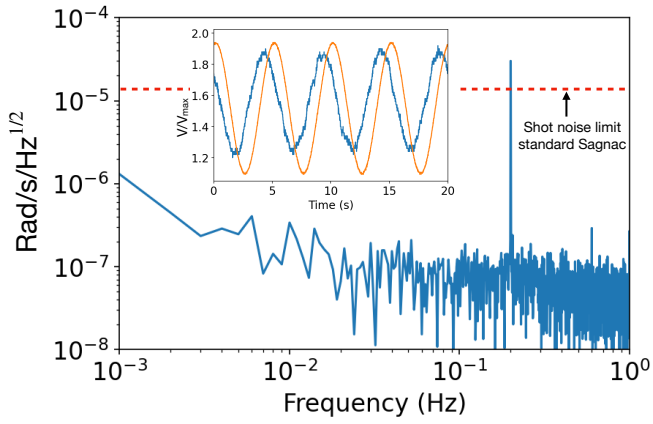


FIG. 3. Amplitude spectral density of the signal response (shown in blue in the inset) of a  $0.003 \text{ m}^2$  interferometer undergoing 200 mHz sinusoidal oscillations from the driven piezoelectric.

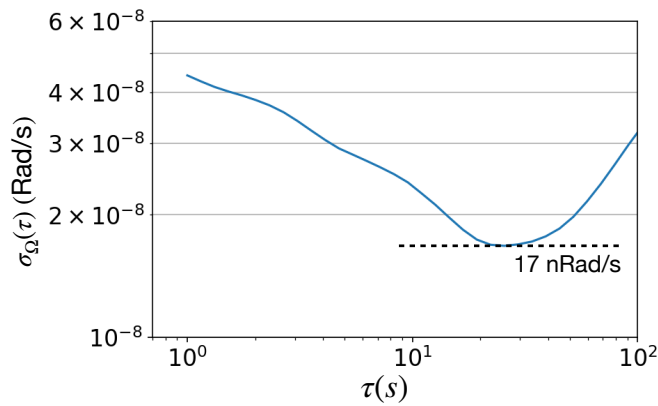


FIG. 4. Allan deviation for  $0.003 \text{ m}^2$  interferometer. The bias drift is observed to be approximately 17 nRad/s.

shows the dependence of the interferometer's sensitivity on the size of the interferometer. We kept one platform fixed and moved the other platform a linear distance  $\Delta L$ . We plot two theoretical behavior lines to show the system sensitivity is linear in the length and not the area, which also differs from other passive optical gyroscopes. Figs. 5c and 5d show that the response of the system is linearly dependent in the amplitude and frequency of the driving oscillation, as expected. It can be seen in Fig. 5c that if the amplitude is too large, when the crystal gain curve is nonlinear, the linear behavior stops. Nonlinear behavior also occurred for oscillation frequencies above 1 Hz (not shown), which is approximately the bandwidth of the gain curve.

A brief discussion of some practical aspects is in order. First, the LCLV, in its current form, is very sensitive to vibrations. Second, the sensitivity and bias of the LCLV is a strong nonlinear function of the laser power. However, with intensities of even just a few 10s of  $\text{mW}/\text{cm}^2$ , the transparent electrodes on the LCLV

can overheat causing a liquid phase transition. This is a technical not a fundamental limitation, which can be improved. Third, air current fluctuations create temporal phase fluctuations, which appear as frequency shifts in the system. Fourth, it is likely that bias drift can be greatly improved. We believe the primary forms of bias drift were from intensity fluctuations and crystal temperature variations, both of which can be actively stabilized. Fifth, we expect the sensitivity of our device can be greatly improved if it is used in a high finesse resonator or loop design. Sixth, there are passive fiber optic gyros with similar sensitivities to ours, but with enclosed areas 10's to 100's of thousands of times larger (see [35] and references therein). Last, we have not yet measured any signal related to the Earth's rotation. This should be manifest by a large frequency offset bias (expected to be several hundred thousand Hertz assuming a linear increase in sensitivity with distance) and a sensitivity to interferometer tilt, neither of which we have observed. We are deeply puzzled by this and are looking for answers to large distance rotations. We note that such a large bias would be so far outside the spectral gain window that the system could not work. We speculate that when the distance to the axis of rotation greatly exceeds the size, the system effectively behaves as if it is undergoing pure translation rather than rotation. We hope this spurs additional theoretical activity in the community.

