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Efficient numerical computations of long-wave run-up and their sensitivity to grid nesting

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1 Abstract

2 Computation of long-wave run-up has been of high interest in the fields of ocean sciences and
3 geophysics - particularly for tsunami and river flood modeling. An accurate calculation of run-up and
4 inundation requires the numerical model to account for a sequence of critical processes - each of them
5 posing a different challenge to the numerical solution. This study presents the strategic development of
6 a numerical solution technique for Shallow Water Equations with a focus on accuracy and efficiency for
7 long-wave run-up. The present model is based on an explicit second-order Finite Volume scheme over
8 a staggered grid that efficiently achieves fundamental properties such as well-balance and preservation
9 of shock fronts without the need for computationally expensive solvers. The streamlined code serves
10 as a foundation for the implementation of nested grids. Computations of commonly used long-wave
11 benchmark tests showcase that accurate predictions of local extreme run-up can often be achieved
12 with highly refined yet spatially focused nested grids. Strategic grid nesting can lead to stable and
13 accurate solutions of run-up at locations of interest and reduce the computational load to a fraction of
14 what is usually necessary for a comparable solution over a single grid.

15 1. Introduction

16 The estimation of run-up from long waves is crucial for the assessment and prediction of
17 hazardous flooding scenarios associated with tsunamis and storm surges. As wave run-up is the final
18 stage a water wave undergoes when it reaches the shore, it depends on multiple processes such as wave
19 transformation, breaking, and interaction with dry land. Consequently, a substantial and continuous
20 effort has been made to better understand and compute the run-up processes of long waves (Liu
21 *et al.* (1991)). This includes studies with respect to the derivation of analytical solutions for simplified
22 geometries (e.g., Carrier and Greenspan (1958), Synolakis (1987), Thacker (1981), Mayer and Kriebel
23 (1995)), laboratory experiments (e.g., Hall *et al.* (1953), Briggs *et al.* (1995, 1996)), and development
24 of new numerical methods (e.g., Liu *et al.* (1998), Titov and Synolakis (1995)). The latter provides
25 approximate yet valid run-up solutions in more general settings suitable for the reconstruction of past
26 events, forecasting, and practical engineering applications.

27 Numerical models for long waves, such as tides, storm surges, and tsunamis, have traditionally
28 been based on Shallow Water Equations (SWE). Despite their simplistic hydrostatic assumptions,
29 the SWE provide a valid basis for many long-wave problems and are often preferred over more
30 complete equations thanks to their hyperbolic nature in which shocks can form as part of the solution.
31 These depth-averaged equations have proven to give a reasonable balance between the accuracy and
32 numerical cost (Brocchini and Dodd (2008)) and serve by far as the most commonly used baseline for
33 run-up calculations (e.g., Titov *et al.* (2016), George and LeVeque (2006), Hervouet (2007)). Various
34 numerical techniques have been proposed for the discretization of the SWE, ranging from conventional
35 mesh-based methods such as Finite Difference (FD), Finite Volume (FV), or Finite Element (FE) to
36 unconventional mesh-free methods such as smooth particle hydrodynamics (SPH) Wei *et al.* (2015).
37 The numerical solutions of SWE have been subject to many trends. Earlier solutions were based
38 on traditional FD schemes solved on a staggered grid (Arakawa and Lamb (1981)). This approach
39 has been successfully employed in many first-generation tsunami models (e.g. TUNAMI Imamura

40 (1989), COMCOT Wang (2009)). Several wetting-drying techniques have been proposed to achieve
41 a reasonable representation of the run-up heights. Shuto and Goto (1978) used a staggered scheme
42 with a Lagrangian description for the moving boundaries. Another approach has been based on the
43 Neumann-type technique, which has been used to extrapolate the velocity at the wet-dry fronts (Titov
44 and Synolakis (1998)), while Liu *et al.* (1995) modeled the run-up based on water-level changes through
45 flooding and drying of the cells.

46 FD methods offer a simplified solution for hyperbolic equations. However, they are known to
47 exhibit deficiencies when dealing with flow discontinuities (Wei *et al.* (2006), Olabarrieta *et al.* (2011)),
48 which particularly require local conservation of both mass and momentum. These conservation
49 properties are necessary for the transport of breaking waves toward the shore and, hence, are important
50 for the accuracy of the run-up computation. FV methods, on the other hand, solve the integral form of
51 the SWE and directly benefit from conservation and shock-capturing capabilities. For this reason, FV
52 methods such as Godunov (1954), and Roe (1986) solvers, which were previously used in gas dynamics,
53 have become increasingly popular for the solution of long wave problems. A new generation of
54 tsunami and flooding models has been developed (Berger *et al.* (2011), Macías *et al.* (2017), Dutykh
55 *et al.* (2011), Yuan *et al.* (2020)) based on a finite volume interpretation of the equations, where the
56 in-going and out-going fluxes over a control volume are computed with approximate Riemann solvers
57 (e.g., Roe (1997), Harten *et al.* (1983), and Toro (1989)). These solvers are designed to preserve the
58 hyperbolicity of the governing equations to allow for the formation of discontinuities in the numerical
59 system. However, hyperbolicity can be a source of problems for the solution of the SWE. One drawback
60 of this property is the well-balance between flux gradient, and source terms (Zijlema (2019)). This
61 means that models based on the FV approach often require computationally expensive techniques to
62 ensure the well-balance of the scheme - especially in the presence of dry cells (Wei *et al.* (2006), LeVeque
63 (1998), Zhou *et al.* (2001), Brufau *et al.* (2002), Toro (2001), Audusse *et al.* (2015)). With respect to run-up
64 and as a way to deal with the numerical problems of the moving shoreline, many FV schemes employ
65 an artificial bed-wetting algorithm. These work through the definition of a minimum value of the
66 water depth in the dry cells adjacent to the wet cells for computation of the numerical flux (Toro (2001),
67 Dodd (1998)). Another difficulty for these schemes lies in the conservation of the non-negativity of
68 the water depth - especially in the case of run-down (Audusse *et al.* (2004)). Nevertheless, several
69 operational models such as FUNWAVE (Shi *et al.* (2012), COULWAVE (Kim *et al.* (2009)), and BOSZ
70 (Roeber and Cheung (2012)) successfully utilize these schemes.

71 Another approach for solving the SWE is linked to the use of conservative staggered schemes.
72 These methods benefit from the efficiency and robustness of the FD approximations while achieving
73 conservative and shock-capturing properties. Such schemes have been successfully applied to flows at
74 high Froude numbers, including hydraulic jumps and inundation of dry areas (e.g., Zhou and Stansby
75 (1999), Stelling and Duinmeijer (2003), Madsen *et al.* (2005), Doyen and Gunawan (2014), Yamazaki *et al.*
76 (2009)). These schemes are based on specific FD approximations, which satisfy the Rankine-Hugoniot
77 jump condition at a discrete level (Zijlema (2019)), and achieve valid solutions for rapidly varying
78 flows. The concept from Stelling and Duinmeijer (2003) has been widely used in many operational
79 wave and run-up models (e.g., SWASH Zijlema *et al.* (2011), NEOWAVE Yamazaki *et al.* (2012) and
80 Xbeach Roelvink *et al.* (2018)). This scheme guarantees the positivity of the water depth under the
81 standard Courant-Friedrichs-Lewy (CFL) condition and therefore is very efficient for the computation
82 of large-scale inundation problems.

83 The design of the numerical solutions of long-wave run-up requires taking the multi-scale nature
84 of the problem into account, i.e., large-scale long-wave propagation in combination with the small-scale
85 run-up and inundation processes. High spatial resolution is necessary for a detailed representation
86 of the run-up process. However, computing a high-resolution grid over the entire domain is often
87 unnecessarily expensive and can hinder the applicability of the model to real problems. With the
88 objective of achieving efficient long-wave run-up computations, it is therefore desirable to utilize
89 different grid sizes - each appropriate for the particular problems in the propagation and the run-up

90 stages. Different approaches have been used to obtain local mesh refinement. For example, traditional
 91 nested grid methods have been implemented in tsunami models (Imamura (1989), Wang (2009),
 92 Yamazaki *et al.* (2011)). These techniques are usually built into structured grids where the refinement
 93 arises from the insertion of a sub-grid with higher resolution. The exchange of information between
 94 the grids is achieved either with one-way or two-way interactions. On the other hand, for unstructured
 95 grids, an adaptive mesh refinement technique has been successfully implemented in a number of
 96 long-wave models (e.g., Berger *et al.* (2011), Sætra *et al.* (2015), Donat *et al.* (2014)). The adaptive mesh
 97 refinement generates locally refined cells adapted to the flow condition without the need to use fixed
 98 sub-grid Liang (2011). The refined region is, therefore, able to move with the area of interest, and
 99 unnecessary refinement is avoided. The disadvantage of these methods lies mainly in the complexity
 100 of the grid generation techniques, which require intensive data storage. In addition, the time step
 101 constraint is bound to the smallest grid cell that can hinder the efficiency of the implementation for
 102 explicit schemes (Debreu and Blayo (2008)).

103 This paper presents the rigorous development of a stable and accurate numerical framework for
 104 the computation of long-wave run-up. We address the details of the numerical scheme and outline the
 105 strategy for grid nesting to achieve a fast and low-cost numerical tool for run-up computations. The
 106 verification process checks off the fundamental properties necessary for the computation of run-up:
 107 shock-capturing capabilities, moving boundaries with bottom friction, and exchange of information
 108 across nested grids. Two standard tsunami benchmark datasets are then employed to demonstrate the
 109 sensitivity of long-wave run-up to the overall grid resolution as well as to the extent of the nested grid
 110 and the refinement factor.

111 2. Methodology

112 2.1. Governing equations

113 The present study considers the two-dimensional, depth-averaged Shallow Water equations
 114 (SWE). These equations provide a powerful baseline for long-wave modeling thanks to their wave-like
 115 hyperbolic structure. Moreover, the SWE serve as the backbone for many numerical models that
 116 address nearshore wave propagation and inundation. This is the case for dispersive Boussinesq-type
 117 and non-hydrostatic models in which the governing equations contain the SWE as a subset.

118 The SWE are derived from the Navier-stokes equations under the following assumptions: (a)
 119 the pressure is hydrostatic, and (b) the vertical distribution of the horizontal velocity is uniform (no
 120 variation). Under these assumptions, the equations take the following differential form in Cartesian
 121 coordinates:

$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} = 0 \quad (1)$$

$$\frac{\partial hu}{\partial t} + \frac{\partial hu^2}{\partial x} + \frac{\partial huv}{\partial y} + gh \frac{\partial \eta}{\partial x} = -gn^2 \frac{u \sqrt{u^2 + v^2}}{h^{1/3}} \quad (2)$$

$$\frac{\partial hv}{\partial t} + \frac{\partial huv}{\partial x} + \frac{\partial hv^2}{\partial y} + gh \frac{\partial \eta}{\partial y} = -gn^2 \frac{v \sqrt{u^2 + v^2}}{h^{1/3}} \quad (3)$$

122 We define t as the time variable, x and y are the space variables, h is the water depth, u and v
 123 are the depth-averaged velocities in the x and y -directions, respectively. η refers to the free surface
 124 elevation: $\eta(x, y, t) = h(x, y, t) - d(x, y)$, where d is the positive bottom topography (1). The constant
 125 g is the gravitational acceleration and n is the Manning roughness coefficient $[s.m^{-1/3}]$.

