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Generalized rotation metric in curved spacetime

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Abstract

Haug and Spavieri have recently introduced a new exact solution to Einstein's field equation that takes into account both the electric field energy of the charge and the gravitational field energy of the mass. Here, we introduce a generalized version of this metric where we take into account rotation.

Key Words: Mass-charge, general relativity, strong gravitational fields, black holes, rotation, charge, angular momentum.

Our recent introduced metric [1] that takes into account both electric field energy and the gravitational field energy is given by (in spherical polar coordinates, geometric units):

$$ds^2 = -\Delta dt^2 + \Delta^{-1} dr^2 + r^2 d\Omega^2, \quad (1)$$

where

$$\Delta = A(r) = 1 - \frac{2M}{r} + \frac{M^2}{r^2} + \frac{Q^2}{r^2} = 1 - \frac{2M}{r} \left(1 - \frac{M}{2r}\right) + \frac{Q^2}{r^2}$$

When generalized to also take into account rotation, we obtain the following metric in Boyer–Lindquist [2] coordinates:

$$ds^2 = \frac{\Delta}{\Sigma} (cdt - a \sin^2 \theta d\phi)^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 - \frac{\sin^2 \theta}{\Sigma} (adt - (r^2 + a^2) d\phi)^2 \quad (2)$$

where $\Delta = 1 - \frac{2M}{r} + \frac{M^2}{r^2} + \frac{Q^2}{r^2} + \frac{a^2}{r^2}$, $\Sigma = 1 + \frac{a^2 \cos^2 \theta}{r^2}$, and a is the Kerr parameter $a = \frac{J}{M}$ (angular momentum per unit mass). Our new metric can simply be seen as a further generalization of the Kerr-Newman [3, 4] metric, with the only difference being that $\Delta = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{a^2}{r^2}$ (see [5]), while our metric in addition has the $\frac{M^2}{r^2}$ term. If $a = 0$ one get our mass-charge metric without rotation.

Horizons

The horizons of the new generalized rotation metric are given by:

$$r_h = M \pm \sqrt{M^2 - M^2 - Q^2 - a^2} = M \pm \sqrt{-Q^2 - a^2} \quad (3)$$

compared to horizons of $r_h = M \pm \sqrt{M^2 - Q^2 - a^2}$, in the Kerr-Newman metric, see [6].

Match to the Planck scale for micro black holes

Ultimately, if general relativity is compatible with future quantum gravity, it is expected that the Planck scale will play a central role. This idea was first suggested by Eddington [7], and today it is the prevailing viewpoint among researchers working on quantum gravity theory (see for example [8, 9]), despite opposition from some researchers [10]. We have recently demonstrated that the Schwarzschild metric can only partially satisfy a few properties of the Planck scale for any micro black hole mass candidate.

See Table 1. The generalized rotational metric can satisfy many more properties of the Planck scale (when $Q = a = 0$) than the Schwarzschild metric. One could attempt using alternative mass candidates other than the Planck mass in the Schwarzschild metric. However, we [11] have recently demonstrated that no mass candidate can match more than a few properties of the Planck scale in the Schwarzschild metric.

Table 1: The table shows predictions for a Planck mass micro black hole under different metrics.

	Schwarzschild metric	Haug-Spavieri metric
Mass micro black hole candiate:	m_p	m_p
Escape radius where $v = c$	$2l_p$	l_p
Reduced Compton wavelength $\frac{\hbar}{mc}$	l_p	l_p
Compton wavelength $\frac{\hbar}{mc}$	$2\pi l_p$	$2\pi l_p$
Time to travel radius	$2t_p$	t_p
Escape velocity at $\bar{\lambda}$	$c\sqrt{2}$	c
Density $\frac{m}{\frac{4}{3}\pi R^3}$	$\frac{1}{8}\rho_p$	ρ_p
Planck energy mc^2	E_p	E_p
Surface area acceleration $\kappa = \frac{Gm}{R_s^2}$		
Number of Planck properties matched	3	7
Reduced Compton wavelength match	no	yes

1 Conclusion

We have extended the Haug-Spavieri metric to include angular momentum (rotation) as well. Our mass-charge metric also seems to fit better the Planck properties for micro black holes than the Schwarzschild metric.

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