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Wet granular flows over a rough inclined plane

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Résumé :

Les écoulements multi-phasiques naturels ou industriels (glissements de terrain, mélanges de poudres, ...) ont des propriétés rhéologiques non triviales, qu'il est utile de mieux comprendre à partir d'expériences modèles. Les matériaux granulaires humides, à mi-chemin entre les matériaux granulaires secs et les suspensions denses concentrées, sont un exemple de matériaux multi-phasiques, dont la rhéologie reste assez peu documentée. Nous réalisons pour la première fois à notre connaissance, des expériences d'écoulement de matériaux granulaires humides, sur plan incliné : celles-ci révèlent un régime d'écoulement stationnaire uniforme pour une certaine gamme de paramètres expérimentaux (angle de la pente, hauteur de l'ouverture du silo). Nous mettons en évidence la pertinence de la contrainte de cohésion τ_c décrivant la limite de stabilité statique via le critère de Mohr-Coulomb avec cohésion $\tau = \tau_c + \mu_0 P$, avec τ et P , les contraintes maximales de cisaillement et de pression et μ_0 le coefficient de frottement interne statique, en écoulement rapide. Ce critère semble ainsi adapté pour décrire la rhéologie des matériaux granulaires humides en écoulement rapide, à condition de caractériser le frottement interne $\mu \neq \mu_0$ en situation d'écoulement rapide et de comprendre ses variations avec les paramètres expérimentaux.

Abstract :

Multi-phase flows encountered in nature or industry (landslides, mudflows, powder mixtures, ...) exhibit non trivial rheological properties, that can be understood better thanks to model materials and appropriate rheometers. Unsaturated wet granular materials, between dry granular and immersed ones (dense granular suspensions), are one example of multi-phase materials, which rheology was not studied much. We realize for the first time to our knowledge, experiments of wet granular flows over a rough inclined plane : our results show steady uniform flows for a wide range of parameters (the inclination angle and the mass flow-rate). We demonstrate the relevance of the cohesion stress τ_c well defined in the cohesive Mohr-Coulomb yield criterion $\tau = \tau_c + \mu_0 P$, with τ and P , the maximal values of the shear and pressure stresses respectively, and μ_0 the static internal friction coefficient, for rapid flows. Thus this yield threshold should be extended to the rheology of rapid wet granular flows, by allowing the internal friction $\mu \neq \mu_0$ to differ from its static value μ_0 , after having understood its variations with the experimental parameters.

Mots clefs : Wet granular flow ; Inclined plane ; Rheology ; Cohesion ; Friction

1 Introduction

Wet (unsaturated) granular materials are triphasic systems composed of an assembly of grains which pore space is partially filled by a liquid and air. These model granular systems may be useful for the understanding of multi-phasic materials encountered in natural (landslides, debris flows, rock or snow avalanches) and industrial fields (civil engineering, powder-process such as wet granulation, fertilizer production). They exhibit striking behaviours and unique rheologies, however the understanding of their flow and even their description are still fragmental, due to the many parameters playing important roles and the difficulty of experimental instrumentations. Thus, a unified approach, successfully comparing theoretical and experimental results in different configurations, is still lacking, appealing for much more experimental studies.

It is commonly known that the addition of liquid among grains, unless saturation of the pores, gives cohesion properties to the wet granular material due to surface tension, when liquid form bridges between grains, that attract them, possibly merging in liquid/grain/air clusters. A traditional description at the macroscopic scale is possible by the cohesive Mohr-Coulomb yield criterion, with τ_c , the cohesion stress and μ_0 , the static internal friction coefficient :

$$\tau = \tau_c + \mu_0 P, \quad (1)$$

where τ and P are the shear stress and the normal confining stress (or pressure) respectively. It successfully describes plastic flows of wet grains, in the quasi-static limit of slow flows, with the same static internal friction coefficient μ_0 for wet and dry grains [1, 2]. This means that in the quasi-static limit, the internal friction is not modified by the presence of liquid, suggesting the absence of modification of the inter-grain friction [3], a question not settled yet [4]. What about the case of non quasi-static but rapid flows [5, 6, 7]? In a non-uniform shear stress configuration, is this criterion useful at the yield plane only or above the yield threshold in dynamic conditions too? Can we extend the yield criterion (1) to rapid flows, even above the yield threshold, by introducing the internal friction μ different from its static value μ_0 :