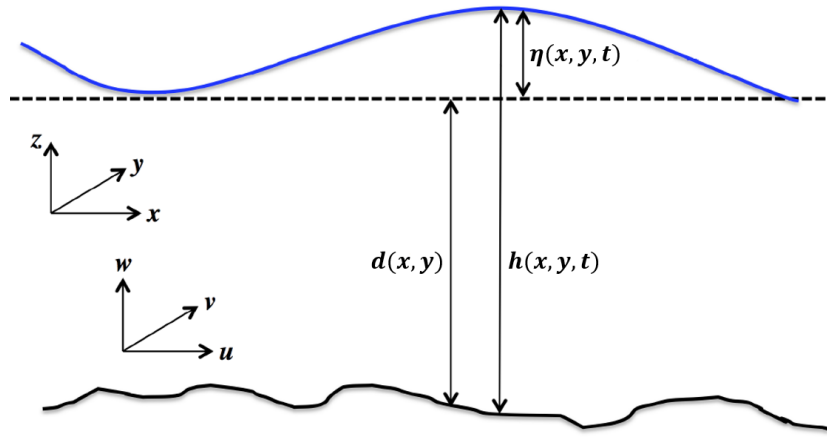


Figure 1. Definition sketch for the free surface flow problem with key variables.

126 We write the SWE, Eqs. 1-3 in a conservative form to ensure the conservation of mass and
 127 momentum across discontinuities. The conserved variables, in this case, are the total water depth h
 128 and its product with the velocity components: hu and hv . In this form of the equations, we avoid the
 129 splitting of the free surface gradient into an artificial flux gradient and a source term that includes
 130 the effect of bed slope. This improves the well-balance properties of the numerical solution. The
 131 preservation of shocks and discontinuities will consequently depend on the numerical approximations
 132 of the scheme, which have to satisfy the Rankine-Hugoniot jump condition at the discrete level (Zijlema
 133 (2019)).

134 We introduce the auxiliary variables p and q , which denote the mass fluxes:

$$p = hu \quad q = hv \quad (4)$$

135 We rewrite the SWE in the following form:

$$\frac{\partial h}{\partial t} + \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y} = 0 \quad (5)$$

$$\frac{\partial hu}{\partial t} + \frac{\partial pu}{\partial x} + \frac{\partial qu}{\partial y} + gh \frac{\partial \eta}{\partial x} = -gn^2 \frac{u \sqrt{u^2 + v^2}}{h^{1/3}} \quad (6)$$

$$\frac{\partial hv}{\partial t} + \frac{\partial pv}{\partial x} + \frac{\partial qv}{\partial y} + gh \frac{\partial \eta}{\partial y} = -gn^2 \frac{v \sqrt{u^2 + v^2}}{h^{1/3}} \quad (7)$$

136 It is worth mentioning that in the momentum equations Eqs. 6 and 7, the variables hu and hv in
 137 the local acceleration and the variables p and q in the convective acceleration play different roles. The
 138 former is a storage quantity, while the latter is a transport quantity. Consequently, these terms are
 139 approximated differently, and in order to avoid confusion, we avoid using the same symbols.

140 2.2. Conservative Staggered scheme

141 As detailed in the introduction, a variety of numerical schemes have previously been developed
 142 for the solution of the SWE. The choice of the numerical scheme depends mainly on the problem being
 143 addressed, which defines the requirements for the scheme properties. For the computation of long
 144 wave run-up, a conservative shock-capturing scheme is crucial for the preservation of momentum and
 145 propagation of shocks at the correct speed and height. Other important properties are well-balance
 146 and non-negativity of the water depth to ensure mass conservation across wet/dry transitions without
 147 parasitic waves. This adds to the stability and robustness of the numerical model - particularly over
 148 irregular bathymetry. Consequently, a scheme that provides these features is suitable for computing
 149 wave-breaking processes and, subsequently, wave run-up estimations.

150 In this study, the objective is to develop a lightweight yet accurate and stable solution structure
 151 that keeps the computational expenses at a low level. For these reasons, we utilize a conservative
 152 scheme on a staggered grid where the numerical fluxes are computed with simple FD approximations
 153 instead of Riemann solvers. The SWE variables, in this case, are approximated on a staggered C-grid:
 154 the total water depth h and the bed topography d are defined at the cell center, and the depth-averaged
 155 velocities (u, v) are stored at the cell interfaces (see Figure. 2).

156 For the discretization, we consider a 2D rectangular computational domain with a uniform grid
 157 spacing of Δx and Δy in the x and y directions, respectively. The variables stored at the cell center
 158 are expressed as $x_{i,j}$, where i and j are the spatial indices in the x and y directions. The variables stored
 159 at the cell interface are denoted by $x_{i\pm\frac{1}{2},j}$ or $x_{i,j\pm\frac{1}{2}}$ in the x- and y- directions, respectively. The time
 160 stepping is based on discrete, non-uniform time intervals $t^n = n\Delta t$, where n is the time index and Δt
 161 is the adaptive time step. The value of each variable a at the time level t^n is denoted with a^n . The water
 162 depth h is evaluated at each time step level $t = n\Delta t$, whereas, the depth-averaged velocities u and v
 163 are evaluated halfway between the present and the following time step $t = \left(n + \frac{1}{2}\right)\Delta t$. This leads to
 164 the staggering of spatial and temporal information and facilitates consistent second-order accuracy in
 165 space and time.

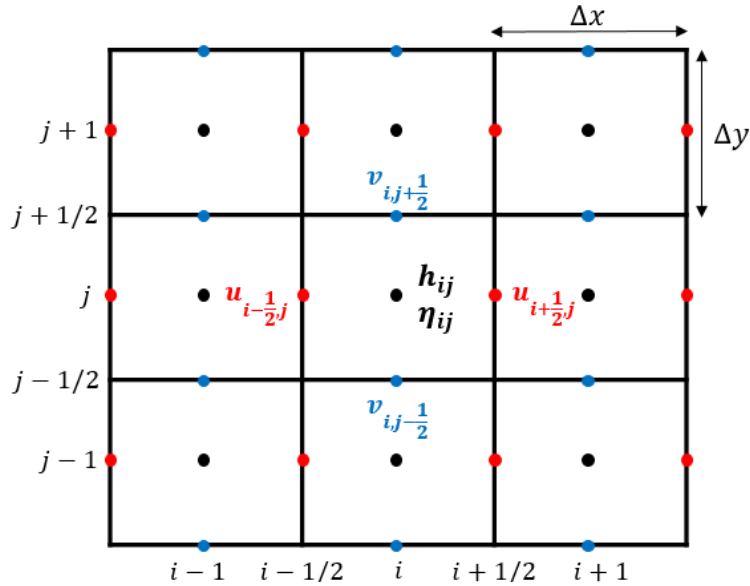


Figure 2. Schematic of the 2D staggered grid

166 The present scheme first requires the solution of the continuity equation, which is subsequently
 167 used in the momentum equation. The discretization of the continuity equation, Eq. 5, is expressed as:

$$\frac{h_{i,j}^{n+1} - h_{i,j}^n}{\Delta t} + \frac{p_{i+\frac{1}{2},j}^n - p_{i-\frac{1}{2},j}^n}{\Delta x} + \frac{q_{i,j+\frac{1}{2}}^n - q_{i,j-\frac{1}{2}}^n}{\Delta y} = 0 \quad (8)$$

168 Where:

$$p_{i+\frac{1}{2},j}^n = \hat{h}_{i+\frac{1}{2},j}^n u_{i+\frac{1}{2},j}^{n+\frac{1}{2}} \quad q_{i,j+\frac{1}{2}}^n = \hat{h}_{i,j+\frac{1}{2}}^n v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} \quad (9)$$

169 $\hat{h}_{i\pm\frac{1}{2},j}^n$ and $\hat{h}_{i,j\pm\frac{1}{2}}^n$ are the water depths at the cell interfaces computed with an upwind
 170 approximation:

$$\hat{h}_{i+\frac{1}{2},j}^n = \begin{cases} h_{i,j}^n & \text{if } u_{i+\frac{1}{2},j}^{n+\frac{1}{2}} \geq 0 \\ h_{i+1,j}^n & \text{if } u_{i+\frac{1}{2},j}^{n+\frac{1}{2}} < 0 \end{cases} \quad \hat{h}_{i,j+\frac{1}{2}}^n = \begin{cases} h_{i,j}^n & \text{if } v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} \geq 0 \\ h_{i,j+1}^n & \text{if } v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} < 0 \end{cases} \quad (10)$$

171 The next step is the solution of the momentum equation, Eq. 6. First, we consider the momentum
 172 equation without the friction term; the approximation of this term will be detailed later. We employ
 173 the FD approximations recommended in Zijlema (2019) to achieve conservation of the momentum flux
 174 across discontinuities, as:

$$\begin{aligned} & \frac{\bar{h}_{i+\frac{1}{2},j}^{n+1} u_{i+\frac{1}{2},j}^{n+\frac{3}{2}} - \bar{h}_{i+\frac{1}{2},j}^n u_{i+\frac{1}{2},j}^{n+\frac{1}{2}}}{\Delta t} + \frac{\hat{u}_{i+1,j}^{n+\frac{1}{2}} \bar{p}_{i+1,j}^n - \hat{u}_{i,j}^{n+\frac{1}{2}} \bar{p}_{i,j}^n}{\Delta x} + \frac{\hat{u}_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} \bar{q}_{i+\frac{1}{2},j+\frac{1}{2}}^n - \hat{u}_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}} \bar{q}_{i+\frac{1}{2},j-\frac{1}{2}}^n}{\Delta y} \\ & = -g \bar{h}_{i+\frac{1}{2},j}^{n+1} \frac{\eta_{i+1,j}^{n+1} - \eta_{i,j}^{n+1}}{\Delta x} \end{aligned} \quad (11)$$

175 Regarding the free surface gradient term, the use of the updated variable $\bar{h}_{i+\frac{1}{2},j}^{n+1}$ is necessary for the
 176 scheme to guarantee the entropy inequality as demonstrated in Doyen and Gunawan (2014). Further,
 177 it is necessary to approximate the convective acceleration with an upwind scheme, where the mass
 178 fluxes p and q are the criteria for upwinding and the velocities u and v are the upwinded quantities:

$$\hat{u}_{i,j}^{n+\frac{1}{2}} = \begin{cases} u_{i-\frac{1}{2},j}^{n+\frac{1}{2}} & \text{if } \bar{p}_{i,j}^n \geq 0 \\ u_{i+\frac{1}{2},j}^{n+\frac{1}{2}} & \text{if } \bar{p}_{i,j}^n < 0 \end{cases} \quad \hat{u}_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} = \begin{cases} u_{i+\frac{1}{2},j}^{n+\frac{1}{2}} & \text{if } \bar{q}_{i+\frac{1}{2},j+\frac{1}{2}}^n \geq 0 \\ u_{i+\frac{1}{2},j+1}^{n+\frac{1}{2}} & \text{if } \bar{q}_{i+\frac{1}{2},j+\frac{1}{2}}^n < 0 \end{cases} \quad (12)$$

179 It is important to note, that a reversed approach where the upwinded quantities are p and q , leads
 180 to errors in the computation of the momentum fluxes across discontinuities as demonstrated in Zijlema
 181 (2019).