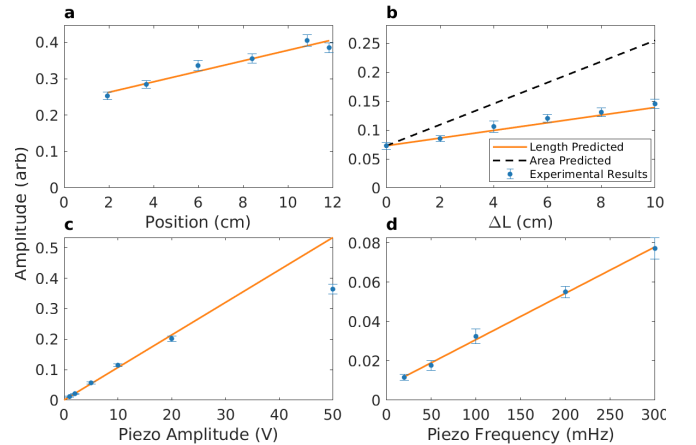


FIG. 5. a) shows the system response as a function of a  $0.003 \text{ m}^2$  interferometer being moved relative to the axis of rotation. b) demonstrates the dependence of the interferometer sensitivity vs a linear change in length. It can be seen that the behavior shows that the system sensitivity is a function of the length and not the area of the interferometer. c) and d) show that the system is linear in the piezo driving amplitude and frequency, respectively.

We believe we have provided compelling evidence of the possibility for enhancing sensitivity relative to some phase estimation problems by considering frequency estimation instead. We have also provided strong evidence of the role of Doppler shifts as being fundamental within

the Sagnac effect. An obvious question is in what other systems such techniques could be applied. For example, it may be possible that ring laser gyros could be improved if the differential spectral estimation of the LCLV can be shown to be better than the typical heterodyne beatnote analysis [25].

JCH acknowledges support from the Hebrew University and H2020-FETOPEN grant 828946. We thank Andrew Jordan for helpful discussions and comments.