$$\tau = \tau_c + \mu P. \quad (2)$$

The influence of a liquid wetting the grains on the wet granular material rheology sensitively depends on the liquid content, its spatial distribution around grains and within pores, and the morphology of the liquid phase [8, 9]. Due to the high solid fraction of grains and close distances between bonded grains, direct inter-grain contacts play an important role in their rheological behaviour, depending on the shape, surface and grain properties (roughness, friction, elasticity, ...). To those micromechanical features, one should add the physical properties of the liquid (viscosity, surface tension, contact angle, ...) and the geometrical microstructures of the granular assembly [8, 9]. Indeed, in addition to the cohesion induced by the presence of liquid interfaces, possible dissipative forces (viscous, capillary, inertial, ...) may occur in the liquid phase. Consequently, the role of some liquid among grains is not obvious at all, thus wet (unsaturated) grains are not as simple as model cohesive materials, with the addition of an inter-grain force interaction [4, 10, 7].

All this suggests us to realize experiments of wet granular flows over a rough inclined plane. First, the experimental methods and the configuration of the inclined plane are presented in sections 2 and 3 respectively. Second, we demonstrate in section 4 that our measurements of the maximal (shear and

normal) stresses in wet granular layers flowing in the steady and uniform regime are compatible with the emergence of a cohesion stress, that is the same than the one appearing in the cohesive Mohr-Coulomb yield criterion (1), suggesting to extend this latter as a rheology (2) for rapid flows. Finally, we conclude in section 5.

2 Experimental methods

Our granular materials consist of glass beads (density $\rho = 2500\text{kg.m}^{-3}$, diameter $d \simeq 250\mu\text{m} \pm 50\mu\text{m}$) either dry or wet by silicon oil (density $\rho_l = 950\text{kg.m}^{-3}$, viscosity $\eta = 20\text{mPa.s}$, surface tension $\Gamma = 20\text{mN.m}^{-1}$). Cohesive forces exerted by liquid bridges on bounded grains become important compared to their weight if the Bond number $Bo = \rho g d^2 / (\pi \Gamma) \ll 1$: here, $Bo \simeq 10^{-2}$, meaning that grain diameter d is much smaller than the capillary length $\sqrt{\pi \Gamma / (\rho g)}$. The liquid content $\epsilon = m_l / m_s$, defined as the mass ratio of liquid over grains, is either 0 or $\epsilon = 0.5\%$, so that we focus on the dry state and on the wet pendular state [8], in which liquid bridges are of small volumes and involve mostly two grains [9]. The solid fraction in steady rapid flows is supposed to be $\phi \approx 0.5$ for wet grains [11], while $\phi \approx 0.6$ for dry grains.

A mass of about 20kg of (wet or dry) grains is stored into the hopper located at the top of the inclined plane, before opening the hopper frontal gate of the height f to release them (figure 1a). The 1.6m-long and 34cm-wide plane is made of a rough bottom plate (sand-paper of roughness $\simeq 350\mu\text{m}$) and two smooth lateral Plexiglass walls, to ensure a no-slip boundary condition at the bottom and negligible sidewall effects [12] : our experimental parameters lead to shear flows deep inside the granular layer up to the bottom plate, and not only at the surface like in a heap flow. The plane can be inclined at the imposed slope angle θ . The hopper is elevated ($\sim 10\text{cm}$) with respect to the inclined plane, so that when opening its gate, the beads first fall down continuously on the rough plane before flowing over it. This thin and dilute ‘rain’ allows a good reproducible initial condition and a constant input mass flow rate Q_m [13].

The mass flow rate Q_m is measured from the linear variations with time of the weighted mass m of (wet or dry) grains flowing out of the inclined plane. A laser sheet is projected on the plane with a small angle with respect to the plane normal to the camera axis, so that it is shifted by the presence of a granular layer proportionally to its thickness $h(x, t)$ at the position x and the time t [14]. The camera positioned at the normal of the plane, record the central region of the inclined plane at a 20Hz frequency, which images are processed using ImageJ and Matlab.