182 Since the mass fluxes p and q are continuous quantities, an averaged approximation of these
 183 quantities can be applied in the computation of the convective acceleration terms:

$$\bar{p}_{i,j}^n = \frac{1}{2} (p_{i+\frac{1}{2},j}^n + p_{i-\frac{1}{2},j}^n) \quad \bar{q}_{i+\frac{1}{2},j+\frac{1}{2}}^n = \frac{1}{2} (q_{i,j+\frac{1}{2}}^n + q_{i+1,j+\frac{1}{2}}^n) \quad (13)$$

184 The flow depth, originally defined at the cell centroid, is approximated at the cell interface with
 185 arithmetic averaging to be used in the computation of the local acceleration:

$$\bar{h}_{i+\frac{1}{2},j}^{n+1} = \frac{1}{2} (h_{i,j}^{n+1} + h_{i+1,j}^{n+1}) \quad (14)$$

186 Finally, the momentum equation, Eq. 7, in the y -direction is solved in an analogous way as:

$$\begin{aligned} & \frac{\bar{h}_{i,j+\frac{1}{2}}^{n+1} v_{i,j+\frac{1}{2}}^{n+\frac{3}{2}} - \bar{h}_{i,j+\frac{1}{2}}^n v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta t} + \frac{\hat{v}_{i,j+1}^{n+\frac{1}{2}} \bar{q}_{i,j+1}^n - \hat{v}_{i,j}^{n+\frac{1}{2}} \bar{q}_{i,j}^n}{\Delta y} + \frac{\hat{v}_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} \bar{p}_{i+\frac{1}{2},j+\frac{1}{2}}^n - \hat{v}_{i-\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} \bar{p}_{i-\frac{1}{2},j+\frac{1}{2}}^n}{\Delta x} \\ & = -g \bar{h}_{i,j+\frac{1}{2}}^{n+1} \frac{\eta_{i,j+1}^{n+1} - \eta_{i,j}^{n+1}}{\Delta y} \end{aligned} \quad (15)$$

187 where:

$$\vartheta_{i,j}^{n+\frac{1}{2}} = \begin{cases} v_{i,j-\frac{1}{2}}^{n+\frac{1}{2}} & \text{if } \bar{q}_{i,j}^n \geq 0 \\ v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} & \text{if } \bar{q}_{i,j}^n < 0 \end{cases} \quad \vartheta_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} = \begin{cases} v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} & \text{if } \bar{p}_{i+\frac{1}{2},j+\frac{1}{2}}^n \geq 0 \\ v_{i+1,j+\frac{1}{2}}^{n+\frac{1}{2}} & \text{if } \bar{p}_{i+\frac{1}{2},j+\frac{1}{2}}^n < 0 \end{cases} \quad (16)$$

188 and:

$$\bar{q}_{i,j}^n = \frac{1}{2} (q_{i,j+\frac{1}{2}}^n + q_{i,j-\frac{1}{2}}^n) \quad \bar{p}_{i+\frac{1}{2},j+\frac{1}{2}}^n = \frac{1}{2} (p_{i+\frac{1}{2},j+1}^n + p_{i+\frac{1}{2},j}^n) \quad (17)$$

189 The flow depth in this case is approximated as:

$$\bar{h}_{i,j+\frac{1}{2}}^{n+1} = \frac{1}{2} (h_{i,j}^{n+1} + h_{i,j+1}^{n+1}) \quad (18)$$

190 2.2.1. Second-order numerical accuracy

191 Staggering of the variables both in space and time and utilization of the Leapfrog scheme lead to
 192 second-order accuracy for both the continuity and the momentum equations, except for the advection
 193 terms, (Stelling and Duinmeijer (2003), Zijlema *et al.* (2011)). The flux terms are responsible for
 194 transporting the conserved quantities, and consequently, the construction of the advection terms with
 195 upwind differencing is necessary for the robustness and stability of the computed solution. However,
 196 first-order upwind methods are diffusive, and it is, therefore, useful to target second-order accuracy
 197 for all terms in the equations. One way to counter unnecessary numerical dissipation is based on
 198 extending the upwind scheme to the second order in combination with a slope limiter.

199 The approximations in Eqs. 10, 12, 16, and 17 can be improved by including two neighbouring
 200 data points instead of only one, as is the case in the first-order upwind approach. The second-order
 201 upwind discretization is shown for Eq. 10 and applied to Eqs. 12, 16, and 17 in the same way.

$$\hat{h}_{i+\frac{1}{2},j}^n = \begin{cases} h_{i,j}^n + \frac{1}{2}\psi\left(r_{i+\frac{1}{2},j}^+\right)\left(h_{i,j}^n - h_{i-1,j}^n\right), & \text{if } u_{i+\frac{1}{2},j}^n \geq 0 \\ h_{i+1,j}^n + \frac{1}{2}\psi\left(r_{i+\frac{1}{2},j}^-\right)\left(h_{i+1,j}^n - h_{i+2,j}^n\right), & \text{if } u_{i+\frac{1}{2},j}^n < 0 \end{cases} \quad (19)$$

202 $r_{i+\frac{1}{2},j}^+$ and $r_{i+\frac{1}{2},j}^-$ are respectively the left and right gradients of the flow depth:

$$r_{i+\frac{1}{2},j}^+ = \frac{h_{i+1,j}^n - h_{i,j}^n}{h_{i,j}^n - h_{i-1,j}^n}, \quad r_{i+\frac{1}{2},j}^- = \frac{h_{i+1,j}^n - h_{i,j}^n}{h_{i+2,j}^n - h_{i+1,j}^n} \quad (20)$$

203 $\psi(r)$ is the slope limiter function, which locally reduces the solution from second to first order.
 204 This is often necessary at locations with opposite slopes, zero gradients, or sharp transitions. Here, a
 205 Generalized MinMod slope limiter is used :

$$\phi(r, \theta) = \max\left(0, \min\left(\theta r, \frac{1+r}{2}, \theta\right)\right) \quad (21)$$

206 θ is a parameter that controls the diffusivity. The generalized MinMod limiter is most dissipative
 207 for $\theta = 1$ when it reduces to the traditional MinMod limiter, and it is least diffusive for $\theta = 2$.

208 A predictor-corrector method can be used to improve the temporal accuracy of the advection
 209 terms to retain second-order accuracy in time. Here, we employ the Total Variation Diminishing (TVD)
 210 Runge-Kutta method. This method enhances the accuracy of the scheme in time while maintaining
 211 the strong stability property of the first-order Euler integration (Gottlieb *et al.* (2001)). It is worth
 212 mentioning that other time integration methods can be combined with the above-described spatial

213 discretization. For example, [Zijlema et al. \(2011\)](#) used a MacCormack approach for the second-order
214 integration in time.

215 We split the SWE equations into a convective acceleration term \mathcal{F} and a free surface gradient
216 term \mathcal{G} , which simplifies the description of the multi-step method:

$$\frac{\partial U}{\partial t} + \mathcal{F}(U) + \mathcal{G}(U) = 0 \quad (22)$$

Where:

$$U = \begin{bmatrix} h \\ hu \\ hv \end{bmatrix} \quad \mathcal{F}(U) = \begin{bmatrix} \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y} \\ \frac{\partial pu}{\partial x} + \frac{\partial qu}{\partial y} \\ \frac{\partial pv}{\partial x} + \frac{\partial qv}{\partial y} \end{bmatrix} \quad \mathcal{G}(U) = \begin{bmatrix} 0 \\ gh \frac{\partial \eta}{\partial x} \\ gh \frac{\partial \eta}{\partial y} \end{bmatrix} \quad (23)$$

217 The discretization described in Eqs. 8, 11, and 15 can be summarized in the following expression:

$$U_{ij}^{n+1} = U_{ij}^n - \Delta t \mathcal{F}(U_{ij}^n) - \Delta t \mathcal{G}(U_{ij}^{n+1}) \quad (24)$$

218 Where

$$U_{ij}^n = \begin{bmatrix} h_{ij}^n \\ \bar{h}_{i+\frac{1}{2},j}^n u_{i+\frac{1}{2},j}^{n+\frac{1}{2}} \\ \bar{h}_{i,j+\frac{1}{2}}^n v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} \end{bmatrix} \quad (25)$$

219 At each time step, the variables (h, hu, hv) are solved using a two-stage time integration with an
220 intermediate solution obtained by the predictor step. In the first step, we solve the equations with only
221 the advection terms on the right-hand side:

$$U_{ij}^* = U_{ij}^n - \Delta t \mathcal{F}(U_{ij}^n) \quad (26)$$

222 This leads to a predictor solution of first-order accuracy for the complete continuity equation
223 and incomplete momentum equations due to the lack of source terms. In the second step, the surface
224 gradient terms are added to the momentum equations, and the predicted variables are corrected to full
225 second-order accuracy in time by:

$$U_{ij}^{n+1} = \frac{\Delta t}{2} (U_{ij}^n + U_{ij}^*) - \frac{\Delta t}{2} \mathcal{F}(U_{ij}^*) - \Delta t \mathcal{G}(U_{ij}^{n+1}) \quad (27)$$

226 It is important to emphasize that the predictor step of the time integration should only involve
227 the convective acceleration terms. The source terms attain second-order accuracy by staggering the
228 flow speed variables in time, and an application of Eq. 26 to the source terms would lead to inaccurate
229 results.

230 The last term on the right-hand side of Eq. 27 applies only to the momentum equations and
231 involves the corrected flow depth value h^{n+1} . This completes the fully explicit time integration where
232 no system of equations with data dependencies has to be solved.

233 2.2.2. Flooding and drying

234 The wetting and drying process requires the model's performance for two fundamental processes.
235 The well-balance and the preservation of positivity of the water depth across wet/dry boundaries.

236 An explicit time integration is bound to the Courant–Friedrichs–Lewy (CFL) condition given by:

$$\left[\frac{\max \left(u_{i+\frac{1}{2},j}^{n+\frac{1}{2}}, 0 \right) - \min \left(u_{i-\frac{1}{2},j}^{n+\frac{1}{2}}, 0 \right)}{\Delta x} + \frac{\max \left(v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}}, 0 \right) - \min \left(v_{i,j-\frac{1}{2}}^{n+\frac{1}{2}}, 0 \right)}{\Delta y} \right] \Delta t \leq 1 \quad (28)$$

237 Under this condition, the scheme preserves the non-negativity of the water depth (Zijlema (2019),
 238 Gunawan (2015)). This has the advantage that the run-up and inundation limits are inherent solutions
 239 of the numerical scheme and are not subject to additional ad-hoc flooding and drying treatments or
 240 require particular restructuring of the flux and source terms.

241 The time step that satisfies Eq. 28 is computed in dependence of the Courant number $Cr \leq 1$, grid
 242 spacing, and maximum flow speed as:

$$\Delta t = \frac{Cr}{\max \left(\frac{|u_{i+\frac{1}{2},j}^{n+\frac{1}{2}}| + \sqrt{g\hat{h}_{i+\frac{1}{2},j}^n}}{\Delta x}, \frac{|v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}}| + \sqrt{g\hat{h}_{i,j+\frac{1}{2}}^n}}{\Delta y} \right)} \quad (29)$$

243 Since the flow depth can become arbitrarily small at the wet-dry transitions and, therefore, can
 244 lead to excessively high-velocity values, it makes sense to limit the minimum flow depth at the run-up
 245 front to a physically and numerically meaningful level. For efficiency reasons, the velocity values can
 246 be set to zero when the local water level falls below a threshold value h_{min} , and the calculation of the
 247 momentum equations can be skipped.

$$u_{i+\frac{1}{2},j}^{n+\frac{3}{2}} = 0 \quad \text{if} \quad \frac{h_{i,j}^{n+1} + h_{i+1,j}^{n+1}}{2} < h_{min} \quad (30)$$

$$v_{i,j+\frac{1}{2}}^{n+\frac{3}{2}} = 0 \quad \text{if} \quad \frac{h_{i,j}^{n+1} + h_{i,j+1}^{n+1}}{2} < h_{min} \quad (31)$$

248 The value of h_{min} should be chosen as small as possible to accurately resolve the wet-dry front
 249 (Toro (2001)), but large enough to avoid physically questionable values in the local flow speed, which
 250 can cause excessively small time steps as shown in Eq. 29. It should be noted that the present scheme
 251 is not particularly sensitive to this threshold, and values between 10^{-8} and 10^{-4} m lead to virtually
 252 identical results. For the sake of quality verification and validation, we are using $h_{min} = 10^{-8}$ m in the
 253 subsequent examples.