- 
- [1] Carlton M Caves, “Quantum-mechanical noise in an interferometer,” *Physical Review D* **23**, 1693 (1981).
- [2] MJ Holland and K Burnett, “Interferometric detection of optical phase shifts at the heisenberg limit,” *Physical review letters* **71**, 1355 (1993).
- [3] Agedi N. Boto, Pieter Kok, Daniel S. Abrams, Samuel L. Braunstein, Colin P. Williams, and Jonathan P. Dowling, “Quantum interferometric optical lithography: Exploiting entanglement to beat the diffraction limit,” *Phys. Rev. Lett.* **85**, 2733–2736 (2000).
- [4] Nicolas Treps, Nicolai Grosse, Warwick P Bowen, Claude Fabre, Hans-A Bachor, and Ping Koy Lam, “A quantum laser pointer,” *Science* **301**, 940–943 (2003).
- [5] Brendon L Higgins, Dominic W Berry, Stephen D Bartlett, Howard M Wiseman, and Geoff J Pryde, “Entanglement-free heisenberg-limited phase estimation,” *Nature* **450**, 393–396 (2007).
- [6] Hugo Cable and Gabriel A. Durkin, “Parameter estimation with entangled photons produced by parametric down-conversion,” *Phys. Rev. Lett.* **105**, 013603 (2010).
- [7] Vittorio Giovannetti, Seth Lloyd, and Lorenzo Maccone, “Advances in quantum metrology,” *Nature photonics* **5**, 222–229 (2011).
- [8] Xiao-Ye Xu, Yaron Kedem, Kai Sun, Lev Vaidman, Chuan-Feng Li, and Guang-Can Guo, “Phase estimation with weak measurement using a white light source,” *Physical review letters* **111**, 033604 (2013).
- [9] Junaaid Aasi, J Abadie, BP Abbott, Richard Abbott, TD Abbott, MR Abernathy, Carl Adams, Thomas Adams, Paolo Addesso, RX Adhikari, *et al.*, “Enhanced sensitivity of the ligo gravitational wave detector by using squeezed states of light,” *Nature Photonics* **7**, 613–619 (2013).
- [10] GJ Pryde, S Slussarenko, MM Weston, HM Chrzanowski, LK Shalm, VB Verma, and SW Nam, “Unconditional shot-noise-limit violation in photonic quantum metrology,” in *Conference on Lasers and Electro-Optics/Pacific Rim* (Optical Society of America, 2018) pp. Th4J–1.
- [11] Benjamin J Lawrie, Paul D Lett, Alberto M Marino, and Raphael C Pooser, “Quantum sensing with squeezed light,” *ACS Photonics* **6**, 1307–1318 (2019).
- [12] Emanuele Polino, Mauro Valeri, Nicolò Spagnolo, and Fabio Sciarrino, “Photonic quantum metrology,” *AVS Quantum Science* **2**, 024703 (2020).
- [13] Benjamin P Abbott, R Abbott, TD Abbott, MR Abernathy, F Acernese, K Ackley, C Adams, T Adams, P Addesso, RX Adhikari, *et al.*, “Gw151226: observation of gravitational waves from a 22-solar-mass binary black hole coalescence,” *Physical review letters* **116**, 241103 (2016).
- [14] Umberto Bortolozzo, Stefania Residori, and John C Howell, “Precision doppler measurements with steep dispersion,” *Optics letters* **38**, 3107–3110 (2013).
- [15] Jon Bernstein, S Cho, AT King, A Kourepenis, P Maciel, and M Weinberg, “A micromachined comb-drive tuning fork rate gyroscope,” in *[1993] Proceedings IEEE Micro Electro Mechanical Systems* (IEEE, 1993) pp. 143–148.
- [16] TL Gustavson, A Landragin, and MA Kasevich, “Rotation sensing with a dual atom-interferometer sagnac gyroscope,” *Classical and Quantum Gravity* **17**, 2385 (2000).
- [17] DS Durfee, YK Shaham, and MA Kasevich, “Long-term stability of an area-reversible atom-interferometer sagnac gyroscope,” *Physical review letters* **97**, 240801 (2006).
- [18] JianCheng Fang and Jie Qin, “Advances in atomic gyroscopes: A view from inertial navigation applications,” *Sensors* **12**, 6331–6346 (2012).
- [19] Yu-Hung Lai, Myoung-Gyun Suh, Yu-Kun Lu, Boqiang Shen, Qi-Fan Yang, Heming Wang, Jiang Li, Seung Hoon Lee, Ki Youl Yang, and Kerry Vahala, “Earth rotation measured by a chip-scale ring laser gyroscope,” *Nature Photonics* **14**, 345–349 (2020).
- [20] WW Chow, J Gea-Banacloche, LM Pedrotti, VE Sanders, Wo Schleich, and MO Scully, “The ring laser gyro,” *Reviews of Modern Physics* **57**, 61 (1985).
- [21] GE Stedman, HR Bilger, Li Ziyuan, MP Poulton, CH Rowe, I Vetharanim, and PV Wells, “Canterbury ring laser and tests for nonreciprocal phenomena,” *Australian Journal of Physics* **46**, 87–102 (1993).
- [22] RB Hurst, GE Stedman, KU Schreiber, RJ Thirkettle, RD Graham, N Rabeendran, and J-PR Wells, “Experiments with an 834 m 2 ring laser interferometer,” *Journal of Applied Physics* **105**, 113115 (2009).
- [23] Karl Ulrich Schreiber, T Klügel, J-PR Wells, RB Hurst, and A Gebauer, “How to detect the Chandler and the annual wobble of the earth with a large ring laser gyroscope,” *Physical Review Letters* **107**, 173904 (2011).
- [24] Karl Ulrich Schreiber and Jon-Paul R Wells, “Invited review article: Large ring lasers for rotation sensing,” *Review of Scientific Instruments* **84**, 041101 (2013).
- [25] Karl Ulrich Schreiber, Heiner Igel, Joachim Wassermann, Andrea Simonelli, André Gebauer, Jan Kodet, and Jon-Paul Wells, “Progress in sagnac interferometry,” in *EGU General Assembly Conference Abstracts* (2018) p. 14308.
- [26] Angela DV Di Virgilio, Andrea Basti, Nicolò Beverini, Filippo Bosi, Giorgio Carelli, Donatella Ciampini, Francesco Fuso, Umberto Giacomelli, Enrico Maccioni, Paolo Marsili, *et al.*, “Underground sagnac gyroscope with sub-prad/s rotation rate sensitivity: Toward general relativity tests on earth,” *Physical Review Research* **2**, 032069 (2020).
- [27] Grigori B Malykin, “The sagnac effect: correct and incorrect explanations,” *Physics-Uspekhi* **43**, 1229 (2000).
- [28] S. Residori, U. Bortolozzo, and J. P. Huignard, “Slow and fast light in liquid crystal light valves,” *Phys. Rev. Lett.* **100**, 203603 (2008).
- [29] Lene Vestergaard Hau, Stephen E Harris, Zachary Dutton, and Cyrus H Behroozi, “Light speed reduction to 17 metres per second in an ultracold atomic gas,” *Nature* **397**, 594–598 (1999).
- [30] U Leonhardt and P Piwnicki, “Ultrahigh sensitivity of slow-light gyroscope,” *Physical Review A* **62**, 055801 (2000).

- (2000).
- [31] F Zimmer and M Fleischhauer, “Sagnac interferometry based on ultraslow polaritons in cold atomic vapors,” *Physical review letters* **92**, 253201 (2004).
  - [32] MS Shahriar, GS Pati, R Tripathi, V Gopal, M Messall, and K Salit, “Ultrahigh enhancement in absolute and relative rotation sensing using fast and slow light,” *Physical review A* **75**, 053807 (2007).
  - [33] Ziv Roi-Cohen, Merav Kahn, Stefania Residori, Umberto Bortolozzo, and John C. Howell, “Reconciling and validating the ashworth-davies doppler shifts of a translating mirror,” 10.48550/ARXIV.2205.03865.
  - [34] DG Ashworth and PA Davies, “The doppler effect in a reflecting system,” *Proceedings of the IEEE* **64**, 280–281 (1976).
  - [35] Yulin Li, Yuwen Cao, Dong He, Yangjun Wu, Fangyuan Chen, Chao Peng, and Zhengbin Li, “Thermal phase noise in giant interferometric fiber optic gyroscopes,” *Optics express* **27**, 14121–14132 (2019).