3 The inclined plane as a rheometer

It is well established that steady and uniform flows are observed over a rough wide inclined plane for saturated (either dry or immersed) granular materials [14, 15, 16]. With our experimental set-up and materials, we obviously observe such steady uniform flows with dry grains, from which we will measure in the following (figure 2) the static internal friction μ_0 appearing in equation (1). The mass flow rates Q_m change between 1kg.s^{-1} and 10kg.s^{-1} .

For wet (unsaturated) grains, for the first time, to our knowledge, we observe experimentally a steady state and uniform regime too with a finite and (quasi-)constant thickness layer of wet grains propagating along the rough inclined plane, which angle of the free surface is equal to the slope angle θ , for high

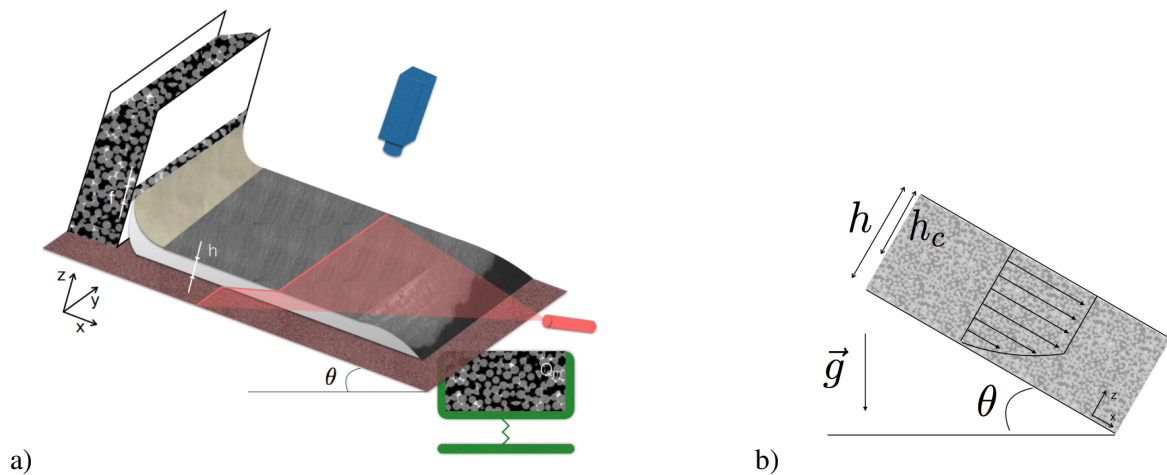


FIGURE 1 – a) The experimental setup consists of a wide rough plane that can be inclined at the imposed slope angle θ . Wet or dry grains, stored in a hopper at the top of the plane, are released through a gate, which aperture thickness f can be adjusted in order to control the mass flow rate Q_m , measured by a scale located underneath. The granular layer free surface is recorded by a camera, allowing for the measurement of the height profile $h(x, t)$ thanks to a LASER diode. b) Schematic representation of the vertical section of a wet granular flow : the longitudinal direction is referred to as x and the direction normal to the plane as z .

enough mass flow rates Q_m and slope angles θ . The mass flow rates Q_m change between $1\text{kg}\cdot\text{s}^{-1}$ and $10\text{kg}\cdot\text{s}^{-1}$.

We refer to the longitudinal direction as x and the direction normal to the plane as z (figure 1b). The momentum equations for a steady free surface flow down an inclined plane at an angle θ give :

$$P = \phi \rho g (h - z) \cos \theta, \quad (3)$$

$$\tau = \phi \rho g (h - z) \sin \theta, \quad (4)$$

with τ and P , the shear and normal stresses. Thus, in case of a steady uniform flow, the inclined plane geometry, whatever the rheology is, allows to control the apparent friction throughout the whole layer thickness :

$$\mu^{app} = \frac{\tau}{P} = \tan \theta. \quad (5)$$

It is interesting to note that with the extended cohesive Mohr-Coulomb criterion in equation (2), the apparent friction coefficient μ^{app} in (5) and the internal friction $\mu = d\tau/dP$ in (2) differ :

$$\mu^{app} = \frac{\tau_c}{P} + \mu. \quad (6)$$

This may imply some confusions, such as this one : the same internal friction μ for dry and wet grains leads nevertheless to a larger apparent friction as soon as a non-zero cohesion stress τ_c appears, leading to larger slope angles for example [17, 18].