254 2.2.3. Friction term

255 The friction terms added to the momentum equations are discretized as:

$$gn^2 \frac{u_{i+\frac{1}{2},j}^{n+\frac{3}{2}} \sqrt{\left(u_{i+\frac{1}{2},j}^{n+\frac{1}{2}} \right)^2 + \left(\bar{v}_{i+\frac{1}{2},j}^{n+\frac{1}{2}} \right)^2}}{\left(\bar{h}_{i+\frac{1}{2},j}^{n+1} \right)^{1/3}} \quad \text{and} \quad gn^2 \frac{v_{i,j+\frac{1}{2}}^{n+\frac{3}{2}} \sqrt{\left(\bar{u}_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} \right)^2 + \left(v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} \right)^2}}{\left(\bar{h}_{i,j+\frac{1}{2}}^{n+1} \right)^{1/3}} \quad (32)$$

256 Where:

$$\begin{aligned} \bar{v}_{i+\frac{1}{2},j}^{n+\frac{1}{2}} &= \frac{1}{4} \left(v_{i,j-\frac{1}{2}}^{n+\frac{1}{2}} + v_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} + v_{i+1,j-\frac{1}{2}}^{n+\frac{1}{2}} + v_{i+1,j+\frac{1}{2}}^{n+\frac{1}{2}} \right) \\ \bar{u}_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} &= \frac{1}{4} \left(u_{i-\frac{1}{2},j}^{n+\frac{1}{2}} + u_{i+\frac{1}{2},j}^{n+\frac{1}{2}} + u_{i-\frac{1}{2},j+1}^{n+\frac{1}{2}} + u_{i+\frac{1}{2},j+1}^{n+\frac{1}{2}} \right) \end{aligned} \quad (33)$$

257 and n , with units [$sm^{-1/3}$], is the Manning roughness coefficient representing the bottom property.
 258 Using the variables $\left(h_{i,j+\frac{1}{2}}^{n+1}, u_{i+\frac{1}{2},j}^{n+\frac{3}{2}}, v_{i+\frac{1}{2},j}^{n+\frac{1}{2}}\right)$ from the next time step in the friction terms improves the
 259 accuracy and the robustness of the solution (Zijlema *et al.* (2011)).

260 Some of the following numerical tests, presented in section 3, are computed with the
 261 Darcy-Weisbach formulation, which requires replacement of the term $\frac{gm^2}{h^{1/3}}$ by $\frac{f}{8}$. f is the dimensionless
 262 Darcy-Weisbach coefficient.

263 2.3. Nested grid method

264 The accuracy and applicability of a numerical model for free surface flows can substantially
 265 benefit from an efficient mesh refinement technique. Here, we concentrate on the nested grid method,
 266 which provides a reasonable trade-off between computational complexity and the gain in accuracy of
 267 the numerical solution for long wave run-up. A simplified approach for mesh refinement involves
 268 the insertion of a high-resolution Child grid into a surrounding Parent grid of coarser resolution. The
 269 grids are herein fixed in space and predefined before the computation is executed. The SWE are solved
 270 independently in each grid. Consequently, the overall solution structure of the governing equations
 271 remains untouched as the exchange between the grids only requires interpolation of the key variables.

272 The staggered C-grid has been widely used in combination with embedded grid models due to
 273 its simplicity and conservative properties (Debreu and Blayo (2008), Liu *et al.* (1995), Herzfeld and
 274 Rizwi (2019)). In this study, we build the nested grid approach on some of the techniques used and
 275 validated by several previously developed tsunami models (Imamura (1989), Wang (2009)). Several
 276 performances are expected from a functioning grid nesting technique:

277 **Data exchange:** The exchange of information between the grids occurs along the boundary of the
 278 inner grid. The Parent grid provides the boundary conditions to the Child grid in a one-way interaction.
 279 The flux variables (i.e., hu and hv) from the coarse grid are linearly interpolated in time and space and
 280 then dynamically imposed in each time step as boundary conditions to the solution of the Child grid
 281 (See Figure. 3). For a two-way interaction, the high-resolution free-surface elevation from the Child
 282 grid is used to update the information in the Parent grid via an averaging operator. The update of the
 283 free-surface only occurs inside the feedback interface in the Parent grid (See Figure. 3), rather than
 284 in the domain occupied by the Child grid. Several authors have proposed separating the feedback
 285 interface from the dynamic interface where the boundary values are interpolated (Phillips and Shukla
 286 (1973), Zhang *et al.* (1986), Oey and Chen (1992)). This separation helps to avoid inconsistencies between
 287 the solutions and stability problems that often arise from forcing the solution of the Parent grid with
 288 the updated values of the inner grid (Debreu and Blayo (2008)).

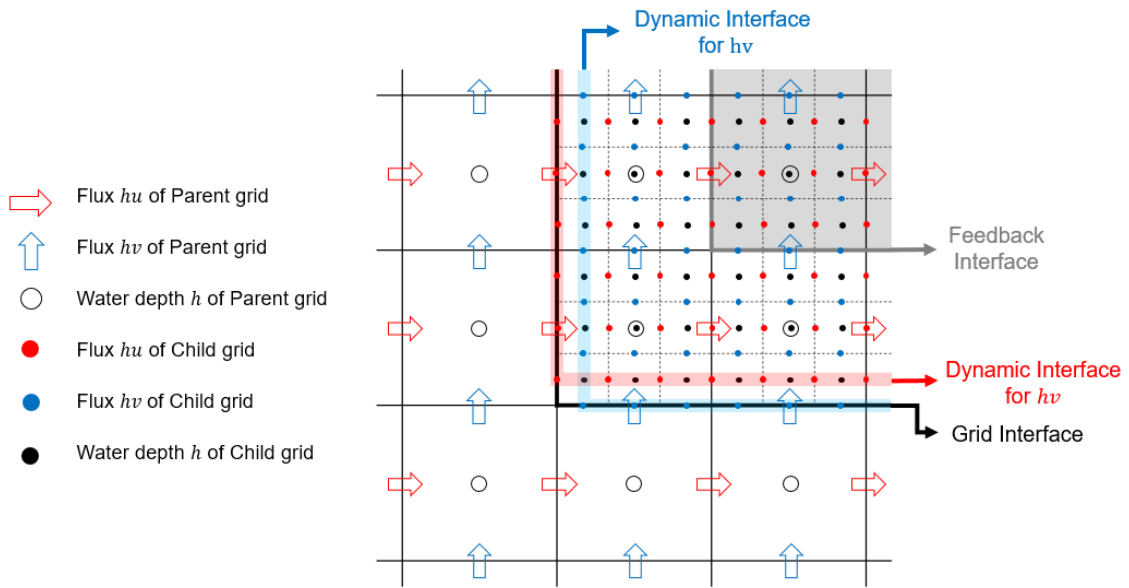


Figure 3. Schematic illustrating the two-way nesting process on a Arakawa C grid

289 **Time synchronization** : The use of an explicit time integration means that the model needs to
 290 verify the CFL stability condition, and the ratio $\Delta t/\Delta x$ must be kept smaller than a given value on
 291 the whole grid hierarchy. Consequently, a temporal refinement must be applied in addition to the
 292 spatial mesh refinement. The integration algorithm for a time refinement of 3 is depicted in Figure. 4.
 293 The model is first integrated on the Parent grid Ω_p with a time step equal to Δt_{p1} , the model is then
 294 advanced multiple times on the Child grid Ω_c to reach the same physical time as the outer grid. To
 295 synchronize the two solutions, the last time step in the inner grid is imposed: $\Delta t_{c3} = \Delta t_{p1} - \sum \Delta t_{ci}$.

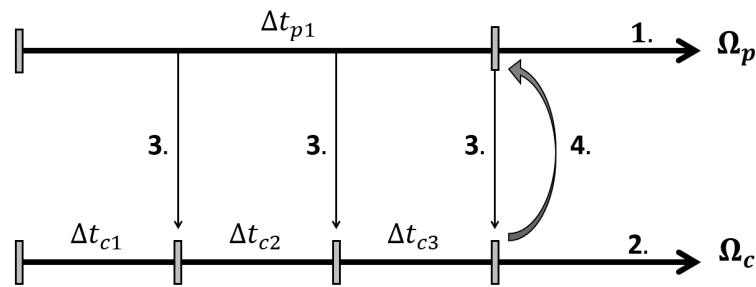


Figure 4. Schematic illustrating the two-way nesting process

- 296 • 1 : Model integration on the Parent grid Ω_p
- 297 • 2 : Model integration on the Child grid Ω_c
- 298 • 3 : Time and space interpolation of the boundary values
- 299 • 4 : Update of the Parent Grid in feedback domain

300 3. Verification

301 A systematical analysis of a numerical solution for long-wave run-up requires benchmarking.
 302 Since the model was developed from scratch and involves a combination of adapted numerical features,
 303 it is required to first verify its performance for idealized flow problems, for which analytical solutions

304 have been derived. These tests examine the model's ability to handle important flow processes, such
 305 as flow discontinuities and wet/dry transitions. These features are particularly critical for the quality
 306 of the computed run-up and can often pose numerical challenges. The implementation of the nested
 307 grid approach is then verified with a 2D moving boundaries problem.

308 3.1. Shock-capturing capabilities

309 Shock-capturing schemes refer to numerical methods that can directly solve wave propagation
 310 with large gradients and rapid changes in the free-surface and velocity regimes. Such nonlinear
 311 phenomena are present in many wave problems (e.g., wave breaking, dam-break wave propagation,
 312 and propagation of wet/dry fronts). Consequently, a lot of effort is made to compute shock waves as
 313 part of the complete solution (Toro (2001)). A stable numerical solution for shock waves targets the
 314 generation and propagation of an oscillation-free discontinuity without excessive smearing across the
 315 shock front.

316 In the following, we examine the solution of the present model in handling discontinuities
 317 and assess the accuracy and quality of the results. Since many shock-capturing flow models are
 318 built around Riemann solvers, we compare the solution from the presented scheme, referred to as
 319 "Present Scheme", with the solution obtained by a 1D HLLC Riemann solver ("HLLC Scheme"). The
 320 HLLC scheme used for comparison was coded based on the techniques given by Toro (2001). For
 321 consistency with the presented scheme, the first-order HLLC scheme is extended to second-order
 322 accuracy through a MUSCL reconstruction (Van Leer (1979)) combined with a generalized MinMod
 323 limiter and a predictor-corrector Runge-Kutta time integration.

The dambreak problem is a widely used test to demonstrate the shock-capturing capabilities of
 numerical schemes. We consider a one-dimensional dambreak over a wet bed of uniform depth. The
 domain is 1 m long and the initial condition is:

$$h(x, 0) = \begin{cases} 1 \text{ m} & \text{if } x \leq 0.5 \text{ m} \\ 0.2 \text{ m} & \text{otherwise} \end{cases} \quad u(x, 0) = 0 \text{ m/s}$$

324 The analytical solution for this test was derived by Stoker (1957) and consists of a shock and a
 325 rarefaction wave moving in opposite directions from the center of the domain. The solutions of the
 326 dambreak test for 100 grid cells ($\Delta x = 1 \text{ cm}$) and at $t = 0.1 \text{ sec}$ are shown in Figure. 5. For both schemes,
 327 we use a constant Courant number of $CN = 0.7$ and a diffusion parameter in the generalized MinMod
 328 limiter $\theta = 1.5$.

329 Both numerical schemes correctly capture the rarefaction and shock waves despite small
 330 discrepancies in comparison to the analytical solution. This small mismatch can be reduced significantly
 331 with a reduction in grid size. In general, the Present scheme achieves slightly sharper solutions around
 332 the flow transitions compared to the HLLC scheme. Consequently, the Present scheme contains
 333 smaller L1-norm errors than the HLLC scheme, as listed in Table 1, albeit the fact that both solutions
 334 converge towards the exact solution with mesh refinement. The presented model is able to compute the
 335 propagation of shocks with the correct wave speed and height, proving its powerful shock-capturing
 336 capability without the need for the computationally expensive sampling of the solution as it is necessary
 337 for the HLLC scheme.

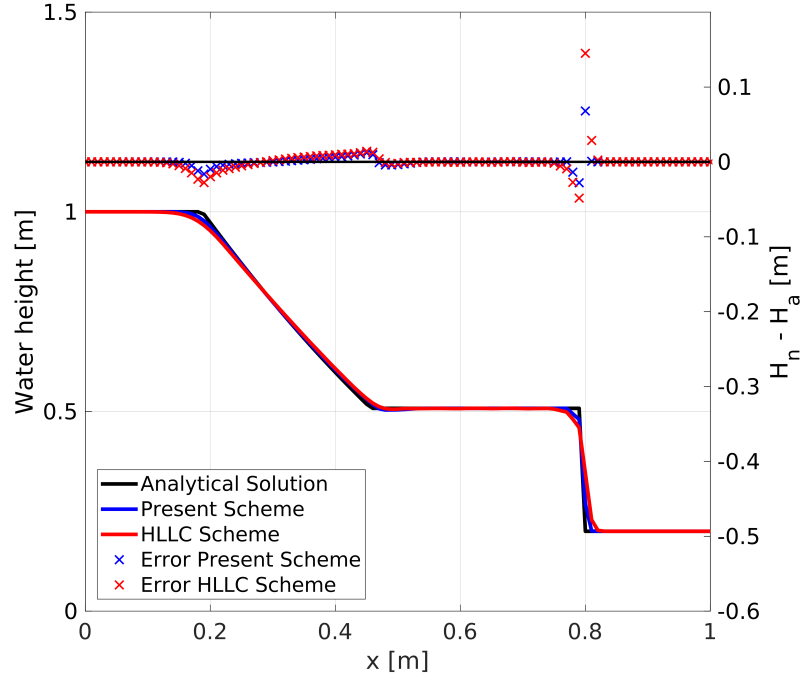


Figure 5. Dambreak over wet bed: Water height profiles from the Present and HLLC schemes at $t = 0.1$ sec for a cell size Δx of 1 cm.

| Number of cells | h | | hu | |
|-----------------|-----------------------|-----------------------|-----------------------|-----------------------|
| | Present | HLLC | Present | HLLC |
| 100 | 3.69×10^{-3} | 5.20×10^{-3} | 6.37×10^{-3} | 1.24×10^{-2} |
| 200 | 1.85×10^{-3} | 2.56×10^{-3} | 3.17×10^{-3} | 6.22×10^{-3} |
| 400 | 7.90×10^{-4} | 1.29×10^{-3} | 1.90×10^{-3} | 3.22×10^{-3} |
| 800 | 4.44×10^{-4} | 6.36×10^{-4} | 7.76×10^{-4} | 1.52×10^{-3} |

Table 1. Dambreak over a wet bed: L1-norm error

338 3.2. Moving boundaries

339 An essential feature of shallow water models used for flood and inundation mapping is the ability
 340 to compute wet-dry transitions and track moving boundaries. The biggest challenges are associated
 341 with the definition of the numerical fluxes and source terms in the presence of dry cells. A clean and
 342 stable representation of the moving boundary is essential for the correct description of run-up and
 343 inundation limits independent of the previous stages of wave propagation and breaking.

344 We investigate the performance of the present model in describing fast sheet flows induced by a
 345 dambreak over a dry bed with and without frictional resistance. This test is also used to verify the
 346 implementation of the friction term.

347 The test case involves a 2000 m long horizontal channel of uniform depth with $\Delta x = 5$ m grid
 348 spacing and the following initial condition:

$$h(x, 0) = \begin{cases} 6 \text{ m} & \text{if } x \leq 1000 \text{ m} \\ 0 \text{ m} & \text{otherwise} \end{cases} \quad u(x, 0) = 0 \text{ m}$$

349 The test is computed with a minimum water depth of $h_{min} = 10^{-8}$ m and a Courant number of 0.7.
 350 Two cases are taken into account:

- 351 1. **Dambreak without friction.** The numerical results are compared with the Ritter solution. The
 352 solution involves a wet-dry front propagating downstream and a rarefaction wave moving
 353 upstream into the reservoir.
- 354 2. **Dambreak with friction.** In this case, the Darcy-Weisbach friction law with a coefficient $f =$
 355 $8g/40^2$ is utilized in the friction source term of the momentum equations. The reference solution
 356 is based on the Dressler/Whitham/Chanson conceptual model (Dressler (1952), Whitham (1955),
 357 Chanson (2009)), which is based on the assumption that near the wavefront, frictional resistance
 358 controls the fluid motion. The exact shape of the wavefront can be found in Chanson (2009). In
 359 contrast to the process at the downstream wave, the frictional resistance in the rarefaction regime
 360 is neglected, and the solution at the front can be described by a modified Ritter's solution as
 361 presented in Delestre *et al.* (2013).

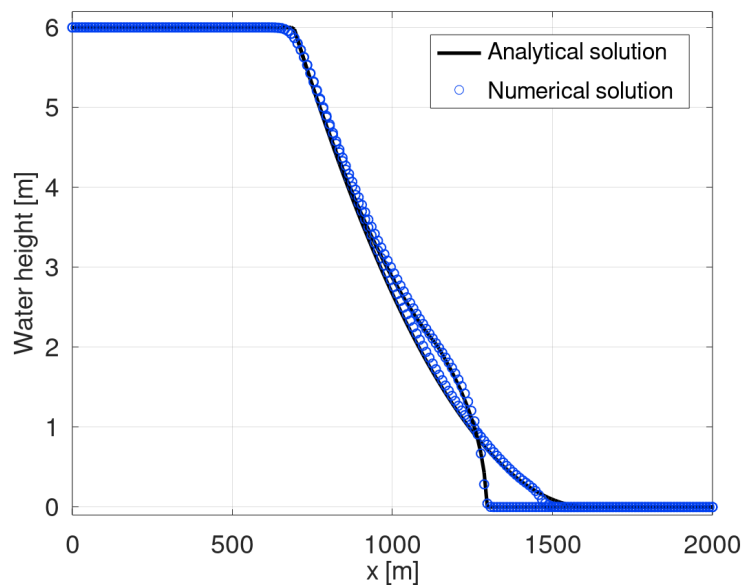


Figure 6. Dambreak on dry bed: Water height profiles after $t = 40$ sec for grid spacing $\Delta x = 10$ m. The wave front around 1250 m corresponds to the solution with a friction coefficient of $f = 8g/40^2$.

362 In both cases, good agreement between the reference and the numerical solutions is obtained
 363 (Fig. 6). In the case of bottom friction, the model accurately captures the deceleration of the wavefront,
 364 which verifies its capability of correctly handling bottom roughness. As before, the results can be
 365 improved through mesh refinement but not through further reduction of the predefined minimum
 366 water depth h_{min} .

367 3.3. Nested grid implementation

368 In this section, we examine the accuracy of the nested grid implementation. This step is important
 369 to scrutinize the model performance with respect to the information exchange across different grid
 370 resolutions, especially in the presence of wet/dry transitions. For applications related to long-wave
 371 run-up, the nested grid approach is expected to deal with moving boundaries and fast flows over
 372 varying topography in two-dimensional settings. A few analytical solutions of the SWE exist for
 373 problems in the 2D horizontal plane. The oscillation in a parabolic basin is one of them, as it addresses
 374 a two-dimensional run-up problem, which helps examine the validity of the numerical structure in the
 375 combined xy -directions.

376 The water oscillation is induced inside a $[0, L] \times [0, L]$ parabolic basin given by:

$$z(r) = -h_0 \left(1 - \frac{r^2}{a^2}\right) \quad \text{where} \quad r = \sqrt{\left(x - \frac{L}{2}\right)^2 + \left(y - \frac{L}{2}\right)^2}$$

377 The value of h_0 represents the still water depth at the basin center, and a is the radius of the wetted
 378 perimeter. The exact solution for this test was derived by [Thacker \(1981\)](#). For a smooth bed with no
 379 friction, the analytical solution for the water depth is described as:

$$h(r, t) = h_0 \left(\frac{\sqrt{1 - A^2}}{1 - A \cos(\omega t)} - 1 - \frac{r^2}{a^2} \left(\frac{1 - A^2}{(1 - A \cos(\omega t))^2} - 1 \right) \right) - z(r)$$

380 where $\omega = \sqrt{8gh_0/a}$ is the frequency of the oscillation, and the coefficient $A =$
 381 $(a^2 - r_0^2) / (a^2 + r_0^2)$ with r_0 the radius of the initial shoreline. For the setup of the dimensions of
 382 the parabola and the initial condition of the free surface, we use $a=1$ m, $r_0 = 0.8$ m, $h_0 = 0.1$ m and L
 383 $=4$ m.

384 The analytical solution is used to verify the symmetry and accuracy of the nested grid
 385 implementation. Here, we place a nested domain off-center, including the moving waterline, with a
 386 refinement factor of 4. The use of an off-center nested grid is critical to verifying the grid exchange for
 387 both the normal and cross fluxes. The Parent grid is computed with a quadratic cell size of 2 cm by
 388 2 cm. The inner Child grid is computed with $\Delta x = \Delta y = 0.5$ cm.

389 The water height evolution at the center of the basin is shown in Fig. 7 after 40 sec corresponding
 390 to over 17 full cycles in the Parent grid. The present solution convinces through the maintenance of
 391 amplitude and phase over multiple oscillation cycles the quality of the second-order numerical scheme.
 392 These results confirm not only the low numerical diffusion but also the smooth transition across the
 393 wet/dry boundary inherent to the model without the need for excessively small grid sizes.

394 Fig. 8 depicts the free surface transect across the basin center line at several stages, $t = T$,
 395 $t = T + T/4$, and $t = T + T/2$, where $T = 2\pi/\omega$ denotes the oscillation period. The run-up is well
 396 described, and no numerical artifact arises from the exchange between the Parent and Child grid. In
 397 addition, Fig. 9 gives a visual impression of the three-dimensional problem and showcases that the
 398 definition of the run-up outline benefits from mesh refinement.

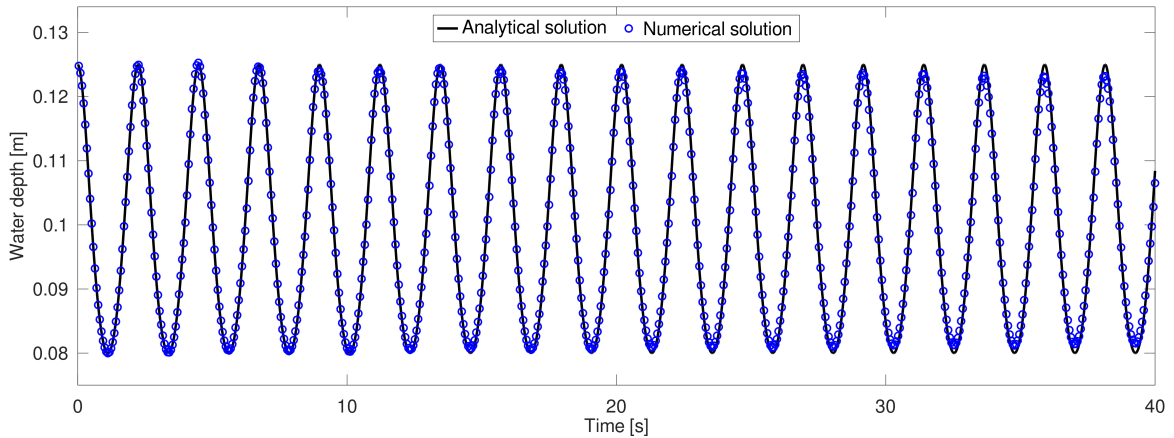


Figure 7. Time series of water height at center point of parabolic basin ($x = L/2, y = L/2$). Grid spacing $\Delta x = \Delta y = 2$ cm. The numerical solution is of low diffusion without requiring excessively fine mesh sizes.