We recall that it is well known that the friction of dry granular flows (the internal friction being equal to the apparent one) is not exactly constant, and its variations are well described by a function of the inertial number [15] : $I = \dot{\gamma}d/\sqrt{P/\rho}$, with $\dot{\gamma}$ the shear rate. The common interpretation of the I -dependence

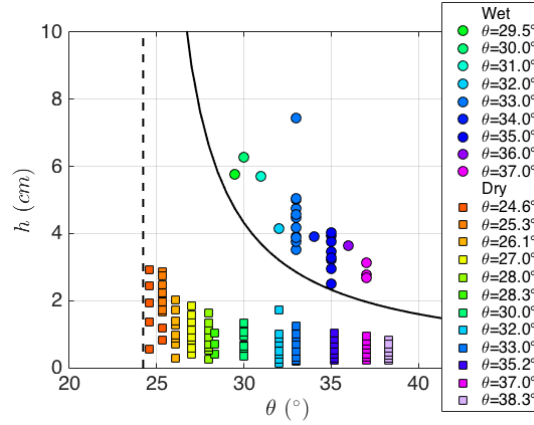


FIGURE 2 – Parameters space (θ, h) of the experiments reported here, with θ the inclined plane slope angle and h the constant height of the wet (circles) and dry (squares) granular flows. The colors encode for the values of θ . The continuous curve $h_c(\theta)$ is computed from equation (7) with $\tau_c = 70\text{Pa}$ and $\mu_0 = 0.45$, for the height threshold for a cohesive steady uniform free-surface gravitational flow. The dashed line corresponds to the angle threshold $\theta_0 = \arctan(\mu_0)$.

lies in the comparison of two time-scales at the grain level : the typical time of deformation $1/\dot{\gamma}$ and the free-fall relaxation time $d\sqrt{\rho/P}$, when the fluid does not apply any drag on the grain. Here the inertial number typically changes between 10^{-2} and 1, corresponding to the inertial flow regime, well above the quasi-static regime, for which the friction tends to its quasi-static value.

When some cohesion is introduced in a granular material, the velocity profile of a gravity-induced uniform steady flow is modified [19, 10, 6] compared to the Bagnold profile (with a $3/2$ power z -dependence). A plug region with no shear appears at the top of the sheared layer, due to the existence of a minimal value for the shear stress to induce some shear flow, as it is for a yield-stress fluid (figure 1b). In general, a yield-stress τ_Y (so that the shear rate $\dot{\gamma} \neq 0$ for a shear stress $\tau \geq \tau_Y$) leads to the emergence of a length-scale h_Y , as a height threshold for shear in a steady uniform gravitational flow of density ρ : $h_Y = \tau_Y/(\rho g \sin \theta)$. In a similar way, cohesion in a granular material induces a yield-stress $\tau_Y = \tau_c + \mu_0 P$, leading to a height threshold h_c for shear in a steady uniform gravitational flow of density $\phi\rho$:

$$h_c = \frac{\tau_c}{\phi\rho g \cos \theta} \frac{1}{(\tan \theta - \mu_0)}, \quad (7)$$

with μ_0 , the static internal friction of the granular material, that was shown to be the same with or without liquid (saturated or unsaturated) [1, 2]. This means that a steady uniform shear flow should occur only for cohesive granular layers thicker than the height threshold h_c ; such a cohesive granular layer (of thickness $h \geq h_c$) will experience some shear at the bottom on the height $h - h_c$ and a plug velocity profile of height h_c above it.

If we wanted to do further predictions on cohesive or wet granular flows, we would need more information about their rheology and thus the measurement of the shear rates within our flows, that is not obvious as soon as shear is localized, which will be devoted to a future work. Instead, here, we will investigate if our experimental measurements are compatible with a cohesive Mohr-Coulomb-type rheology, with some cohesion stress relevant both for the yield criterion and for rapid flows.

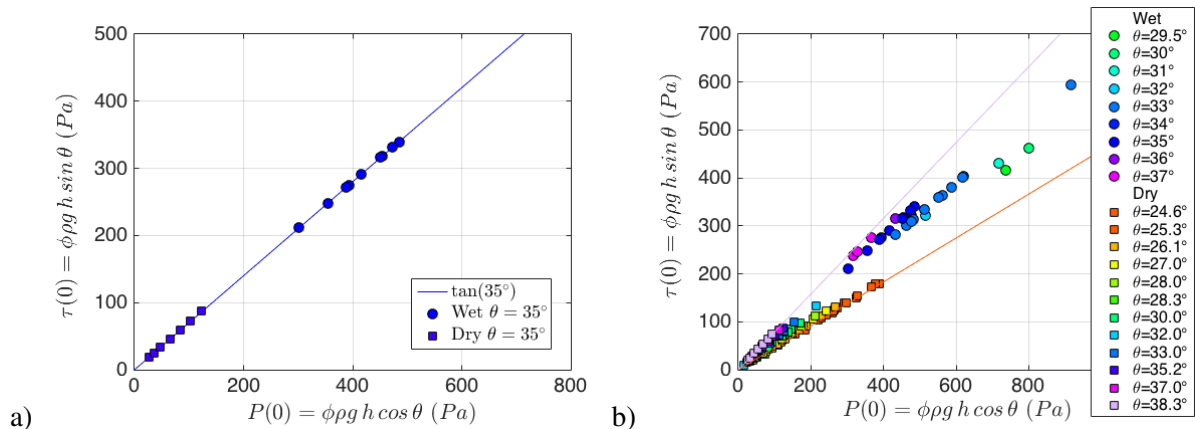


FIGURE 3 – Shear and normal stresses $\tau(0)$ and $P(0)$ computed at the the bottom ($z = 0$) where they are maximal : a) for our dry and wet flows at $\theta = 35^\circ$; b) for all of our wet flows at $\theta \in [29-37]^\circ$ and for all of our dry flows at $\theta \in [24-39]^\circ$, having neither the same apparent friction μ^{app} nor the same internal friction μ . The colors encode for the inclined plane slope angle θ as in figure 2.

4 The cohesion stress

4.1 The cohesion stress at the yield threshold

Figure 2 shows the thickness h of steady-uniform flows for the different slope angles θ explored for both the dry (squares) and the wet (circles) granular samples ($\epsilon = 0.5\%$). Steady-uniform flows are achievable for large enough values of h (or f) and θ . One can see that wet granular flows are much thicker than dry ones. Indeed minima of h (and θ) for wet granular flows are much larger than the ones for dry granular flows.

As we did not observe any steady uniform dry granular flows for slopes smaller than $24.6 \pm 0.5^\circ$, we identify the angle threshold $\theta_0 = 24^\circ$ and get for the static internal friction $\mu_0 = 0.45$, represented by the dashed line in figure 2. The relation (7) for the height threshold is drawn as a continuous line for $\tau_c = 70\text{Pa}$ and $\mu_0 = 0.45$: $h_c(\theta)$ is below the values (θ, h) of the steady uniform wet granular flows reported here, seeming to describe well the yield threshold. As the minimal flowing height $h_c(\theta)$ increases and diverges when the slope approaches the angle threshold θ_0 , getting steady uniform wet flows at smaller θ would need much larger thicknesses h , gate heights f and mass flow rate Q_m , not achieved in our set-up. That is why minima of θ for our wet granular flows seem larger than the ones for our dry granular flows. Now we will check whether or not this cohesion stress $\tau_c = 70\text{Pa}$ describing the yield threshold for our steady uniform wet granular flows ($\epsilon = 0.5\%$) over an inclined plane is useful to describe our rapid flows.

4.2 The cohesion stress relevant for rapid wet granular flows

In our configuration of a gravity-induced uniform steady flow, under the assumption of a no-slip boundary condition at the bottom, the material is sheared from the bottom ($z = 0$) to the interface with the plug ($z = h - h_c$), we choose to plot the maximal values of the pressure and the shear stress ($\max(P)$, $\max(\tau)$) as coordinates ; each point (both for dry and wet flows) is obviously computed from equations (3) and (4) for $z = 0$: $(P(0), \tau(0))$. Thus, each set of data realized at a constant slope angle θ is aligned according a straight line of slope $\tan \theta$ passing through the origin, as shown in figure 3a for $\theta = 35^\circ$, by wet (circles) and dry (squares) flows. As expected from figure 2 showing steady uniform