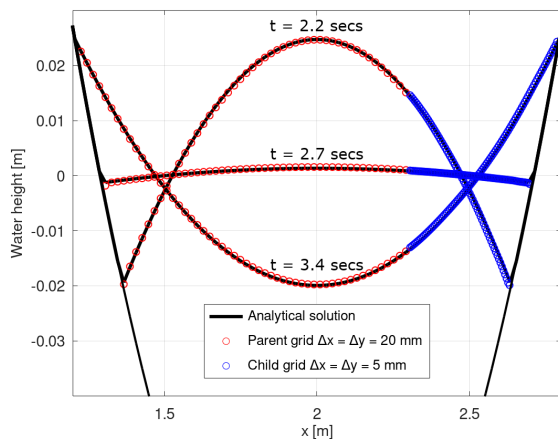


Figure 8. Oscillation in a parabolic basin: Cross-section of water height after one full oscillation. Each circle represents the solution from the numerical model at each grid cell across the transect. The solution from the nested grid is indicated with blue circles.

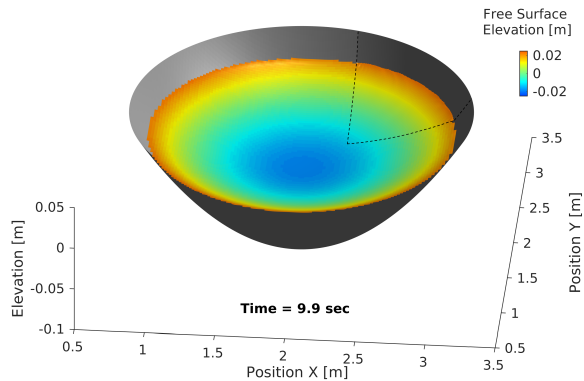


Figure 9. Free surface elevation of oscillation in parabolic basin with grid nesting after 4.5 cycles (9.9 sec). The refined Child grid is denoted by the dashed line and shows a more detailed run-up limit than the coarse Parent grid.

399 4. Effect of grid nesting on wave run-up

400 Previous verification efforts have ensured that the present model correctly handles the
 401 fundamental features that are essential for the accurate computation of long-wave run-up. The
 402 following tests examine the sensitivity of the computed results to grid nesting for efficient computation
 403 of local run-up problems. For this purpose, we utilize two standard experimental benchmark tests
 404 that have been widely used in the tsunami community and that highlight the complexity of the local
 405 long-wave run-up. The two tests present common long-wave features such as the increase in local
 406 wave run-up from the collision of two or more waves as well as extreme run-up over highly detailed
 407 terrain. We will present an analysis of the sensitivity of the computed run-up to the general mesh size
 408 and further investigate the sensitivity of the maximum run-up extent to the size of the nested grid and
 409 the refinement factor.

410 4.1. Solitary wave run-up around a conical island

411 The transformation of long waves around islands has attracted a lot of attention in the past -
 412 especially among tsunami researchers. A common observation is that long waves can refract and
 413 diffract around an island from both sides and collide in the back. In some cases, the maximum
 414 run-up occurs counter-intuitively at the island's lee side due to a superposition effect when the
 415 refracted/diffracted waves from both sides run into each other and double up. The problem of the
 416 conical island is exemplary since the high run-up and inundation at the lee side cannot be approximated
 417 with empirical formulae or computationally cheap 1D calculations. Instead, the problems require a full
 418 2D solution that naturally exhibits a substantial computational effort.

419 [Briggs et al. \(1995\)](#) conducted a large-scale laboratory experiment to investigate solitary wave
 420 transformation around a conical island. The basin is 25 m by 30 m with a circular island in the shape of
 421 a truncated cone constructed of concrete with a diameter of 7.2 m at the bottom and 2.2 m at the top.
 422 The island is 0.625 m high and has a side slope of 1:4. A 27.4-m long directional wavemaker consisting
 423 of 61 paddles generated the input solitary waves for three laboratory tests. Wave absorbers at the three
 424 remaining sidewalls reduced reflection in the basin. Further details about the laboratory model setup,
 425 the location of the wave gauges, and the numerical setup can be found in [NTHMP \(2012\)](#).

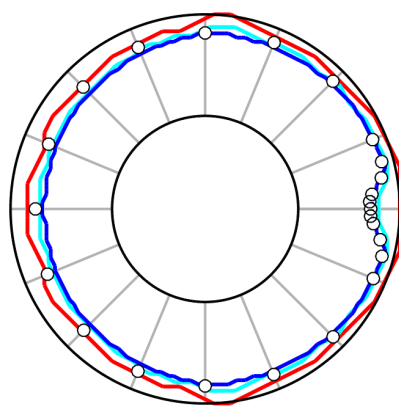
426 The present study focuses on experiments with a water depth $h = 0.32m$ and solitary wave
 427 heights of $A/h = 0.1$. Consistent with NTHMP benchmark problem 6 ([NTHMP \(2012\)](#)), our numerical

428 test uses the measured wave heights of $A/h = 0.096$ from the laboratory experiment instead of the
429 target wave heights as they better represent the recorded data and thus the incident wave conditions to
430 the conical island. A reflective boundary condition is imposed at the lateral sides. The wave absorbers
431 from the laboratory layout are not considered since their absorbing performance is unknown. The
432 model is set up with a reference grid of $\Delta x = \Delta y = 5$ cm. A Manning roughness coefficient of $n = 0.012$
433 $s/m^{1/3}$ accounts for the smooth concrete finish according to [Chaudhry \(2007\)](#). The Courant number
434 is set to $Cr = 0.5$. The model setup is comparable to earlier work and will be used as a reference as
435 it is expected to return solutions of similar and comparable quality to previously published studies.
436 The results from the free surface elevation observed at five wave gauges are omitted here as they are
437 comparable to previously published results.

438 **Sensitivity to grid resolution:**

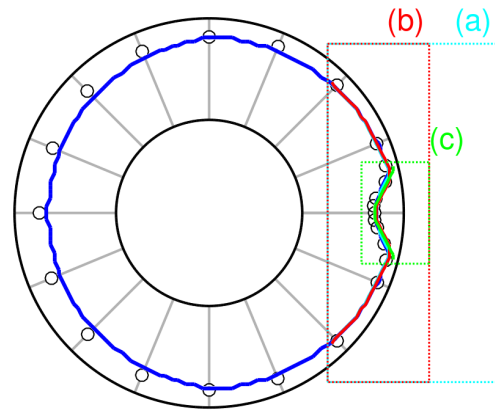
439 In view of sensitivity to the grid size, Fig. 10 shows the run-up limits for the reference scenario
440 with $\Delta x = \Delta y = 5$ cm, as well as the solutions of the model with coarser grid sizes of $\Delta x = \Delta y = 10$ cm
441 and $\Delta x = \Delta y = 20$ cm, respectively. The reference mesh size of 5 cm returns the closest agreement
442 overall with the run-up data - particularly at the lee side of the island. Nevertheless, a numerical
443 domain with four times fewer cells, i.e., uniform 10 cm grid spacing, still provides a decent estimate
444 of the run-up, albeit with less precision at the lee side. It is not really surprising that a grid size of
445 $\Delta x = \Delta y = 20$ cm is too coarse to represent the details of the run-up outline, and virtually no run-up is
446 recorded in the lee of the island.

447 Careful examination of the temporal evolution of the wave field at $\Delta x = \Delta y = 5$ cm resolution
448 shows that the colliding waves in the back of the island locally and momentarily augment the water
449 level, but then pass through each other and continue the refraction/diffraction process. The locally
450 high run-up in the back of the island results to a great extent from the two waves that shoal and spill
451 up on either flank of the leeward side. It often goes unnoticed that the steepened refracted waves
452 then meet head-on over the leeward topography, i.e. the initially dry beach, from where a substantial
453 portion of maximum run-up and inundation originates. For a relatively steep slope, this wrapping
454 process requires rather fine resolution to properly account for the flooding process, and insufficient
455 grid cells over the beach can lead to an under-representation of the run-up.



— Single grid, $\Delta x = \Delta y = 20$ cm
 — Single grid, $\Delta x = \Delta y = 10$ cm
 — Single grid, $\Delta x = \Delta y = 5$ cm

Figure 10. Grid size sensitivity of maximum run-up outlines for the test with $A/h = 0.096$ of Briggs *et al.* (1995). Black dots denote experimental data, solid lines represent results from the present model.



— Single grid, $\Delta x = \Delta y = 5$ cm
 — Nested grid (a), $\Delta x = \Delta y = 5$ cm
 — Nested grid (b), $\Delta x = \Delta y = 5$ cm
 — Nested grid (c), $\Delta x = \Delta y = 5$ cm

Figure 11. Maximum run-up outlines for the test with $A/h = 0.096$ of Briggs *et al.* (1995). Each model run uses a Parent grid with $\Delta x = \Delta y = 20$ cm resolution and one Child grid of $\Delta x = \Delta y = 5$ cm. The three individual nested grid set-ups (a), (b), and (c) and their corresponding maximum run-up limits are color-coded and denoted by the dashed rectangle and the solid lines within.

Sensitivity to grid nesting:

It is understood that any reduction in the total cell count will reduce the computational load. A nested grid approach caters to lowering the computational effort without compromising too much on the quality of the results. A question of practical interest is whether the overall wave transformation around the island could potentially be computed over a coarse grid, from which information is fed into a nested inner grid of higher resolution that is placed only over a local area of interest. Fig. 10 demonstrates that a grid resolution of 5 cm is an adequate choice for the resolution of the wave run-up along the beach of the conical island and that coarser mesh sizes, in particular the 20 cm resolution, are insufficient to resolve most of the run-up.

The solitary input wave has a length of several meters. As shown in the previous benchmark tests, e.g. 3.3, the present model computes long waves with minimal numerical diffusion and hence is expected to handle the general processes of the solitary wave transformation around the island even over a rather coarse mesh. Inspection of the full free surface evolution has shown that even a grid of 20 cm mesh size can account for the overall wave processes in the vicinity of the conical island and that it only fails in computing the detailed run-up.

Fig. 11 shows the results from three nested grid approaches - all with an inner grid of 5 cm resolution placed into an outer grid of 20 cm resolution. The individual inner domains are color-coded and of 2 m by 4 m, 1.2 m by 4 m and 0.8 m by 1.2 m. The additional computational load arising from the inner grid is associated with 3200, 1920, or 384 cells, respectively. It can be seen that the run-up outline in the nested grid (a) denoted by the cyan line in Fig. 11 is nearly identical to the outline of the uniform 5 cm reference grid. This implies that the overall wave processes are sufficiently resolved by the coarse outer grid up to the boundary of the nested grid (a), which subsequently takes care of the detailed wave transformation and run-up processes at the back side of the conical island. The

479 domain size of the nested grid is then reduced behind the island, as illustrated by the red dashed
480 rectangle. The corresponding run-up limit (red line) remains nearly identical to the run-up outline
481 from the largest nested grid setup. The run-up at the lee side, therefore, depends only minimally on
482 the higher resolution in the area behind the island where the wave collision process occurs. The nested
483 grid extends to a very small area just around the hotspot of run-up, as denoted by the green dashed
484 rectangle. Surprisingly, the run-up along the center lee side remains qualitatively very similar to the
485 run-up computed by the larger nested grids.

486 The effect of the nested grid approaches can be seen in Fig. 12 in more detail. Row 1 shows the
487 free surface evolution over the 5 cm uniform reference grid. The corresponding alternate solutions,
488 denoted by the black dashed rectangles, illustrate the nested grid solutions. As the wave is moving
489 around the island, the nested grid (a) (second row) picks up its energy and resolves the wrap of the
490 run-up tongue in detail, though with slightly less steepness at the leading edge compared to the
491 reference solution. The maximum run-up after 9.2 sec in the nested grid is nearly identical to the
492 uniform reference solution. The third row shows the free surface elevation from the red rectangle
493 and the run-up limit from Fig. 11. The high-resolution inner grid extends only marginally over the
494 bathymetry behind the island and mostly covers the topography. The detailed solution of the colliding
495 waves behind the island is less critical for the maximum run-up than a high-resolution computation of
496 the two refracted run-up tongues that meet each other over the dry slope. The last row shows that
497 a representative run-up limit is achievable even by only using an extremely small inner grid of high
498 resolution at the location where the refracted waves collide over the beach.