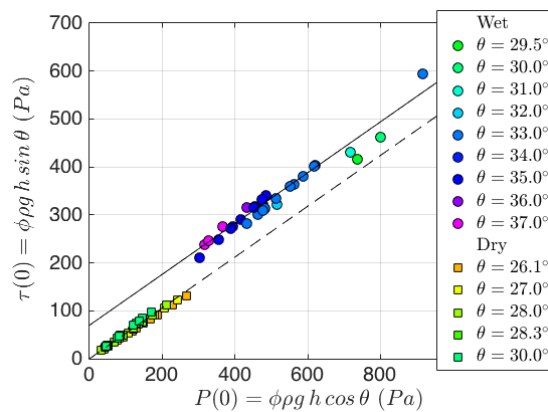


FIGURE 4 – Shear and normal stresses $\tau(0)$ and $P(0)$ computed at the the bottom ($z = 0$) where they are maximal, for all of our wet flows and for dry flows at $\theta \in [26-30]^\circ$, equivalent to $\mu \in [0.5-0.6]$, such that they have approximately both the same internal friction μ . The colors encode for the inclined plane slope angle θ as in figure 2.

dry flowing layers thinner than wet ones, $(P(0), \tau(0))$ data of the dry glass beads have smaller values than the wet ones : dry data are closer to the origin than wet ones, these latter must have the shear stress larger than the yield stress (see Section 4.1).

However, when we plot all of our experimental wet data, for slope angles $\theta \in [29-37]^\circ$, as shown by the circles in figure 3b and 4, surprisingly, they all lie along a single line, demonstrating that the wet material can be straightforwardly described by an affine law, indicating a finite cohesion stress τ_c and an approximately constant internal friction $\mu \simeq 0.53$. In this figure 3b, all of our dry granular flows, for slope angles $\theta \in [24-39]^\circ$, are also plotted (squares), but they have not the same friction (the internal friction being equal to the apparent one) $\mu = \mu^{app} \in [0.45-0.81]$, as shown by the non parallel lines delimited by the colored lines and squares : this is a consequence of the dependence $\mu(I)$ with the inertial number. That's why we restrict data from our dry granular flows to a small range of slopes $\theta \in [26-30]^\circ$ in order to get approximately the same internal friction $\mu \simeq 0.53$ in figure 4.

Finally, our major result is visible in figure 4, where are plotted the maximal values of shear $\tau(0)$ and pressure $P(0)$ stresses for our rapid granular flows, either dry or wet, but having approximately the same internal friction, so that the two sets of data lie along two parallel lines. Whereas dry data pass through the origin indicating no cohesion ($\tau_c = 0$), all of our wet data while steadily and uniformly flowing over the inclined plane lie along a single line, whose shift with respect to the origin demonstrate the emergence of a cohesion stress τ_c , even above the yield threshold. The measurement of this cohesion stress $\tau_c \simeq 70\text{Pa}$ is compatible with the one characterizing the yield threshold (see Section 4.1). Besides, this shift between dry and wet data (at the same internal friction) indicate that the apparent friction coefficient of the wet beads is higher than the dry ones, only because of a finite cohesion stress.

5 Conclusion

For the first time to our knowledge, we realized experiments of wet (unsaturated) granular flows over a wide rough inclined plane and we observe experimentally a steady state and uniform regime with a finite and (quasi-)constant thickness propagating at a constant velocity for high enough mass flow rates Q_m and slope angles θ . As expected from other configurations [1, 2, 20], the minimal thickness h_c for a

wet granular layer to be sheared uniformly and steadily is well described thanks to the cohesive Mohr-Coulomb yield criterion, in equation (1), with τ_c , the cohesion stress and μ_0 , the static internal friction coefficient, being the same for wet and dry grains. Surprisingly, extended equation (1) by allowing some variations of the internal friction to equation (2) allows to describe a whole set of data of wet granular flows (at a constant liquid content ϵ and for a given liquid), even if far above the yield threshold, and without needing to be at the yield plane [6]. This demonstrates the relevance of the cohesion stress τ_c in equation (2) for rapid wet granular flows. Furthermore, the same value of the cohesion stress τ_c holds for these two regimes.

In the future, we need to characterize the internal friction of wet granular flows and to understand its variations, especially when changing the liquid content and the properties of the liquid. To this aim, we will have to be able to measure the shear rate characteristic of such localized flows.

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