499 The long-wave refraction and collision processes do not necessarily require high grid resolution
500 given that a low-diffusive coarse solution captures the main energy flux. Counter-intuitively, the
501 locally high run-up of long waves, as illustrated in this example, is often driven by wave processes in
502 the immediate vicinity of the shoreline and over the beach. Accurate run-up results can potentially be
503 obtained with locally very small nested grids as long as they cover the entire run-up zone over the
504 beach. This is particularly true for locations with steep beach slopes.

505 As for the results from Figs. 10 to 12, the computed wave field is symmetric to machine precision
506 with respect to the horizontal center line at 15 m in the y-direction. This supports the quality of the
507 numerical results as any instability arising from the interface at the boundary of the nested grids would
508 have eliminated the perfect symmetry.

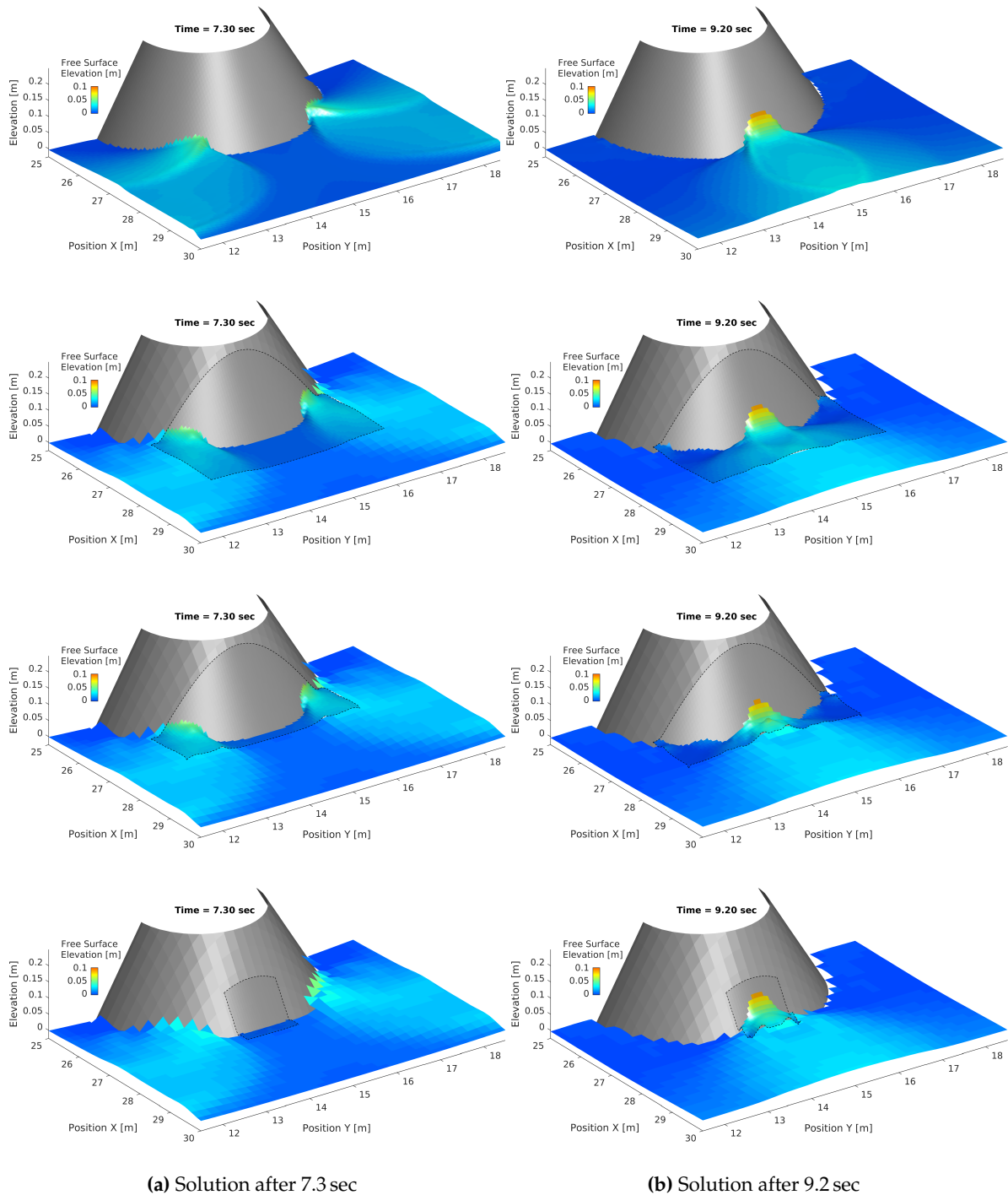


Figure 12. Free surface elevation at lee side of conical island computed over the single reference grid of $\Delta x = \Delta y = 5$ cm (first row) and with three separate grid nesting approaches each combining a coarse outer grid of $\Delta x = \Delta y = 20$ cm mesh size with inner fine grids of $\Delta x = \Delta y = 5$ cm resolution (2nd, 3rd, 4th row). First column: Refraction/diffraction of solitary wave around flank of conical island. Second column: Maximum run-up from superposition of refracted/diffracted waves. The extent of the nested grid is outlined by the black dashed line in row 2 to 4.

509 4.2. Long-wave run-up at Monai Valley

510 The second benchmark is testing the sensitivity of the present model to the mesh size and
 511 refinement of the solution with a nested grid for the computation of nonlinear wave processes over an

512 irregular terrain that favors extreme run-up. The 1993 Hokkaido Nansei-Oki tsunami is a well-studied
 513 event thanks to the laboratory experiments conducted by Matsuyama and Tanaka (2001) at the Central
 514 Research Institute for Electric Power Industry (CRIEPI) in Japan. The down-scaled laboratory test
 515 examined the extreme run-up of over 30 m at Monai Valley, located between two headlands and
 516 sheltered by the small Muen Island. The area around Monai Valley was reconstructed with a plywood
 517 model at 1:400 scale based on bathymetric and topographic data as shown in Fig. 13.

518 A wave gauge near the wavemaker recorded the initial low amplitude N-wave used in the
 519 present numerical model as boundary input with the free surface elevation interpolated from the
 520 data according to the model time step. As in the previous test, we first examine the sensitivity of the
 521 numerical solution to the grid size over a single domain with uniform resolution. Again, the Courant
 522 number is kept constant at $Cr = 0.5$. A Manning coefficient of $n = 0.012 \text{ sm}^{-1/3}$ accounts for the surface
 523 roughness of the plywood model (Chaudhry (2007)).

524 Sensitivity to grid resolution:

525 Fig. 14 shows the comparison between the computed and recorded data at the wave gauges placed
 526 in the numerical and experimental setup between Muen Island and Monai Valley. The computed
 527 results are of similar quality as the solutions from previous studies. The wave regime at the locations
 528 of the gauges is still reasonably well resolved with a rather coarse mesh. Even with a 10 cm grid size,
 529 the general shape of the free surface time series is captured, and the overall energy of the wave field
 530 behind Muen island is accounted for.

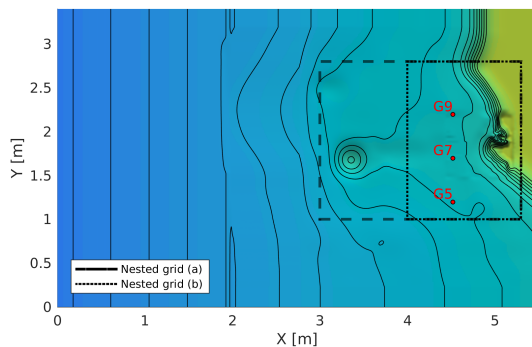


Figure 13. Outline of the bathymetry from the 1:400 scaled model used by Matsuyama and Tanaka (2001). The black dashed and dotted lines denotes the boundaries of two individual inner nested grids (a) and (b).

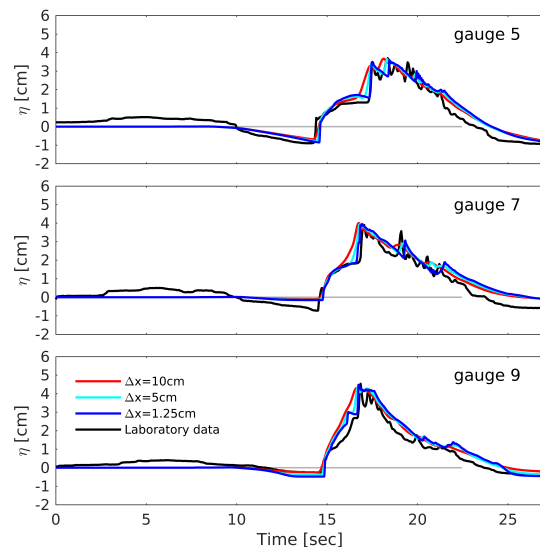


Figure 14. Free surface time series at the gauges shown in the left panel from computations over the entire domain with different uniform mesh sizes.

531 Fig. 15(A) illustrates the sensitivity of the computed maximum run-up to different uniform grid
 532 sizes of 1.25 cm, 2.5 cm, 5 cm, and 10 cm. The local run-up in Monai Valley is more sensitive to the grid
 533 resolution than the nearshore wave field in front of the beach. Since the terrain is steep and narrow, the
 534 computations with the present model show that a rather fine grid of 1.25 cm is necessary to obtain a
 535 proper outline of the run-up envelope. NTHMP (2012) confirms that most previous numerical studies
 536 utilized a mesh size of $\Delta x = \Delta y < 1.5 \text{ cm}$ to obtain a consistent definition of the wave run-up in the
 537 narrow and steep valley. The fine grid of 1.25 cm in the second row of Fig. 16 resolves the details
 538 of wave refraction and collision in front of the steep cliff, whereas a coarser option of 10 cm resolves
 539 neither the flow details nor the small-scale flow features over the topography and consequently leads
 540 to a significant underestimation of the run-up in the Monai Valley.

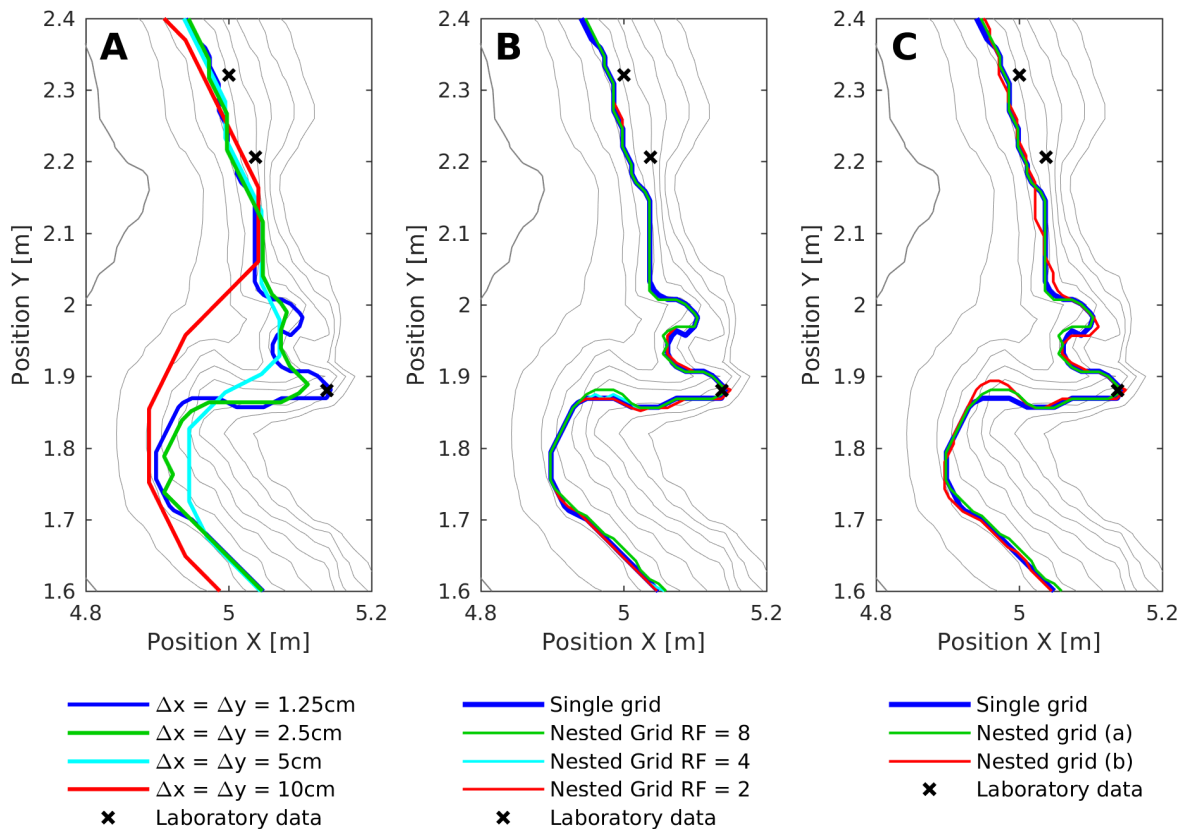


Figure 15. Maximum run-up limits around Monai Valley.

Left (A): Uniform grid with different resolutions.

Center (B): Nested grid (a) with 1.25 cm resolution and different Parent grid resolutions of 2.5 cm, 5 cm, and 10 cm leading to refinement factors of $RF = 2$, $RF = 4$, and $RF = 8$

Right (C): Nested grid (a) and (b) with 1.25 cm resolution and Parent grid resolution 10 cm.

541 **Sensitivity to grid nesting:**

542 Similar to the previous benchmark test the question arises whether it is possible to utilize a coarse
 543 mesh for the overall flow field in combination with a fine nested grid for the detailed run-up in an area
 544 of interest like Monai Valley. Knowing that the run-up over terrain with irregular and steep slopes
 545 requires small grid sizes, we utilize a 1.25 cm nested grid (a) inside a Parent grid as outlined in Fig. 13.
 546 The inner nested grid starts offshore of Muen island, similar to what Yamazaki *et al.* (2011) have used.
 547 The Parent grid is of 5.5 m by 3.4 m size. It contains only 1870 cells with a 10 cm resolution. The two
 548 nested grid options (a) and (b) have dimensions of 2.3 m by 1.8 m and 1.3 m by 2.3 m and consequently
 549 add 26,496 or 14,976 grid cells, respectively, to the computation. Hence, the two nested grid options
 550 reduce the total cell count by 76% and 86% in comparison to a single grid of uniform 1.25 cm resolution
 551 with 119,680 cells.

552 The sensitivity of the results with respect to the refinement factor is analyzed by increasing the
 553 Parent grid resolution by factors of 2, 4, and 8 with respect to the nested grid. Consequently, the
 554 individual run-up limits of Fig. 15(B) refer to the results from a 1.25 cm nested grid in combination
 555 with different Parent grids of 2.5 cm, 5 cm, and 10 cm. The refinement factor hardly influences the
 556 run-up limit with a nested grid domain that covers most of the nearshore area (dashed line of the
 557 domain (a) in Fig. 13). Again, a basic requirement for the utilization of a coarse Parent grid is a low
 558 diffusivity of the numerical scheme. It is understood that the interpolation in the nesting process
 559 between the individual grids can lead to small discrepancies in comparison to a uniform grid with

560 high resolution. This can be seen in Fig. 15. The grid nesting strategy should therefore always be seen
561 as method to primarily reduce the computational load by still retaining an acceptable quality of the
562 solution.

563 It is finally shown how the computed results are sensitive to the nested domain size. This is
564 analyzed through reduction of the area covered by the nested grid (see dotted line (b) in Fig. 13).
565 The resolutions of the Parent and Child grid are identical to the setup with nested grid (a). The two
566 scenarios only differ in the domain size of the nested grids. Fig. 15(C) highlights that the run-up limit
567 from the two scenarios varies only at some locations. Though the flow details of the overtopping and
568 refraction processes around Muen island are resolved in detail with a fine grid as shown in rows two
569 and three of Fig. 16, they do not have a substantial influence on the run-up. It is sufficient that the
570 outer grid resolves the overall wave energy and the inner grid accounts for the run-up process.

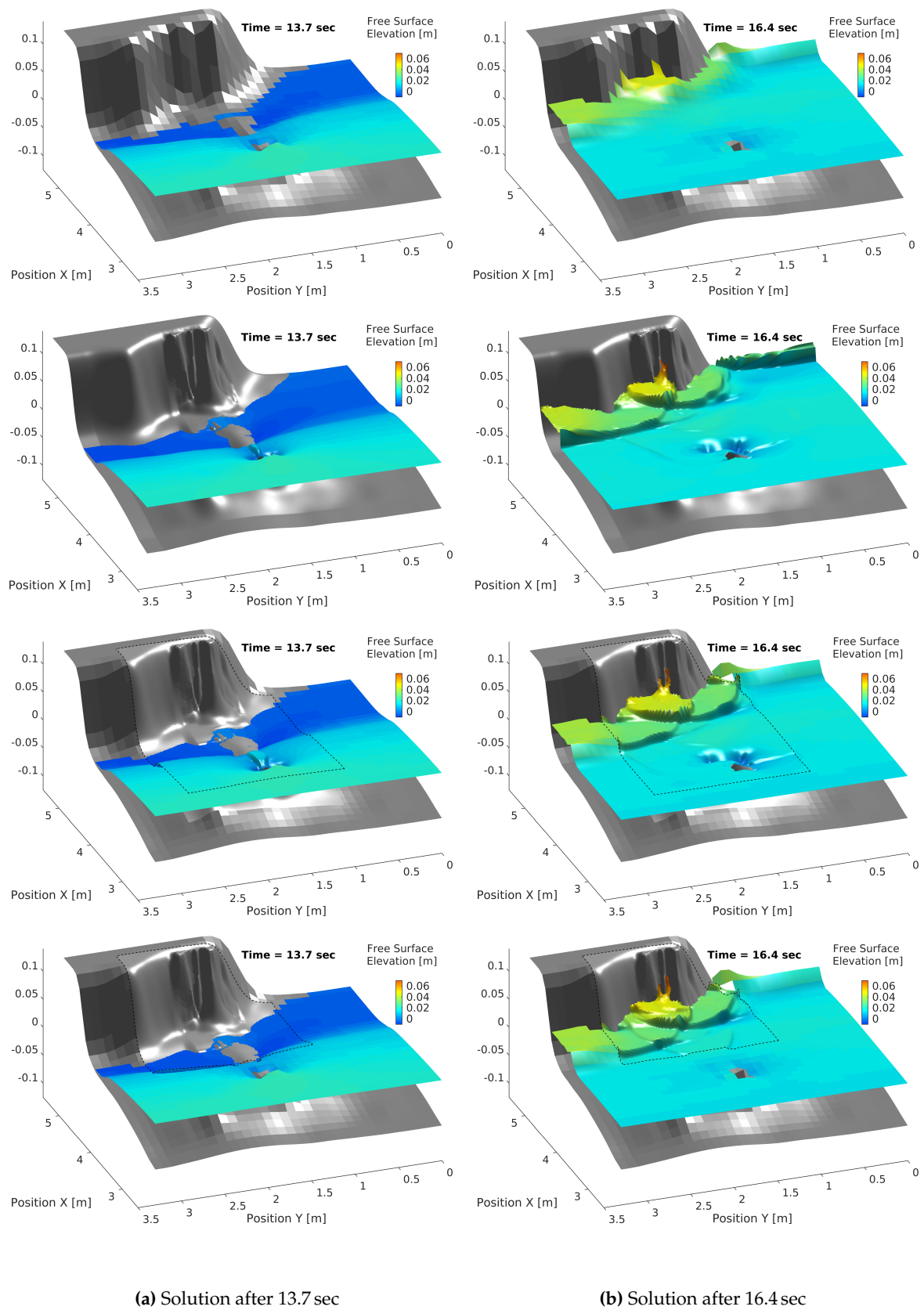


Figure 16. Free surface elevation in front of Monai Valley with single Parent grid of $\Delta x = \Delta y = 10$ cm (first row) and $\Delta x = \Delta y = 1.25$ cm (second row) mesh size. Results from embedded nested Child grids (a) and (b) of $\Delta x = \Delta y = 1.25$ cm in a Parent grid of $\Delta x = \Delta y = 10$ cm are shown in the second row and third row. The extent of the respective Child grid is outlined by the black dashed lines. First column: Drawdown from leading depression of N-wave and approaching wave crest upstream of Muen island. Second column: Maximum run-up from superposition of refracted and reflected waves.

571 5. Conclusions and Perspectives

572 We have shown the performance of a newly developed model for long-wave run-up with respect
573 to standard analytical solutions and laboratory experiments. The model was demonstrated to be
574 shock-capturing, well-balanced, and water-depth positivity preserving, which are crucial properties
575 for the correct estimation of long-wave-driven run-up. The model was proven to be stable and efficient
576 in dealing with wet/dry transitions without the need for computationally expensive treatment of
577 the moving boundary. The numerical scheme is based on a Finite-Volume staggered approximation
578 with second-order accuracy in space and time. The accuracy in time arises from a combination of the
579 Runge-Kutta method for the convective acceleration and the Leapfrog method for the surface gradient
580 and friction terms. Similarly, the spatial accuracy comes from a second-order upwinded advection
581 along with a second-order central difference scheme for the remaining terms.

582 The model performs consistently for shock-driven problems and compares to established Riemann
583 solver-based TVD methods. The wet-dry interface is stable and well-defined without the need for
584 additional treatment of the moving boundary. The model contains a two-way grid nesting scheme
585 that allows for local refinement of the solution. The implementation has been verified and proven to
586 be accurate and stable for moving boundaries and was shown to be applicable to long-wave run-up
587 problems.

588 The performance and sensitivity of long-wave run-up was then investigated in dependence of
589 the nested grid's domain size and the level of its refinement. Two standard benchmark tests from the
590 tsunami community were chosen for the investigation. Though there are no universal rules for the size,
591 position, and refinement factor of nested grids, our results from the two benchmark tests reveal that
592 computations of high quality can be achieved with small nested grids placed strategically at a location
593 of interest such as in areas where locally high run-up occurs. The refinement factor was found to have
594 only small influence on the run-up limit, if the solution in the Parent grid is representative of the wave
595 envelope and the grid nesting method accounts for the correct exchange of the total wave energy flux.

596 Further, it was demonstrated that it is possible to place a nested grid rather close to the initial
597 still-water level as long as the long-wave flow regime prevails across nested grid's offshore boundary.
598 Long-wave run-up is often more subject to the resolution of the local topography than it is influenced
599 by the detailed wave processes over the bathymetry. This is line with commonly used empirical run-up
600 formulae for swell waves where the maximum run-up envelope is controlled by the overall wave
601 energy and the slope.

602 The quality of the computed results encourages to expand the development of the model with
603 respect to frequency dispersion. This will allow for a further investigation of how grid nesting
604 can affect the run-up from swell waves. In the same context, the model can be optimized through
605 implementation of massive parallelization techniques commonly used to reduce the computation
606 time associated with large flow problems. It is evident that a low overall cell count reduces the
607 model's computation time and that the insights gained from the present study can be used to efficiently
608 decrease the computational load for computations of long waves by retaining accuracy and quality of
609 the solutions.

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 627 Project administration: [Volker Roeber];
 628 Funding acquisition: [Volker Roeber];
 629 Supervision: [Volker Roeber, Denis Morichon].

630 Data availability

631 The routines for replication of the numerical results can be provided upon reasonable request.

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